$$\mathcal{S}_R = -(N^2/3)\Sigma_{i,\hat{n}}\Sigma_{R,R'}^{e_R+e_{R'}=0}\ \beta_{R,R'}\ \mathrm{Re}\ \ell_R(i)\ell_{R'}(i+\hat{n})\ .$$