$$= e^{ie_R\theta}|\langle \ell_N \rangle|^{p_+ + p_-} + O(N^{-1})$$