$$\mathrm{tr}\ \mathbf{L}_R = \#(\mathrm{tr}\ \mathbf{L}_N)^{p_+}(\mathrm{tr}\ \mathbf{L}_N^\dagger)^{p_-} + \ldots$$