# The Big Picture:

Effective theories are like Mom, apple pie...

# And the Higgs!

# QCD at nonzero temperature

$T \sim 0$: effective hadronic models Gallas, Giacosa & Richske, 0907.5084

$T \to \infty$: "perturbative" QCD Andersen, Leganger, Strickland, & Su, 1105.0514

*How to meld the two?*

There's always *some* effective theory.

Matrix Model of "Semi"-QGP:
*simple*, based upon lattice simulations

*Moderate* coupling. *Versus* AdS/CFT = strong coupling

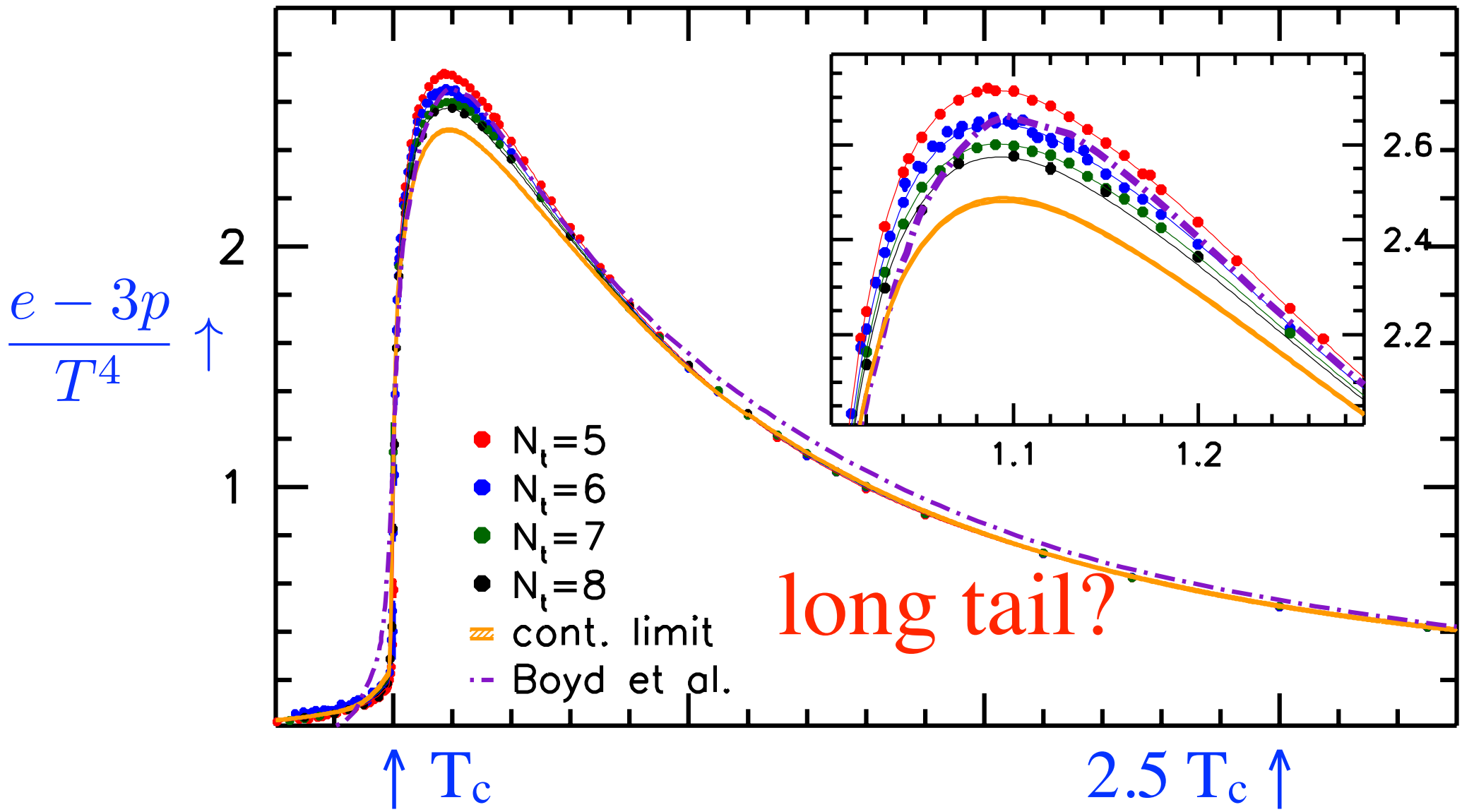
# The Little Picture:

## Lattice and (resummed) perturbation theory

# Lattice: what you know

"Pure" SU(3), no quarks. Peak in (e-3p)/$T^4$, just above $T_c$.

Borsanyi, Endrodi, Fodor, Katz, & Szabo, 1204.6184

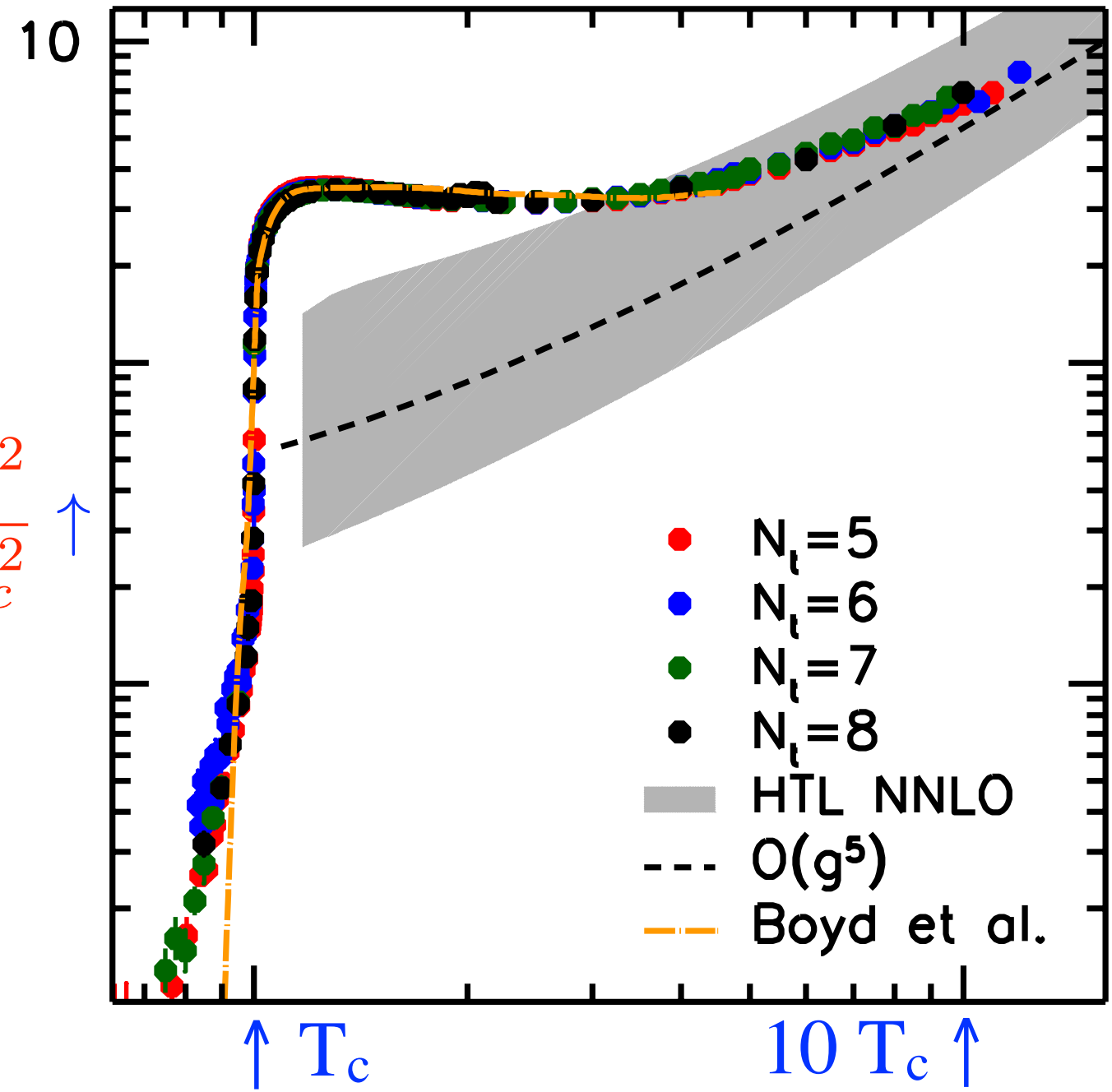


# Lattice: what you should know

$T_c \rightarrow 4\ T_c$:

For pressure, leading corrections to ideality, $T^4$, are *not* a bag constant, $T^0$, but ~ $T^2$ - ? Take as *given*.

$$\frac{e-3p}{T^4}\ \frac{T^2}{T_c^2} \uparrow$$




Borsanyi +... 1204.6184

In 2+1 dim.s, $T^3$ & $T^2$, *not* T

Caselle +... 1111.0580

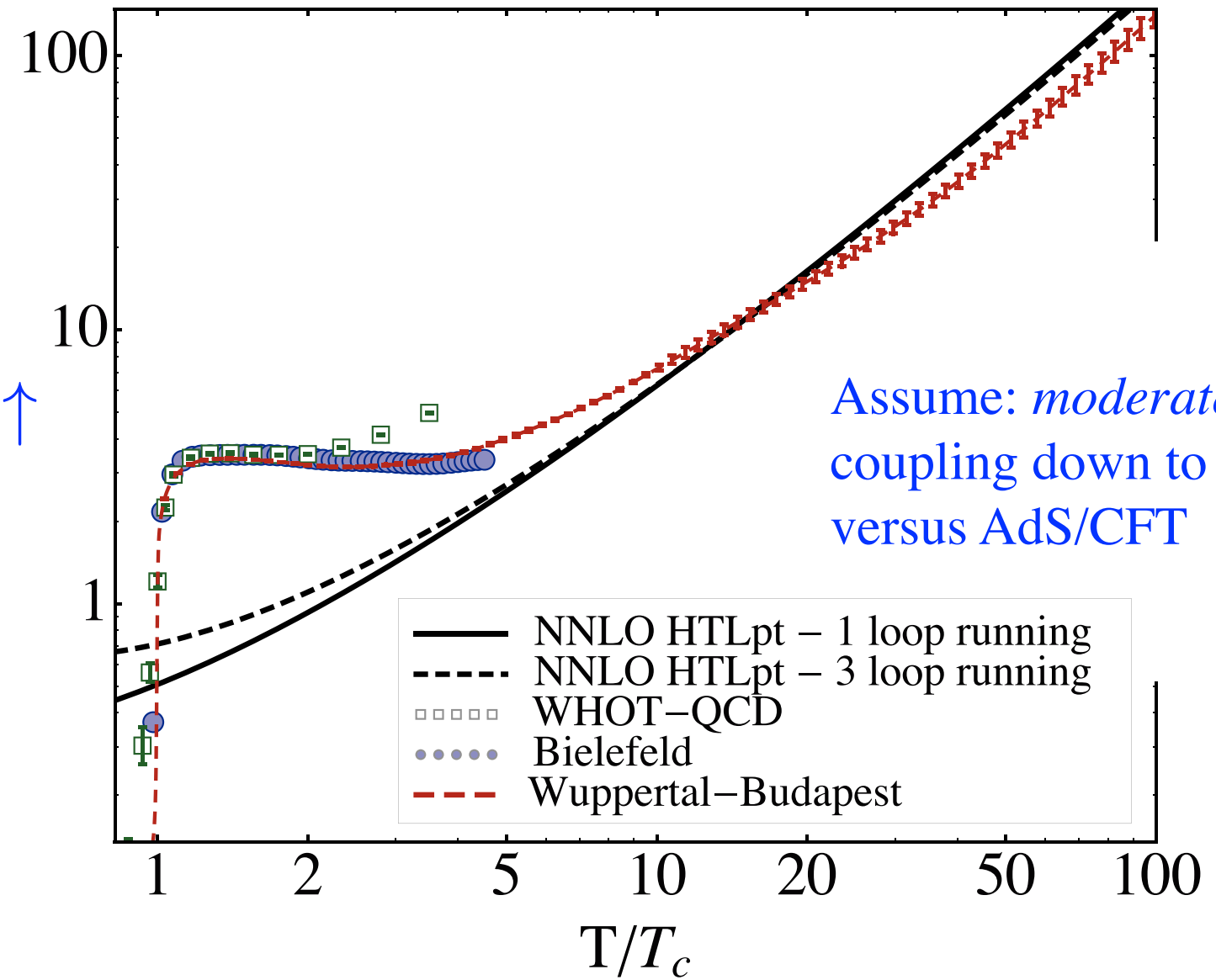
*Not* a gluon "mass"

# *Moderate* coupling, even at $T_c$

QCD coupling is *not* so big at $T_c$, $\alpha(2\pi T_c) \sim 0.3$ (runs like $\alpha(2\pi T)$ )

HTL perturbation theory at NNLO: Andersen, Leganger, Strickland, & Su, 1105.0514

$$\frac{e-3p}{T^4}\,\frac{T^2}{T_c^2}\uparrow$$




Assume: *moderate* coupling down to $T_c$ versus AdS/CFT

# The Competition:

Models for the “s”QGP, $T_c$ to ~ 4 $T_c$

”s” = strong...or...semi?

# Unrelated

Massive gluons: Peshier, Kampfer, Pavlenko, Soff '96...Castorina, Miller, Satz 1101.1255
Castorina, Greco, Jaccarino, Zappala 1105.5902

Mass decreases pressure, so adjust m(T) to fit p(T) with *three* parameters.

$$p(T) = \# T^4 - m^2 T^2 + \ldots$$

Polyakov loops: Fukushima ph/0310121...Hell, Kashiwa, Weise 1104.0572

Effective potential of Polyakov loops. Potential has *five* parameters

$$V_{eff}(T) \sim m^2 \ell^* \ell + T \log f(\ell^* \ell)$$

$$m^2 = T^4 \sum_{i=0}^{3} a_i (T_c/T)^i$$

AdS/CFT: Gubser, Nellore 0804.0434...Gursoy, Kiritsis, Mazzanti, Nitti, 0903.2859

Add potential for dilaton, φ, to fit pressure. Only infinite N, *two* parameters

$$V(\phi) \sim \cosh(\gamma\phi) + b\,\phi^2$$

# Related

Linear model of Wilson lines: Vuorinen & Yaffe, ph/0604100;
de Forcrand, Kurkela, & Vuorinen, 0801.1566; Zhang, Brauer, Kurkela, & Vuorinen, 1104.0572

$$V_{eff}(\mathbf{Z}) = m^2 \, \mathrm{tr}\mathbf{Z}^\dagger\mathbf{Z} + \kappa \left(\det \mathbf{Z} + c.c.\right) + \lambda \, \mathrm{tr}(\mathbf{Z}^\dagger\mathbf{Z})^2 + \ldots$$

**Z** is not unitary; *four* parameters. 't Hooft loop *approximate*.

Above four models, with three, five, two, and four parameters,
are comparable to a Matrix Model with *one* free parameter.

Because (e-3p)/$T^4$ is flat above 1.2 $T_c$, a *very narrow* transition region, $T < 1.2T_c$!
Like Func. Ren. Group analysis of Schwinger-Dyson eqs.:
Braun, Gies, Pawlowski, 0708.2413; Marhauser & Pawlowski, 0812.1444;
Braun, Eichhorn, Gies, & Pawlowski, 1007.2619

See, also: Borsanyi, Endrodi, Fodor, Katz, & Szabo, 1204.6184

# *Not* from 1st principles

In Matrix Model, *very* simple ansatz. Add terms, *by hand*, to *fit* the lattice data.

We do *not* derive the effective theory from QCD. Who does:

Monopoles: Liao & Shuryak, ... + 0804.0255, 1206.3989; Shuryak & Sulejmanpasic, 2012

Dyons: Diakonov & Petrov, ... + 1011.5636: explain 1st order for SU(N) > 4 & G(2)

Bions: Unsal + .. + Poppitz, Schaefer, & Unsal 1205.0290: term ~ q(1-q) in SUSY limit

Both semi-QGP & these 3 approaches expand about background classical fields:

Semi-QGP: $$A_0 = \frac{\pi T}{g}\, q\, \sigma_3$$

$A_0$ is a constant, diagonal matrix. *Simplest* possible background field

Monopoles, dyons, & bions: $$A_0^a(x) = \frac{1}{g}\, q^a(x)$$

$A_0$ is a *function* of space *and* color space. Must integrate over gas of monopoles, dyons, or bions

# *Why* is such a simple ansatz ok?

loop vs. eigenvalues

# Order parameter(s) for deconfinement

Thermal Wilson line: $$\mathbf{L} = \mathcal{P}\, \mathrm{e}^{ig \int_0^{1/T} A_0\, d\tau}$$

Under global Z(3) rotations: $$\mathbf{L} \to e^{2\pi i/3}\, \mathbf{L}$$

Wilson line gauge variant.
Trace = Polyakov loop gauge *in*variant $$\ell = \frac{1}{3}\, \mathrm{tr}\, \mathbf{L}$$

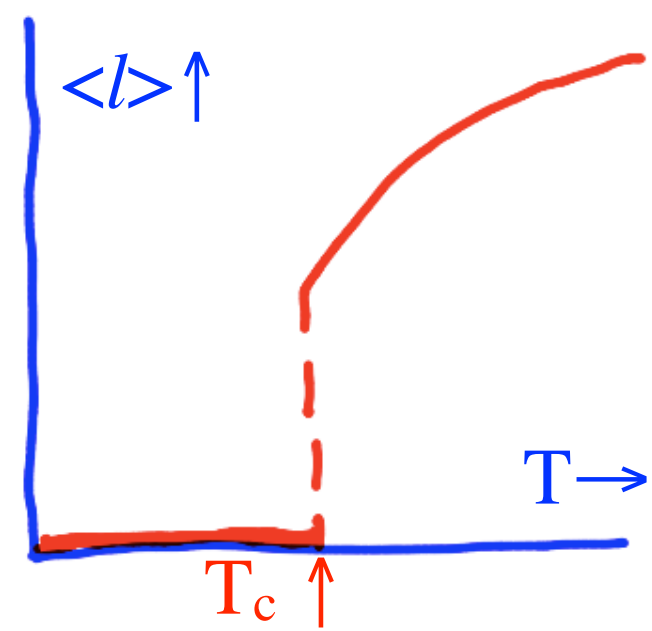
*Eigenvalues* of **L** are also gauge *in*variant:
basic variables of matrix model

$\langle$ *loop* $\rangle$ measures *partial* ionization of color:
when $0 < \langle$ *loop* $\rangle < 1$ , "semi"-QGP

*Must* model the change in the loop/eigenvalues!

(Confinement in Loop Model: Z(3) symmetry

In Matrix Model: *complete eigenvalue repulsion*)



# Matrix Model:

*Simplest* possible model that generates confinement

# Matrix Model: SU(2)

*Simple* approximation: *constant* $A_0 \sim \sigma_3$ .
*Must* have this to model change in loop

$$A_0^{cl} = \frac{\pi T}{g}\, q\, \sigma_3$$

For SU(2), single field q

Wilson line **L**:

$$\mathbf{L}(q) = \begin{pmatrix} e^{i\pi q} & 0 \\ 0 & e^{-i\pi q} \end{pmatrix}$$

Polyakov loop *l*:

$$\ell = \cos(\pi q)$$

Z(2) symmetry: q → 1 - q, L → - L

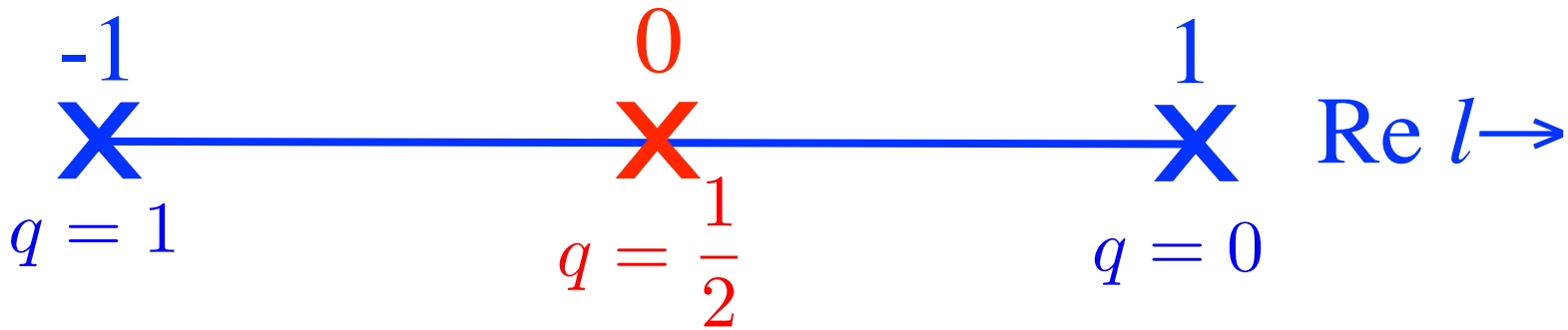
Perturbative vacua: q = 0 and 1, **L** = ± **1**

Point *half*way in between: q = ½ :
*Confined* vacuum, $\mathbf{L}_c$, $l = 0$ .

$$\mathbf{L}_c = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

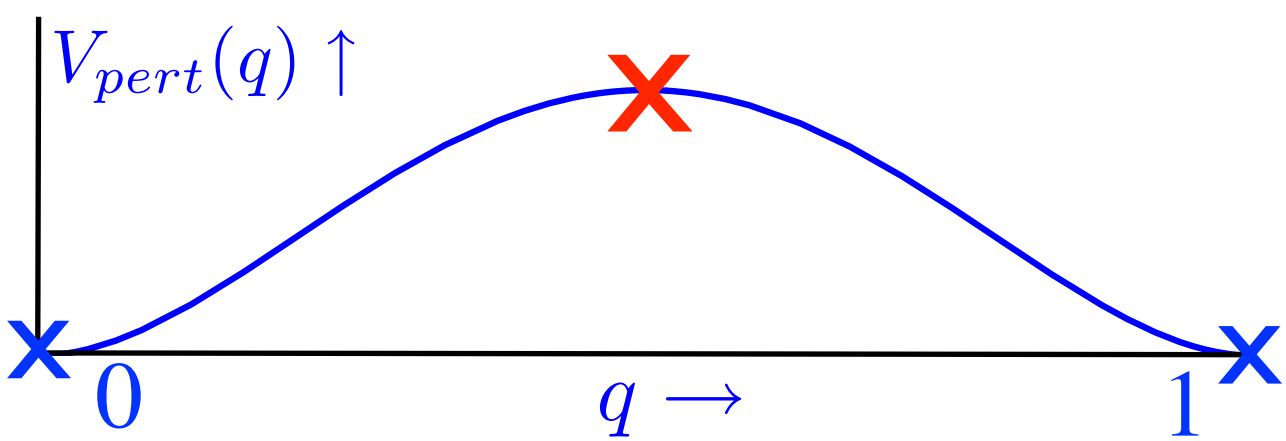
# Perturbative potential for q

Classically, *no* potential:




One loop order: pert. q-potential (Gross, RDP, & Yaffe, '81; Weiss '81)

$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 q^2 (1-q)^2$$

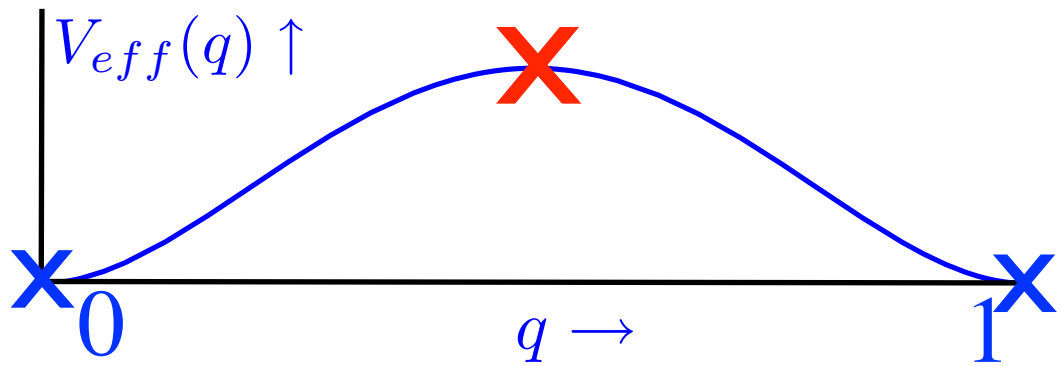


# Non-perturbative potential

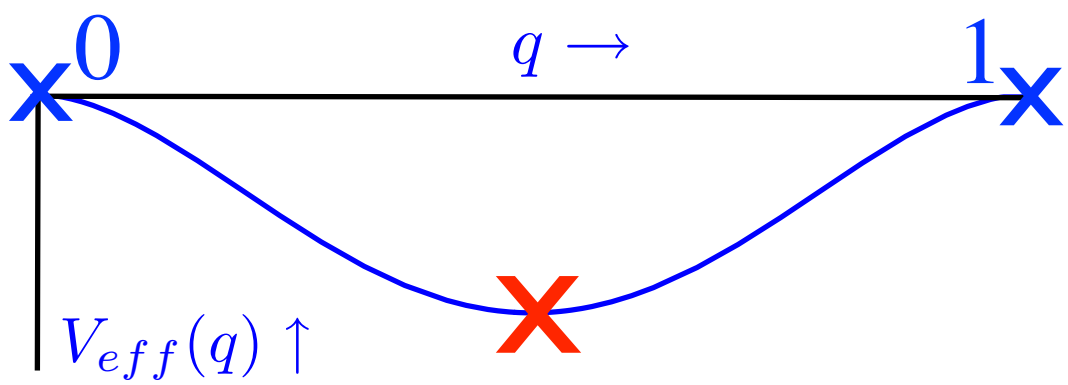
By *fiat*, add *non*-perturbative q-potentials to generate <q> ≠ 0 and confinement:

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q)$$

T >> $T_c$: $\langle q \rangle = 0,1 \rightarrow$



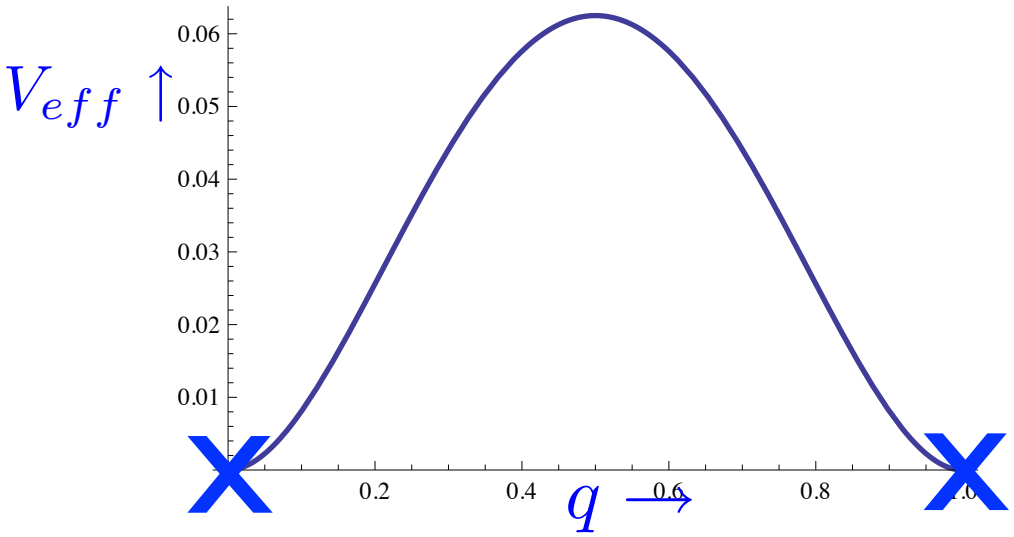

T < $T_c$: $\langle q \rangle = ½ \rightarrow$



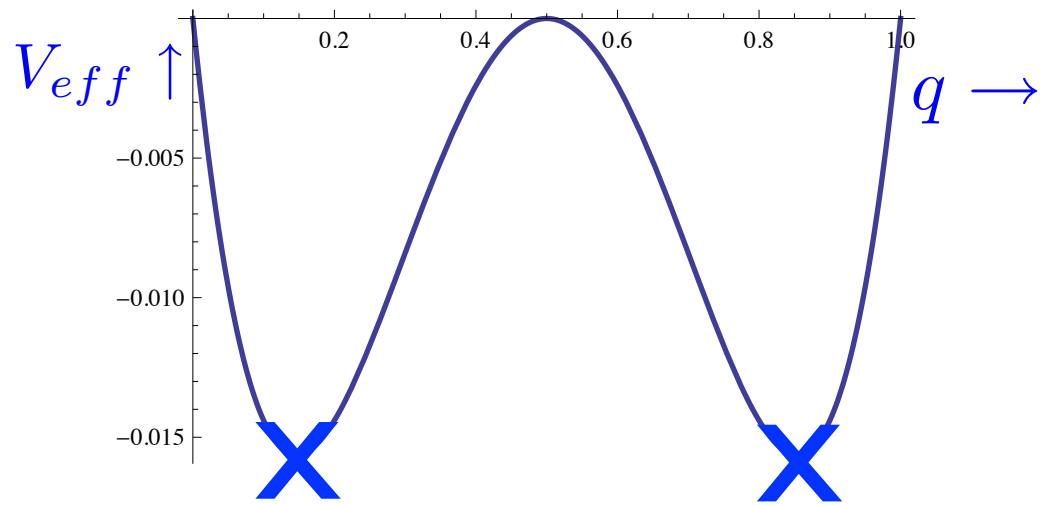
# Cartoons of deconfinement

$$V_{eff} = q^2(1-q)^2 - a\,q(1-q)\,,\; a \sim T_c^2/T^2$$

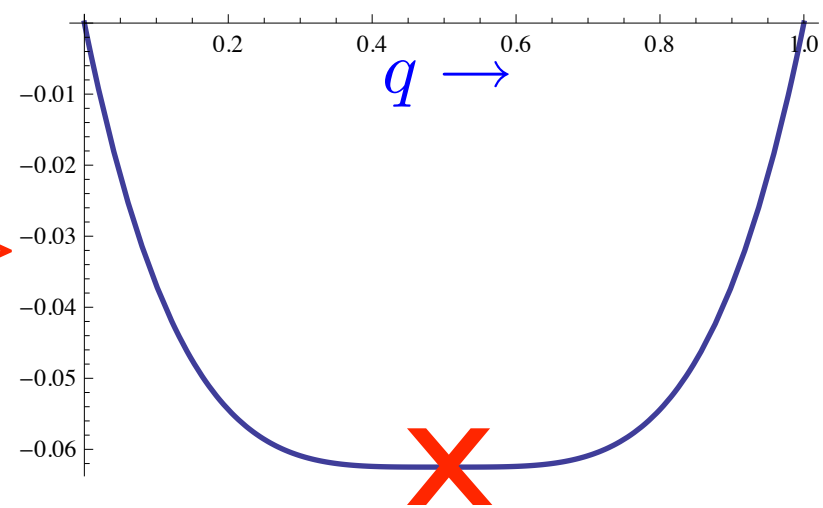
⇓ T >> $T_c$: complete QGP




⇓ T > $T_c$: semi QGP




T = $T_c$ =>



# Matrix Models, two colors

Zero parameter model: Meisinger, Miller, & Ogilvie, ph/0108009

1 parameter: Dumitru, Guo, Hidaka, Korthals-Altes, & RDP, 1011.3820; 2 parameter: 1205.0137

Effective potential sum of perturbative and non-perturbative potentials for q:

$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \left( -\frac{1}{20} + q^2(1-q)^2 \right)$$

$$V_{non}(q) = \frac{4\pi^2}{3} T^2 T_c^2 \left( -\frac{c_1}{5} q(1-q) - c_2 q^2(1-q)^2 + \frac{c_3}{15} \right) + B T_c^4$$

Typical mean field theory: $\left. \frac{d}{dq} V_{eff}(q) \right|_{q=\langle q \rangle} = 0$

Pressure: $p(T) = -V_{eff}(\langle q \rangle)$

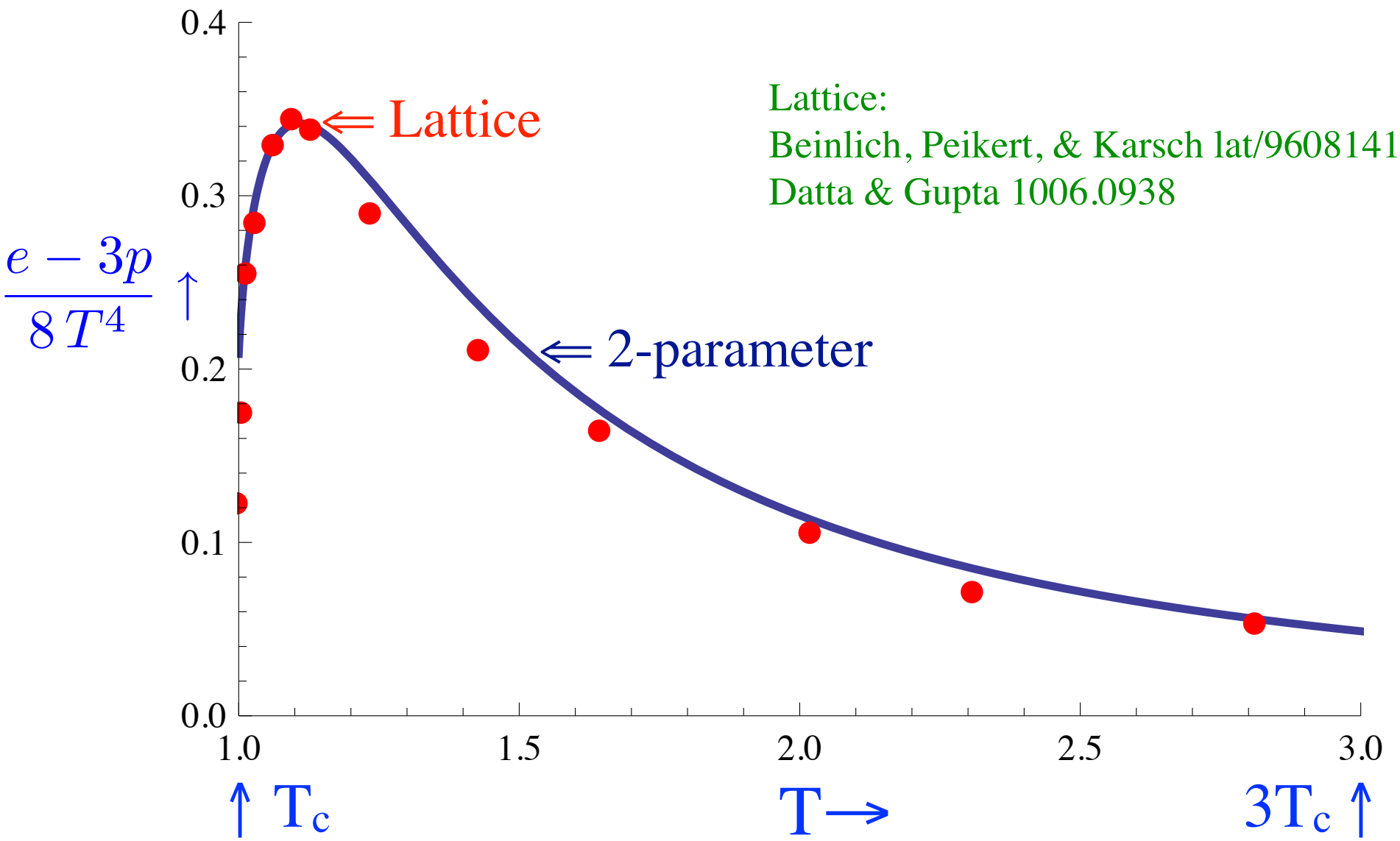
Start with four parameters: $c_1$, $c_2$, $c_3$, & MIT bag constant B.

Require: transition at $T_c$; pressure($T_c$) = 0. End up with *two* free parameters.

# Matrix Model, three colors

Fix two parameters by fitting to latent heat and e-3p:

$$c_1 = .83 \ , c_2 = .55 \ , c_3 = 1.3 \ , B = (262 \text{ MeV})^4 \ .$$

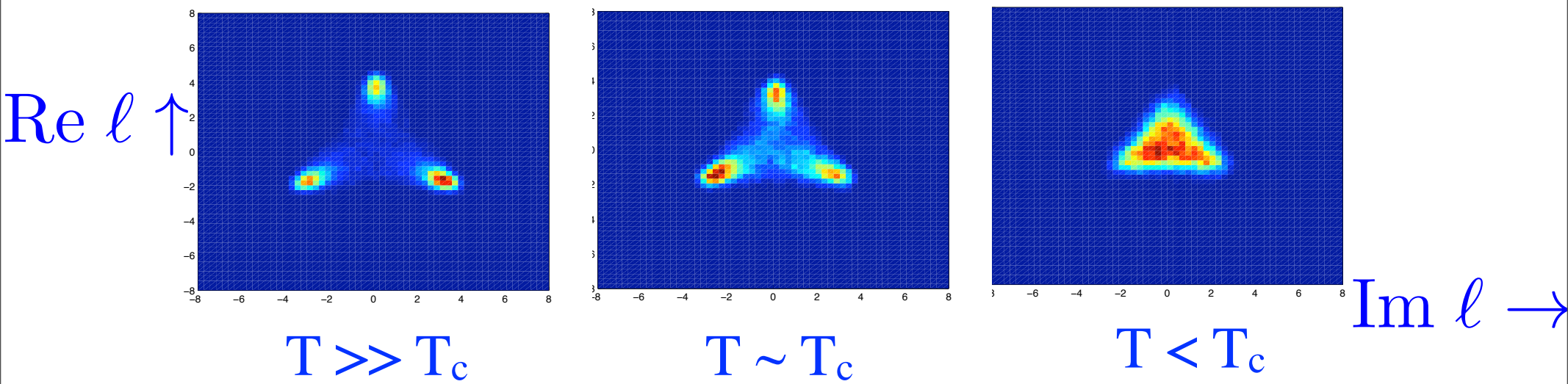


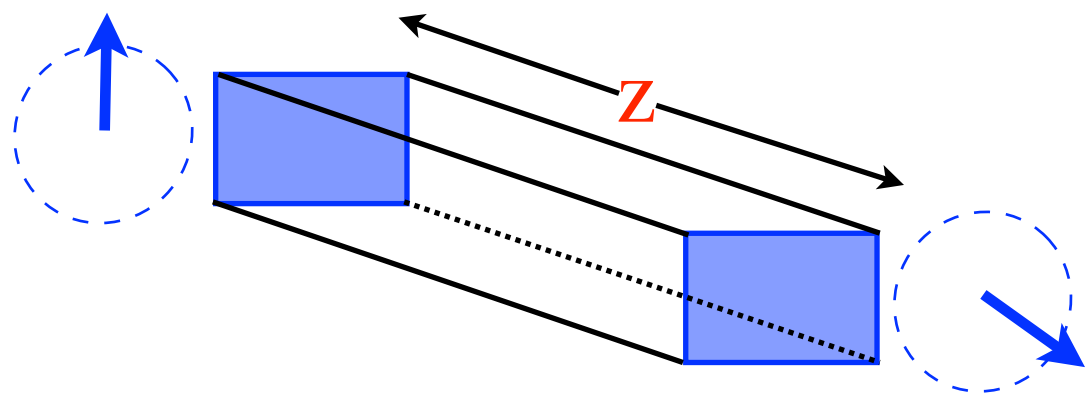

Lattice:
Beinlich, Peikert, & Karsch lat/9608141
Datta & Gupta 1006.0938

# ‘t Hooft loop

Lattice, A. Kurkela, unpub.’d: 3 colors, loop $l$ complex.

Distribution of loop shows Z(3) symmetry:




Interface tension: take long box.

Each end: distinct but *degenerate*

In between: interface, action ~ interface tension, σ:

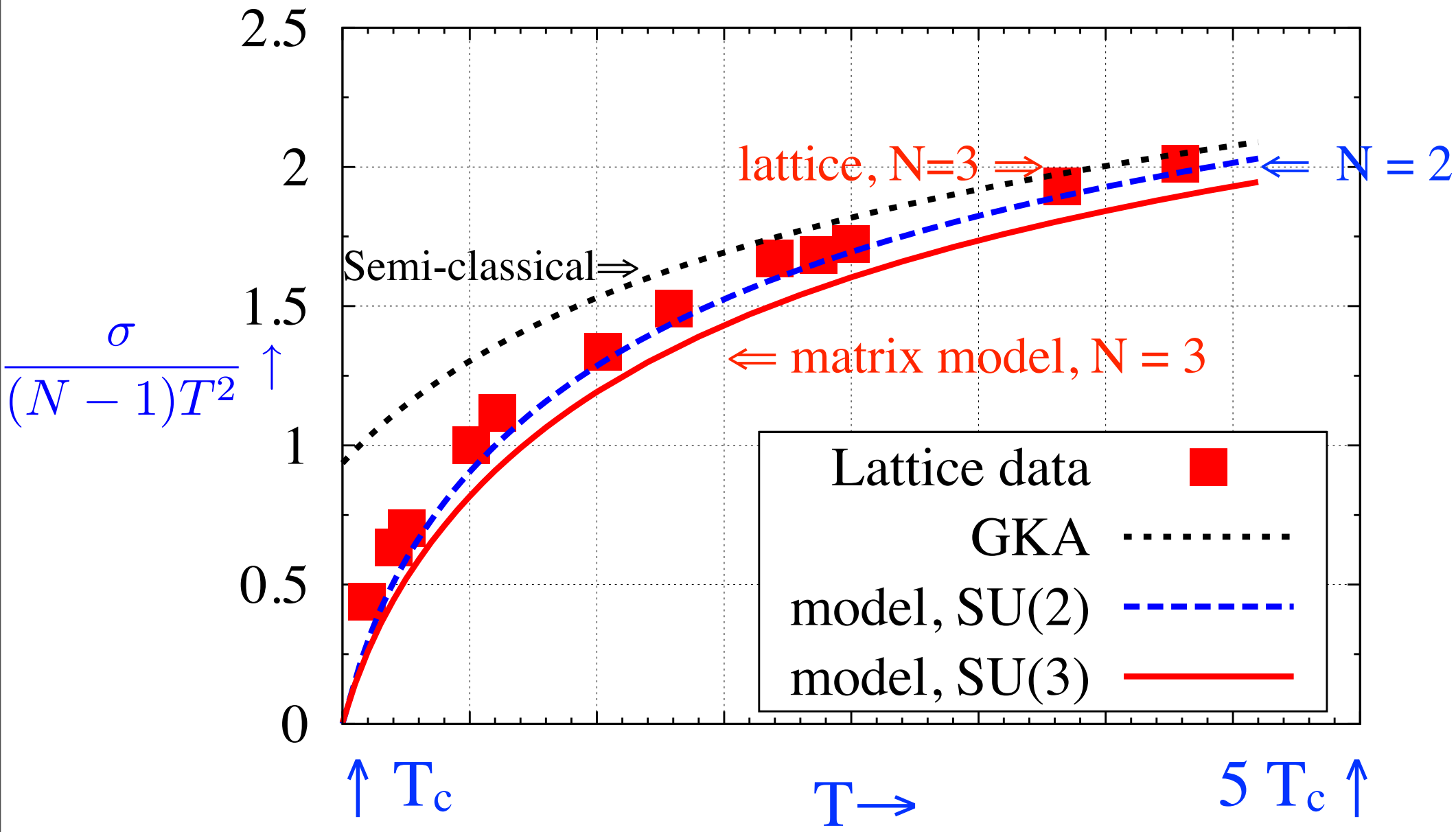
$$Z \sim e^{-\sigma V_{tr}}$$

T > $T_c$: order-order interface = ‘*t Hooft loop*:

# Success: ‘t Hooft loop

Matrix Model works well:

Lattice: de Forcrand, D’Elia, & Pepe, lat/0007034; de Forcrand & Noth lat/0506005



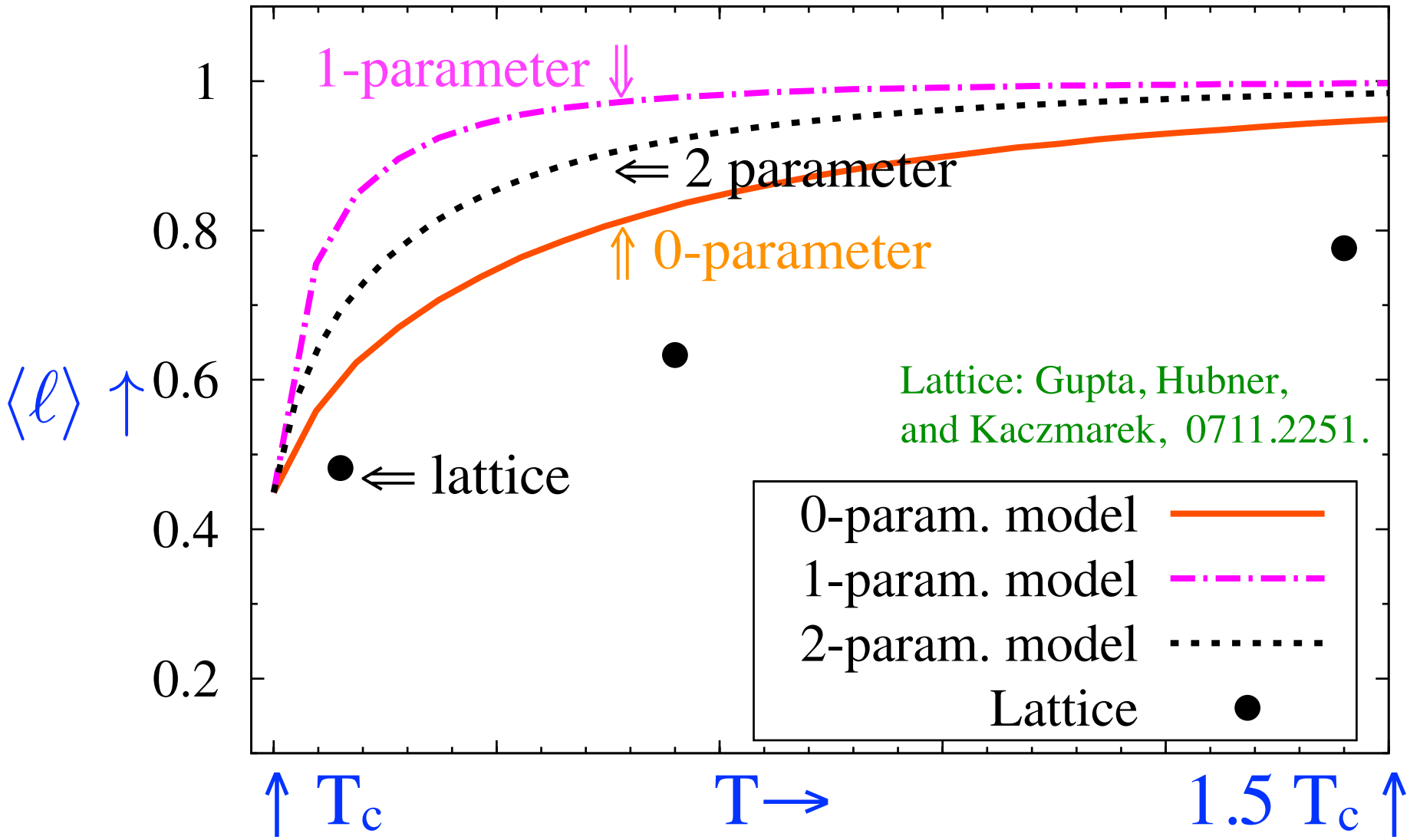
# Failure: Polyakov loop

*Renormalized* Polyakov loop from lattice *nothing* like Matrix Model

Model: transition region *narrow*, to ~ 1.2 $T_c$. Lattice: loop *wide*, to ~ 4.0 $T_c$.

Does the Matrix Model fail, or...:

Does the ren.'d Polyakov loop reflect the eigenvalue distribution?



# G(2) gauge group:

Confinement is *not* due to center symmetry;

the “law” of maximal eigenvalue repulsion

# G(2) group: confinement with*out* a center

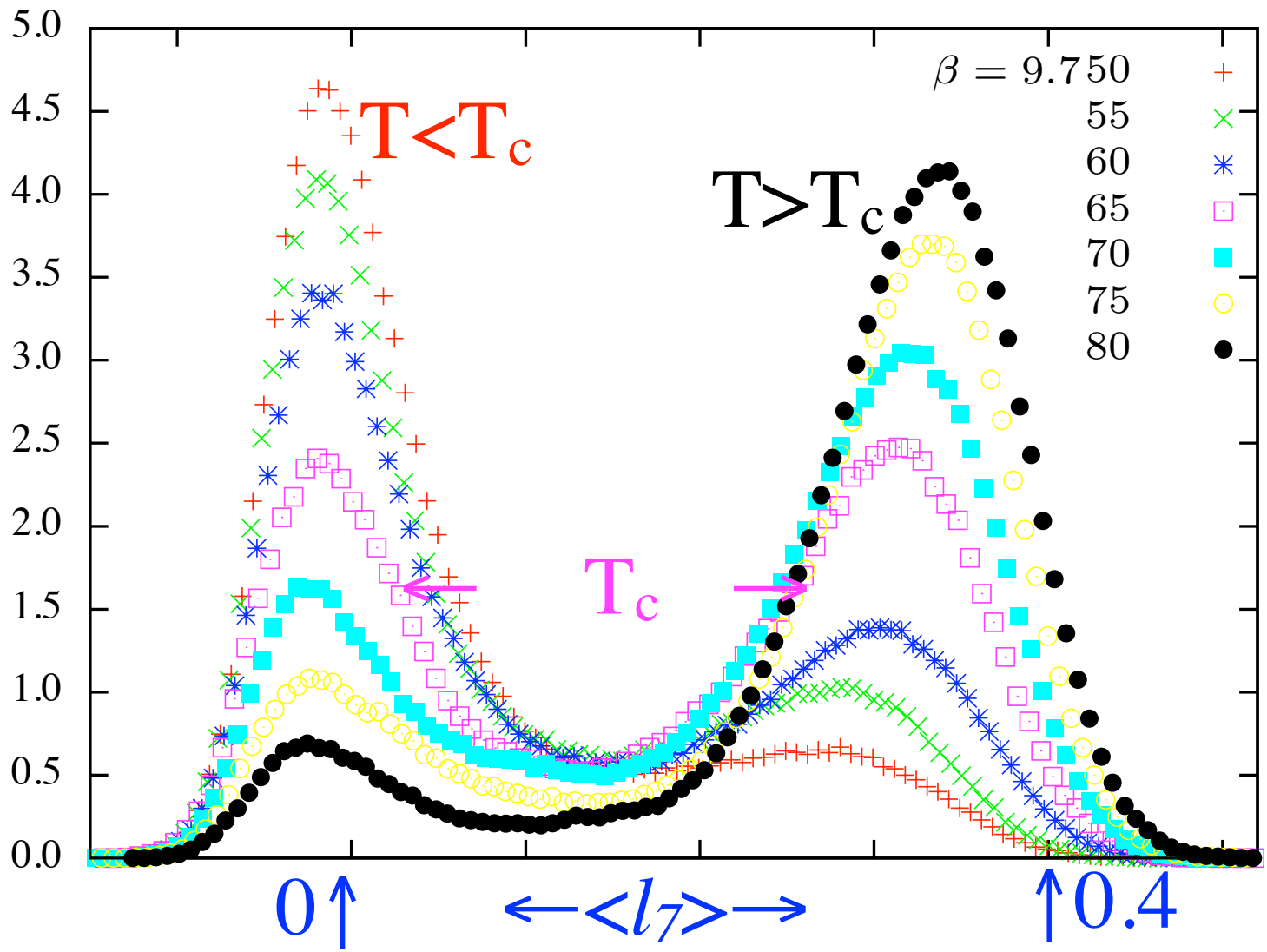
Holland, Minkowski, Pepe, & Wiese, lat/0302023...

Exceptional group G(2) has *no* center, so in *principle*, *no* "deconfinement"

With no center, $\langle loop_7 \rangle$ can be *non*zero at *any* $T > 0$.

Lattice: 1st order transition, $\langle l_7 \rangle \sim 0$ for $T < T_c$, $\langle l_7 \rangle \neq 0$ for $T > T_c$: deconfinement!

Welleghausen, Wipf, & Wozar 1102.1900.



# G(2): perturbative potential

Natural SU(3) embedding: fundamental **7 = 1 + 3 + 3***

$$\mathbf{L}_7 = \mathrm{e}^{2\pi i \mathbf{q}_{G(2)}} \ , \ \mathbf{q}_{G(2)} = (0, q_1, q_2, -q_1 - q_2, -q_1, -q_2, q_1 + q_2))$$

Only two parameters, $q_1$ and $q_2$.

Adjoint = **14 = 3 + 3* + 8**. Perturbative potential: $V_2(q) = q^2(1-q)^2$

$$V_2^{G2} = V_2(q_1) + V_2(q_2) + V_2(q_1 + q_2)$$
$$+V_2(q_1 - q_2) + V_2(2q_1 + q_2) + V_2(q_1 + 2q_2)$$

Looks like a SU(3) gluon potential *plus* fundamental fields.

Hard to get confinement with G(2) potential: **3** and **3***'s give *non*-zero loop

$q_{G(2)}$ looks like a special case of SU(7):

$$\mathbf{q}_{SU(7)} = (0, q_1, q_2, q_3, -q_1, -q_2, -q_3)$$

By taking $q_3 = q_1 + q_2$ and permuting the order of the eigenvalues.

# G(2): non-perturbative potentials

To one loop order, perturbative V:

$$V_{pert}(q) = \pi^2 T^4 \left( -\frac{14}{45} + V_2^{G2}(q_1, q_2) \right)$$

Without Z(N) symmetry, have *many* possible terms:

$$V_{non}(q) = T^2 T_c^2 \left( -c_1^{G2} V_1^{G2} - c_2^{G2} V_2^{G2} - c_1^{SU7} V_1^{SU7} - c_2^{SU7} V_2^{SU7} - d\, V_2^{\ell_7} \right)$$

All V's = V($q_1$,$q_2$). Generally,

$$V_n(q_1, q_2) = \sum_{q_1, q_2} |q|^n (1 - |q|)^n$$

The $V^{G2}$'s have the symmetry of the perturbative G(2) term.

The $V^{SU7}$'s have the symmetry of the SU(7) theory, fixing $q_3 = q_1 + q_2$.

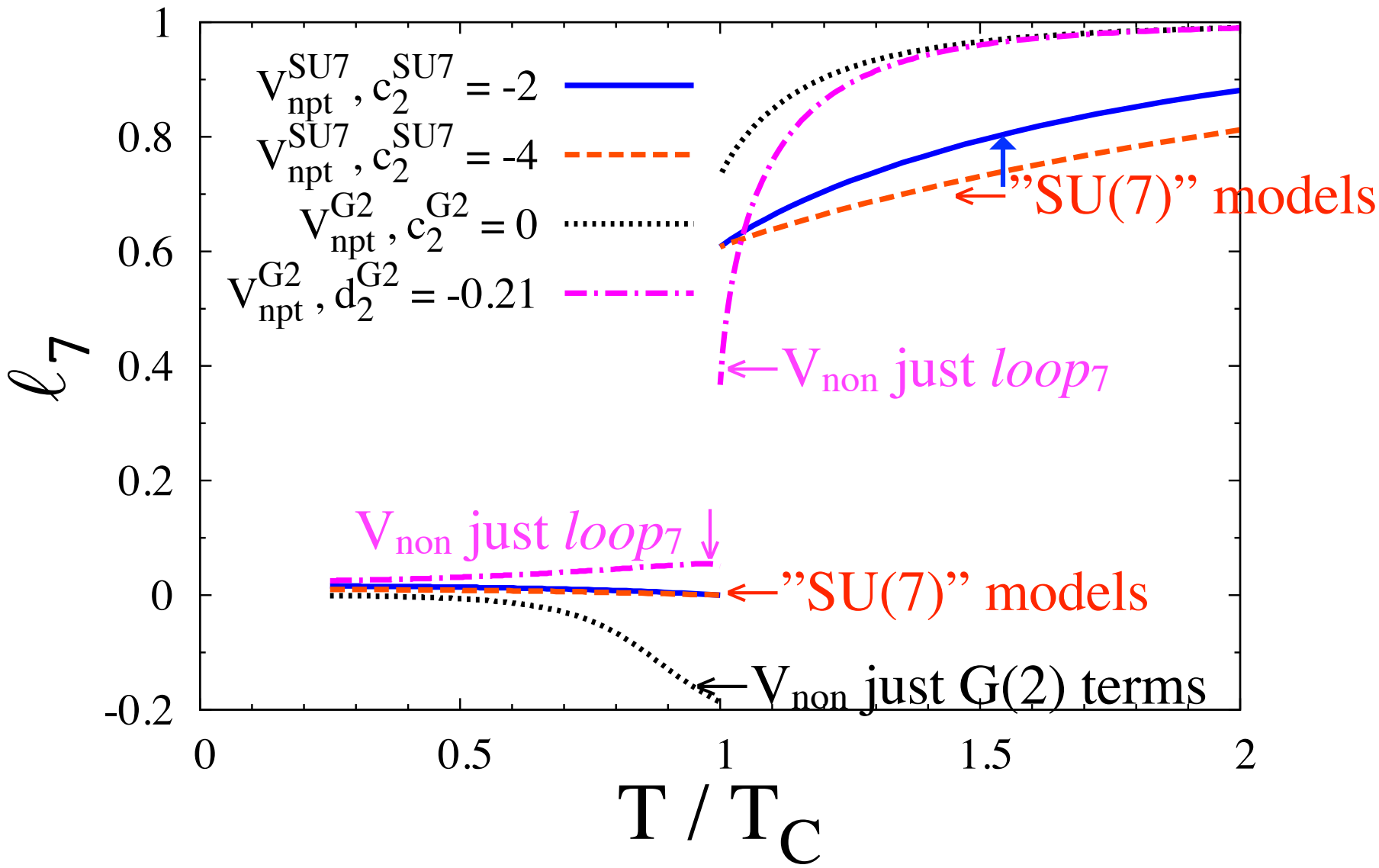
Note a term ~ *loop₇* is allowed in the potential, unlike for SU(N). No center!

Both $V^{SU7}$ and *loop₇* generate eigenvalue *repulsion*, and so *small* $\langle loop_7 \rangle$.

# Law of maximal eigenvalue repulsion

Generically, *easy* to find 1st order transitions. *Most* have $\langle l_7 \rangle$ *non*zero below $T_c$.

To get $\langle l_7 \rangle \sim 0$ below $T_c$, *must* add terms to generate maximal eigenvalue repulsion

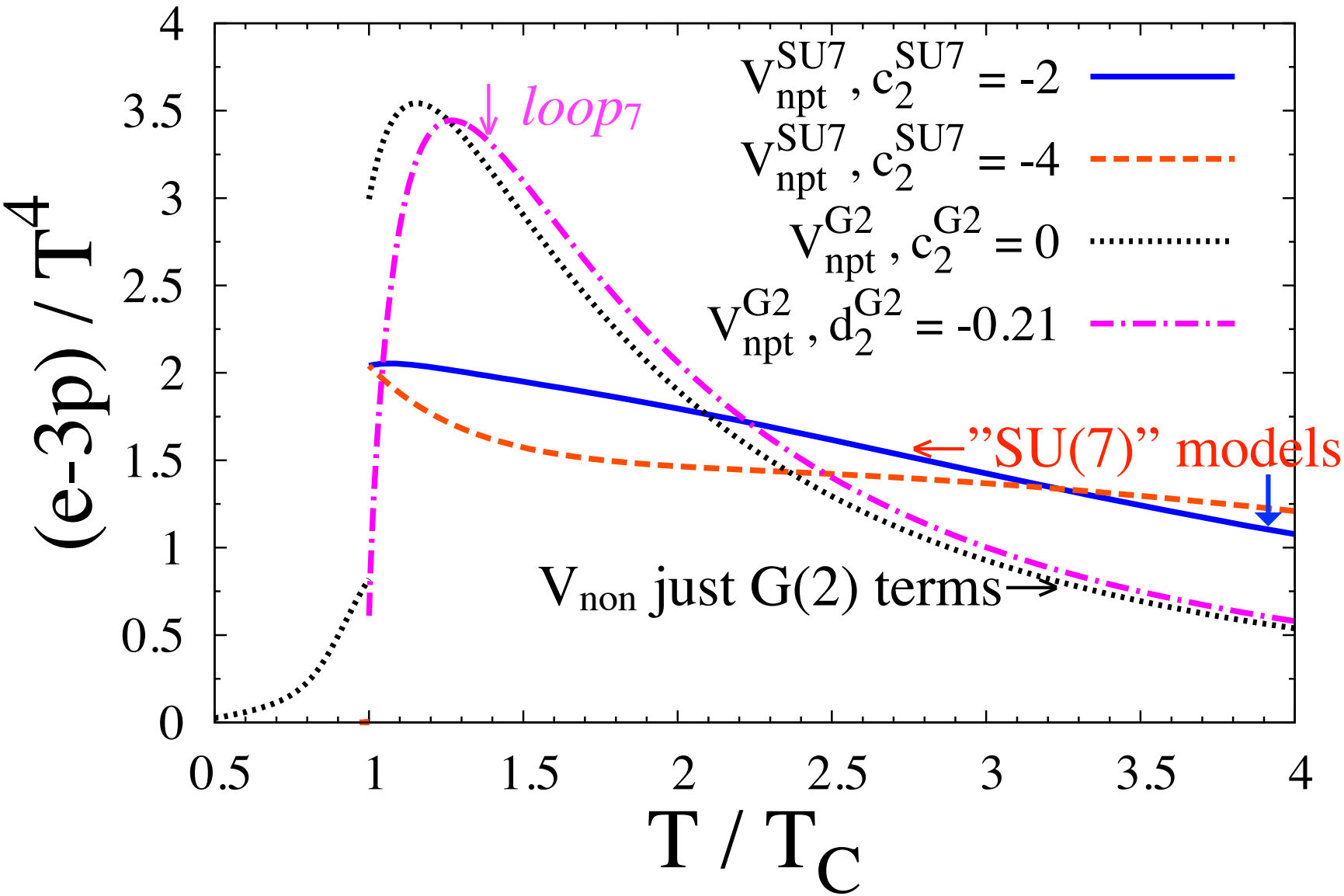


# Predictions for G(2)

Start with model with 3 parameters

Requiring $\langle l_7 \rangle \sim 0$ below $T_c$ *greatly* restricts the possible parameters.

Yields *dramatic* differences in the behavior of $(e-3p)/T^4$.



# Beyond pure glue:

## heavy quarks and one corner of the Columbia plot

Kashiwa, RDP, & Skokov 1205.0545

# Adding heavy quarks

Quarks add to the perturbative q-potential,

$$V^{qk}_{pert}(q) = -\,\mathrm{tr}\ \log(\not{D}^{cl} + m) \sim -\frac{\sqrt{2}}{\pi^{3/2}}\, T^{5/2} m^{3/2}\, \mathrm{e}^{-m/T}\ \mathrm{Re}\,\mathrm{tr}\,\mathbf{L} + \ldots$$

Plus terms ~ $e^{-2m/T}$ Re tr $\mathbf{L}^2$, etc. Quarks act like background Z(3) field.
Heavy quarks wash out deconfinement at deconfining critical endpoint, "DCE".

For the DCE, first term works to ~ 1% for all quantities.

Add $V^{qk}_{pert}(q)$ to the gluon potential, and change *nothing* else! $T_c$ the same!

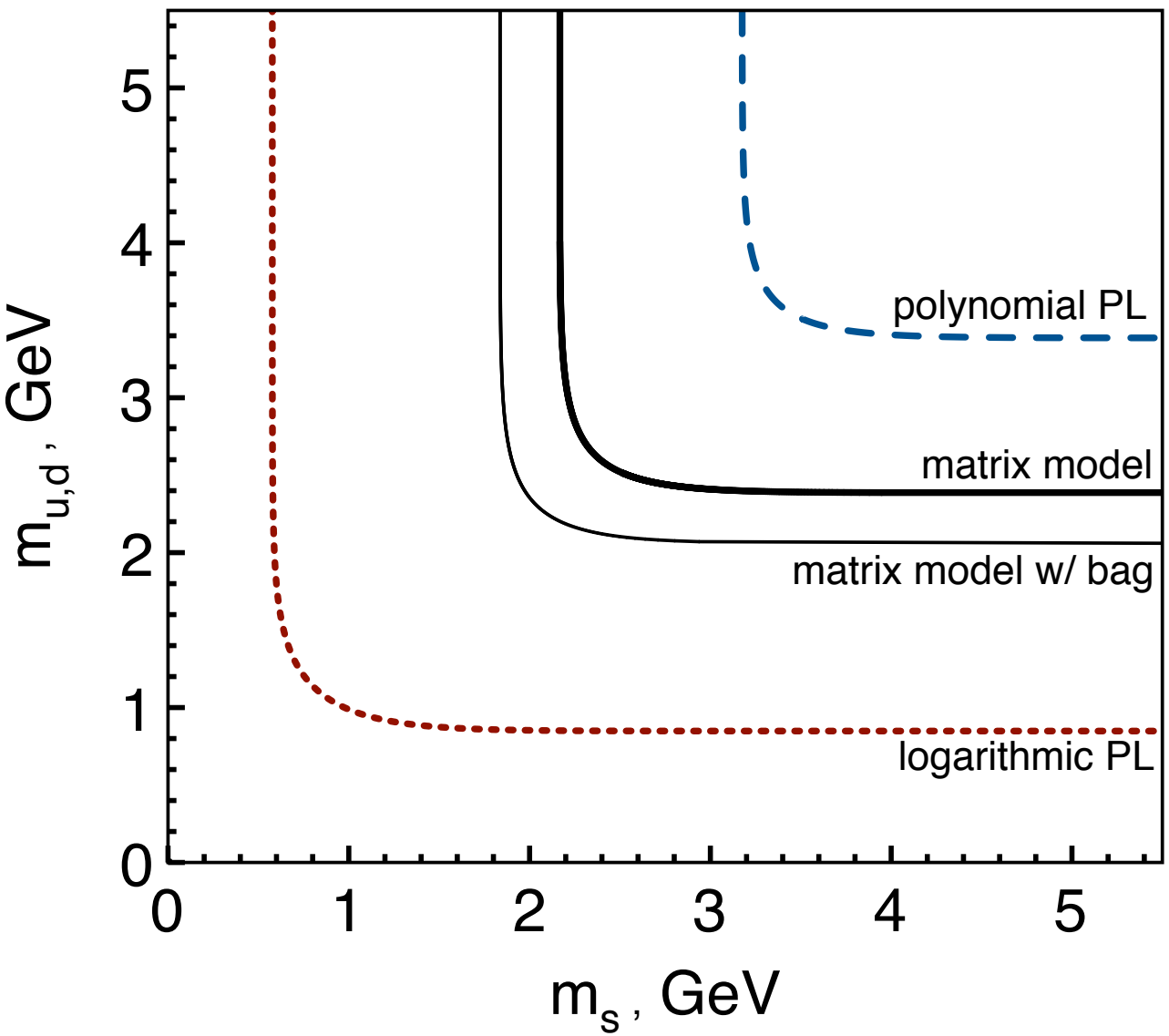
Dumbest thing possible to do. Different flavors: no problem!

N.B.: Quarks generate v.e.v for $\langle loop \rangle$ below Tc, and so become sensitive to details of pressure in the confined phase. Have to modify the potential by hand to avoid unphysical behavior (negative pressure)

# Predictions for upper right hand corner of Columbia plot

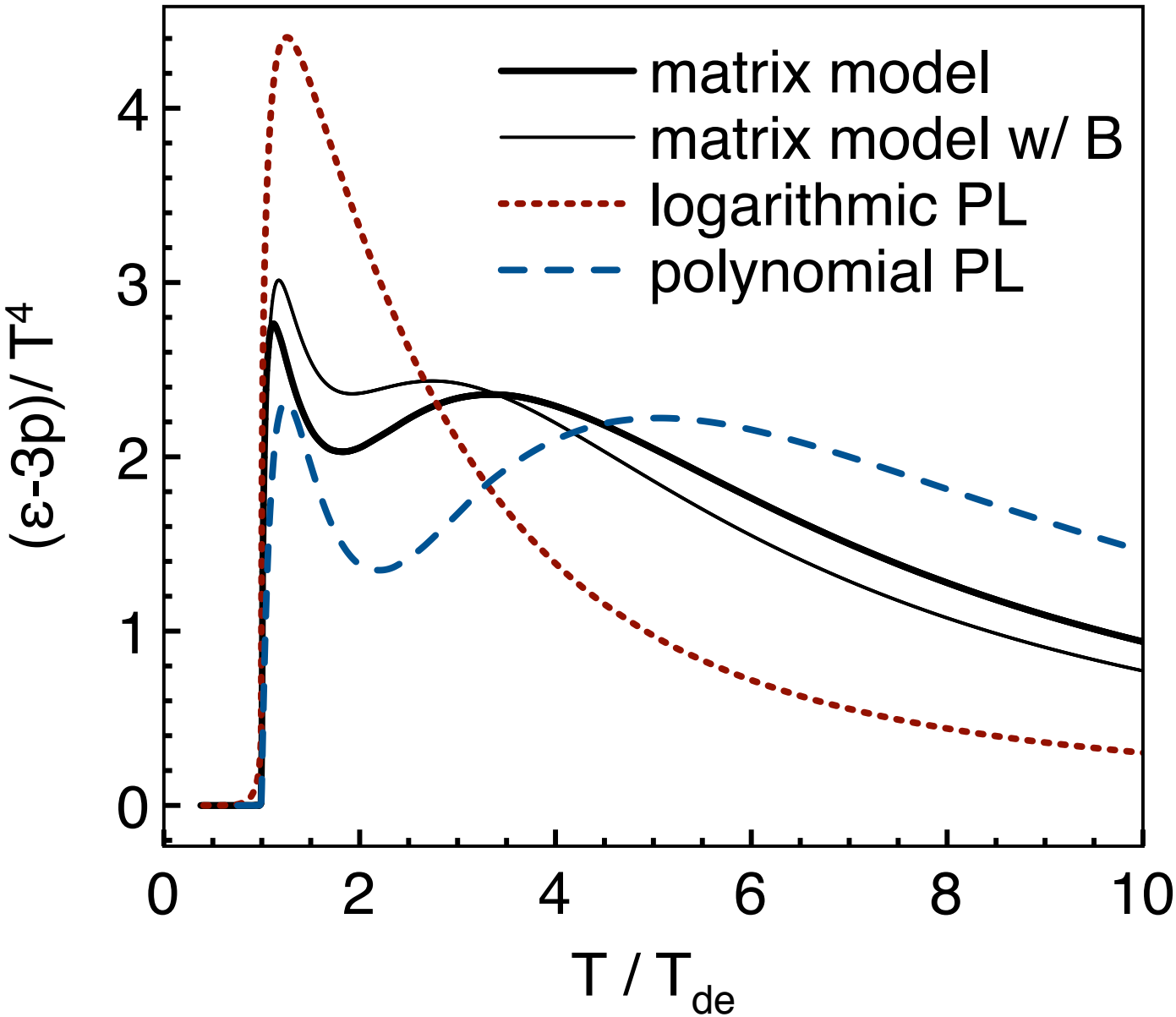
Matrix Model: $T_{DCE} \sim 0.99\ T_c$. Polyakov loop models: $T_{DCE} \sim 0.90\ T_c$.

Mass heavy: $m_{DCE} \sim 2.4$ GeV. Mass light: $m_{DCE} \sim 1$ GeV



# Predict two bump structure for three flavors

For three flavors, Matrix Model gives one bump in $(e-3p)/T^4$ just above $T_{DCE}$, from gluons, plus another at ~ 4 $T_{DCE}$, from quarks. Origin trivial, because $m_{DCE}$ heavy. Does not happen for log. loop potential, because $m_{DCE}$ light.



# Infinite N:

"Gross-Witten" transition = critical first order

RDP & Skokov 1206.1329

# Gross-Witten transition at infinite N

Solve at N=∞: Find "critical first order" transition:

Latent heat *non*zero ~ $N^2$, *and* specific heat diverges ~ $1/(T-T_c)^{3/5}$

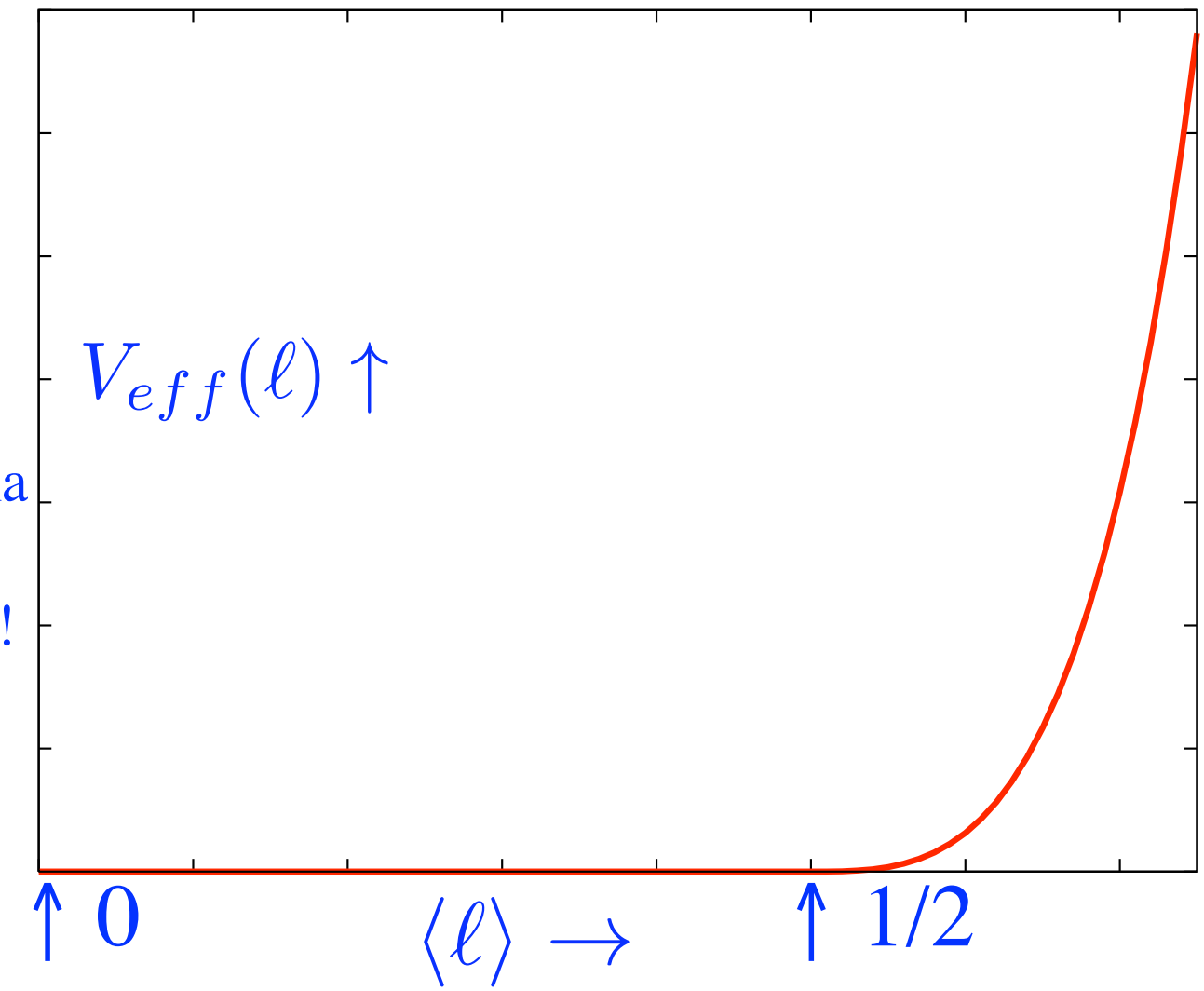
Like femtosphere: ... + Aharony... th/0310825; Dumitru, Lenaghan, RDP, ph/0410294

$$\ell(T_c^-) = 0$$

$$\ell(T_c^+) = \frac{1}{2}$$

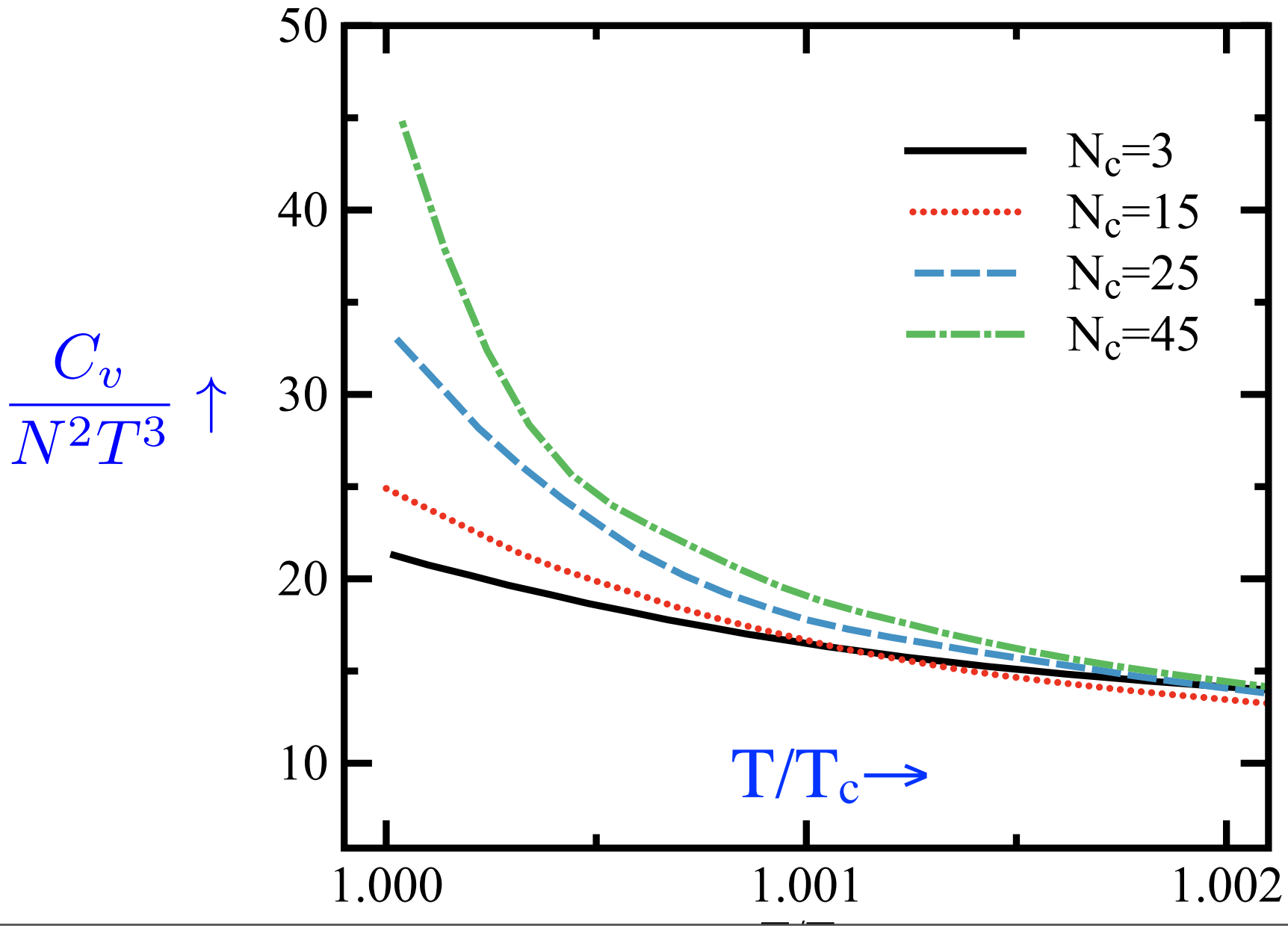
At $T_c$, 2 degenerate minima

But $V_{eff}$ *flat* between them!



# Signs of Gross-Witten transition at finite N?

See increase in specific heat only *very* near $T_c$, ~ .1 %, for *very* large N > 40



# Summary

"Miles to go, before I sleep..."

RDP & Skokov, Lin, Rischke +....

Pure gauge: T: 1.2 to 4.0 $T_c$, pressure dominated by *constant* ~ $T^2$ : *stringy?*

Tests: discrepancy with Polyakov loop; heavy quarks; large N

*Need to include quarks! Is there a single "$T_c$"?*

Standard kinetic theory: strong coupling gives *small* $\eta$ and *large* $\hat{q}$

Majumder, Muller, & Wang, ph/0703082; Liao & Shuryak, 0810.4116

Semi-QGP: naturally *small* $\eta$ near $T_c$:

$\sigma$ ~ *loop*$^2$, but $\varrho$ = density ~ *loop*$^2$ $T^3$:

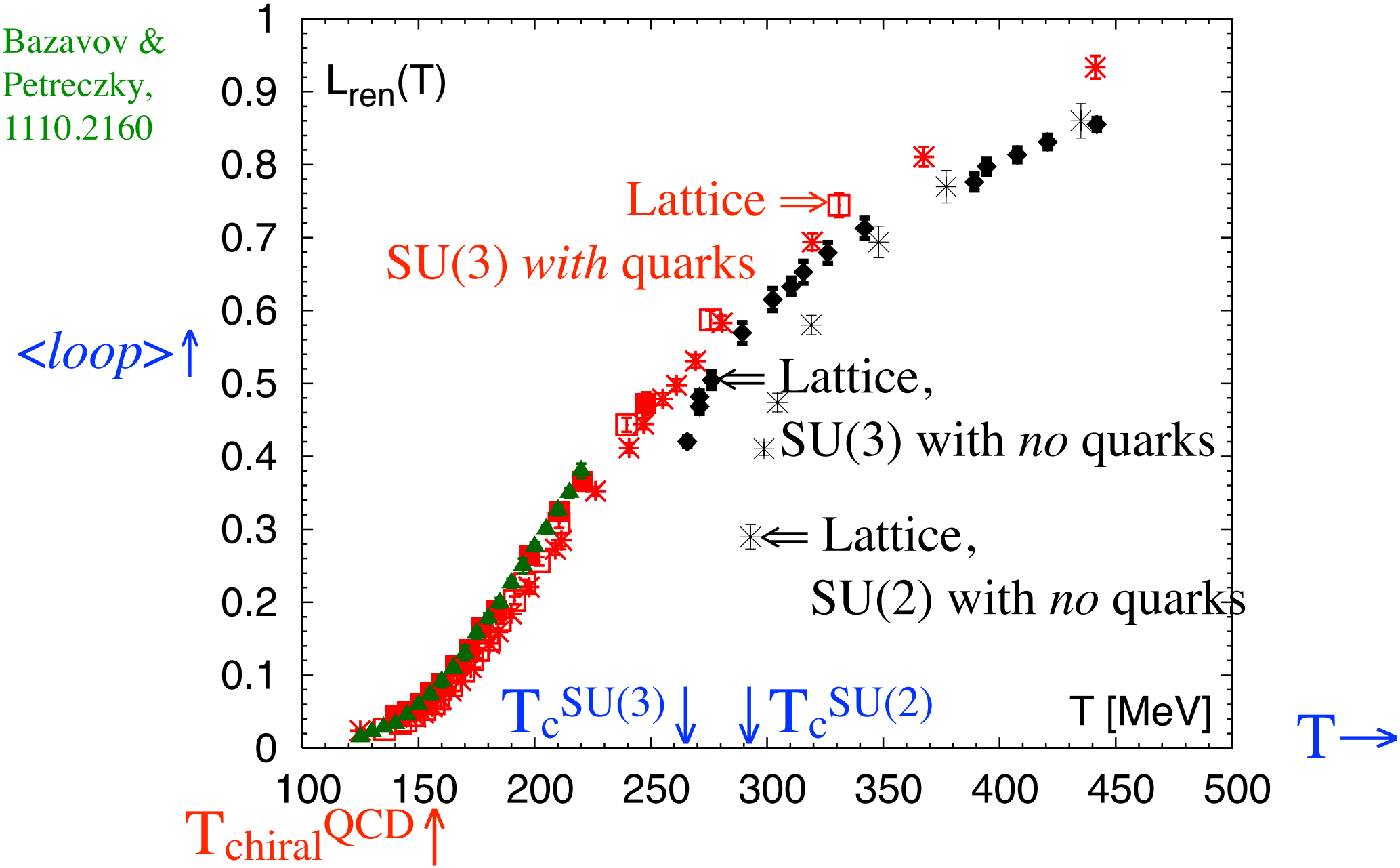
Y. Hidaka & RDP, 0803.0453, 0906.1751, 0907.4609, 0912.0940:

$$\eta \sim \frac{\rho^2}{\sigma} \sim \ell^2$$

(Relation to anomalous viscosity? Asakawa, Bass, & Muller, ph/0603092 & ph/0608270)

But energy loss also *small*: $\hat{q} \sim \ell$ for quarks

# Loop with, and without, quarks

Matrix Model: use *same* $T_c$ with quarks. Loop turns on below $T_c$.
Chiral transition is *not* tied to deconfinement. Like lattice results:

Bazavov & Petreczky, 1110.2160