$$\ell_{adj} = |\ell_N|^2 + O\left(\frac{1}{N^2}\right)$$