$$\ell_{(N^2\pm N)/2} \equiv \frac{2}{N^2 \pm N} \mathrm{tr}\mathbf{L}_{(N^2\pm N)/2} \sim_{N\to\infty} \ell_N^2 + O\left(\frac{1}{N}\right)$$