Some puzzles in the Quark Gluon Plasma

How do the phase transition(s) *change* as a function of:
SUSY, $N_c = 3\ldots\infty$, pure glue vs + quarks....

Puzzles:

Pure glue: strings persist in the *de*confined phase, *above* T_d?

Is the deconfining transition at $N_c = \infty$ *special* ($T \neq 0$, $\mu = 0$)?

Baryons at large N_c: di-quark pairing or coherent field?

Neutron stars at 2 $M_\odot$ and "stiff" Equations Of State?

Quantum thermalization in *small* systems?

Strings in the *de*confined phase for *pure* SU(N)?

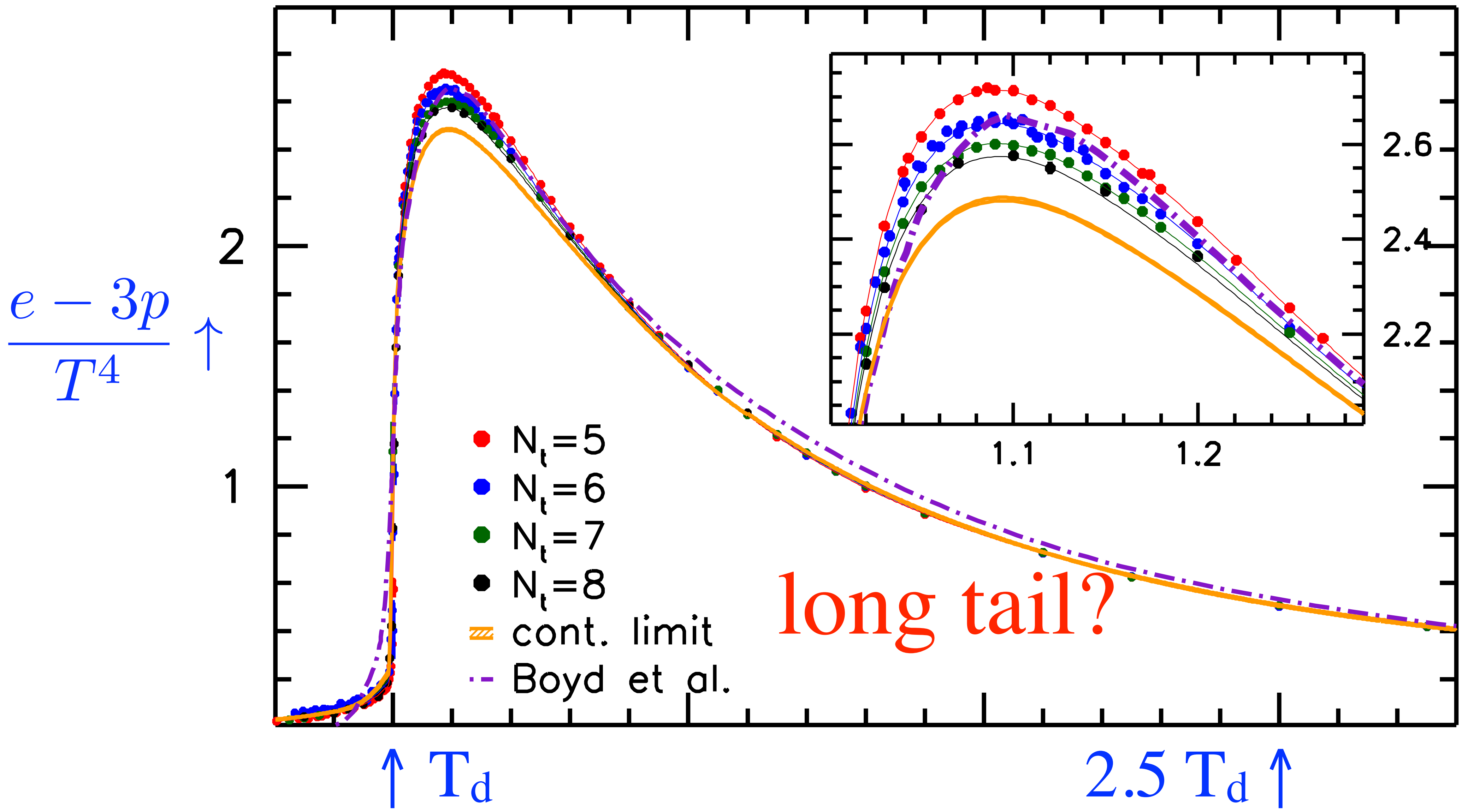
"Hidden" scaling of the pressure near T_d (M. Panero)

Must be strings.

Lattice: usual thermodynamics, pure SU(3)

Pure SU(3): *no* quarks. Peak in (e-3p)/T^4, just above T_d.

Borsanyi, Endrodi, Fodor, Katz, & Szabo, 1204.6184



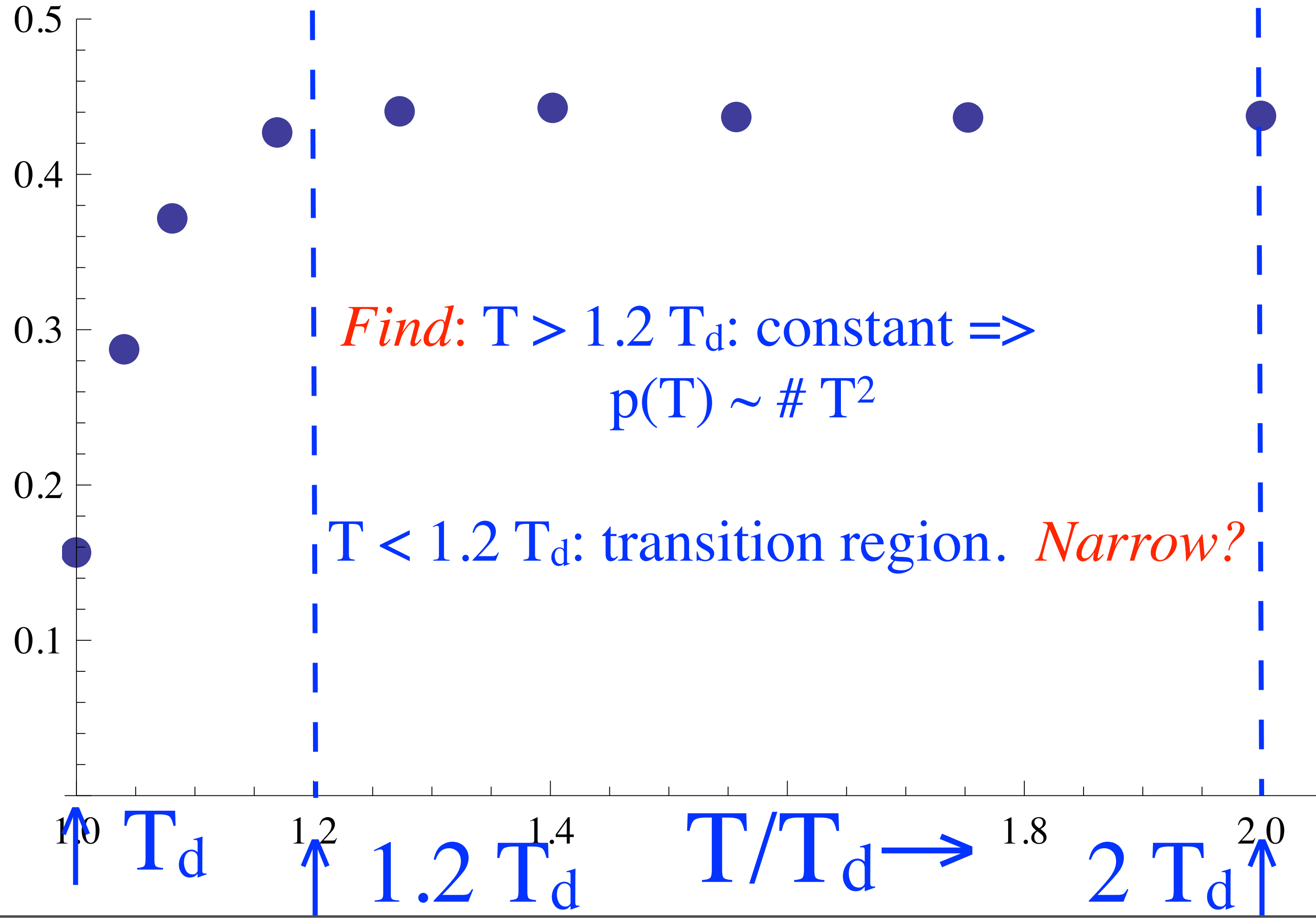
Lattice: *hidden* scaling of the pressure, pure SU(3)

$(e-3p)/T^4 \times (T^2/T_d^2)$ *approximately* constant near T_c:

Meisinger, Miller, & Ogilvie, ph/0108009; RDP, ph/0608242

$$p(T) \approx \# \left(T^4 - c\, T_d^2\, T^2 \right) ,\; c = 1.00 \pm .01$$

$$\frac{1}{8}\; \frac{e-3p}{T^4}\; \frac{T^2}{T_d^2} \uparrow$$

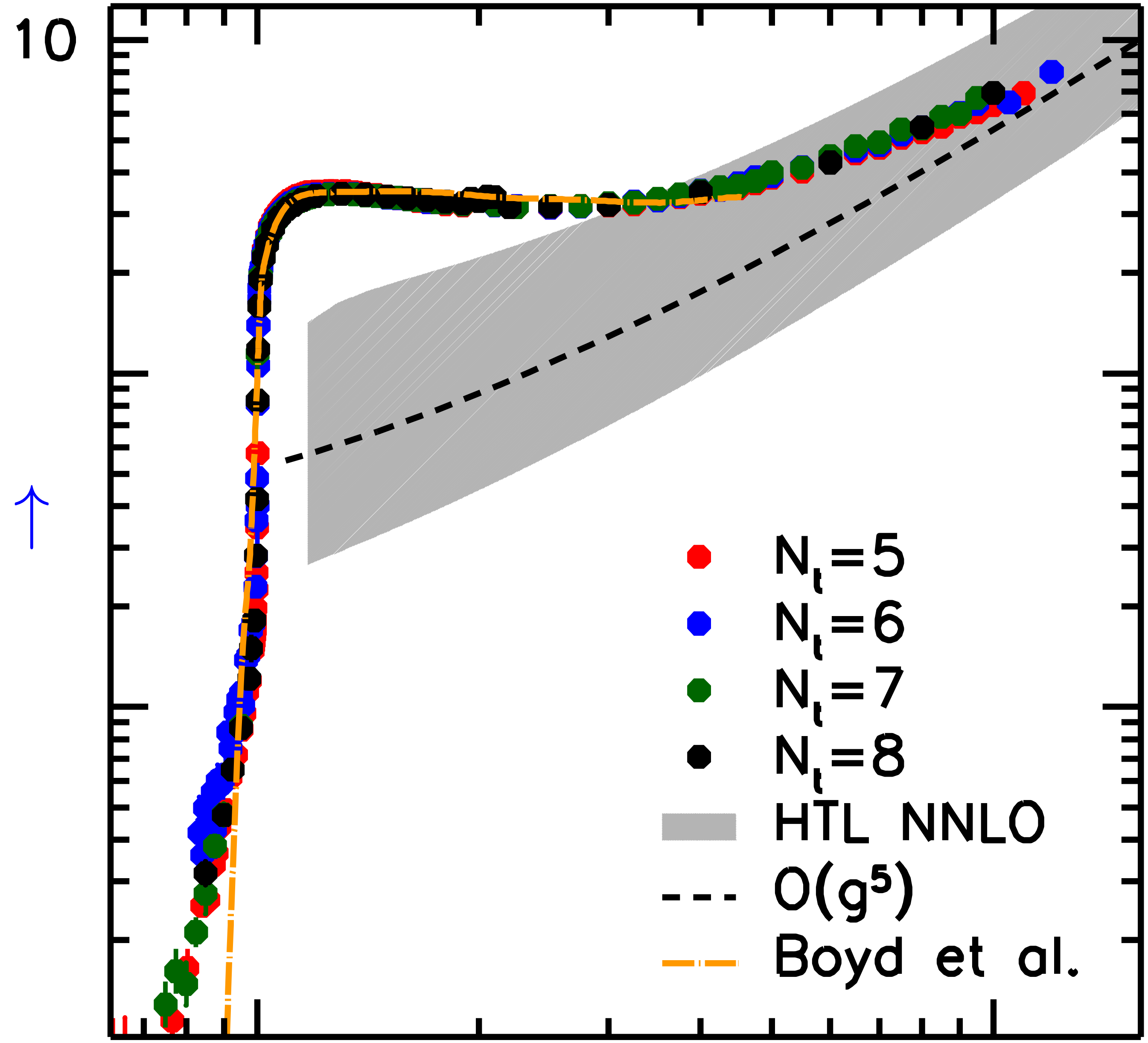


WHOT: Umeda, Ejiri, Aoki, Hatusda, Kanaya, Maezawa, Ohno, 0809.2842

Lattice: hidden scaling, pure SU(3)

$T_d \rightarrow 4\ T_d$:
For pressure, leading corrections to ideality, T^4, are T^2. Not as flat as WHOT.

$$\frac{e-3p}{T^4}\ \frac{T^2}{T_d^2} \uparrow$$



↑ T_c $T/T_d \rightarrow$ $10\ T_d$ ↑

Borsanyi, Endrodi, Fodor, Katz,& Szabo, 1204.6184

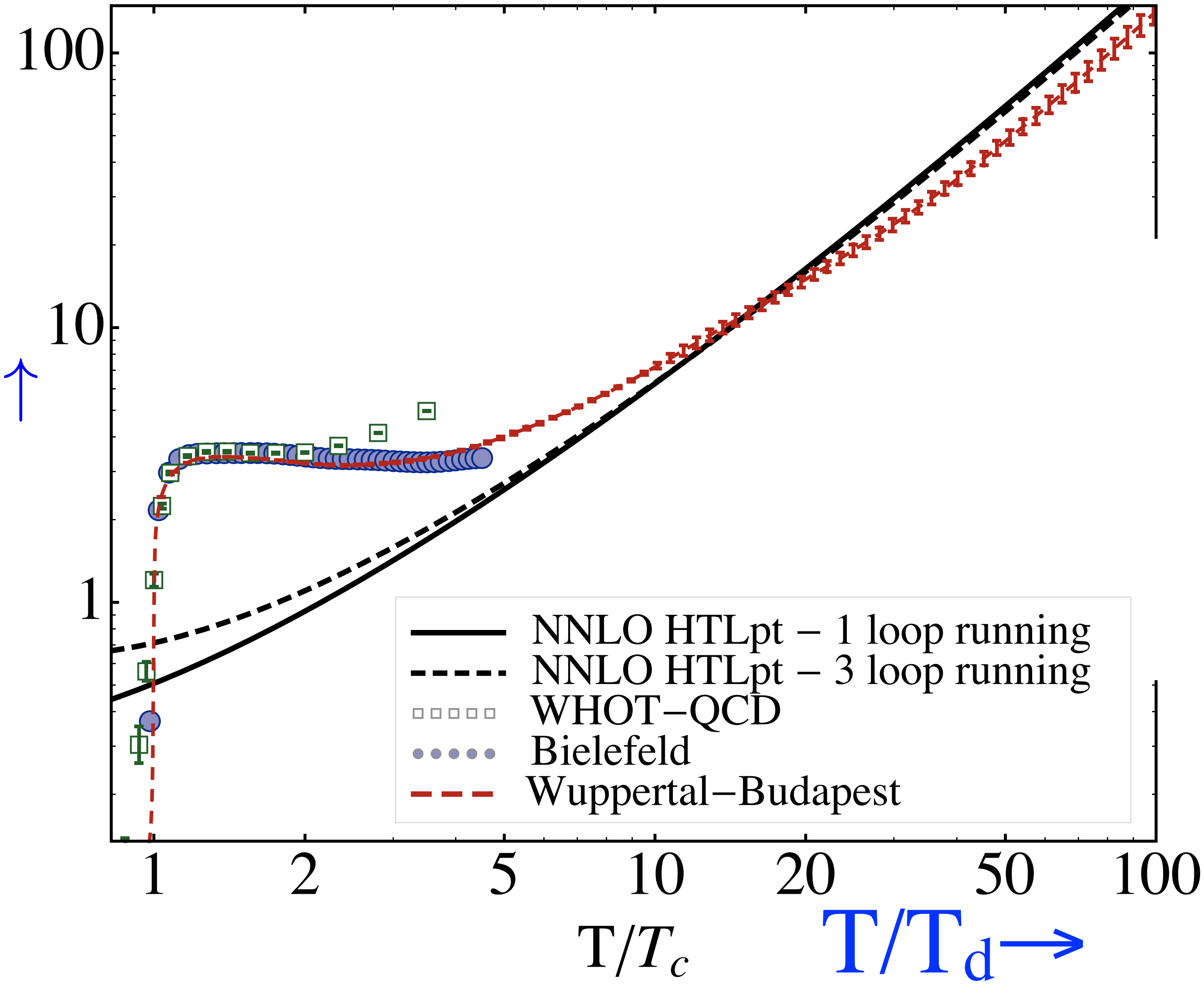
Comparison to (resummed) perturbation theory

Perturbative contribution to e-3p ~ g^4 , from trace anomaly

HTL pert. theory at NNLO: Andersen, Leganger, Strickland, & Su, 1105.0514

Clear excess above perturbative below 5 T_d.

$$\frac{e-3p}{T^4}\ \frac{T^2}{T_d^2}\uparrow$$

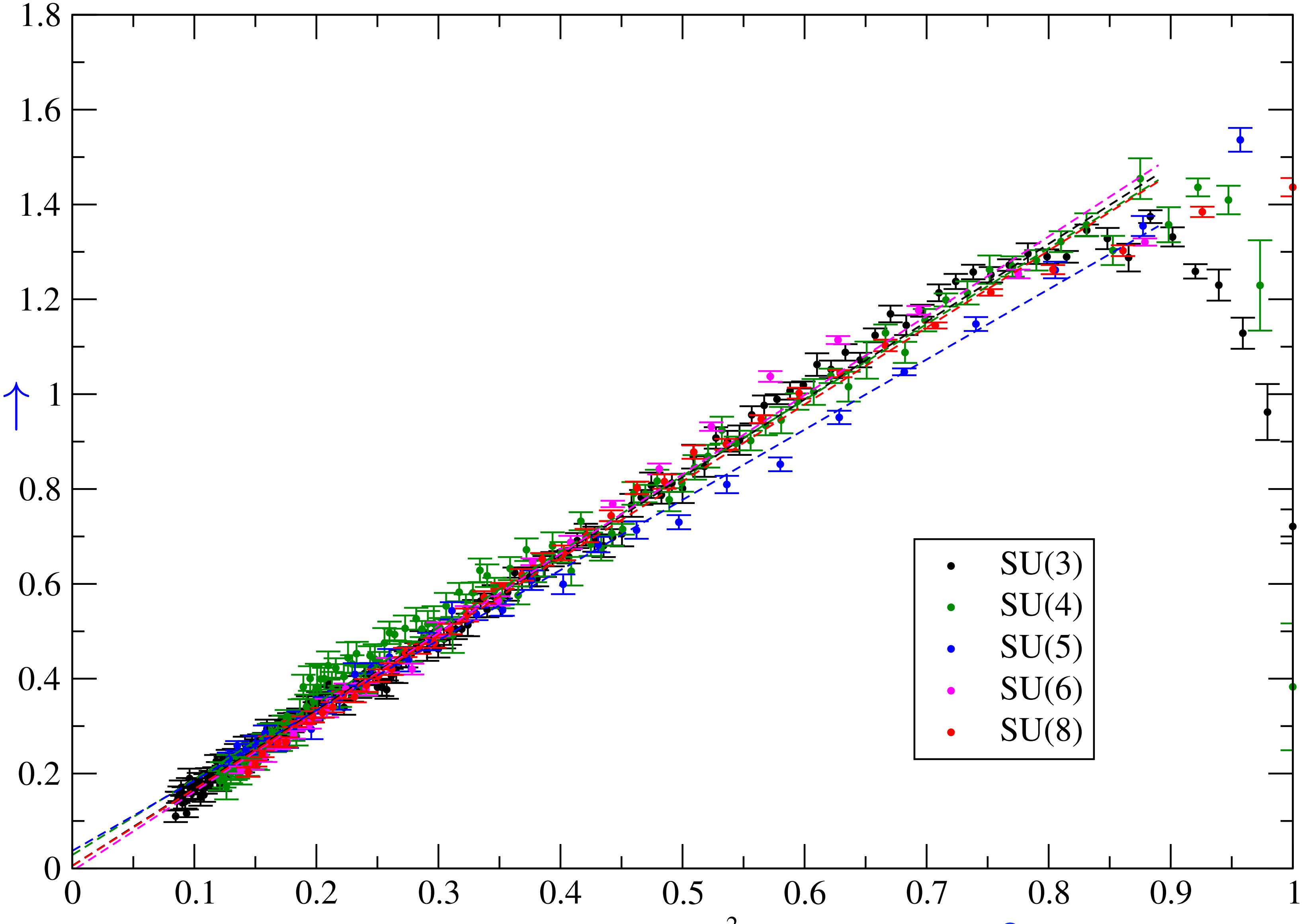


$T/T_d \rightarrow$

Lattice: hidden scaling, pure SU(N), N=3...8

Pure SU(N): $1/(N^2-1)(e-3p)/(T^2\, T_d^2) \sim$ constant near T_d, *in*dependent of N

$$\frac{1}{p_{SB}/T^4}\ \frac{e-3p}{T^4}\ \uparrow$$



$3T_d \uparrow$ $(T_d/T)^2 \longrightarrow$ $T_d \uparrow$

Panero, 0907.3719; Datta & Gupta, 1006.0938; Lucini & Panero, 1210.4997

Just mass expansion?

For a free massive boson, if m << T, $$p(T) \approx \frac{\pi^2}{90}\left(T^4 - \frac{15}{4\pi^2}\, m^2\, T^2 + \ldots\right)$$

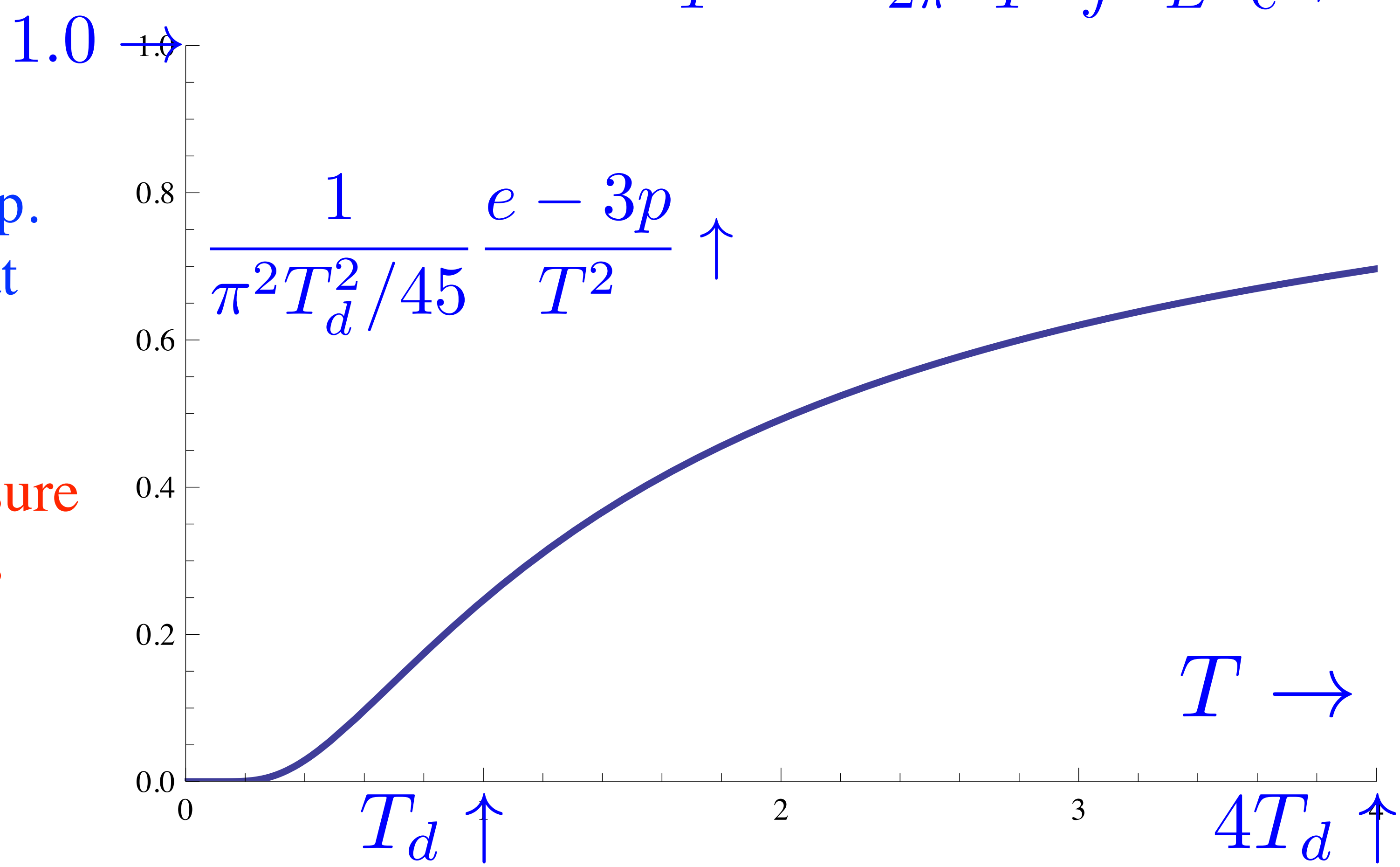
Choose $m^2 = \dfrac{4\pi^2}{15} T_d^2$ $$p(T) \approx (\pi^2/90)(T^4 - T_d^2 T^2 + \ldots)$$

For general m, (e-3p)/T^2 (E = $\sqrt{k^2 + m^2}$) :

$$\frac{e-3p}{T^2} = \frac{m^2}{2\pi^2}\,\frac{1}{T^2}\int \frac{dk}{E}\,\frac{k^2}{e^{E/T}-1}$$

For above m^2,
test small mass exp.
(e-3p)/T^2 is *not* flat
for T ~ T_d.

Term ~ T^2 in pressure
in 3+1 dimensions
is *non*trivial

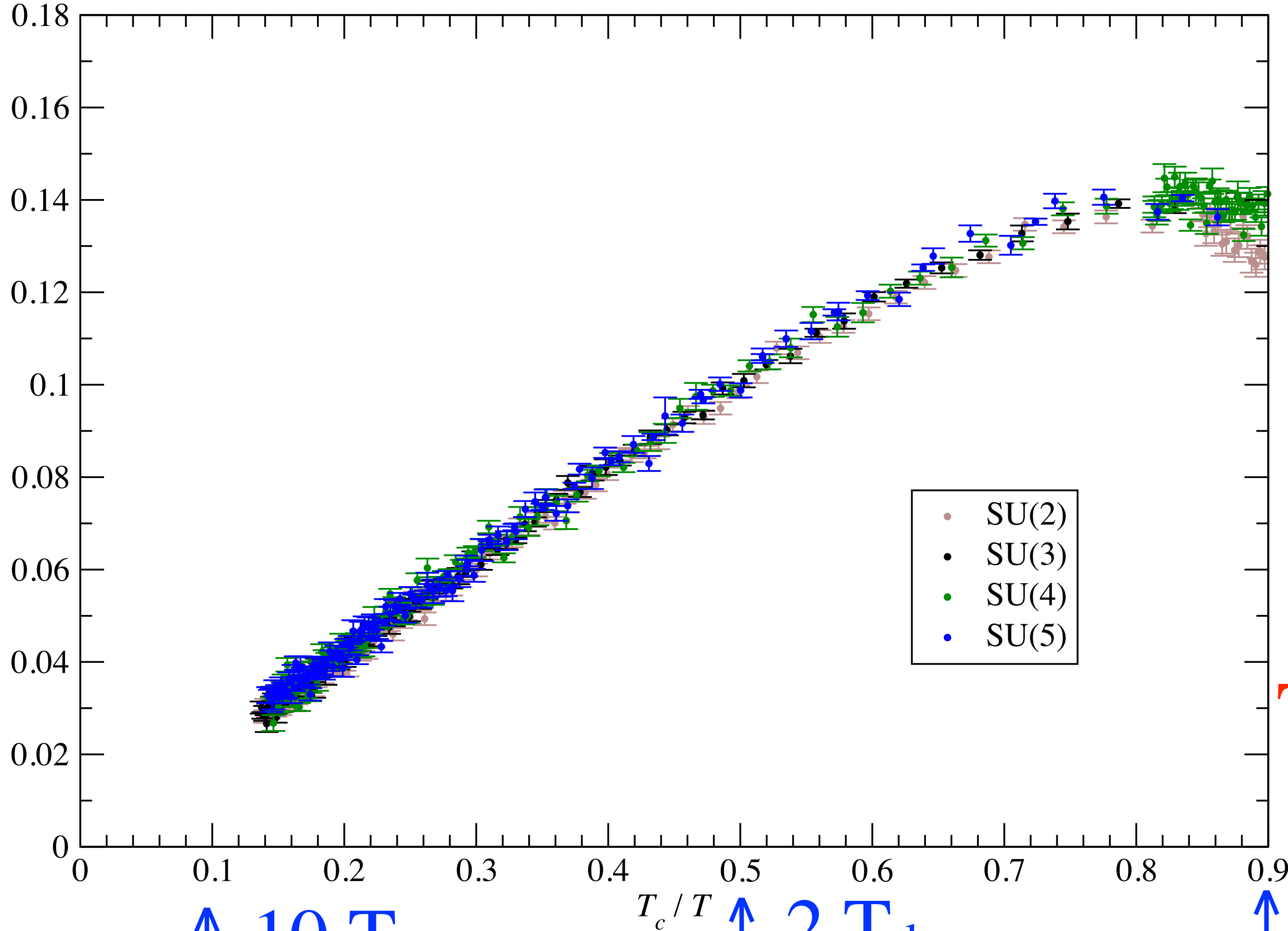


Lattice: hidden scaling, pure SU(N) in 2+1 dimensions

In 2+ 1 dimensions, hidden scaling again ~ T^2: *not* a mass term, ~ m^2 T:

$$p(T) \approx \#\,(T^3 - c\,T_d\,T^2)\,,\; c \approx 1.$$

$$\frac{1}{N^2-1}\;\frac{e-2p}{T^3}\uparrow$$



$T_d/T\rightarrow$

$\uparrow$ 10 T_d $\uparrow$ 2 T_d $\uparrow$ 1.1 T_d

Caselle, Castagnini, Feo, Gliozzi, Gursoy, Panero, Schafer, 1111.0580

Do stringy effects persist *above* T_d?

For pure SU(N) gauge, lattice finds corrections to ideality are *always* ~ T^2 :

Stringy: ~ T^2 is the free energy of *massless* fields in two dimensions

Not stringy: in *de*confined phase, *above* Hagedorn temperature

For SU(N), contributes to pressure ~ N^2, and *not* ~ N^0: strings color singlets?

With dynamical quarks: data not clear.

N.B.: only clear with *precise* lattice data

Pure SU(∞) at the deconfining transition:

Gross-Witten-Wadia transition?

Exact solution at T_d from AdS/CFT?

QCD on a femtosphere

Consider pure SU(∞) on a spatial sphere so small that coupling is small

Sundberg, th/9908001;

Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk, th/0310285; th/0508077

Dumitru, Lenaghan, RDP, ph/0410294

Integrate out modes with $J \neq 0$, obtain eff. theory for static modes, matrix model

Consider eigenvalues of Wilson line, $\mathbf{L} = \exp(2\pi i\, \mathbf{q})$

Take $A^i_0 \sim q^i$, $i = 1...N$. discrete sum $\Sigma_i \Rightarrow \int dq\, \varrho(q)$.

$$\# \left| \int dq\, \rho(q)\, e^{2\pi i\, q} \right|^2 + \int dq \int dq'\, \rho(q)\, \rho(q') \log \left| e^{2\pi i q} - e^{2\pi i q'} \right|$$

Solve by usual large N tricks. At T_d, eigenvalue density is

$$\rho(q) = 1 + \cos(2\pi\, q) \quad , \quad q : -1/2 \to 1/2$$

N.B. in 2-dim.'s, Gross, Witten, & Wadia found 3rd order transition in lattice β.

Here, at any temperature, find 3rd order transition when

$$\ell = \frac{1}{N} \operatorname{tr} \mathbf{L} = \frac{1}{2}$$

Gross-Witten-Wadia transition at N=∞

Solution at N=∞: "critical first order" transition - both first *and* second order

Latent heat *nonzero* ~ N^2. *And* specific heat diverges, $C_v \sim 1/(T-T_c)^{3/5}$

Potential function of *all* tr $\mathbf{L}^n$, n = 1, 2.... But at T_d^+, only *first* loop is nonzero:

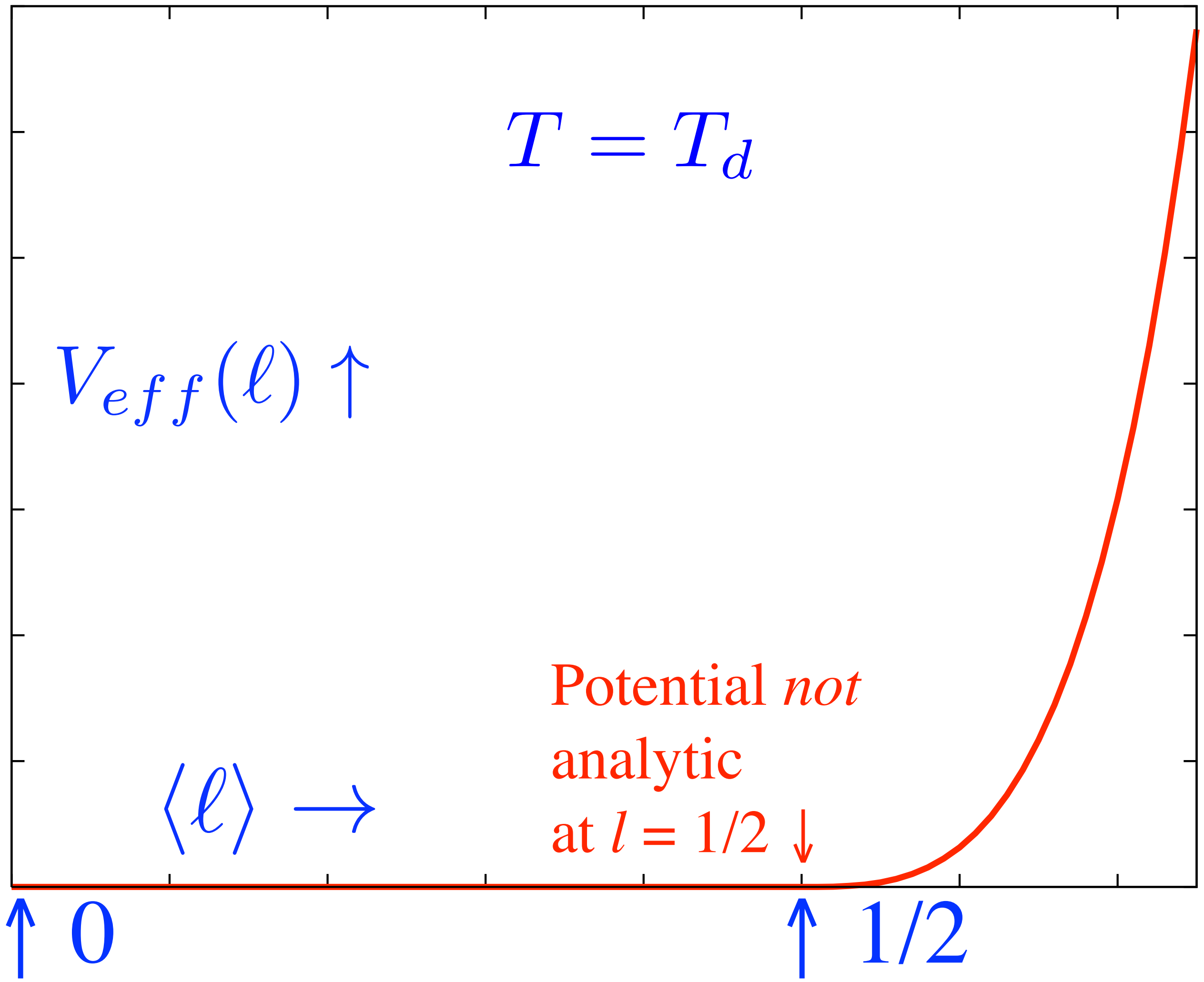
$$\ell = \frac{1}{N}\,\mathrm{tr}\,\mathbf{L}$$

$$\ell(T_c^-) = 0$$

$$\ell(T_c^+) = \frac{1}{2}$$

But V_{eff} *flat* between them!

$$\mathrm{tr}\,\mathbf{L}^n\,(T_d) = 0 \;,\; n \geq 2$$



Above *only* for g=0: to ~ g^4, standard 1st order transition. So GWW curiosity?

Matrix models in infinite volume

Construct effective matrix model for deconfinement:

Meisinger, Miller, & Ogilvie, ph/0108009.

A. Dumitru, Y. Guo, Y. Hidaka, C. Korthals-Altes & RDP, 1011.3820, 1205.0137;

K. Kashiwa, V. Skokov & RDP, 1205.0545; K. Kashiwa & RDP, 1301.5344.

(Gauge invariant) variables eigenvalues of (thermal) Wilson line:

Simple ansatz: constant, diagonal A_0:
$$A_0^{ij} = \frac{2\pi T}{g} q_i \, \delta^{ij} \; , \; i, j = 1 \ldots N$$

At 1-loop order, perturbative potential

$$V_{pert}(q) = \frac{2\pi^2}{3} T^4 \left(- \frac{4}{15}(N^2 - 1) + \sum_{i,j} q_{ij}^2 (1 - q_{ij})^2 \right) \; , \; q_{ij} = |q_i - q_j|$$

Assume non-perturbative potential $\sim T^2 \, T_d^2$:

$$V_{non}(q) = \frac{2\pi^2}{3} T^2 T_d^2 \, (- \frac{c_1}{5} \sum_{i,j} q_{ij} (1 - q_{ij}) - c_2 \sum_{i,j} q_{ij}^2 (1 - q_{ij})^2 + \frac{4}{15} c_3) + B T_d^4$$

Matrix models in infinite volume, N = ∞

Solve at N=∞: RDP & V. Skokov, 1206.1329;

Interface tensions: S. Lin, RDP, & V. Skokov, 1301.7432

$$V_{\text{eff}}(q) = d_1\, V_1 + d_2\, V_2$$

$$V_n(q) = \int dq \int dq'\ \rho(q)\ \rho(q')\ |q-q'|^n (1-|q-q'|)^n$$

Take derivatives of equation of motion, at T_d solution

$$\rho(q) = 1 + \cos(2\pi\, q) \quad , \quad q : -1/2 \to 1/2$$

At T_d, solution *identical* to GWW model on a femtosphere!

Solution differs away from T_d. But why same solution at T_d? V_{eff} *very* different.

On a femtosphere, coupling is fixed, $g^2(1/R)$, couplings don't run.

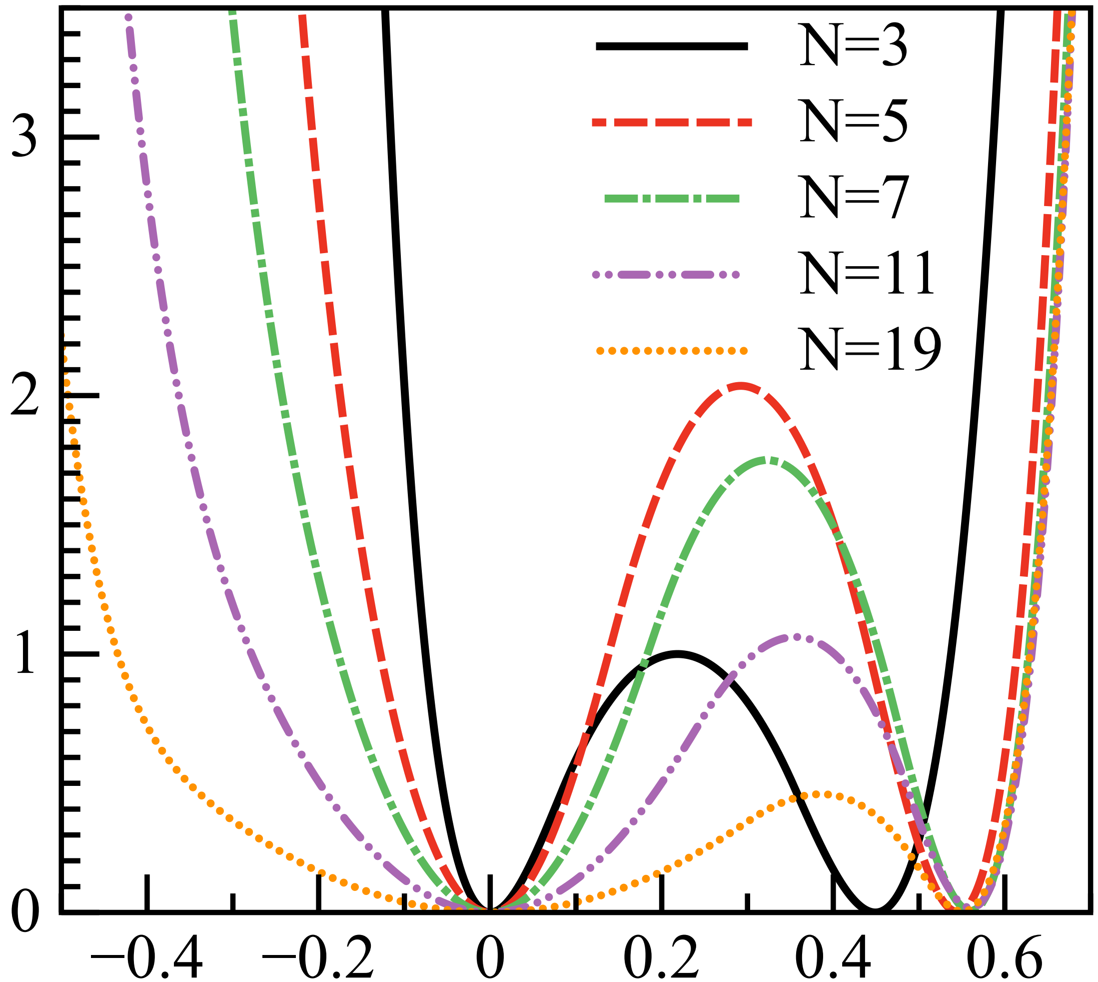
But in infinite volume, coupling constants can flow:

Is Gross-Witten-Wadia an infrared stable fixed point for pure gauge SU(∞)?

Remnants of Gross-Witten-Wadia at finite N?

At finite N, solve model numerically. Find two minima, at 0 and ~ 1/2.
Standard first order transition, with barrier & interface tension *non*zero
Barrier disappears at infinite N: so interface tensions *vanish* at infinite N
Below: potential /(N^2-1), versus tr **L** .

$$\frac{V_{eff}(\ell)}{N^2-1} \uparrow$$



$\langle \ell \rangle \rightarrow$

$0 \uparrow$ L $\uparrow 1/2$

Signs of GWW at finite N: interface tensions *small* at T_d?

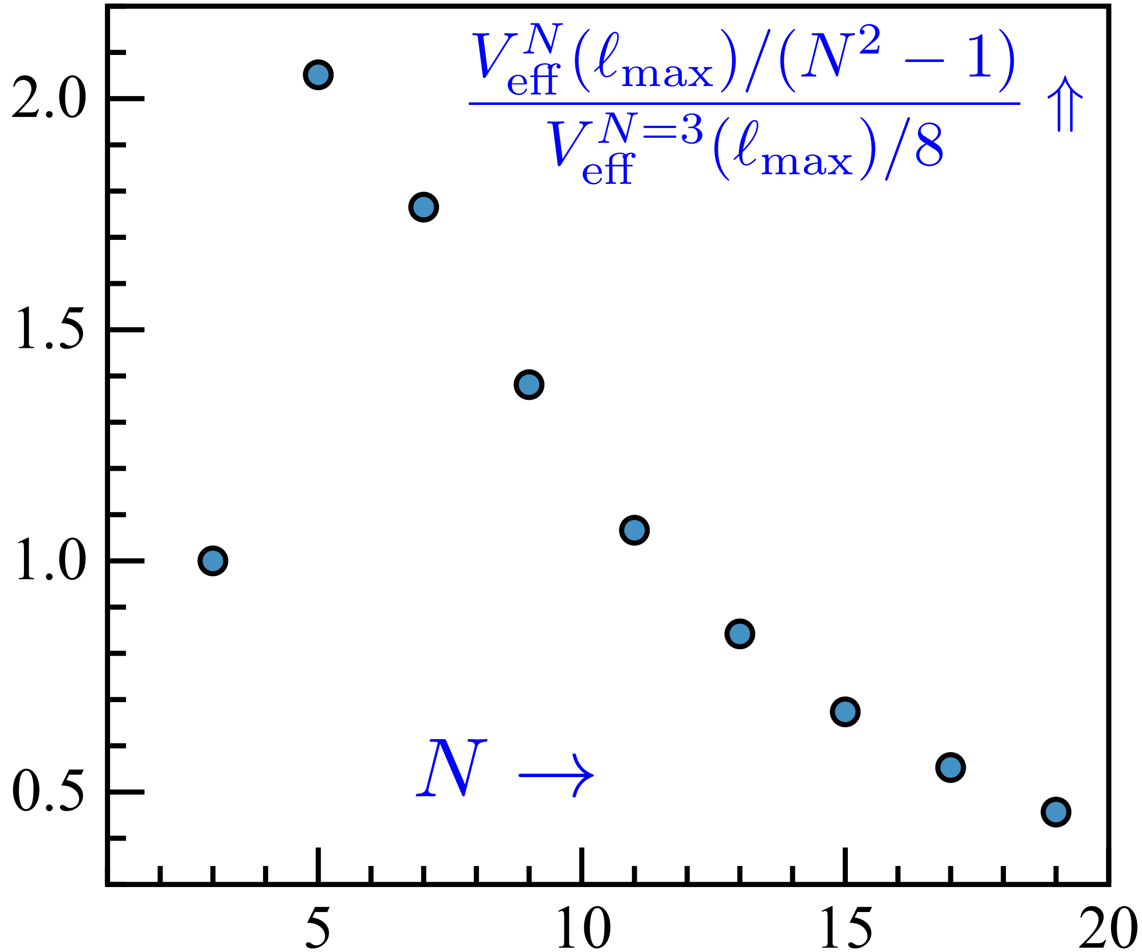
Consider maximum of previous figure, versus number of colors:
increases by ~ 2 from N = 3 to 5, then *decreases* monotonically as N increases
Perhaps: non-monotonic behavior of order-disorder interface tension with N?
Below: maximum in potential /(N^2-1), versus tr **L** .

Lattice: order-disorder
interface tension α^{od} at T_d:
Lucini, Teper, Wegner, lat/0502003

$$\frac{\alpha^{od}}{N^2 T_d^3} = .014 - \frac{.10}{N^2}$$

Coefficients *small*, χ^2 *large*, ~ 2.8.
Non-monotonic behavior of α^{od}/N^2?
't Hooft loops also *small* near T_d

Remnants of Gross-Witten-Wadia
fixed point at finite N?



Effective theory for deconfinement?

There's always *some* effective theory:

AdS/CFT?

Matrix model: parameters from the lattice

Matrix model: choose variables as eigenvalues of thermal Wilson line

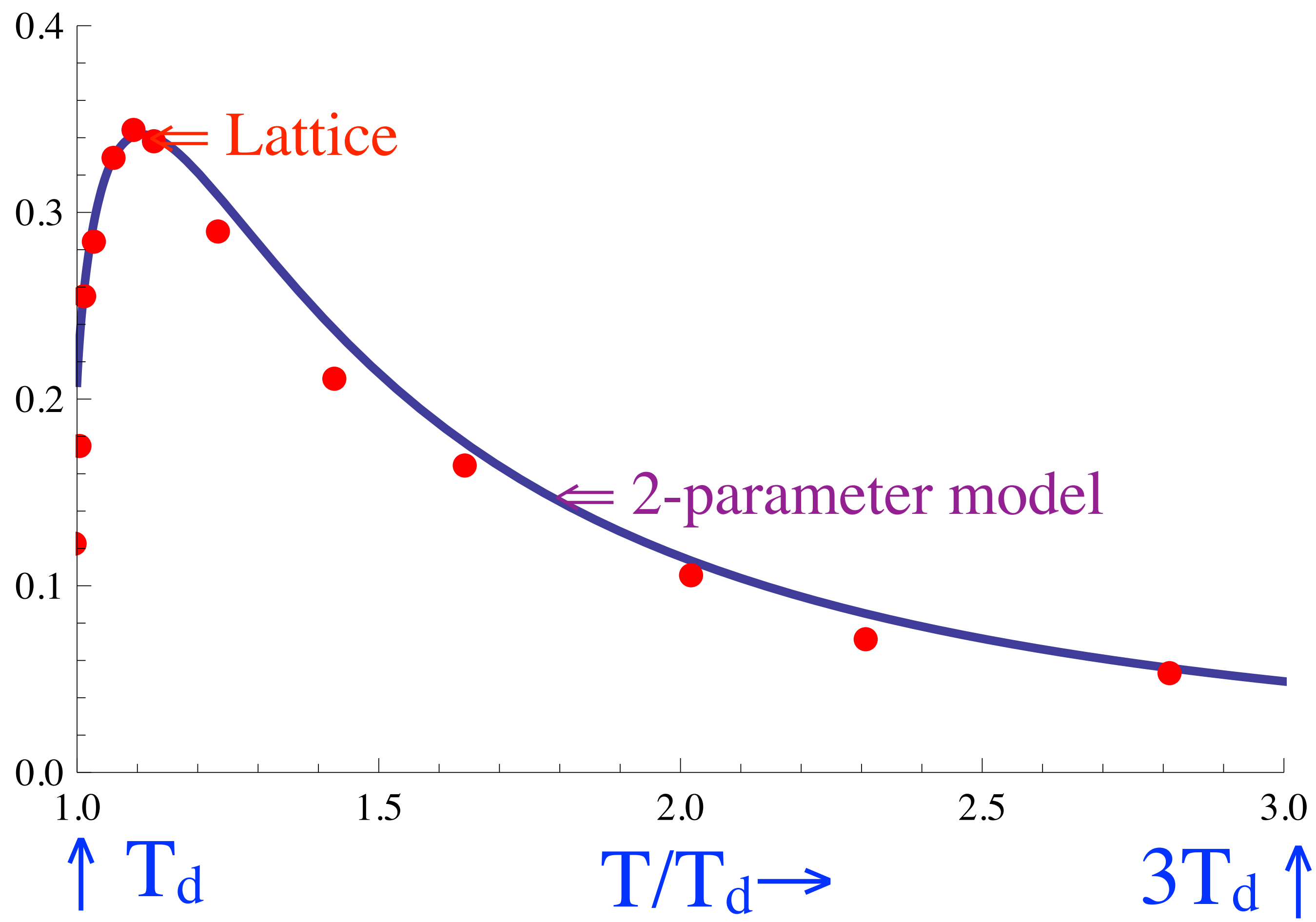
Choose 2 free parameters to fit:
latent heat at T_d, $(e-3p)/T^4$ at large T

$c_1 = .88, c_2 = .55, c_3 = .95$

Reasonable value for bag constant B:

$T_d = 270$ MeV, B~ $(262 \text{ MeV})^4$

$$\frac{e-3p}{8\,T^4} \uparrow$$

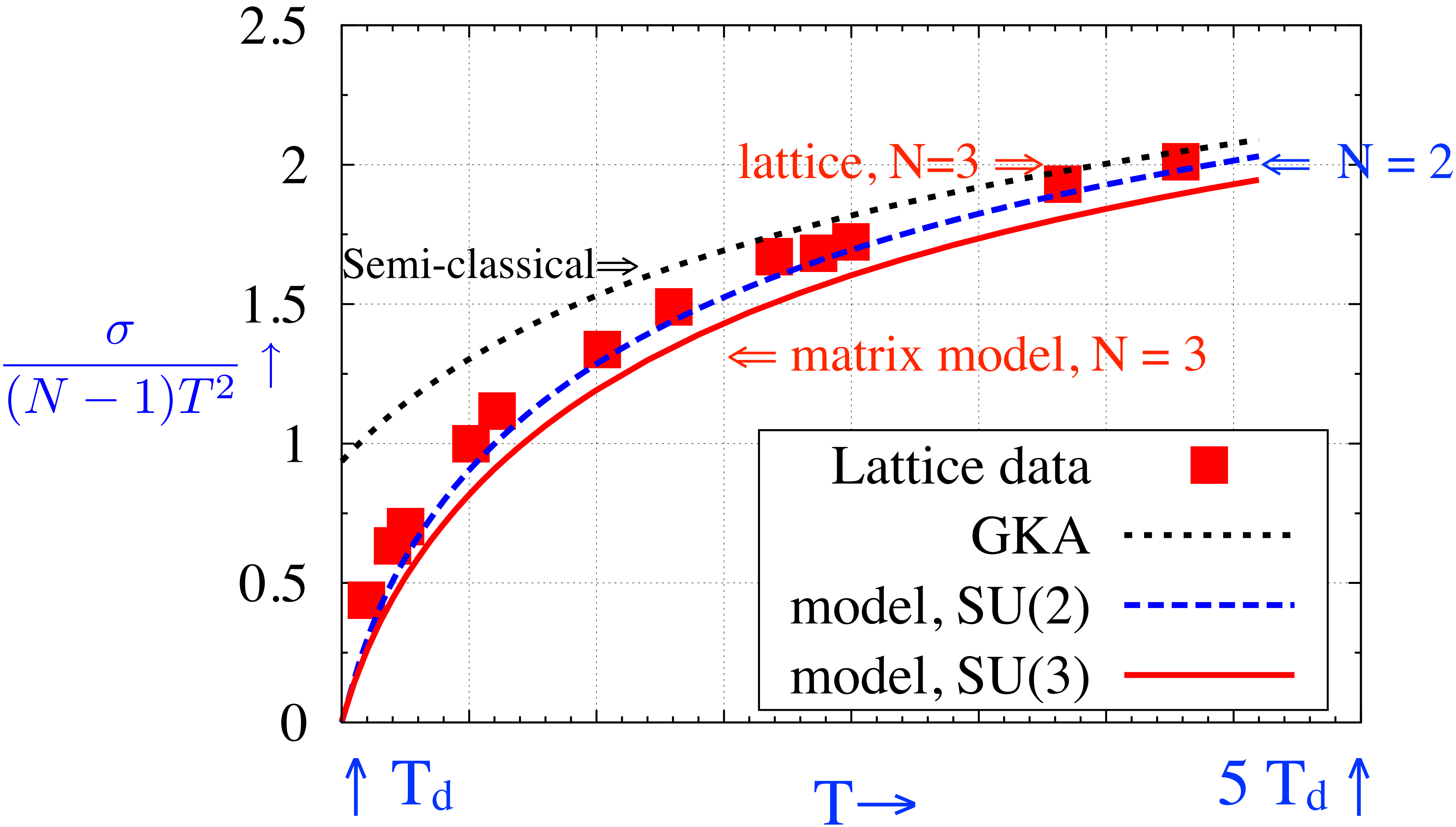


Latent heat, lattice:
Beinlich,
Peikert, Karsch
lat/9608141
Datta, Gupta
1006.0938

Matrix model: 't Hooft loop vs lattice

Matrix model works well:

Lattice: de Forcrand, D'Elia, & Pepe, lat/0007034; de Forcrand & Noth lat/0506005



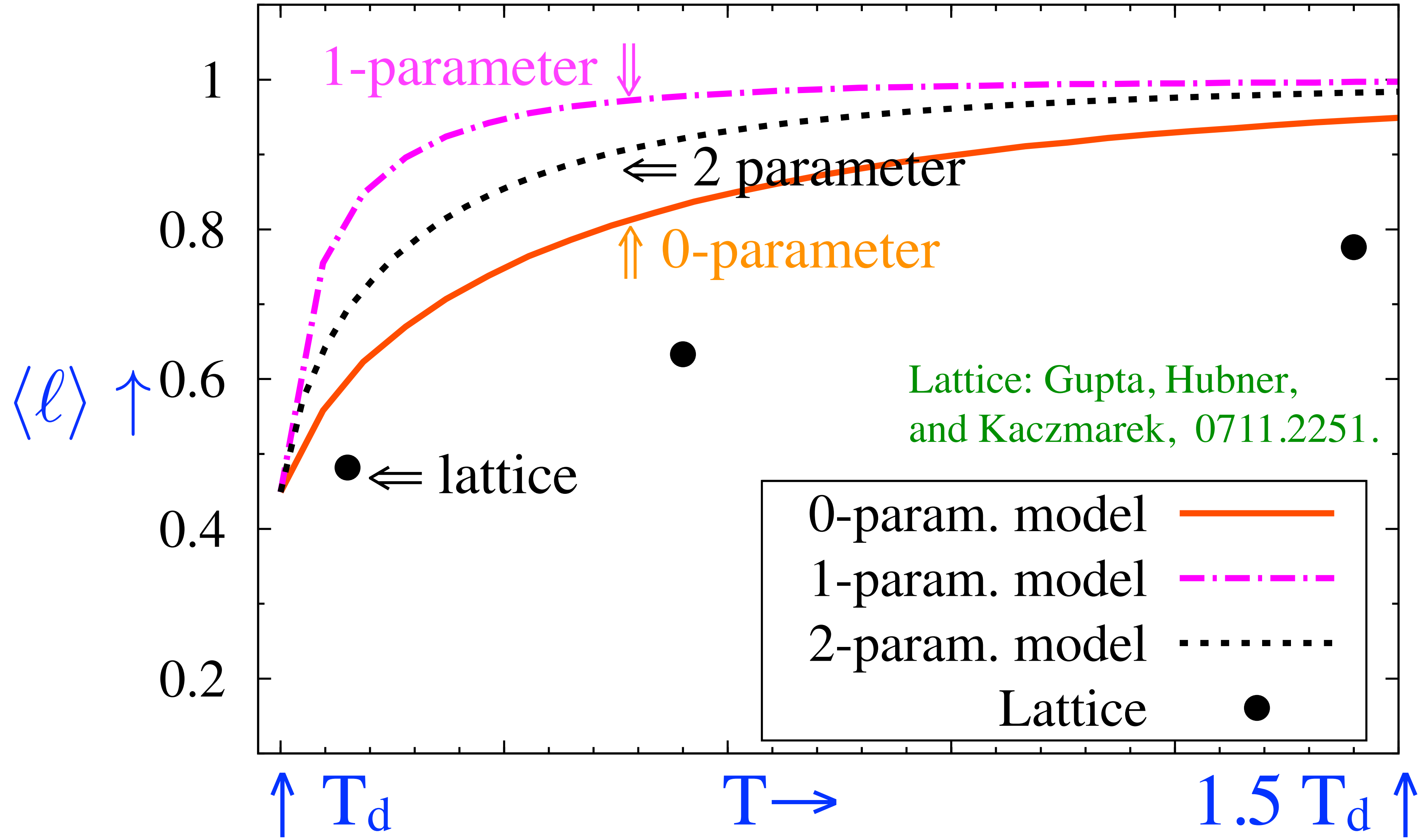
Polyakov loop: vs lattice: huge discrepancy

Renormalized Polyakov loop from lattice *nothing* like Matrix Model

Model: transition region *narrow*, to ~ 1.2 T_c. Lattice: loop *wide*, to ~ 4.0 T_c.

Can alter parameters to fit Polyakov loop; do not fit latent heat with 2 parameters

Are the eigenvalues of the (thermal) Wilson line enough? Other variables?



Axial charge for heavy quarks:

Is $g_A \sim N$ at large N?

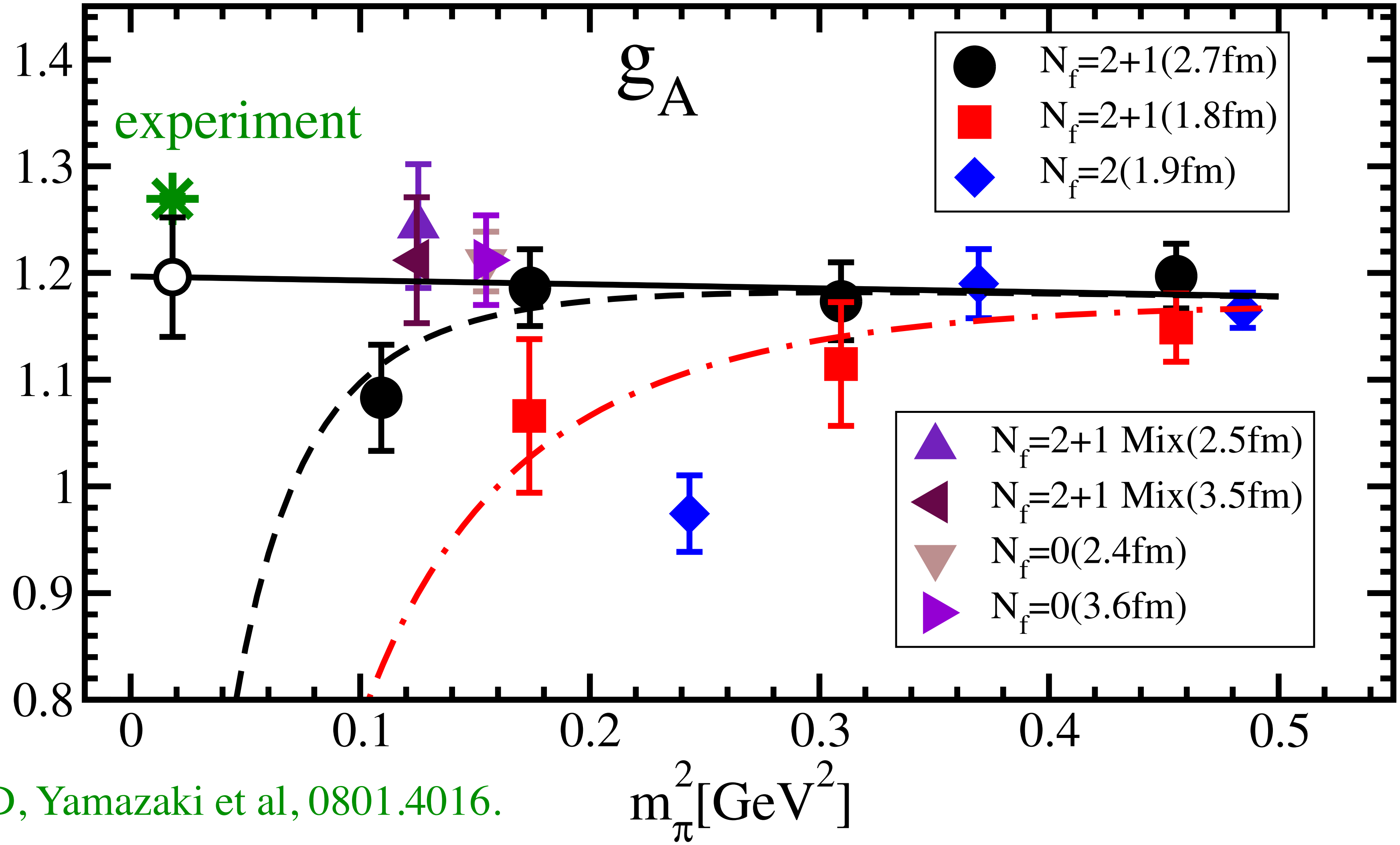
Baryons at large N: $g_A \sim N$?

Non-relativistic quark model: $g_A = (N+2)/3 \sim N$ at large N. N = 3: $g_A = 5/3$.

But even for *heavy* quarks, $g_A \sim 1$. As $m_\pi \to 0$, approaches $g_A^{exp} \sim 1.2$ from *below*

Is $g_A \sim N$ at large N? Diquark pairing? Hidaka, Kojo, McLerran, RDP, 1004.0261

Diquark pairing vs pion cloud? T. Kojo 1208.5661



RBC + UKQCD, Yamazaki et al, 0801.4016.

Equation Of State for cold, dense quarks:

severe constraints from neutron stars

EOS from AdS/CFT?

Neutron stars

"Most" neutron stars:

masses ~ 1.4 $M_\odot$,

max. density ~ 2-3 ρ_0 (nucl. matter)

Recently:

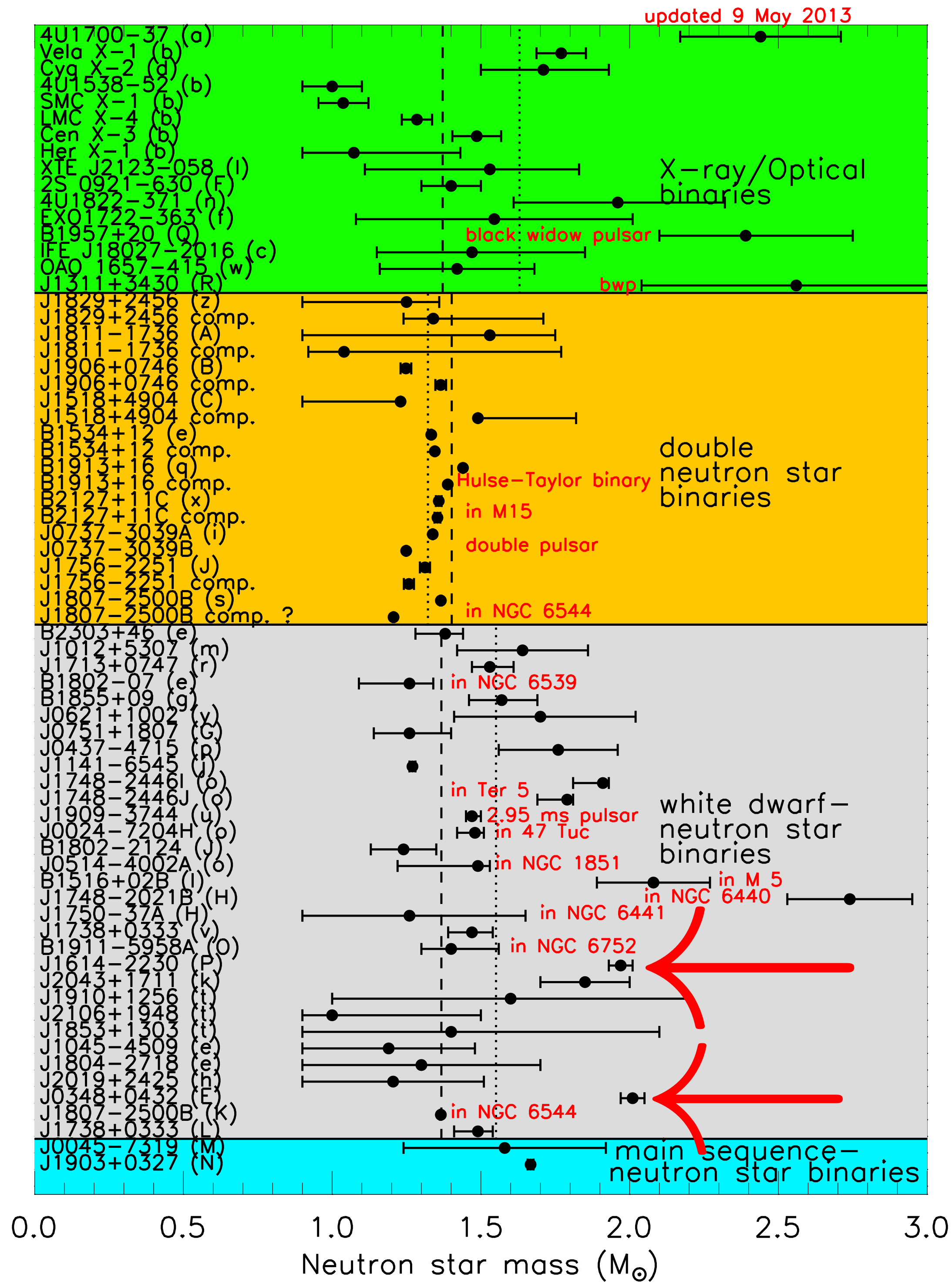
PSR J1614-2230 : $1.97 \pm 0.04\ M_\odot$

PSR J0348+0432: $2.01 \pm 0.04\ M_\odot$

These use a GR effect, Shapiro time delay, to determine the mass

Other candidates with large masses, but much larger errors.

Lattimer, 1305.3520; Prakash, 1307.0397



Neutron stars and "stiff" Equations Of State

To obtain $M \sim 2\, M_\odot$, need "stiff" Equation Of State

EOS "stiff" if speed of sound $c_s > c/\sqrt{3}$. Stiffest possible: $c_s = c$.

For $M \sim 2\, M_\odot$, need $c_s^2 > 0.4\, c^2$. Lattimer, 1305.3520; Prakash, 1307.0397

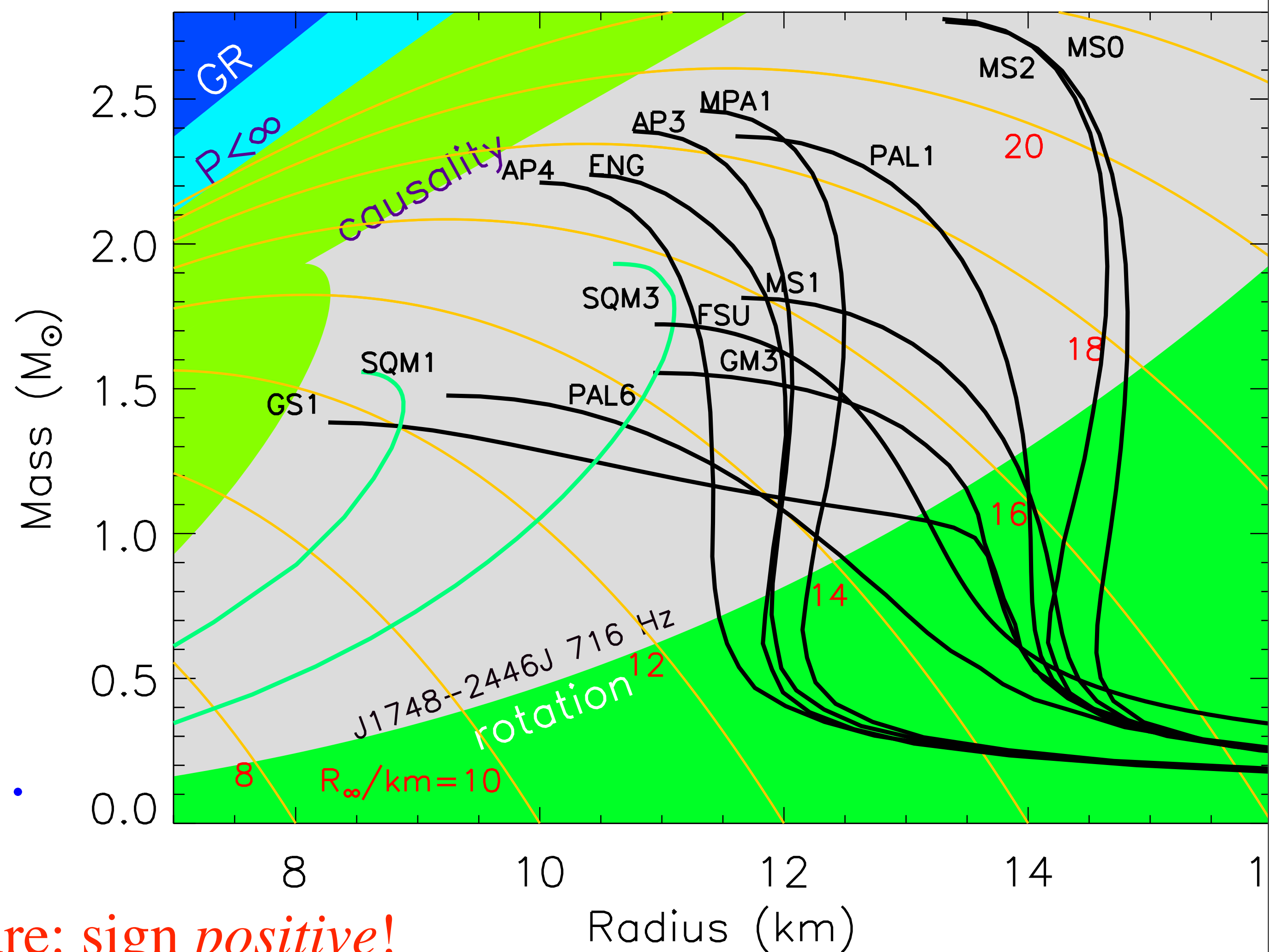
Severe constraint on EOS.

1st order transition:
$c_s = 0$ in mixed phase

Strange quarks:
new degrees of freedom
soften EOS.

Stiffest EOS:

$$p(\mu) = +\,\Lambda^2\,\mu^2 + \ldots$$



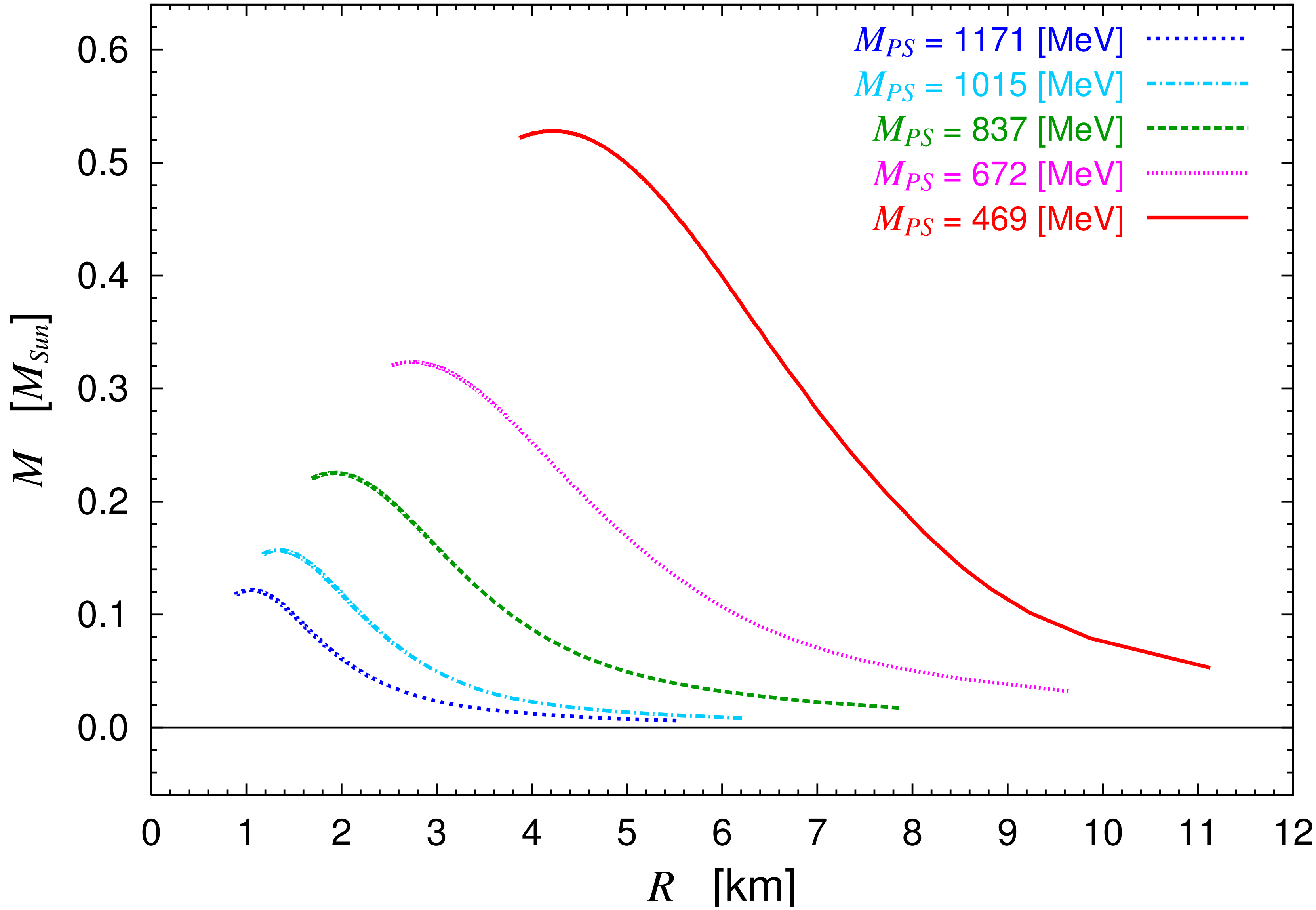
Not like the T^2 term in pressure: sign *positive*!

Lattice & neutron stars

HAL QCD, Inoue et al, 1307.0299: Determine V_{NN} from lattice...input into EOS

Assume $M_{max} \sim a/(m_\pi + b) + c$. *Not* m_π^2. Can obtain $M_{max} \sim 2.2\ M_\odot$, *if:*

No 1st order transition, *no* strange quark matter.



Quantum thermalization in *small* systems

AdS/CFT?

Eigenstate thermalization hypothesis

In quantum mechanics, consider evolution in time for some operator A:

$$|\psi(t)\rangle = \Sigma_\alpha\, C_\alpha\, \mathrm{e}^{-iE_\alpha t}|\Psi_\alpha\rangle\ ;\ A_{\alpha\beta} = \langle\Psi_\alpha|A|\Psi_\beta\rangle$$

At large times, only the diagonal elements survive:

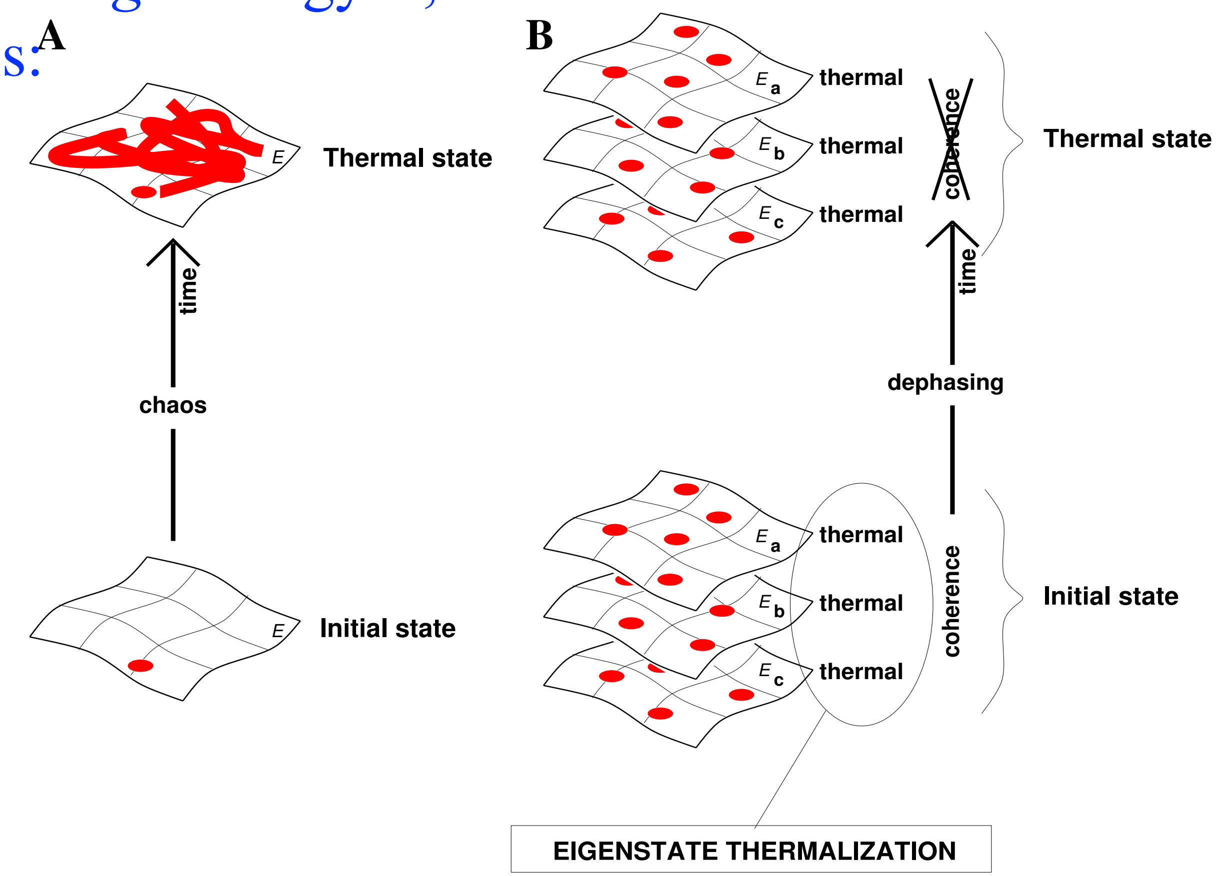
$$\langle A(t)\rangle = \Sigma_{\alpha,\beta}\, C^*_\alpha C_\beta\, \mathrm{e}^{-i(E_\alpha - E_\beta)t} A_{\alpha\beta} \approx_{t\to\infty} \Sigma_\alpha |C_\alpha|^2\, A_{\alpha\alpha}$$

Assume that one probes the system at *high* energy E, in a *narrow* interval ΔE:
eigenstate thermalization hypothesis:
each state close to thermal,

$$\langle A(\infty)\rangle \approx \Sigma_\alpha A_{\alpha\alpha}$$

≈ microcanonical ensemble.
"*Quantum*" thermalization for *isolated* quantum systems.

von Neumann, '29
Deutsch, '91,
Srednicki, cond-mat/9403051;
Rigol, Dunjko, Olshanii, 0708.1324 =>



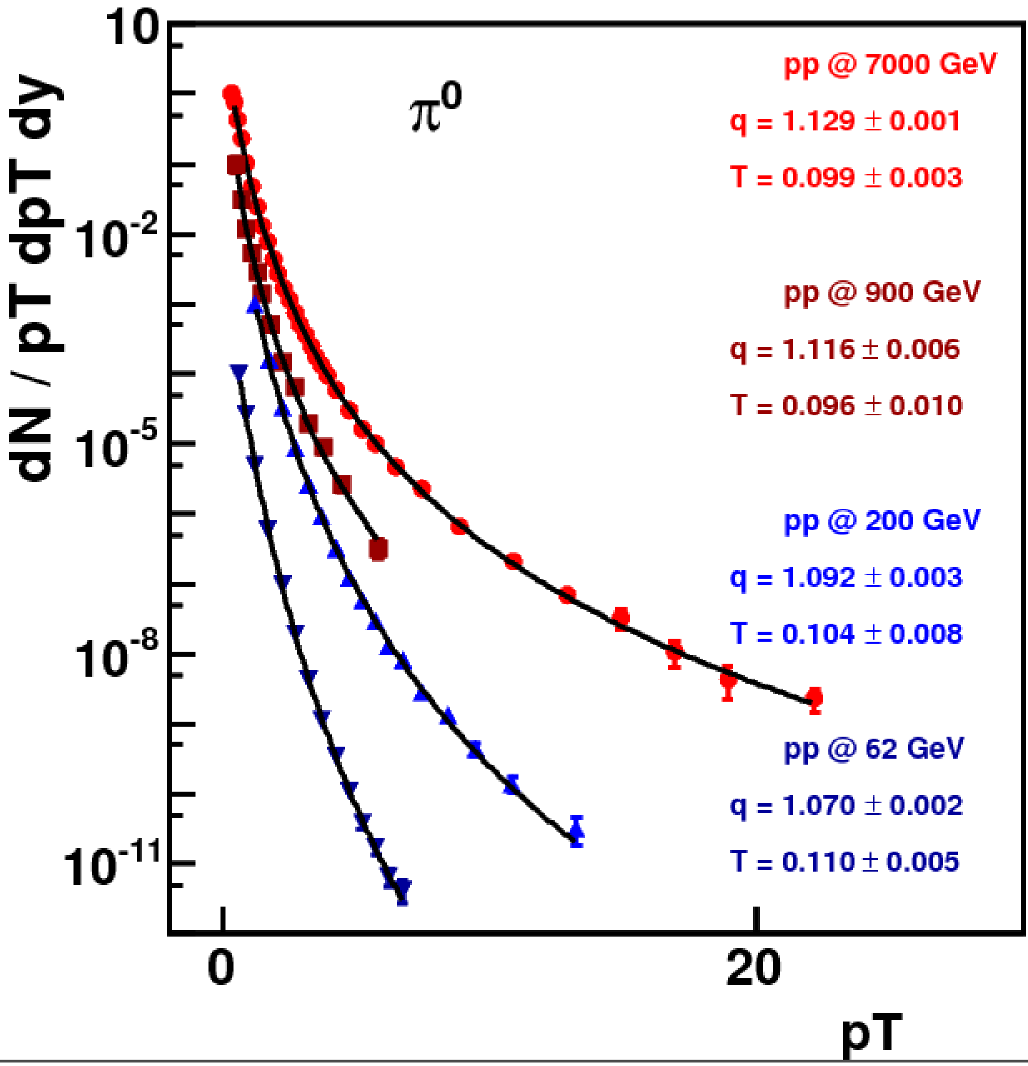
Quantum thermalization in hadronic collisions

Assume quantum wave function has *spread* in temperature, with *power law tail*: related to Tsallis statistics,

$T_0 = T_{Tsallis}/(1\text{-}q)$, $\alpha = 1/(1\text{-}q)$

$$n(p_t) = \int dT\, \frac{e^{-T_0/T}}{T^{\alpha+1}}\, e^{-p_t/T} \sim \frac{1}{(1+p_t/T_0)^{\alpha}}$$

Urmossy, 1212.0260



Extremely successful fit to *all* hadronic collisions: e^+e^-, pp, pA, AA. For pp:

$\sqrt{s}$	α	T_0
62	14	1600
200	11	1130
900	8.6	830
7000	7.75	767

Await results from AdS/CFT!