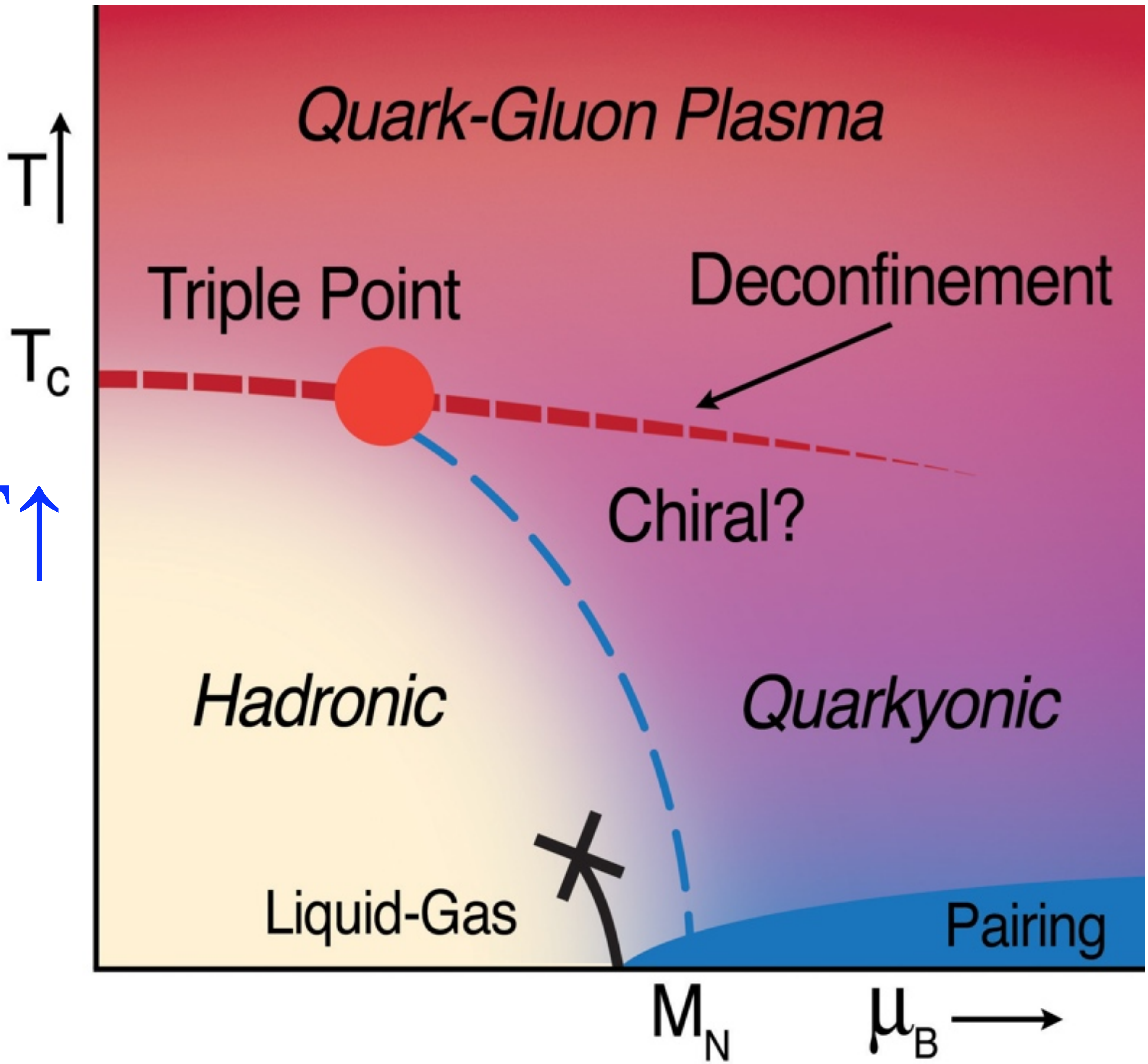
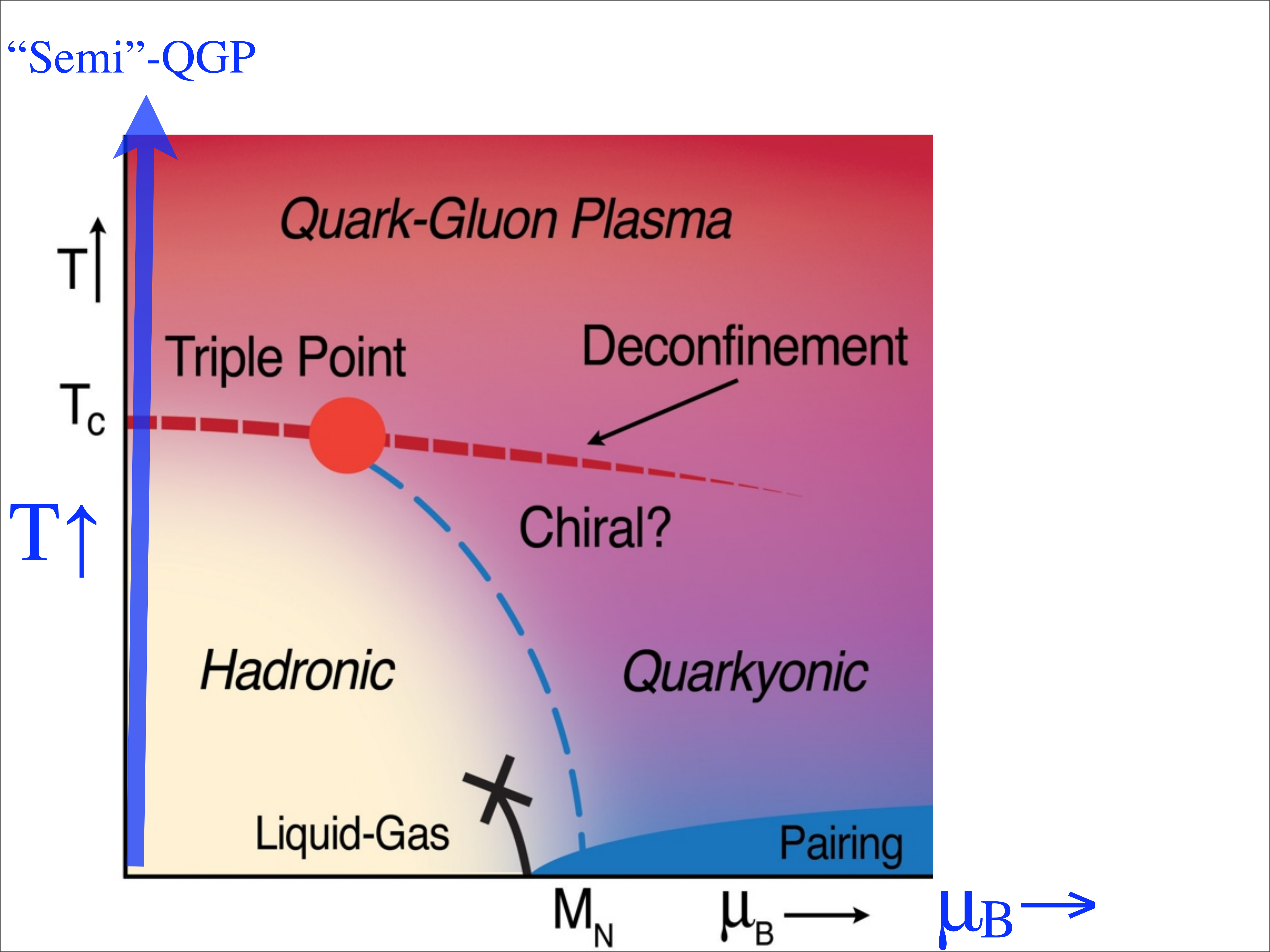
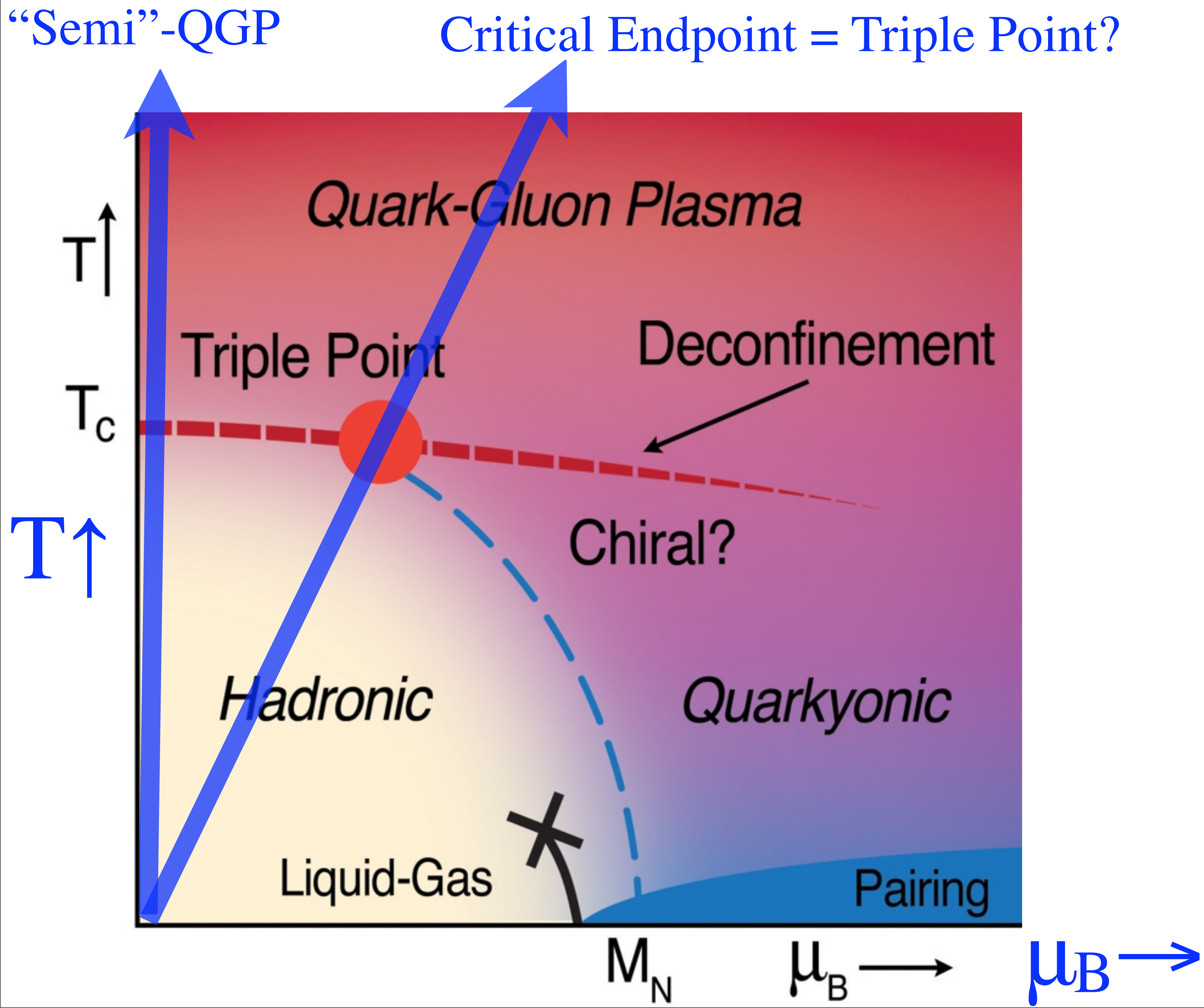


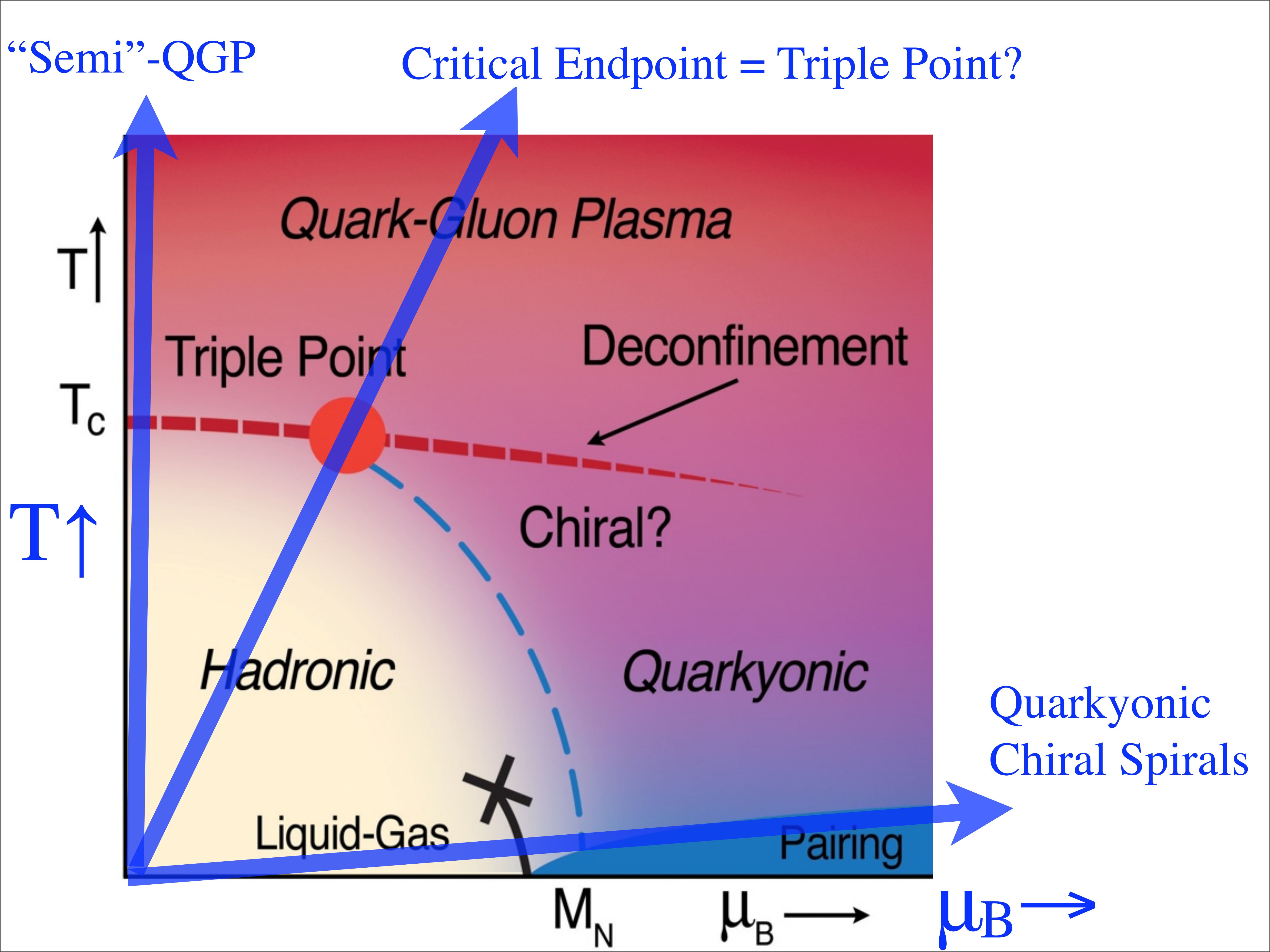
$T\uparrow$

$\mu_B \rightarrow$


"Semi"-QGP
T↑
Quark-Gluon Plasma
T
Triple Point
Deconfinement
T_c
Chiral?
Hadronic
Quarkyonic
Liquid-Gas
Pairing
M_N
$\mu_B \longrightarrow$
$\mu_B \longrightarrow$



“Semi”-QGP
Critical Endpoint = Triple Point?
Quark-Gluon Plasma
T
Triple Point
Deconfinement
T_c
T↑
Chiral?
Hadronic
Quarkyonic
Liquid-Gas
Pairing
M_N
μ_B
μ_B



"Semi"-QGP
Critical Endpoint = Triple Point?
Quark-Gluon Plasma
T
Triple Point
Deconfinement
T$_c$
Chiral?
Hadronic
Quarkyonic
T↑
Quarkyonic
Chiral Spirals
Liquid-Gas
Pairing
M$_N$
μ_B
μ_B

Semi-QGP, a Triple Point, and Quarkyonic Chiral Spirals

Extreme QCD, temperature T, chemical potential $\mu_{qk} \neq 0$:

1. Effective Theory near T_c : Semi-QGP ($\mu_{qk} < T$)

2. Quarkyonic Matter: at large N_c, quark Fermi sea with a *confined* Fermi surface

3. Triple Point at Large N_c = Critical Endpoint in QCD?

4. Chiral Spirals in Quarkyonic Matter

5. The Unbearable Lightness of Being (Nuclear Matter)?
   "Purely pionic" effective Lagrangians and nuclear matter

1. Y. Hidaka and RDP, 0803.0453, 0906.1751, 0907.4609, 0910.xxxx
2. L. McLerran & RDP, 0706.2191; Y. Hidaka, L. McLerran, & RDP 0803.0279
3. Blaschke, Braun-Munzinger, Cleymans, Fukushima, Oeschler, RDP, McLerran, Redlich, Sasaki, & Stachel 0910.xxxx
4. T. Kojo, L. McLerran, & RDP 0910.xxxx

Effective Theory near T_c :
Semi-QGP

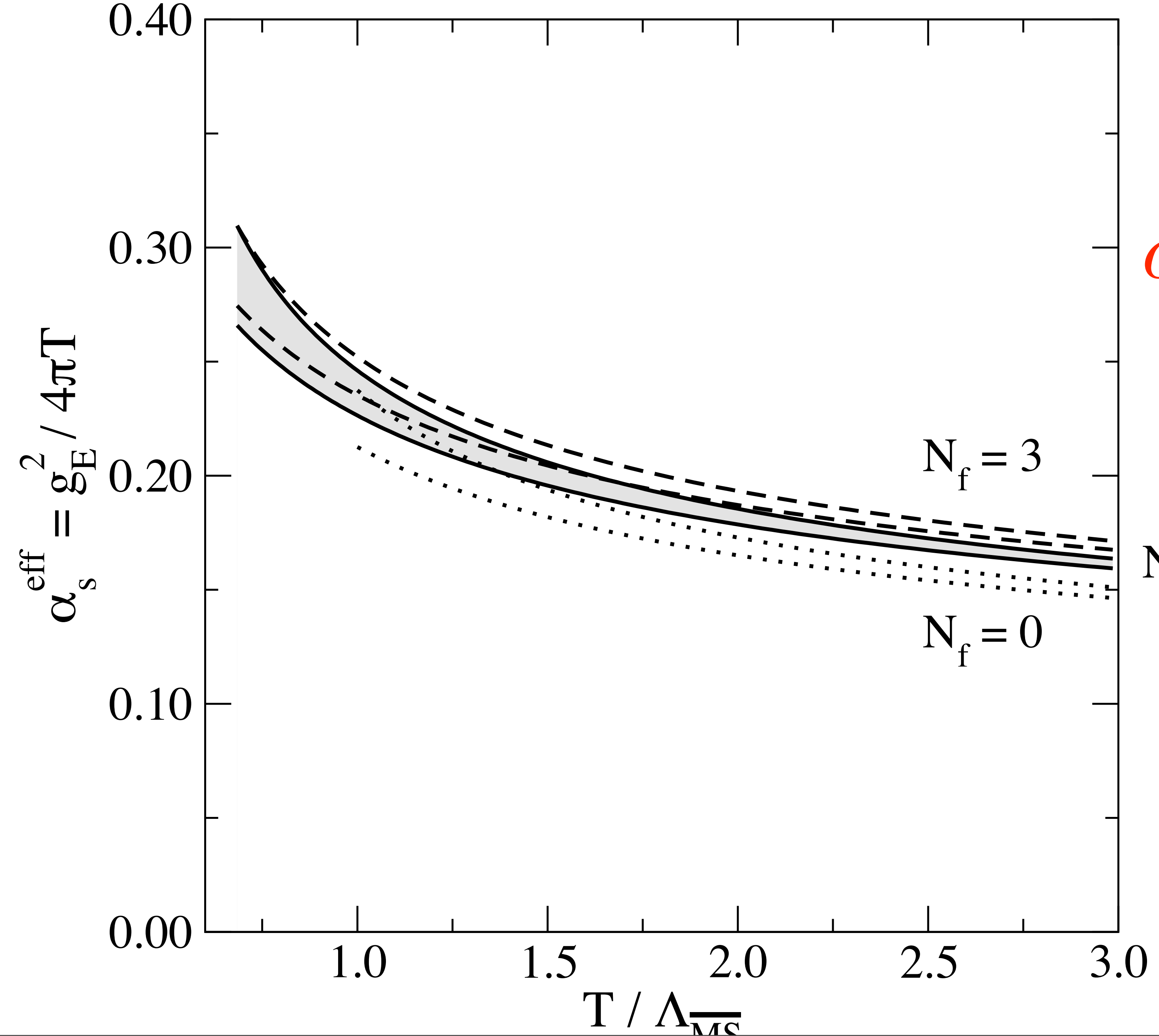
Maybe α_s is *not* so big at T_c

Braaten & Nieto, hep-ph/9508406...........Laine & Schröder, hep-ph/0503061 & 0603048:

$T_c \sim \Lambda_{MS} \sim 200$ MeV. But $\alpha_s^{eff}(T) \sim \alpha_s^{eff}(2\pi T) \sim 0.3$ at T_c: *not* so big

Two loop calculation in effective theory; grey band changing Λ_{MS} by factor 2

$\alpha_s^{\rm eff}(T) \uparrow$



$\alpha_s^{\rm eff}(T_c) \approx 0.3$

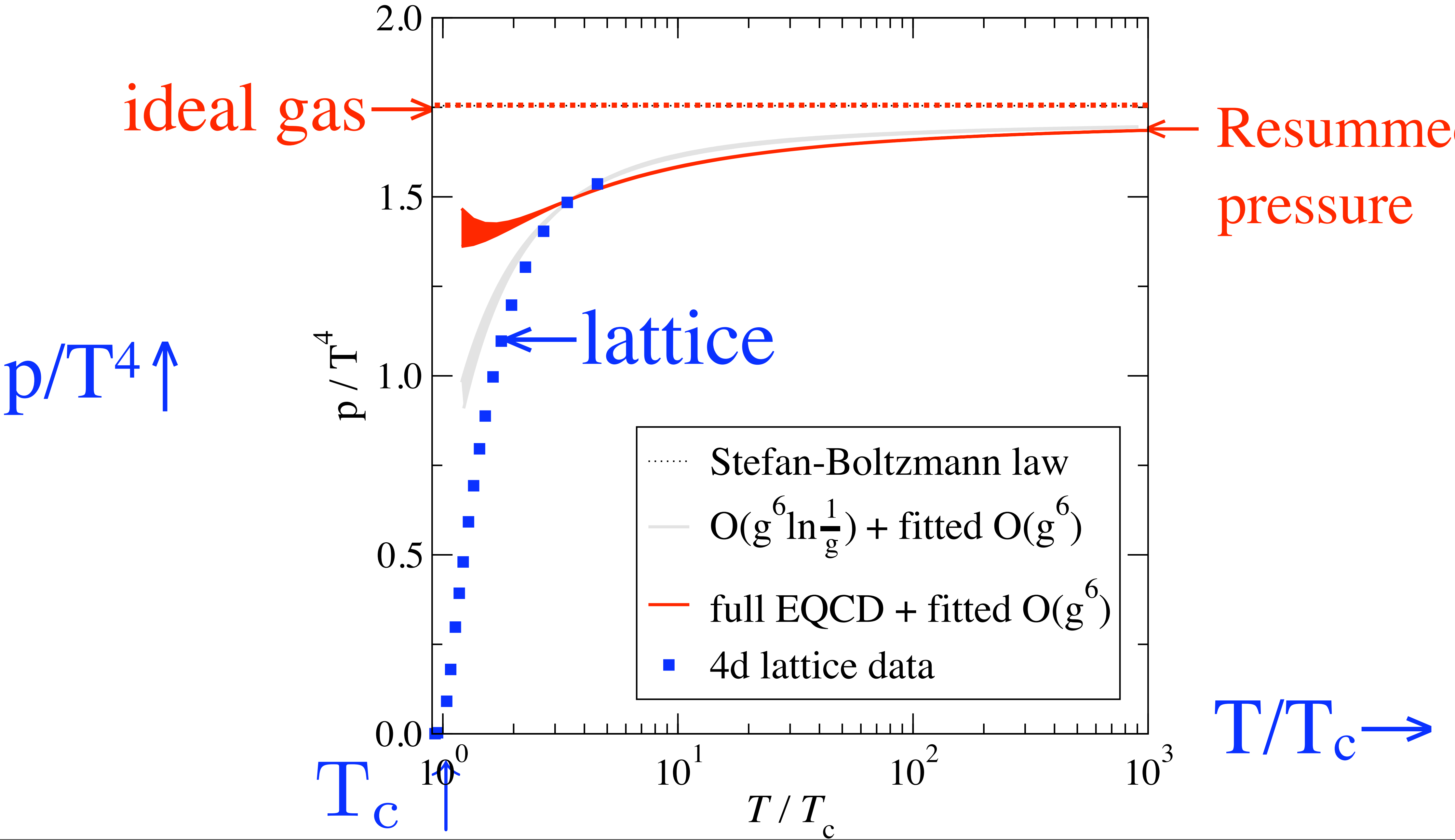
$T/\Lambda_{\overline{\rm MS}} \rightarrow$

Perturbative Resummation of the Pressure

"Helsinki" resummation: Hietanen, Kajantie, Laine, Rummukainen, Schröder, 0811.4664:

$$\mathcal{L}^{eff} = \frac{1}{2}\,\mathrm{tr}\, G_{ij}^2 \; + \; \mathrm{tr}\, |D_i A_0|^2 + \; m_D^2\, \mathrm{tr}\, A_0^2 \; + \; \kappa\, \mathrm{tr}\, A_0^4$$

Now to 4 loop, $\sim g^6$. *Works* to $\sim 3\,T_c$, *fails* below. *Why*, if $\alpha_s^{eff}(T_c)$ is *not* so big?

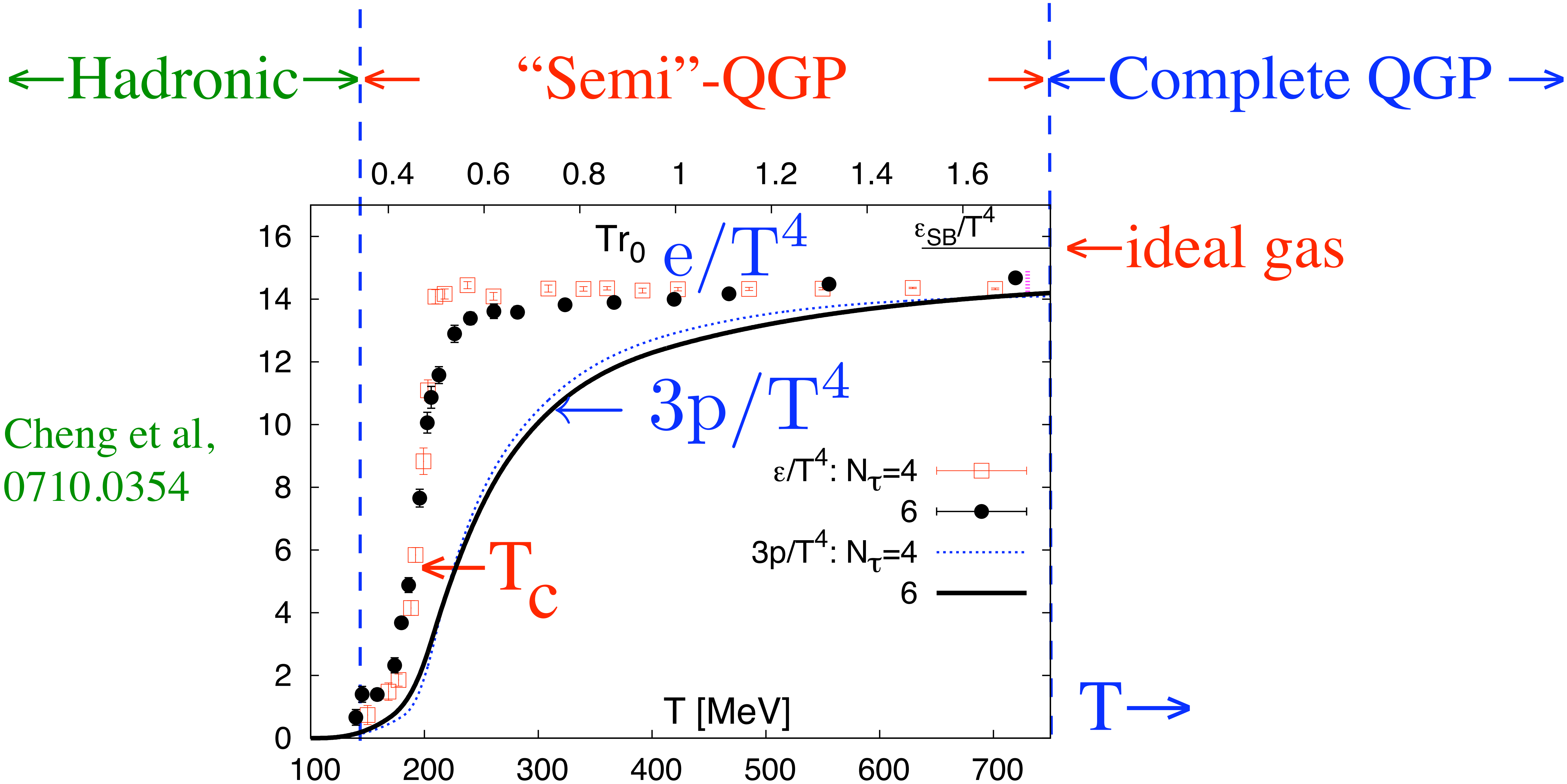


Ionizing Color in the QGP: Complete and Partial

T > 3-4 T_c: *complete* ionization of color, perturbative QGP, pressure ~ ideal.

T: T_c to ~ 3-4 T_c: *partial* ionization of color. “*Semi*” QGP. Pressure/ideal < 1.

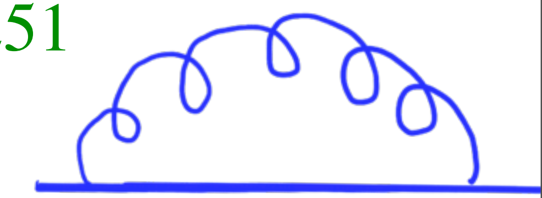
T < T_c: no ionization of color, *confined*. Pressure << ideal.



Renormalized Wilson Loops

Gervais & Neveu '80. Polyakov '80.......Gupta, Hübner & Kaczmarek 0711.2251

"Mass" renormalization of loops without cusps:



$$< \mathcal{W}_R^{\rm bare} > -1 = - \mathcal{C}_R g^2 \, L \int^{1/a} \frac{d^3 k}{k^2} = -(\mathcal{C}_R g^2 + \ldots) \frac{L}{a}$$

Loop in representation R, length L, lattice spacing "a". Vanishes with dim. reg.
With lattice reg., multiplicative renormalization of loop:

$$< \mathcal{W}_R^{\rm bare} >= \mathcal{Z}_R \; < \mathcal{W}_R^{\rm ren} > \;\; ; \;\; \mathcal{Z}_R = {\rm e}^{-f_R(g^2) N_t}$$

Determine $f_R(g^2)$ numerically, *not* perturbatively. Consider potential at T = 0:

$$< \mathcal{W}_{\rm fund} >= {\rm e}^{-V(R) t} \; , \; V(R \to \infty) \sim \sigma R + E_0 - \frac{\pi}{12 \, R} + \ldots$$

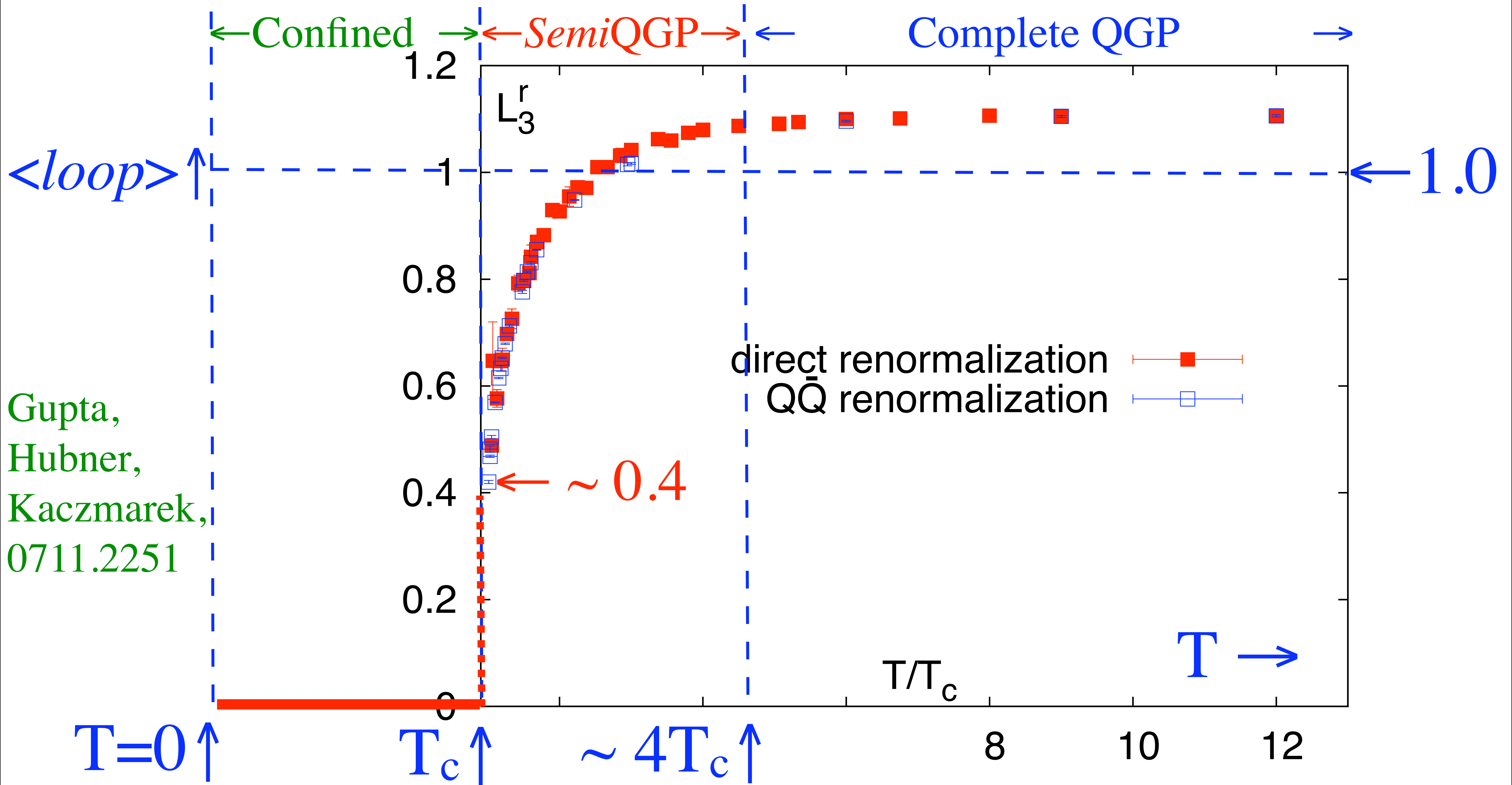
Zero point energy $E_0 = E_0^{\rm pert} + E_0^{\rm non\text{-}pert}$: with above, $E_0^{\rm pert} = 0$, $E_0^{\rm non\text{-}pert}$ is physical.

E.g., in string models:
Nambu: $E_0^{\rm non\text{-}pert} = 0$. Smooth or confining strings: $E_0^{\rm non\text{-}pert} \sim \sqrt{\sigma} < 0$. *Testable*.

Lattice: Ren.'d Polyakov Loop, Glue c/o Quarks

Lattice: renormalized Polyakov loop. Looks like pressure:

Semi-QGP: <*loop*> < 1, T: T_c → 4 T_c Complete QGP: <*loop*> ~ 1, T > 4 T_c

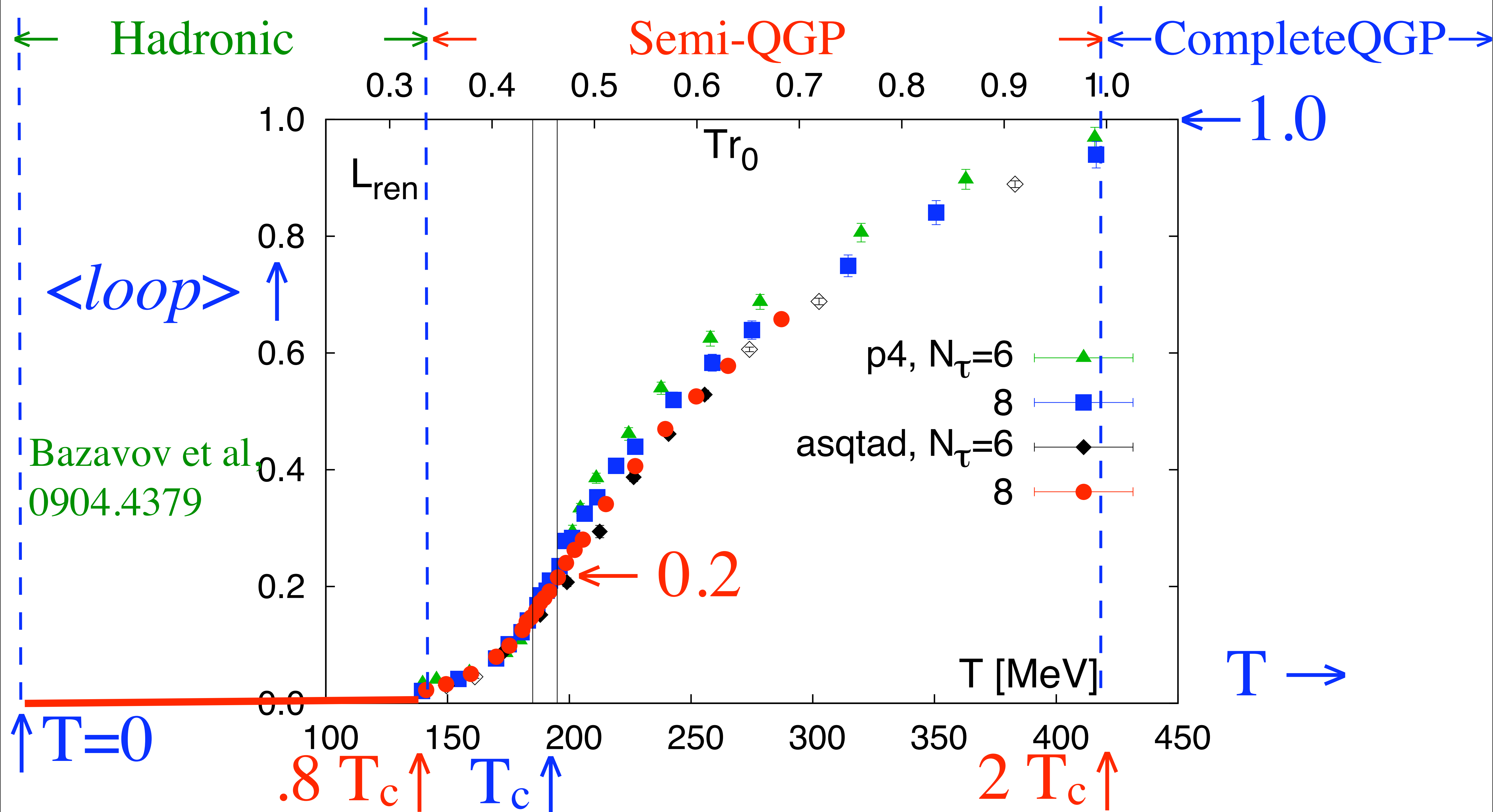


Gupta, Hubner, Kaczmarek, 0711.2251

Lattice: Ren.'d Polyakov Loop, Glue plus Quarks

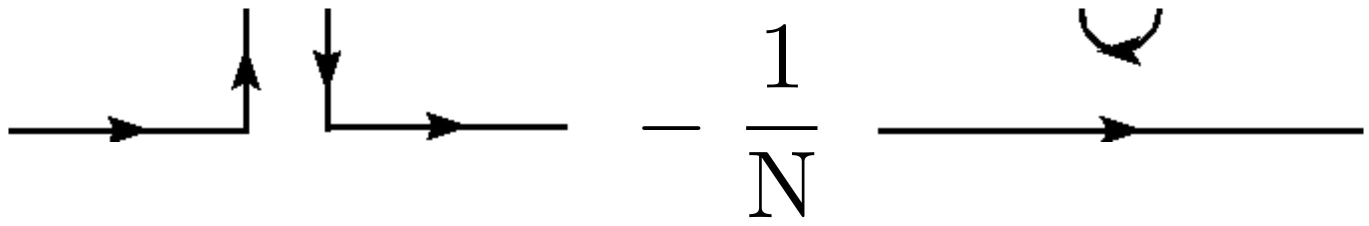
Quarks ~ background Z(3) field. *Lattice*: quarks do not wash out loop.

Semi-QGP: <*loop*> nonzero above 0.8 T_c (< T_c), < 1 up to ~ 2-3 T_c .



Semi-QGP in Weak Coupling

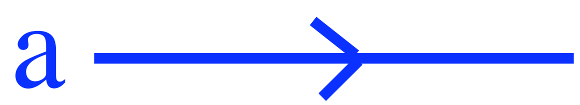
Double line notation at *finite* N:
"birdtracks", Cvitanovic '76, '09.

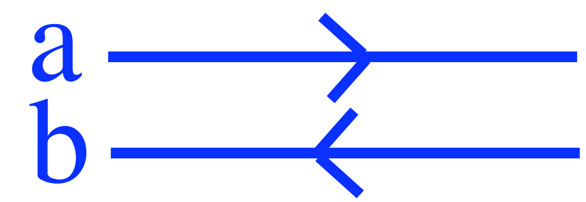


Semi-classical expansion:
Q = Q^a diagonal matrix

$$A_\mu = A_\mu^{\rm cl} + B_\mu \ , \quad A_0^{\rm cl} = Q/g \ .$$

Propagators, large N_c:





$$i D_0^{\rm cl} = p_0 + Q^a = p_0^a \qquad\qquad i D_0^{\rm cl} = p_0 + Q^a - Q^b = p_0^{ab}$$

Propagators just with "shifted" p_0's.
Real time: $p_0^a \rightarrow i\,\omega$, etc. Furuuchi, hep-th/0510056

Q (imaginary) chemical potential
for (diagonal) color charge.
e.g., for quarks:

$$\widetilde{n}(E - iQ^a) = \frac{1}{{\rm e}^{(E - iQ^a)/T} + 1}$$

Bleaching of Color in the Semi-QGP

Aharony, Marsano, Minwalla, Papadodimas, &Van Raamsdonk, hep-th/0310285, 0502149

$$\mathrm{tr}\ \frac{1}{\mathrm{e}^{(E-iQ^a)/T}-1} = \mathrm{tr}\ \sum_{j=1}^{\infty} \mathrm{e}^{-j(E-iQ^a)/T} = \sum_{j=1}^{\infty} \mathrm{e}^{-jE/T}\ \mathrm{tr}\ \mathbf{L}^j$$

L = $e^{i\,Q/T}$ = Wilson line. Obtain expressions in terms of moments of L, L^j.

Complete effective theory for Q's? Correlate pressure & ren.'d loop. Not yet, so:

Take first moment, l = <*loop*> = < tr L>/N, from lattice for N = 3.
For higher moments, given l, assume either: Gross-Witten, or step function.

Wilson line L ~ propagator of *infinitely* heavy (test) quark.

In a semi-cl. expansion, quark propagator ~ e^{iQa} ~ l ; gluon prop. ~ $e^{i(Qa-\,Qb)}$ ~ l^2 :

Universal "bleaching" of color as <*loop*> → 0, for *any* momentum & mass

Shear Viscosity in the Semi-QGP

Perturbative shear viscosity, η: Arnold, Moore & Yaffe, hep-ph/0010177 & 0302165

Generalize to $Q \neq 0$: Boltzmann equation in background field.

$$\eta = \frac{\mathcal{S}^2}{\mathcal{C}}$$

S = source, C = collision term. *Two* ways of getting small η:

"Strong" QGP, *large* coupling $S \sim 1$, $C \sim (\text{coupling})^2 >> 1$.

$\mathcal{N} = 4$ SU(N), $g^2 N = N = \infty$: Kovtun, Son & Starinets hep-th/0405231

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

"Semi" QGP: small *loop* at *moderate* coupling:

Pure glue: $S \sim \langle loop \rangle^2$, $C \sim g^4 \langle loop \rangle^2$

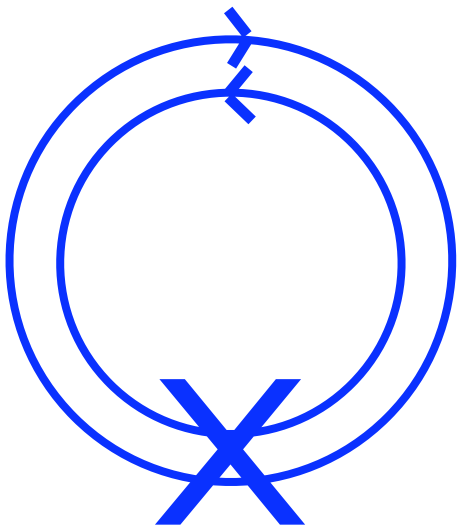
With quarks: $S \sim \langle loop \rangle$, $C \sim g^4$

Both: $\eta \sim \langle loop \rangle^2$

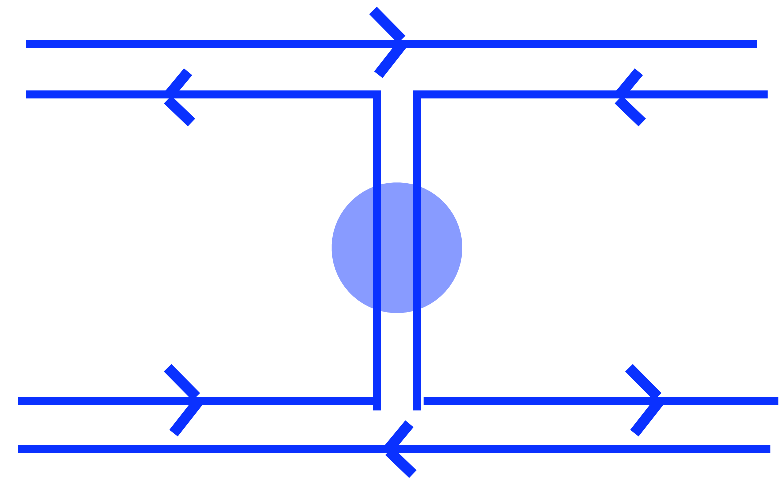
To leading log order: # from AMY, constant "c" beyond leading log

$$\frac{\eta}{T^3} = \frac{\#}{g^4 \log(c/g)} \mathcal{R}(\ell) \quad ; \quad \mathcal{R}(\ell \to 0) \sim \ell^2$$

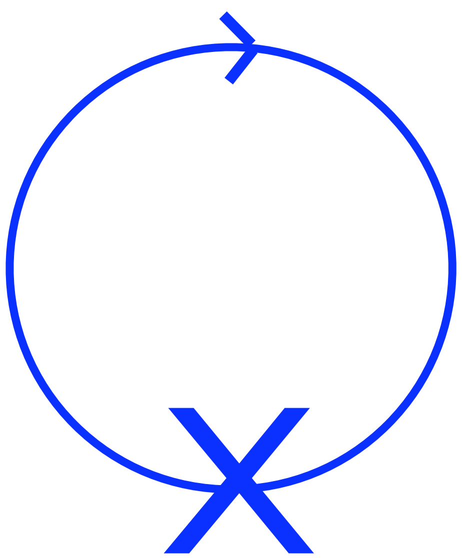
Counting Powers of <*loop*> = $l \to 0$



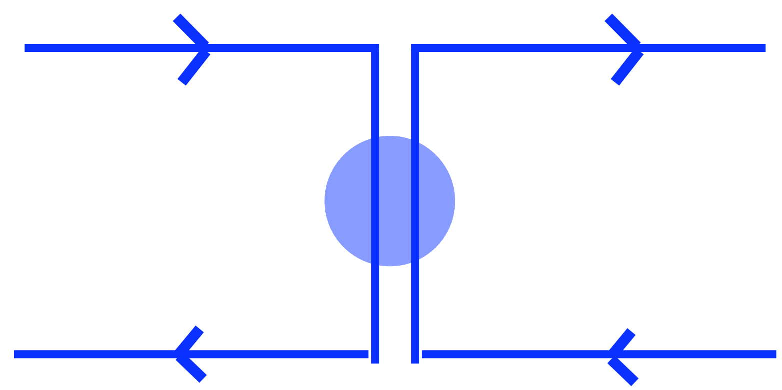
$\mathcal{S} \sim \ell^2$



$\mathcal{C} \sim \ell^2$



$\mathcal{S} \sim \ell$



$\mathcal{C} \sim 1$

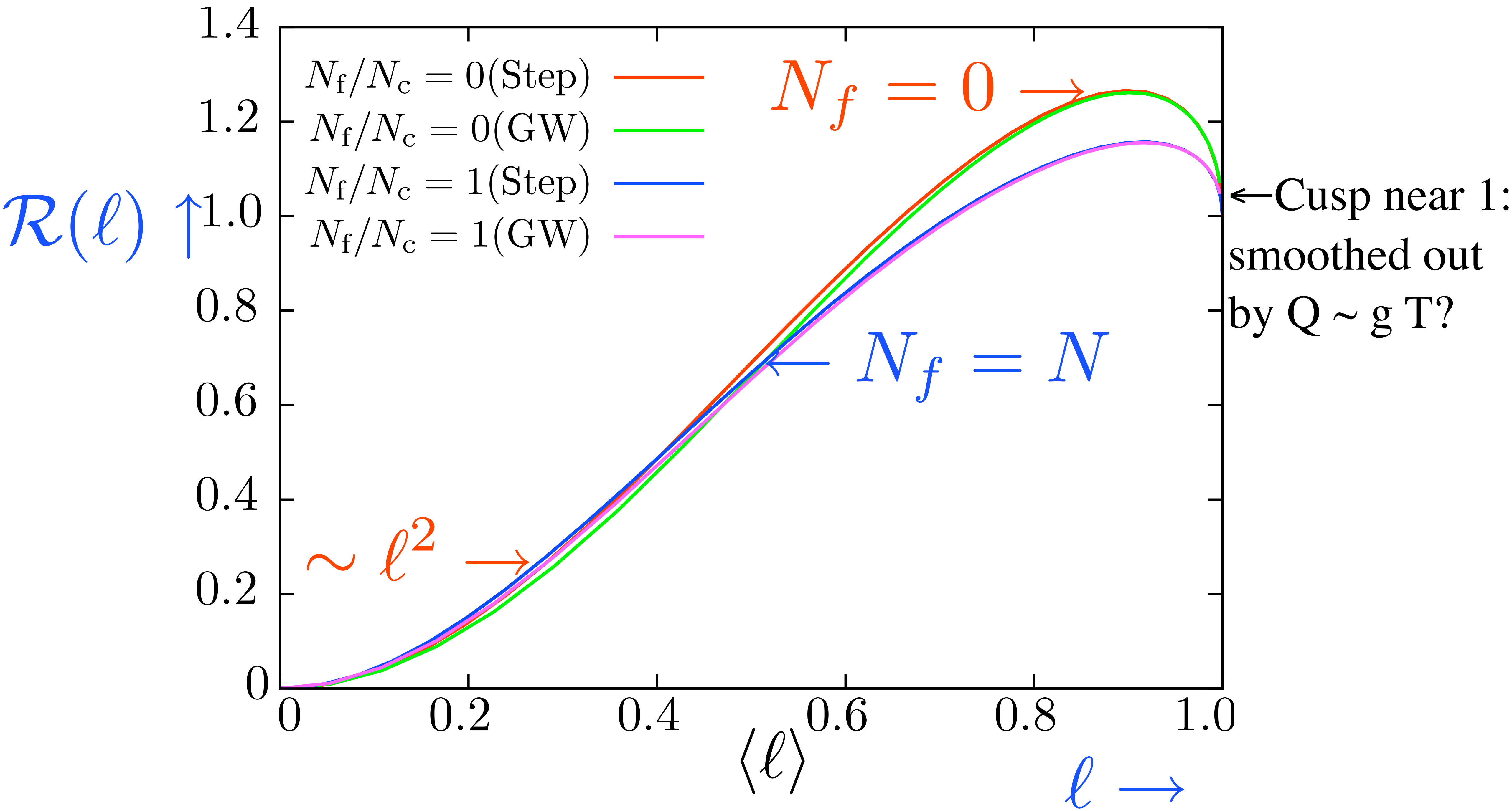
$\longrightarrow \quad \sim e^{+iQ^a/T}$

$\longleftarrow \quad \sim e^{-iQ^a/T}$

Suppression of Shear Viscosity

R = ratio of shear viscosity in semi-QGP/complete-QGP at *same* g, T.
Two different eigenvalue distributions give *very* similar results!

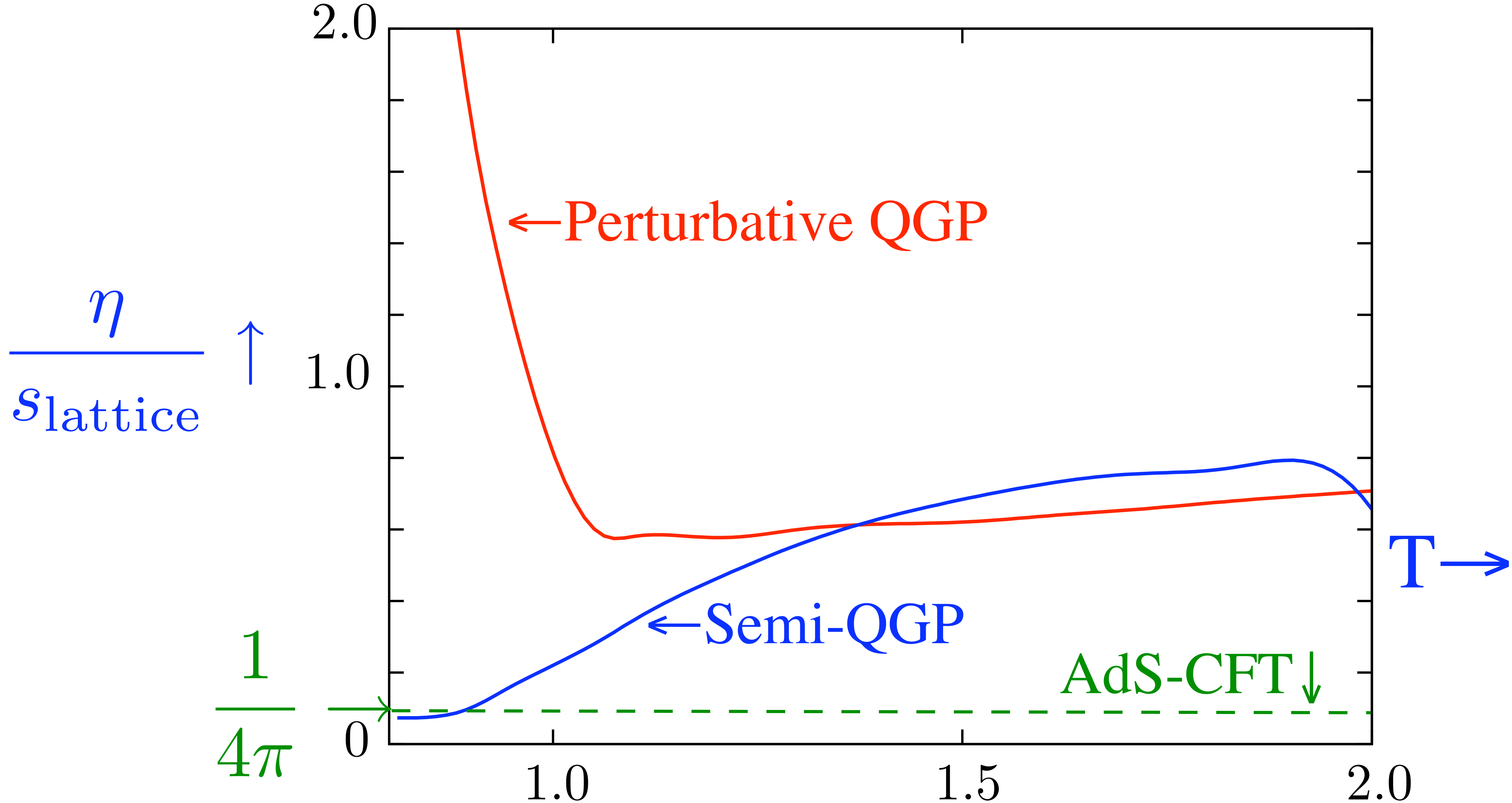
When <*loop*> ~ 0.3, *R* ~ 0.3.



Shear Viscosity in the Semi-QGP

Semi-QGP: <*loop*> small near T_c, ~ 0.2 : $\eta/s \sim \langle loop \rangle^2$

Entropy from lattice. Estimates only leading log, so overall scale *will* change



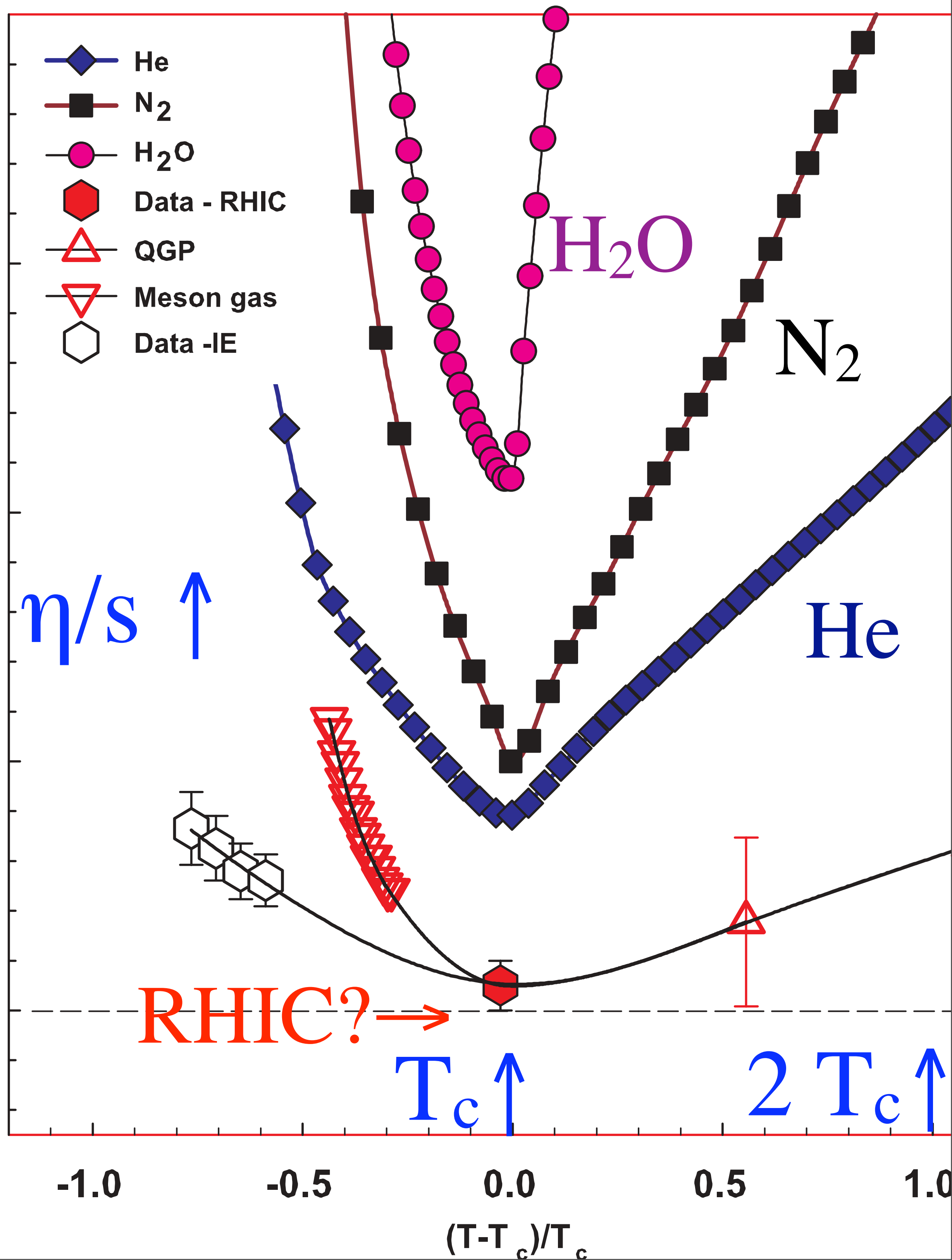
For Heavy Ions, is LHC ≈ RHIC?

At RHIC, $\eta/s \sim 0.1 \pm 0.1$
Luzum & Romatschke, 0804.4015
Close to N = 4 SU(∞), $\eta/s = 1/4\pi$.

Strong-QGP: in $\mathcal{N} = 4$ SU(∞),
add scalar potential to fit lattice pressure
But η/s remains $= 1/4\pi$
Liu, Rajagopal & Shi, 0803.3214
Evans & Threlfall, 0805.0956
Gubser & Nellore, 0804.0434
Gursoy, Kiritsis, Mazzanti & Nitti
0804.0899, 0812.0792, 0903.2859
QGP stays nearly ideal,
LHC ≈ RHIC

Semi-QGP: η/s changes from T_c to 2 T_c.
At early times, QGP viscous,
LHC ≠ RHIC

Lacey....Stocker, nucl-ex/0609025 ↓



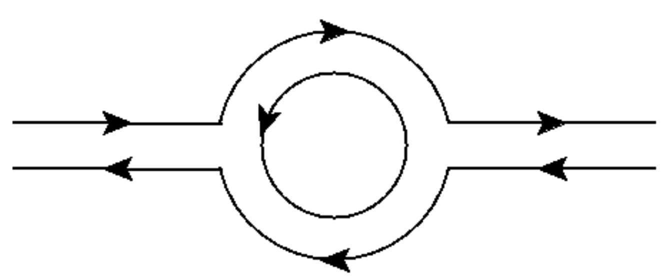
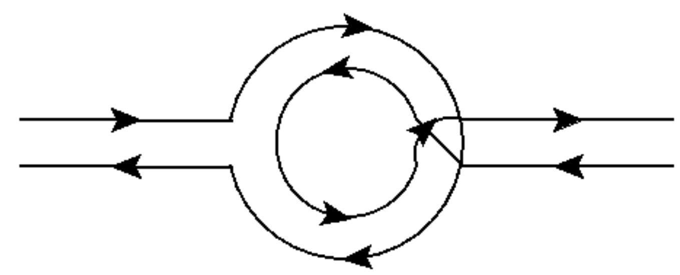
Quarkyonic Matter

QCD at Large N_c , small N_f

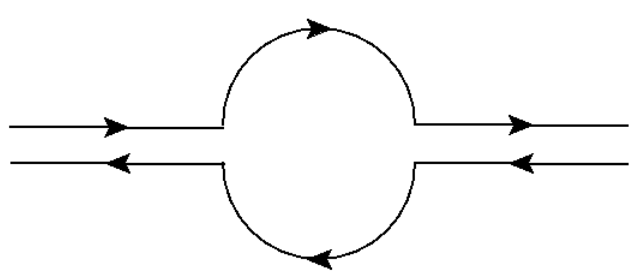
In $SU(N_c)$, gluons matrices, N_c x N_c, quarks column vectors.

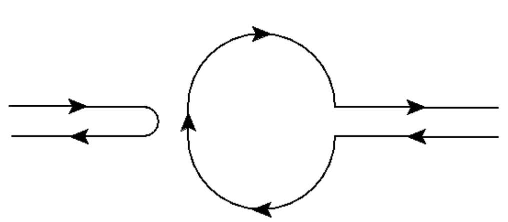
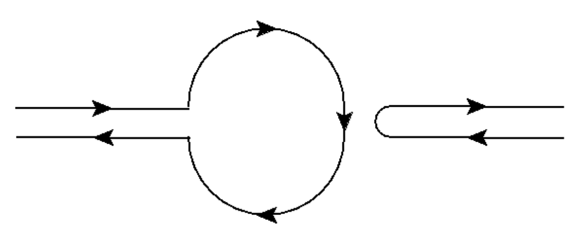
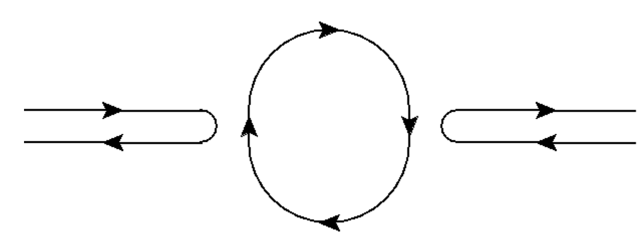
't Hooft '74: let N_c = # colors $\rightarrow \infty$, $\lambda = g^2 N_c$ *fixed*. Keep N_f = # flavors *finite*.

Gluon self energy at 1 loop order. At any N_c, color structure:

$\sim g^2 N_c \sim \lambda$  $-$  $\sim g^2 \sim \dfrac{\lambda}{N_c}$

Planar diagrams dominate. For quarks:

 $\sim g^2 N_f \sim \dfrac{1}{N_c} N_f \lambda$

$-\dfrac{1}{N}$  $-\dfrac{1}{N}$  $+\dfrac{1}{N^2}$ 

If $N_f/N_c \rightarrow 0$ as $N_c \rightarrow \infty$, loops dominated by gluons, blind to quarks.

Phases at Large N_c : *Pressure* as an Order Parameter

$T = \mu_{qk} = 0$: confined, only color singlets. Glueballs, meson masses ~ 1.
Baryons *very* heavy, masses $\sim N_c$, so no virtual baryon anti-baryon pairs.

$T \neq 0$, $\mu_{qk} = 0$:

$T < T_c$: Hadrons. $T_c \sim$ mass ~ 1. # hadrons ~ 1, so pressure = $p \sim 1$: *small*.

$T > T_c$: Quark-Gluon Plasma. Deconfined gluons & quarks.
gluons $\sim N_c^2$, so $p \sim N_c^2$: *big*. Dominated by gluons.

$T \neq 0$, $\mu_{qk} \neq 0$: usual mass threshold, baryons only when $\mu_{qk} > M_N/N_c = m_{qk} \sim 1$.

$T < T_c$, $\mu_{qk} < m_{qk}$: Hadronic "box" in T-μ_{qk} plane: *no* baryons.

$T > T_c$ any μ_{qk} : Quark-Gluon Plasma. Some quarks, so what, $p_{qk} \sim N_c$.

$T < T_c$, $\mu_{qk} > m_{qk}$: # quarks $\sim N_c$, so $p \sim N_c$: *dense* nuclear matter (*not* dilute)
Confined phase! But Fermi sea of *quarks*? "*Quark-yonic*"

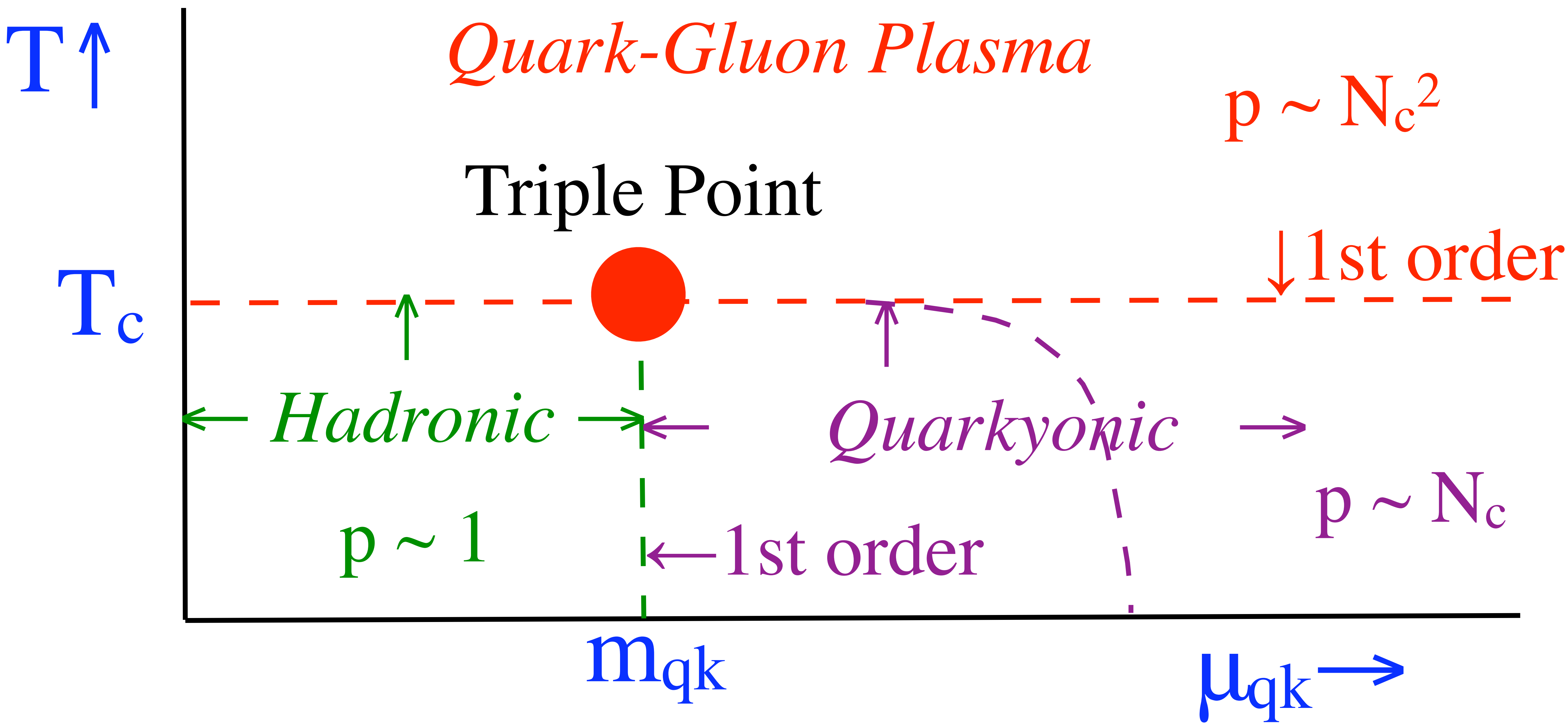
Phase Diagram at Large N_c , Small N_f

Lattice (Panero, 0907.3719): deconfining transition 1st order at $T \neq 0$, $\mu_{qk} = 0$.
must remain so when $\mu_{qk} \neq 0$. *Straight* line in T - μ_{qk} plane.

Hadronic/Quarkyonic transition: energy density jumps by N_c, => 1st order ?

Chiral transition: where in Quarkyonic phase?

True triple point!



Lattice, Pure Glue: SU(3) close to SU(∞)

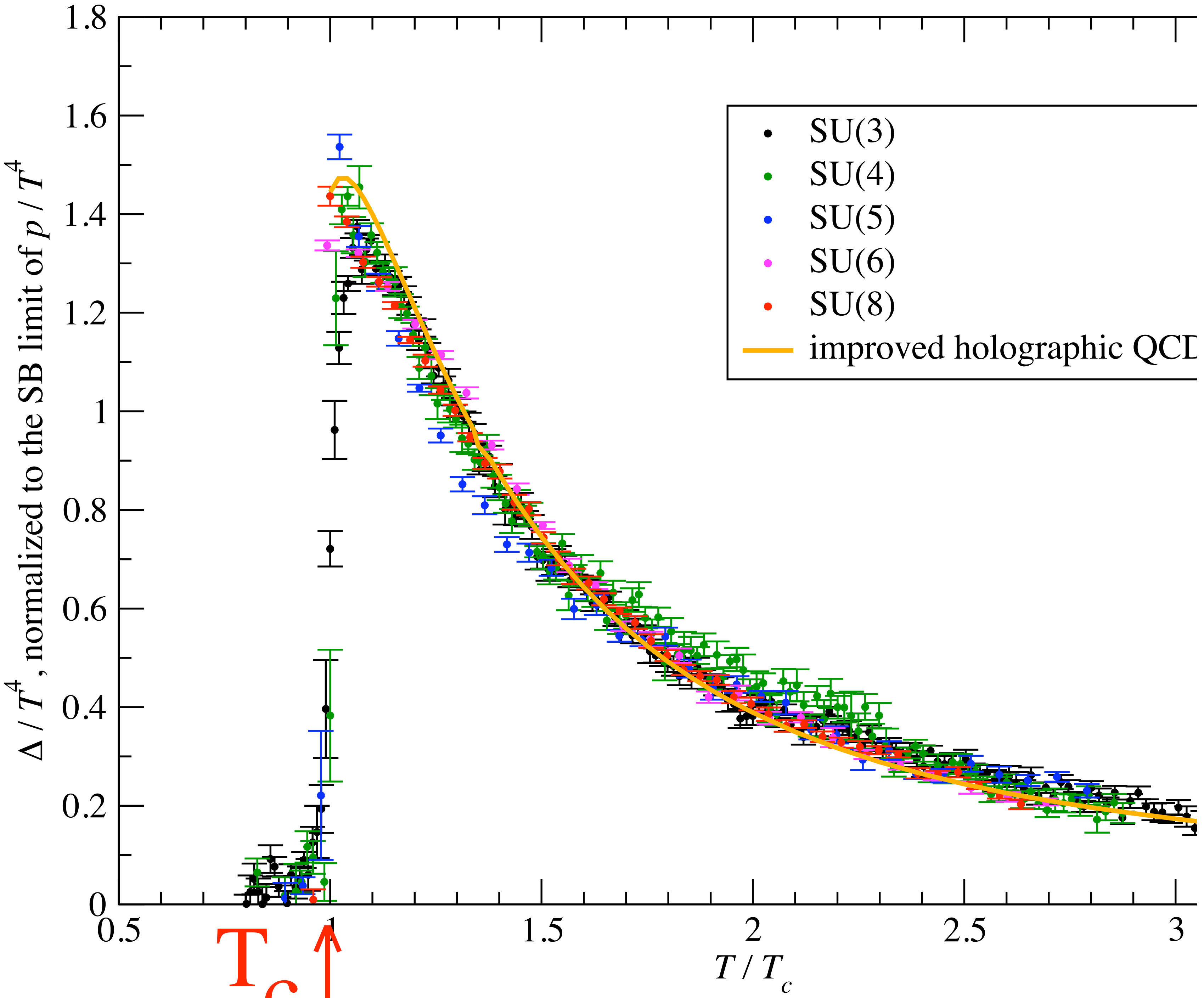
Panero, 0907.3719: SU(N_c), *no quarks*, $N_c = 3, 4, 5, 6, 8$.

Deconfining transition first order, $N_c = 3$ close to $N_c = \infty$

Improved holographic: *fit* of scalar potential

$$\frac{e - 3p}{N^2\, T^4} \sim \text{const.}$$

$$\frac{e - 3p}{N^2\, T^4} \uparrow$$



$T_c \uparrow$

$T/T_c \rightarrow$

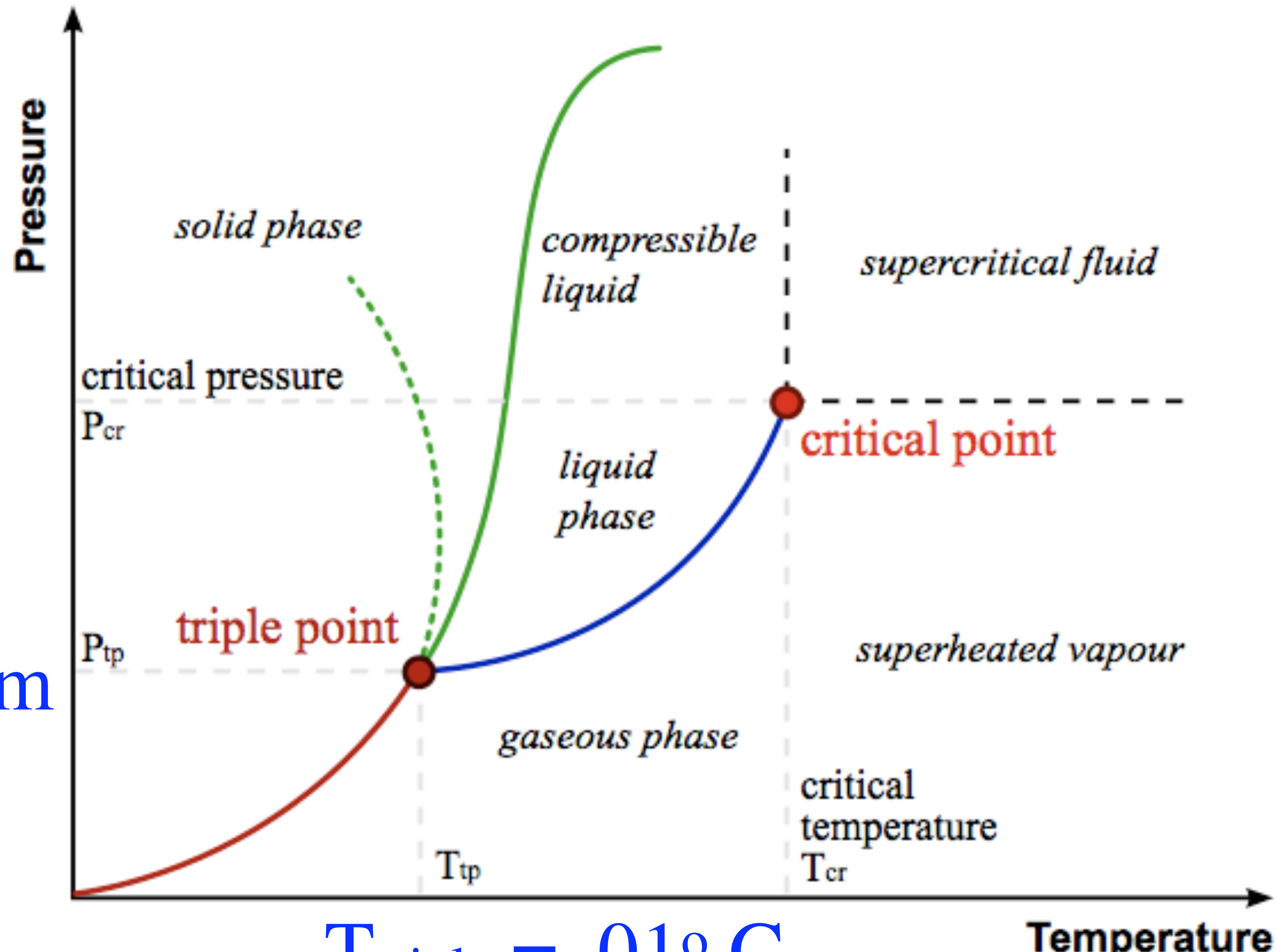
Triple Point for Water

Triple point where three lines of first order transitions meet.

E.g., for ice/water/steam, in plane of temperature and pressure.

(Generalizes: four lines of first order transitions meeting is a quadruple point.)

Generically, *distinct* from critical (end) point, where one first order line ends.



P↑

P_{triple} =
.006 atm

T →

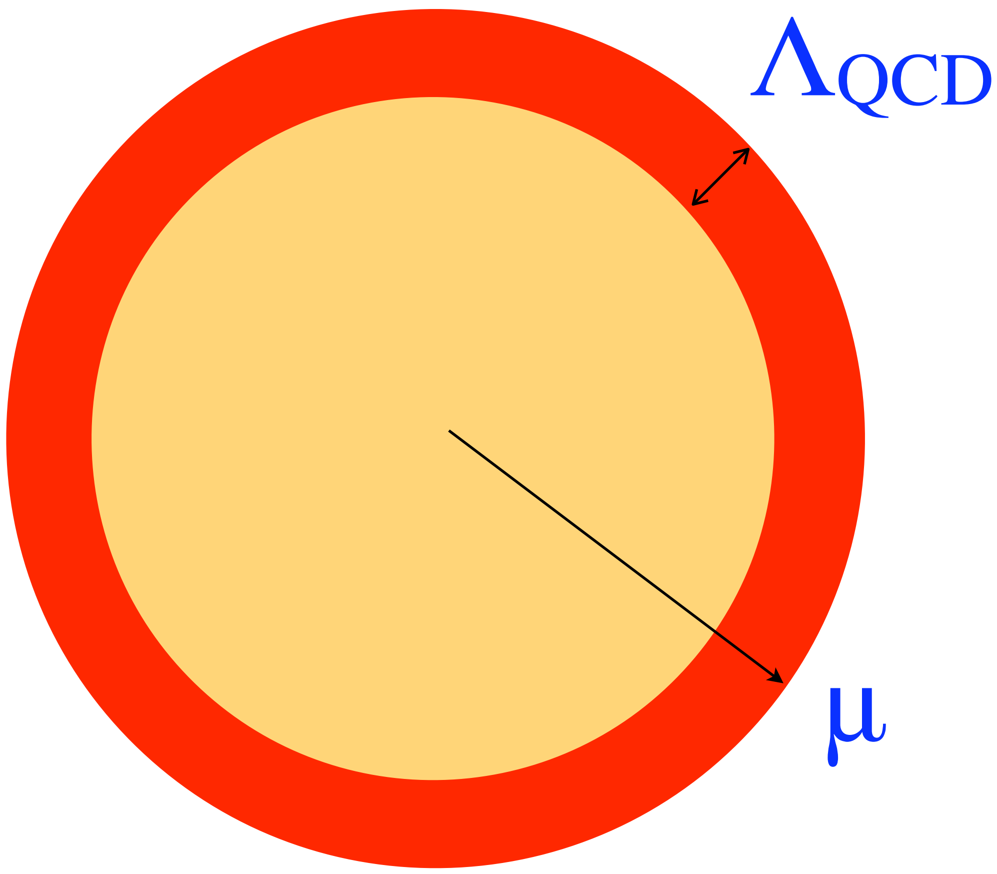
$T_{triple} = .01^{o}$ C

Quarkyonic Matter at Large μ

Let $\mu \gg \Lambda_{QCD}$ but $\sim N_c^0$. Coupling runs with μ, so pressure $\sim N_c$ is close to perturbative! How can the pressure be (nearly) perturbative in a confined theory?

Pressure: dominated by quarks far from Fermi surf.: perturbative,
$p_{qk} \sim N_c\, \mu^4 (1 + g^2(\mu) + g^4(\mu) \log(\mu) + \ldots)$

Within Λ_{QCD} of Fermi surface: *confined* states.
$p_{qk} \sim N_c\, \mu^4 (\Lambda_{QCD}/\mu)^2$, *non*-perturbative.
Within skin, only confined states contribute.



Fermi sea of quarks + Fermi surface of bar-yons
= "quark-yonic". *N=3?*

Pressure dominated by quarks.
But transport properties *dominated* by confined states near Fermi surface.

QCD: cold nuclear matter at high density?

Deconfining Critical Endpoint at Large $\mu_{qk} \sim N_c^{1/2}$

Semi-QGP:

for large μ: compute one loop determinant in background field. $A_0 = \frac{T}{g}\,Q$

Korthals-Altes, Sinkovics, & RDP hep-ph/9904305

$$S_{qk} = \mathrm{tr}\,(\mu + i\,T\,Q)^4 \;,\; T^2\;\mathrm{tr}\,(\mu + i\,T\,Q)^2 \;,\; N_c^2\,T^4\,V(Q)$$

Expand for large μ:

$$S^{qk}_{\mu\sim\sqrt{N_c},T\sim 1} \sim N_c\,\mu^4 - 6\,\mu^2\,T^2\,tr\,Q^2 + \ldots \sim N_c^3 \;,\; N_c^2\;(tr\,Q^2/N_c)$$

Consider $\mu \sim N_c^{1/2}$, $T \sim 1$: gluons *do* feel quarks.

Term $\mu^4 \sim N_c^3$ dominates, but *independent* of Q *and* T. Term $\sim \mu^3\,T\,tr\,Q = 0$

Term $\mu^2 \sim N_c^2$ Q-dependent. Breaks $Z(N_c)$ symmetry, so washes out 1st order deconfining transition. Critical endpoint for deconfinement.

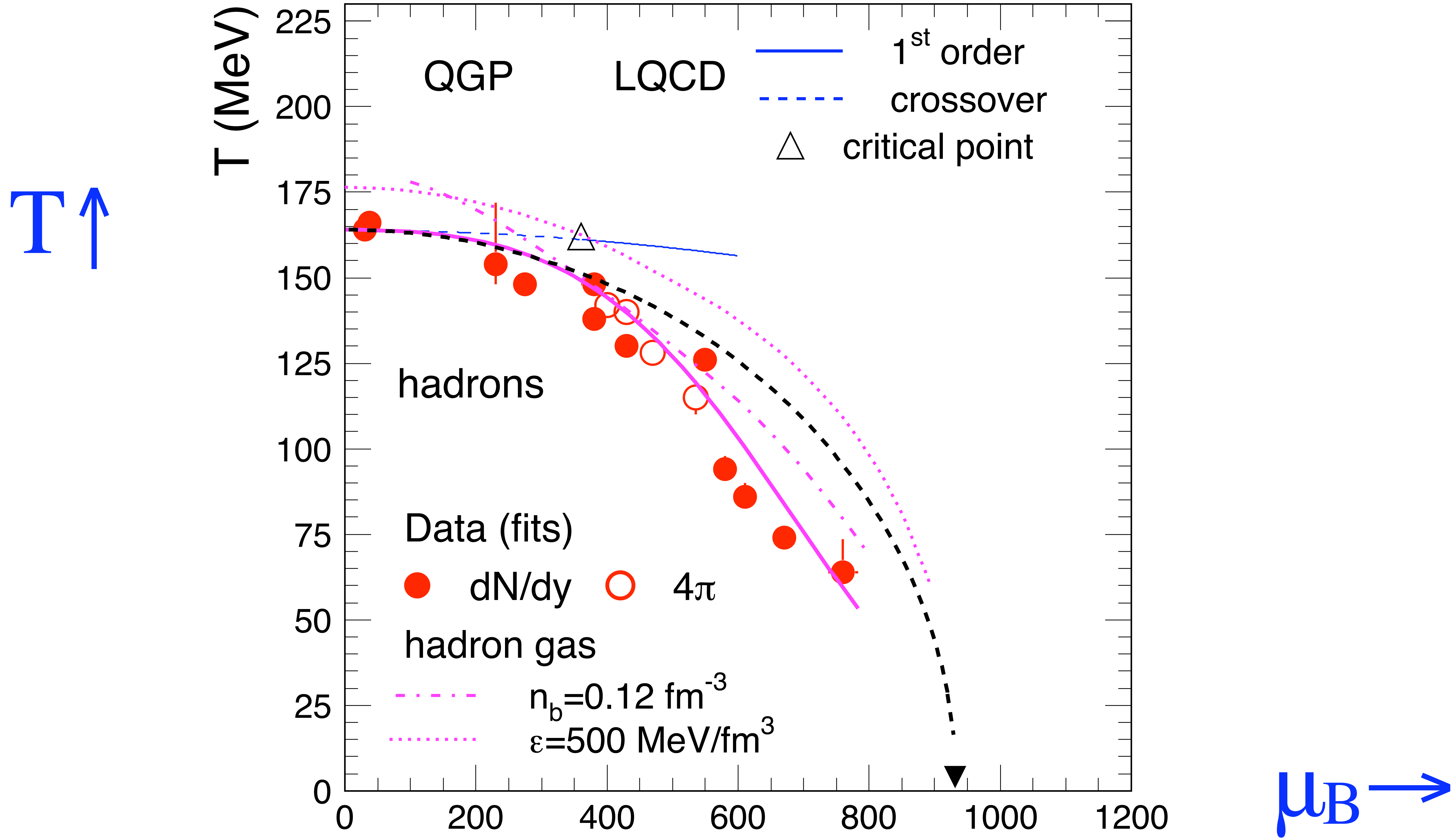
Triple Point at Large N_c:
= Critical Endpoint in QCD?

Wonderous Utility of Statistical Models

Chemical equilibriation at SIS, AGS, SPS, RHIC, and onto NICA and FAIR:

Braun-Munzinger, Cleymans, Oeschler, Redlich, Stachel

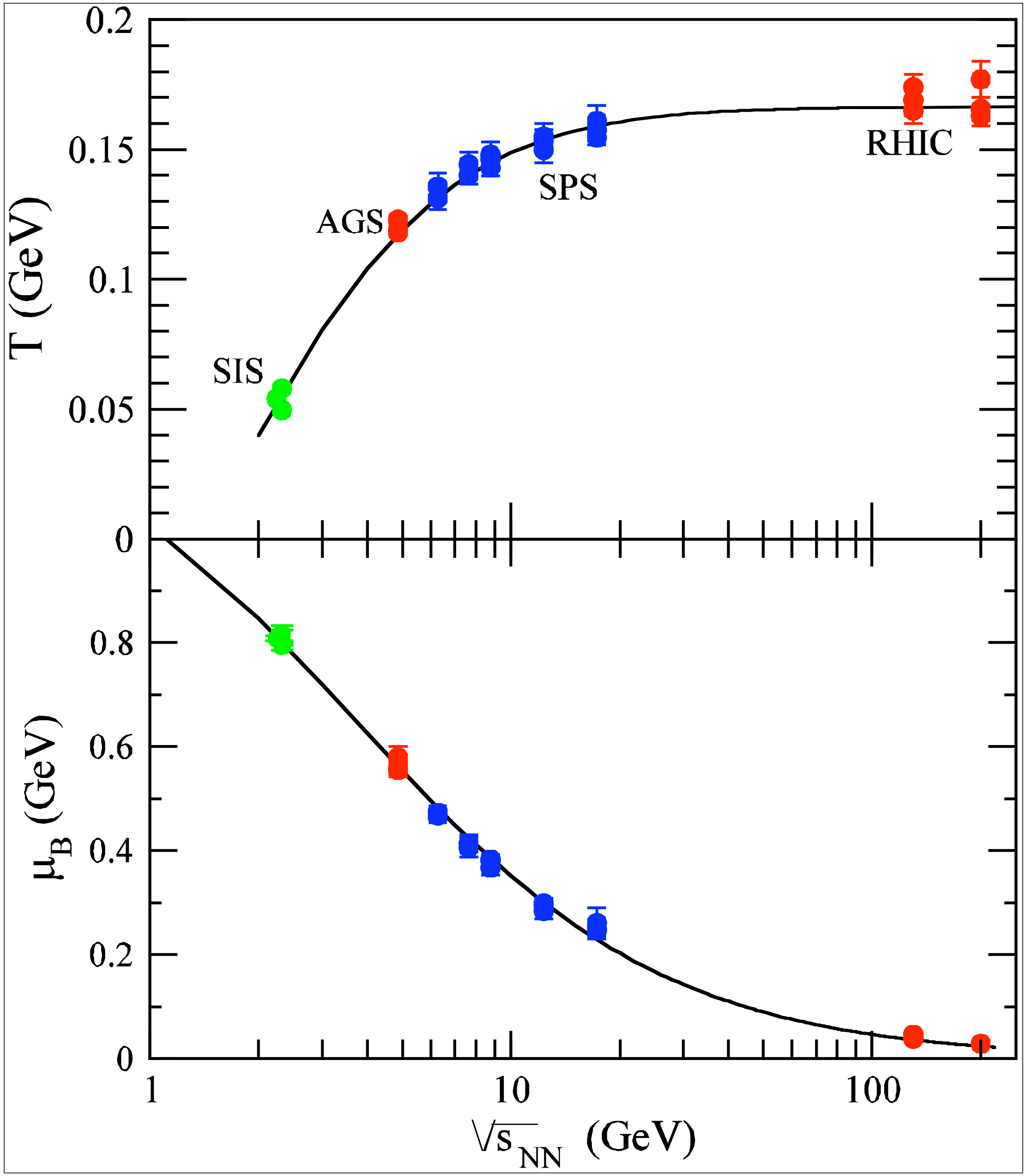
plus: Bialas, Biro, Broniowski, Florkowski, Levai, Ko, Satz + ...



Smooth Evolution in T, μ_{Baryon} with $\sqrt{s_{NN}}$

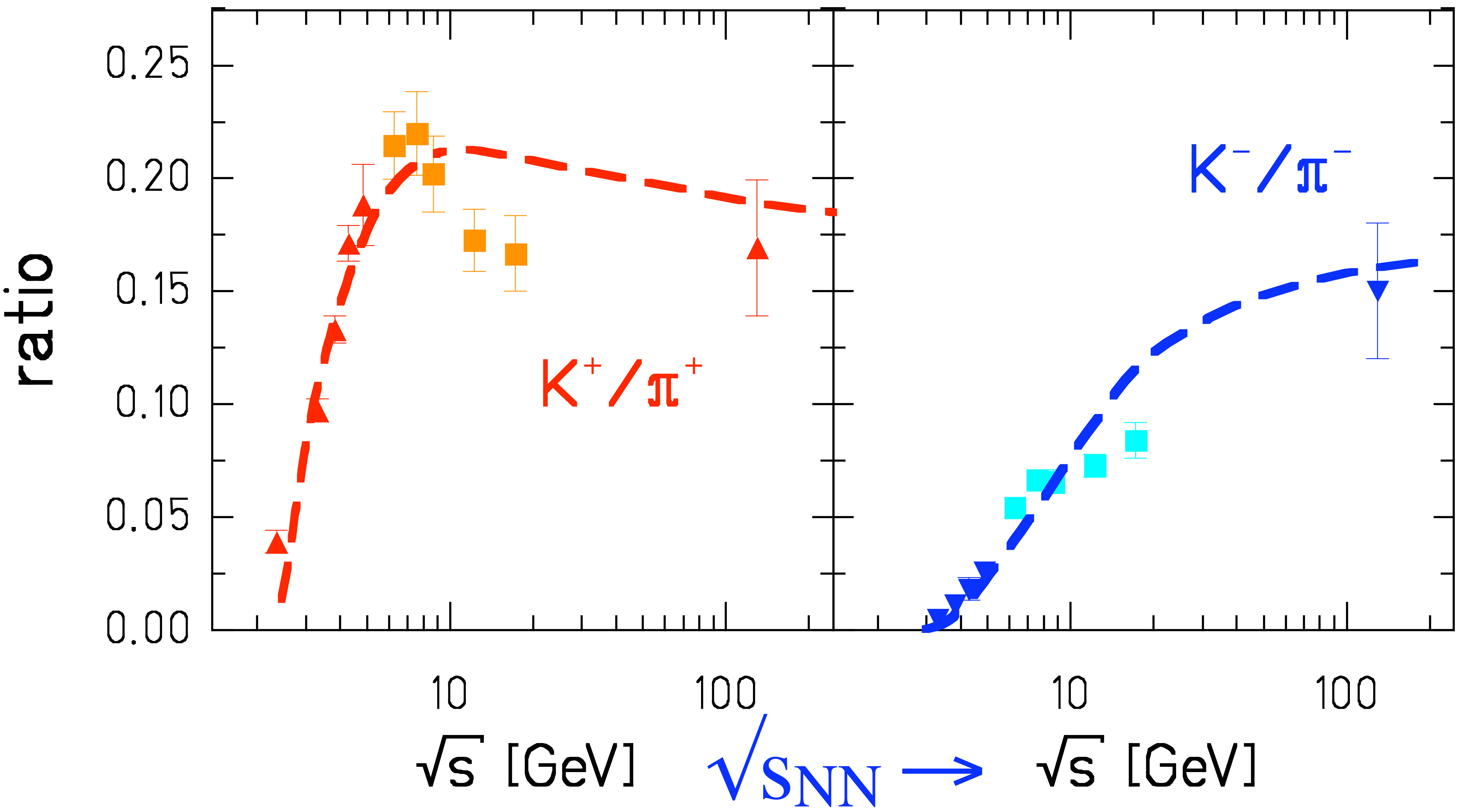
$T\uparrow$

$\mu_B\uparrow$



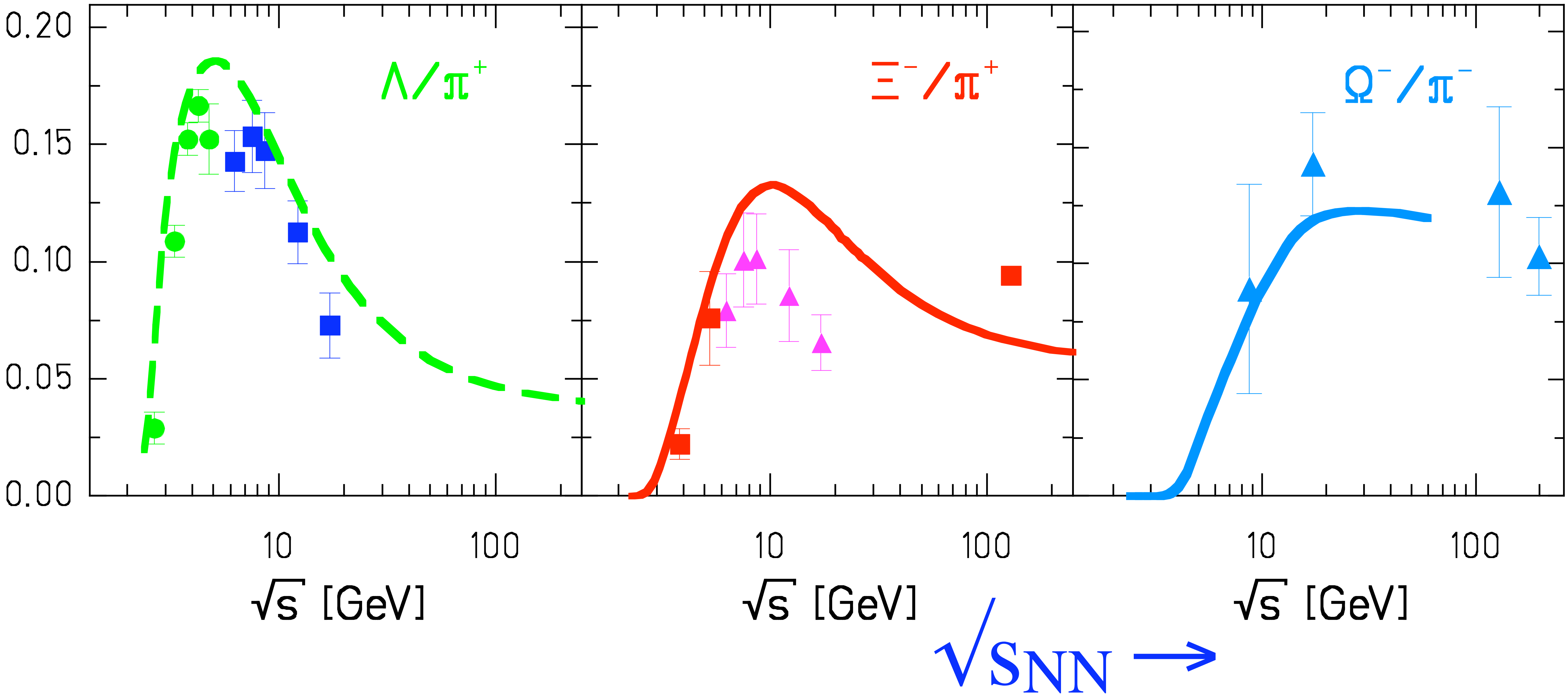
$\sqrt{s_{NN}} \rightarrow$

Strange MatterHorn: Peak in K^+/π^+, *not* K^-/π^-



Strange MatterHorn: also in Baryons

Natural to have peaks in K^+/π^+, strange baryons: start with (s s-bar) pairs.
At $\mu \neq 0$, strange quarks combine into baryons, anti-strange into pions.
For different baryons, peaks do not occur at same energy, but nearby, so
not true phase transition, at best approximate.

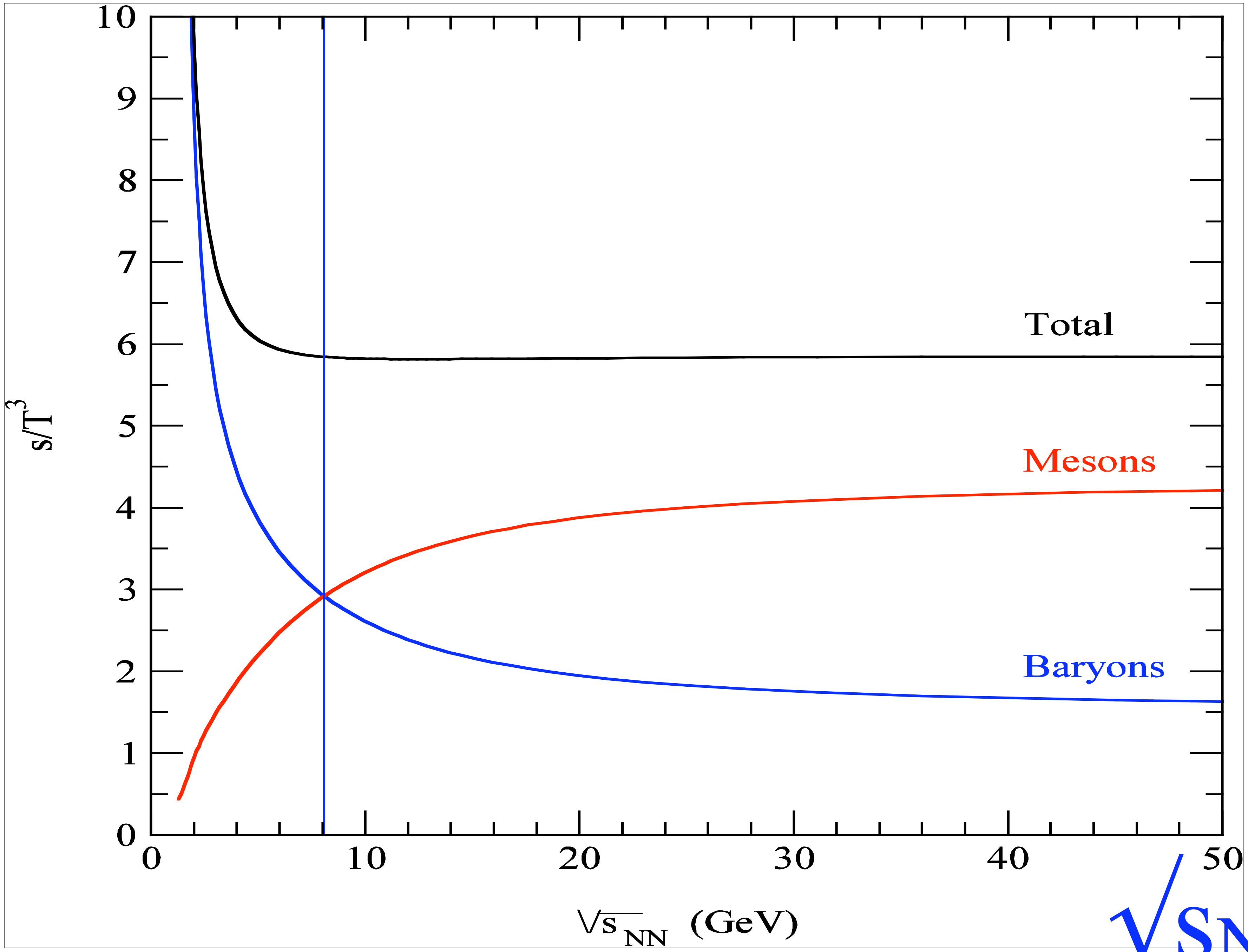


Strange MatterHorn and the Triple Point?

Usual explanation of MatterHorn: transition from baryons to mesons at freezeout.

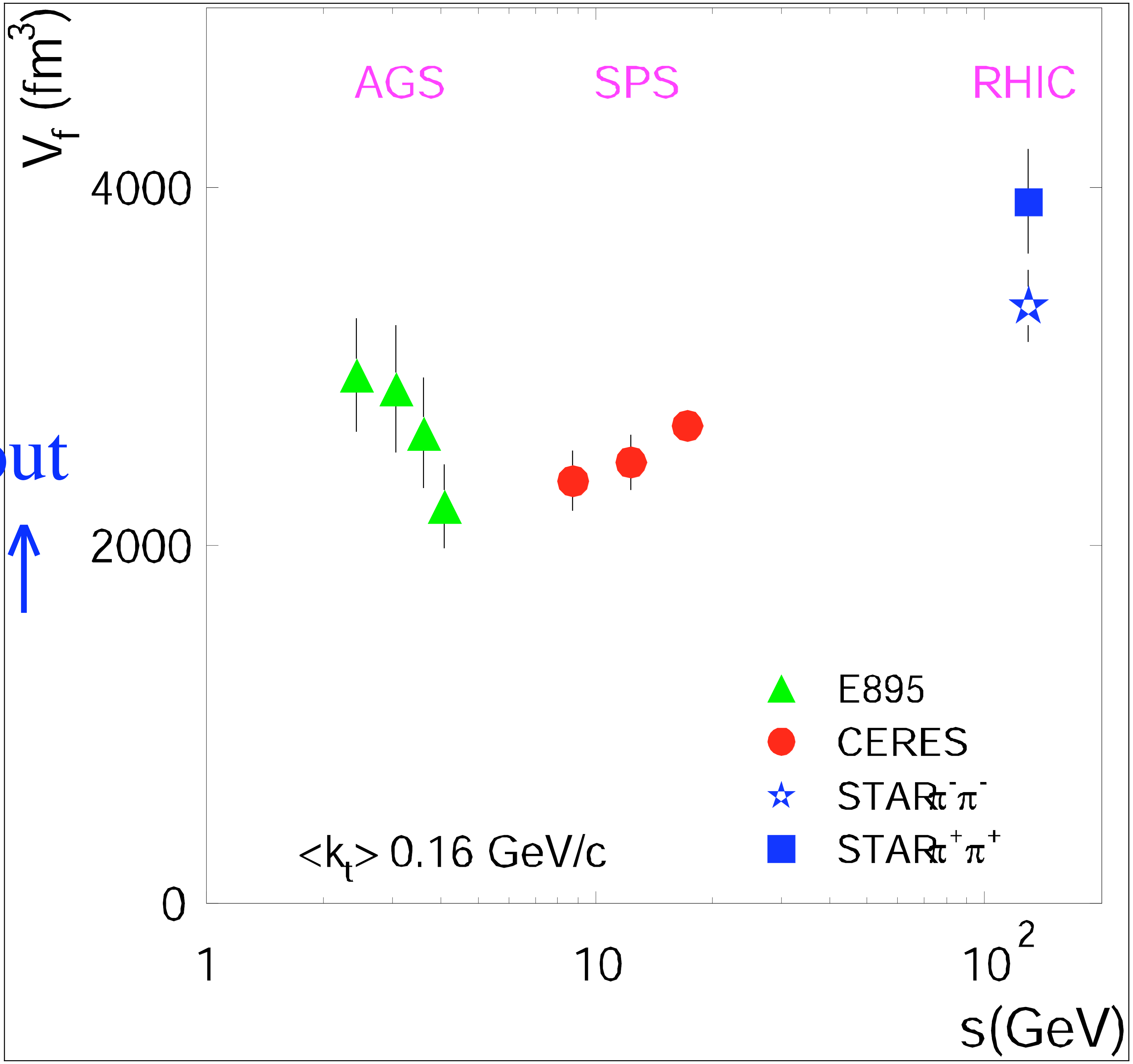
Or: changing from Hadronic/Quarkyonic boundary to Hadronic/QGP boundary: i.e., (approximate) triple point.

entropy density/T^3 ↑



$\sqrt{s_{NN}}$ →

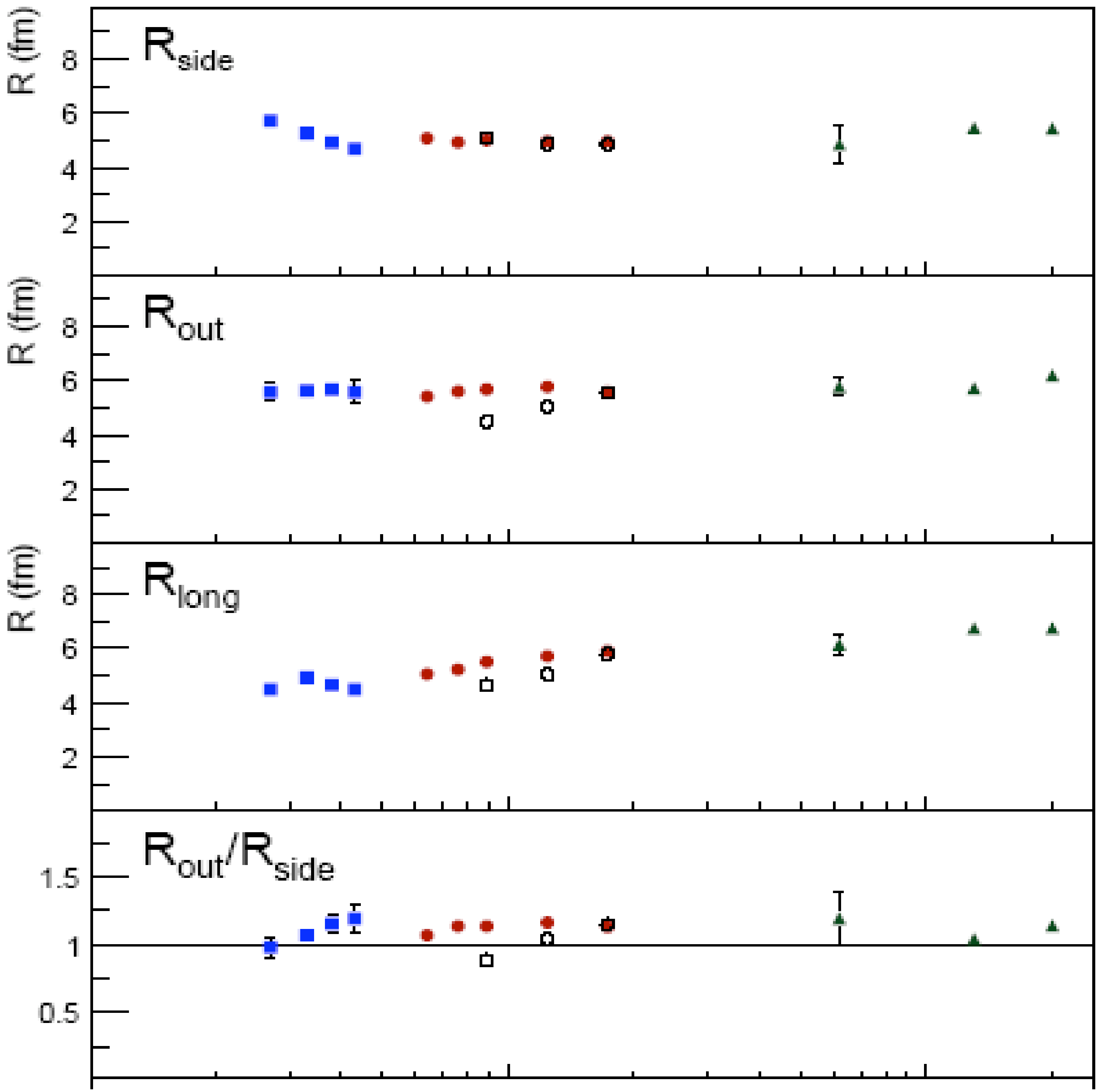
HBT Radii: Minimum near Strange MatterHorn?



freeze out volume ↑

$\sqrt{s_{NN}}$ →

HBT Radii: Flat from NA49.



$\sqrt{s_{NN}} \rightarrow$

Triple Point = Critical Endpoint?

Rajagopal, Shuryak, & Stephanov hep-ph/9806219, 9903292:
Critical endpoint in phase diagram of QCD. Correlation lengths *diverge*.

Hence: HBT radii should increase, affect light particles more than heavy.
K^+/π^+ should decrease, not increase. Strange MatterHorn?

Assume that at triple point, chiral transition splits from deconfining.
Leading operator which couples the two transitions is
Mocsy, Sannino, & Tuominen, hep-ph/0301229, 0306069, 0308135, 0403160:

$$c_1 \, \ell \, \mathrm{tr}\, \Phi^\dagger \Phi \sim c_1 \, \ell \, (\pi^2 + K^2 \ldots)$$

If this coupling c_1 flips sign, transitions diverge. Hence $c_1 = 0$ at triple point?
If so, leading coupling then becomes

$$c_2 \, \ell \, \mathrm{tr} M \, \Phi \sim c_2 \, \ell \, (m_\pi^2 \pi^2 + m_K^2 K^2 + \ldots)$$

This coupling is proportional to mass squared: *bigger* for kaons than pions!
Enchancement of K^+/π^+, strange baryons due to dense environment.
Hand waving argument; needs detailed analysis.

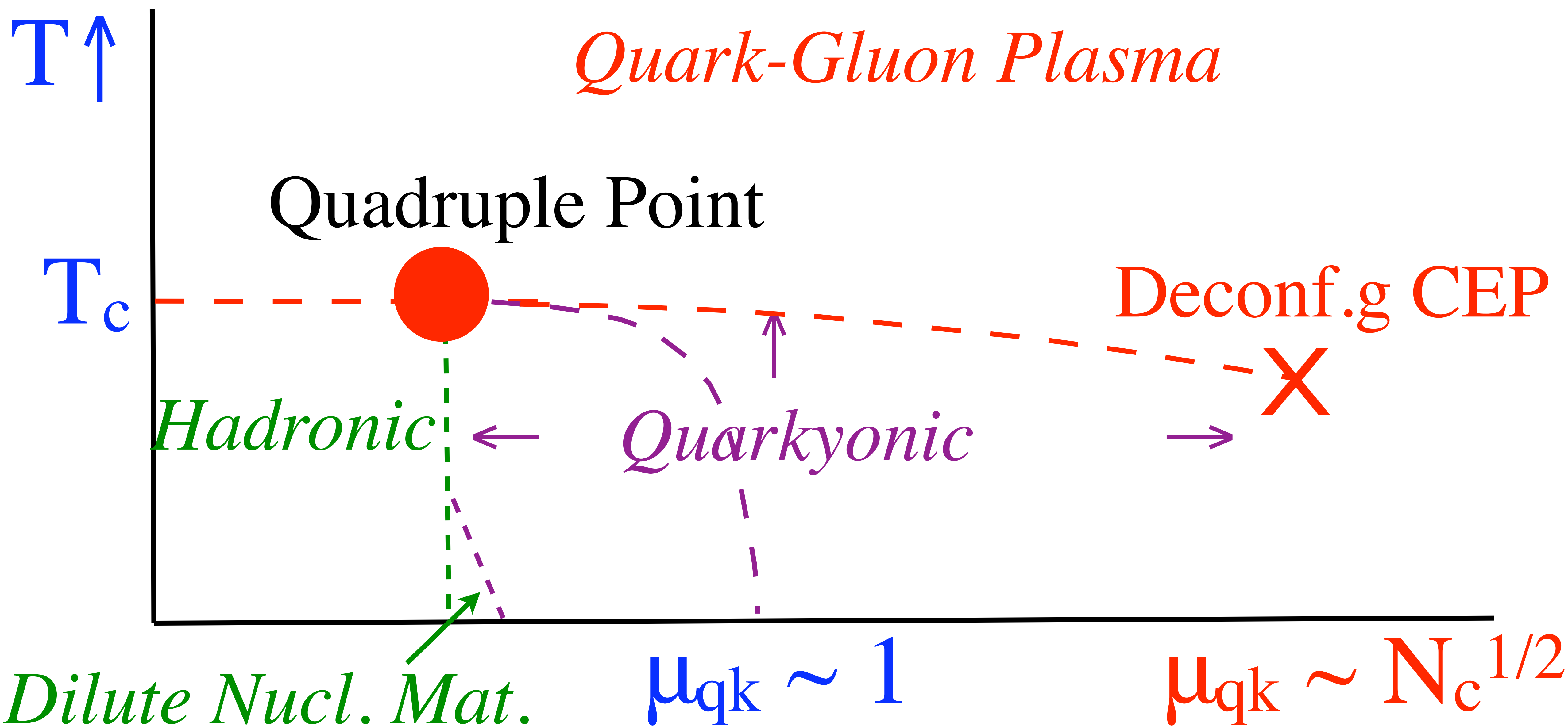
Phase Diagram at Large N_c and Small N_f, II

About critical endpoint for deconfinement,
smooth transition between deconfined and quarkyonic phases.

Since gluons are sensitive to quarks for such large μ, expect curvature in line.

Triple point still well defined, as coincidence of three 1st order lines.

Chiral transition?



Chiral Spirals in Quarkyonic Matter

Chiral Spirals in the Chiral Gross-Neveu Model

Schon & Thies, hep-th/0003195; 0008175; Thies, 06010243: coined term
Basar & Dunne, 0806.2659, Basar, Dunne, & Thies, 0903.1868

Chiral Gross-Neveu model
in 1+1 dimensions:

$$\mathcal{L} = \overline{\psi}\, i \,\partial\!\!\!/ \,\psi + G\left((\overline{\psi}\psi)^2 + (\overline{\psi}\gamma_5\psi)^2\right)$$

Continuous chiral symmetry:

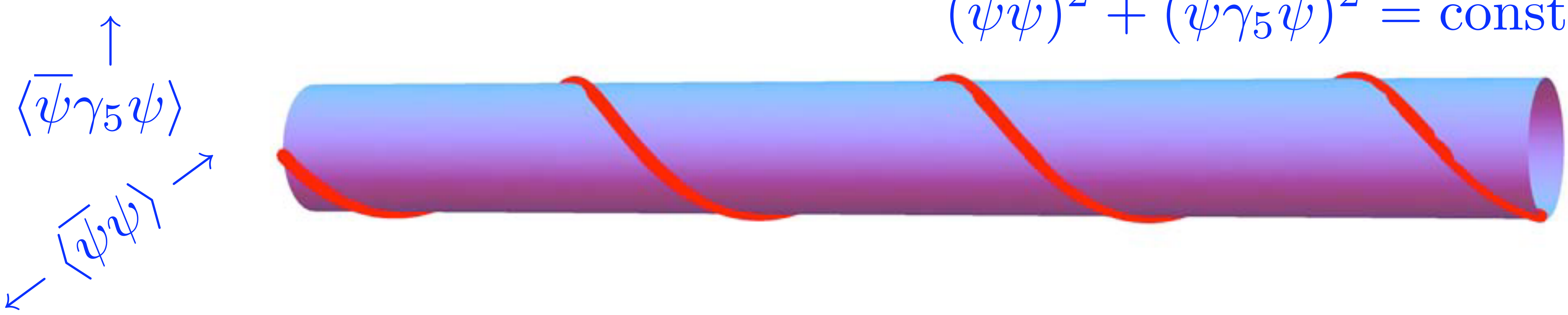
$$\psi \rightarrow \mathrm{e}^{i\theta\gamma_5}\,\psi$$

Model *exactly* soluble as # flavors → ∞.

At μ ≠ 0: periodic structure (crystal) which *oscillates* in space: "*chiral spiral*"

In chiral limit, oscillations symmetric about zero.

$$(\overline{\psi}\psi)^2 + (\overline{\psi}\gamma_5\psi)^2 = \mathrm{const.}$$



Why crystal? Bosonization in 1+1 dim.'s:

Fermion current gives spatially varying scalar field

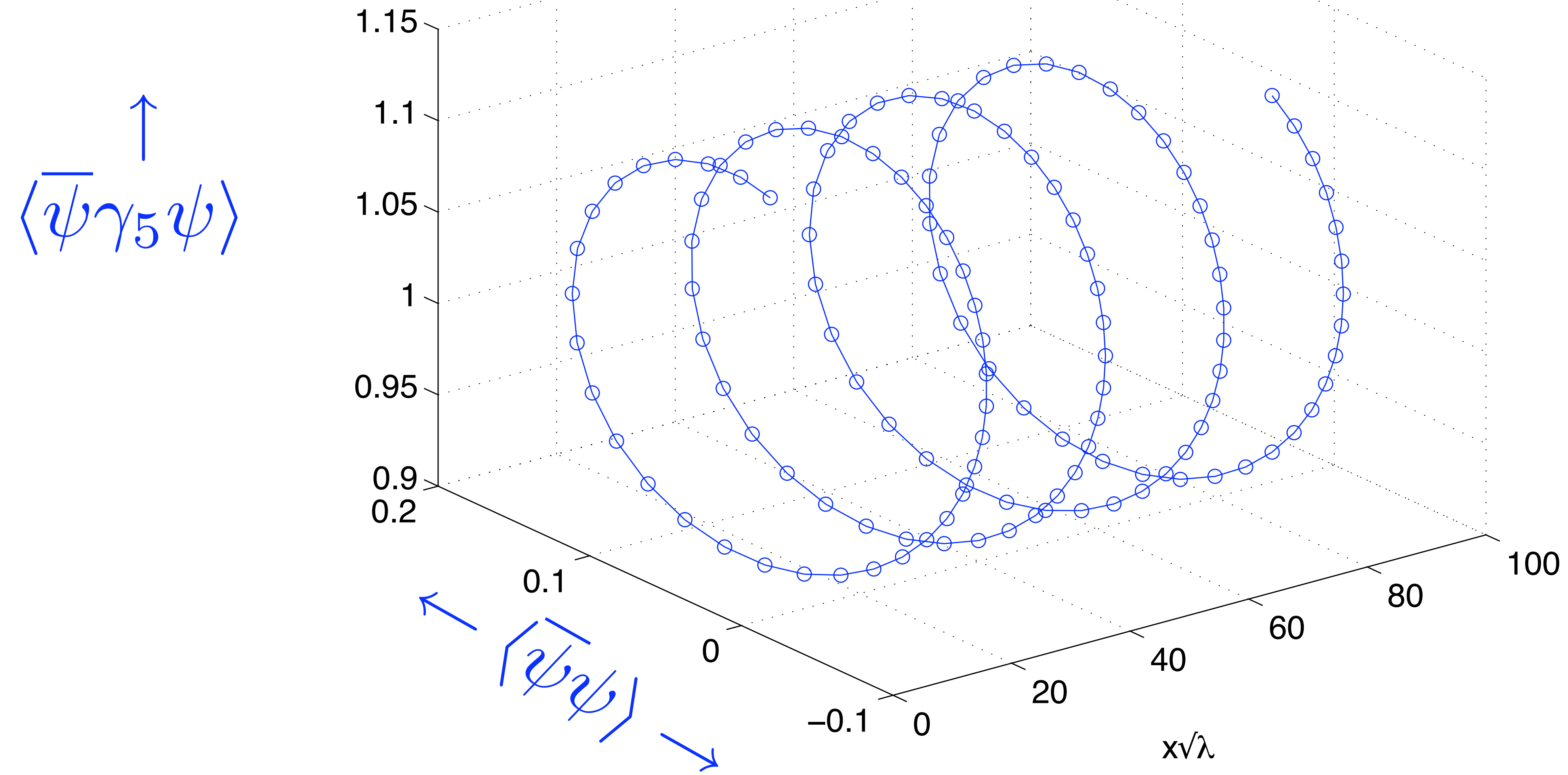
$$\overline{\psi}\,\gamma^0\psi = \partial_1\phi$$

Chiral Spiral for QCD in 1+1 dimensions

Salcedo, Levit, & Negele, '91...Bringoltz, 0901.4035:
't Hooft model, QCD in 1+1 D, with *massive* quarks.

Coulomb gauge in a finite box, canonical ensemble with fixed baryon number.
Numerically solve equations of motion under constaint of nonzero baryon #

Finds "chiral spiral", with oscillations about *non*zero value:
magnitude of the oscillations decreases as $g^2 N_c/m^2$ increases.



Quarkyonic Matter: from 4D to 2D

Simplest model of confinement: $$\Delta^{\mu\nu}_{\text{gluon}} = \frac{\sigma}{(P^2)^2} \left(\delta^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} \right)$$

Bethe-Saltpeter kernel for a meson:

For quarks near the Fermi surface, can neglect transverse momenta, $p_\perp$:

$$i p_0 - \mu + \sqrt{(p_F + p_1)^2 + p_\perp^2} \approx i p_0 + p_1 + p_\perp^2/2\mu + \ldots$$

So can integrate over $p_\perp$
in the gluon propagator:

$$\int d^2 p_\perp \, \frac{1}{(p_0^2 + p_1^2 + p_\perp^2)^2} = \frac{1}{p_0^2 + p_1^2}$$

End up with an effective model for QCD (at nonzero density) in 1+1 dimensions.

SU(2 N_f) symmetry for N_f flavors of 2D quarks

Start with a chiral basis: $$\psi_{R,L} = (1 \pm \gamma_5)\,\psi/2$$

and introduce helicity projections: $$\psi_{R\pm,L\pm} = (1 \pm \gamma_0\gamma_1)\,\psi/2$$

Effective quark Lagrangian in 1+1 D: Shuster & Son, hep-ph/9905448

$$\mathcal{L}_{\rm qk} = \sum_{R,L}\sum_{\pm} \overline{\psi}_{R\pm,L\pm}(\gamma^0(\partial_0 - i\mu) + \gamma^1\partial_1)\psi_{R\pm,L\pm}$$

For N_f flavors, have SU(2N_f) symmetry in 1+1 D. Like heavy quark symmetry.

1+1 D Dirac matrices = Γ^μ : $\Gamma_5 = \Gamma^0\,\Gamma^1$

For one flavor: $$\Psi = (\,\psi_{R+}, \psi_{R-}, \psi_{L-}, \psi_{L+})$$

QCD Lagrangian maps directly: $$(\overline{\psi}\,\gamma^0\,\psi, \overline{\psi}\,\gamma^1\,\psi) \rightarrow (\overline{\Psi}\,\Gamma^0\,\Psi, \overline{\Psi}\,\Gamma^1\,\Psi)$$

Condensates map as: $$(\overline{\psi}\psi, \overline{\psi}\,\gamma^0\gamma^1\,\psi) \rightarrow (\overline{\Psi}\Psi, \overline{\Psi}\,\Gamma_5\,\Psi)$$

Many terms break SU(2N_f) symmetry: $$\overline{\psi}\,\gamma_5\tau_3\,\psi \rightarrow \overline{\Psi}\,\Gamma_5\tau_3\,\Psi$$

Chiral Spirals in 1+1 D and 3+1 D

Effective quark Lagrangian in 1+1 D: 2D gluons plus 2D quarks:

$$\mathcal{L}_{\rm qk} = \overline{\Psi} \left(i \not{D}_{2D} + \mu \Gamma^0 \right) \Psi \; ; \; g^2_{2D} = g^2 \, \sigma$$

Perform anomalous chiral rotation *linear* in x: $\Psi \rightarrow \exp(i \, \mu \, \Gamma_5 \, x) \, \Psi$

Find: anomalous chiral rotation *removes* μ from quark Lagrangian.

Fischler, Kogut & Susskind '79 : $\Gamma^1 \, \Gamma_5 = \Gamma^0$

In 1+1 dimensions, Fermi sea => vacuum.

Where is the Fermi density? From the anomaly. Chiral condensate:

$$< \overline{\Psi} \, \Psi >_{\mu=0} \rightarrow \cos(2\mu x) \; < \overline{\Psi} \Psi >_\mu + i \sin(2\mu x) \; < \overline{\Psi} \, \Gamma_5 \, \Psi >_\mu$$

$< \overline{\Psi} \, \Psi >_{\mu=0} \neq 0$ => Chiral Spiral (= Chiral Density Wave) in Fermi sea.

For 4D quarks, chiral spiral = condensate in helicity:

$$< \overline{\psi} \psi > = \cos(2\mu x) \, c \; ; \; < \overline{\psi} \, \gamma^0 \gamma^1 \, \psi > = i \sin(2\mu x) \, c$$

(Many) Massless Modes about Chiral Spirals

Excitations near the Fermi surface:

Witten '84: non-Abelian bosonization for QCD. a, b= 1...N_c. i,j = 1... N_f.

$$J_+^{ij} = \overline{\psi}^{a,i}\psi^{a,j} \sim g^{-1}\partial_+ g \; ; \; J_+^{ab} = \overline{\psi}^{a,i}\psi^{b,i} \sim h^{-1}\partial_+ h \; .$$

Steinhardt '80. Affleck '86. Frishman & Sonnenschein, hep-th/920717...
Armoni, Frishman, & Sonnenschein , hep-th/0011043...

QCD in 1+1 D: "fractionalization" of color and flavor.

Flavor currents: Wess-Zumino Witten model. *Many* massless excitations!
Color currents: gauged WZW model. Massive excitations of 't Hooft model

1+1 D: only quasi long range order. At large N_c, disorders at scales exp(-N_c).

Effective 1+1D model embedded in 3+1D.

Transverse dimensions: break SU(2 N_f) to SU(N_f), produce true long range order.

Quarkyonic Chiral Spirals versus ...

Chiral Density Waves (CDW) in *perturbative* regime:
Deryagin, Grigoriev, & Rubakov '92. Shuster & Son, hep-ph/9905448.
Rapp, Shuryak, and Zahed, hep-ph/0008207.
Shuster & Son: in perturbative regime, CDW only wins for $N_c > 1000\, N_f$
Large N_c: Quarkyonic Chiral Spirals until $\mu \sim \sqrt{N_c}$.

QCD: *certainly* color superconductivity for asymptotically high density.
Quarkyonic Chiral Spirals (QCS) for intermediate density? Both pionic & kaonic

QCS analogous to pion condensation: (Migdal '71, Sawyer & Scalapino '72...)

$$< \overline{\psi}\psi >= \cos(2\mu x) < \overline{\psi}\psi >_0 \; ; \; < \overline{\psi}\, \gamma_5\, \psi >= i\sin(2\mu x) < \overline{\psi}\psi >_0$$

Pion condensation *like* chiral spiral in 1+1 D, *differs* from QCS in 3+1 D.

3+1 D NJL models (Nickel 0906.5295): flavor sym in 3+1 D ≠ 1+1 D so *no* QCS

Kaon condensation (Kaplan & Nelson '86) constant $<K^->$, *not* QCS.
Do expect kaonic QCS in QCD (if pionic QCS exist)!

The Unbearable Lightness of Being (Nuclear Matter)

Nucleon-Nucleon Potentials from the Lattice

Ishii, Aoki & Hatsuda, PACS-CS, 0903.5497

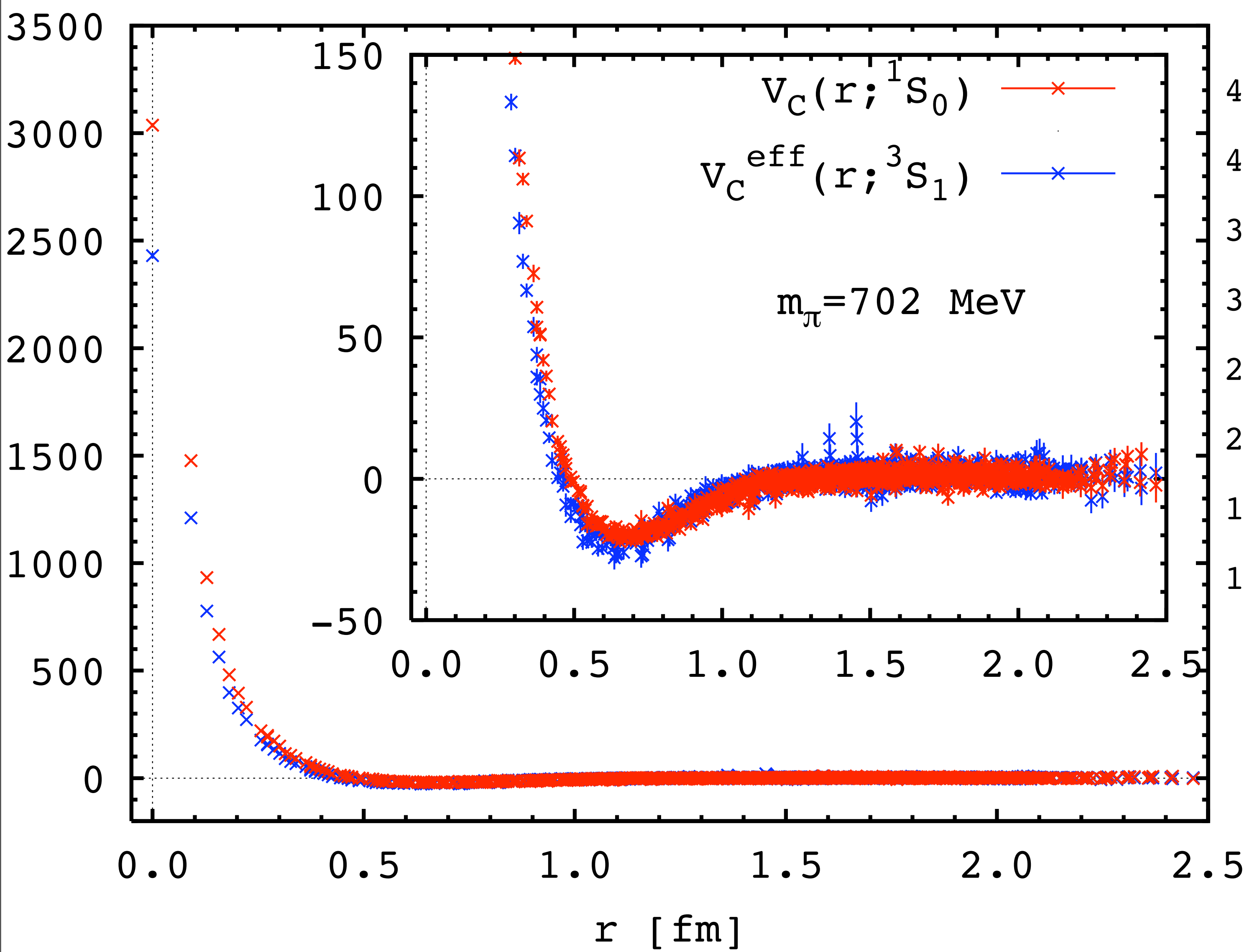
Nucleon-nucleon potentials from quenched and 2+1 flavors.

Pions heavy: 700 MeV (left) and 300 MeV (right)

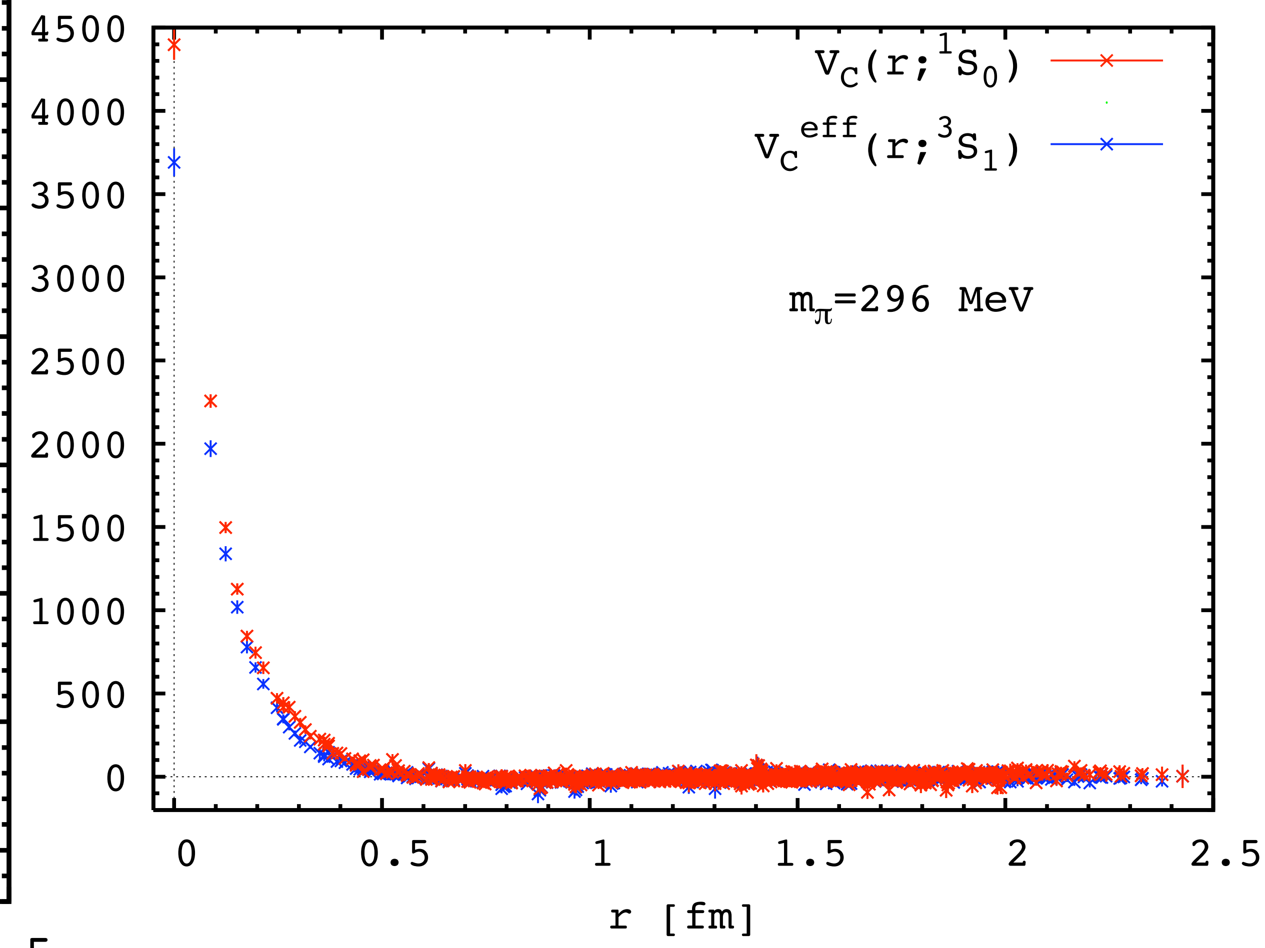
Standard lore: delicate cancellation. *So why independent of pion mass?*

Essentially *zero* potential plus strong hard core repulsion

$m_\pi = 702$ MeV



$m_\pi = 296$ MeV



Purely Pionic Nuclear Matter

At infinite N_c, integrate out *all* degrees of freedom *except* pions:

Lagrangian power series in $U = \mathrm{e}^{i\pi/f_\pi}$, $V_\mu = U^\dagger \partial_\mu U$

Infinite # couplings: Skyrme *plus* complete Gasser-Leutwyler expansion,

$$\mathcal{L}_\pi = f_\pi^2 \, V_\mu^2 + \kappa \, [V_\mu, V_\nu]^2 + c_1 \, (V_\mu^2)^2 + c_2 \, (V_\mu^2)^3 + \ldots$$

All couplings ~ N_c , every mass scale ~ typical hadronic.

Need *infinite* series, but nothing (special) depends upon exact values

Valid for momenta < f_π, masses of sigma, omega, rho...

Large N_c version of in-medium chiral perturbation theory:

W. Weise & ...: 0808.0856, 0802.2212,0801.1467, 0712.1613, 0707.3154 +...

Higher time derivatives, but no acausality at low momenta.

Purely Pionic Nuclear Matter: *Free* Baryons

From purely pionic Lagrangian, take baryon as stationary point.

Find baryon mass ~ N_c, some function of couplings.

Couplings of baryon dictated by chiral symmetry:

$$\overline{\psi}\left(i\partial\!\!\!/ + M_B\, \mathrm{e}^{i\tau\cdot\pi\gamma_5/f_\pi}\right)\psi$$

By chiral rotation, $W = \exp(-i\pi\gamma_5/2f_\pi)$

$$\mathcal{L}_B = \overline{\psi}\left(iW^\dagger \partial\!\!\!/ W + M_B\right)\psi \sim \frac{1}{f_\pi}\overline{\psi}\gamma_5 \partial\!\!\!/ \pi\psi + \ldots$$

At large ~ N_c , $f_\pi \sim N_c^{1/2}$ is *big*. Thus for momenta k < hadronic, interactions are *small*, ~ $1/f_\pi^2 \sim 1/N_c$.

Thus: baryons from chiral Lag. free at large N_c , down to distances $1/f_\pi$.

Manifestly special to chiral baryons. True for u, d, s, but *not* charm?

The Unbearable Lightness of Being (Nuclear Matter)

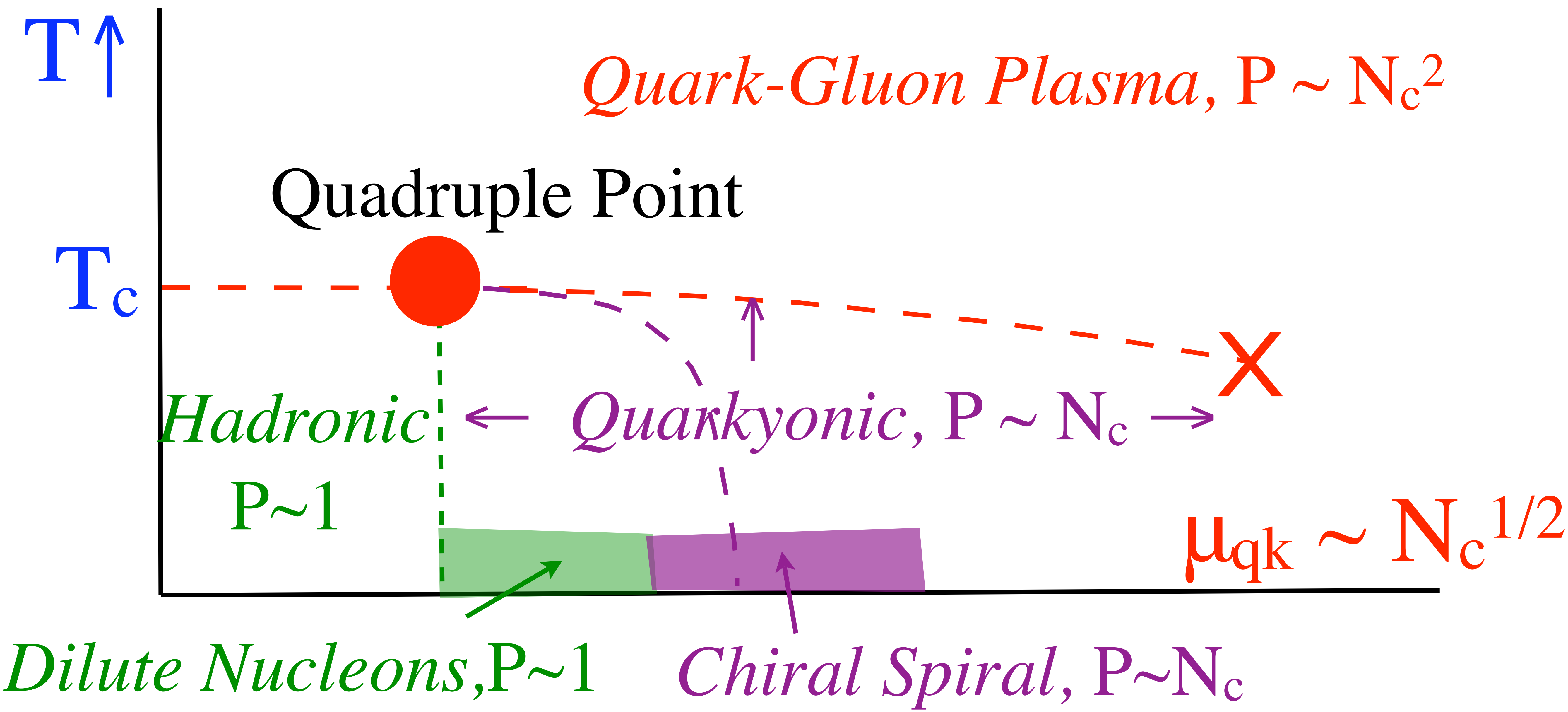
Use purely pionic Lagrangian for all of nuclear matter?

Then pressure ~ 1, and *not* N_c. Like hadronic phase, *not* quarkyonic.

*Un*like standard lore, where pressure(nucl mat) grows quickly, ~ N_c

Red line: 1st order. Green line: Baryons condense. Purple: chiral trans.

IF chiral transition 1st order, etc, Quadruple Point where *four* phases coexist.



Today's Phase Diagram for QCD