$$\ell_{\pm} \sim \ell_N^2 + O\left(\frac{1}{N}\right)$$