Matrix models for the "semi" Quark-Gluon Plasma

"Semi" QGP: in QCD, near the chiral phase transition, T: 130 -> 300 MeV

Matrix model: *simple* mean field for Polyakov loop

1. What the lattice tells us about deconfinement in pure glue SU(3)

2. Matrix model for pure glue SU(3): T_c = deconfinement, T_d

   Dumitru, Guo, Hidaka, Korthals-Altes & RP, 1011.3820 & 1205.0137 + ….

3. Chiral matrix model for QCD (2+1 flavors): T_c = chiral transition, T_χ

   V. Skokov & RP: 1604.00022

4. Tetraquarks and (maybe) a *second* chiral transition (for light quarks)

   V. Skokov & RP: 1606.04111

The Quark-Gluon Plasma *near* T_c

T = 0 to T ~ 130 MeV: hadronic resonance model, χ perturbation theory….

T> 300 MeV: *resum* perturbation theory

Hard Thermal Loop perturbation theory at *three* loop order

Haque, Bandyopadhyay, Andersen, Mustafa, Mike Strickland, Nan Su, 1402.6907

But: in heavy ion collisions, most time is spent *near* T_c.

Assume Bjorken hydrodynamics: in the central plateau, $T \sim \dfrac{1}{\tau^{1/3}}$

T_f = 160 MeV. RHIC, T_i = 400 MeV. LHC, T_i = 600 MeV.

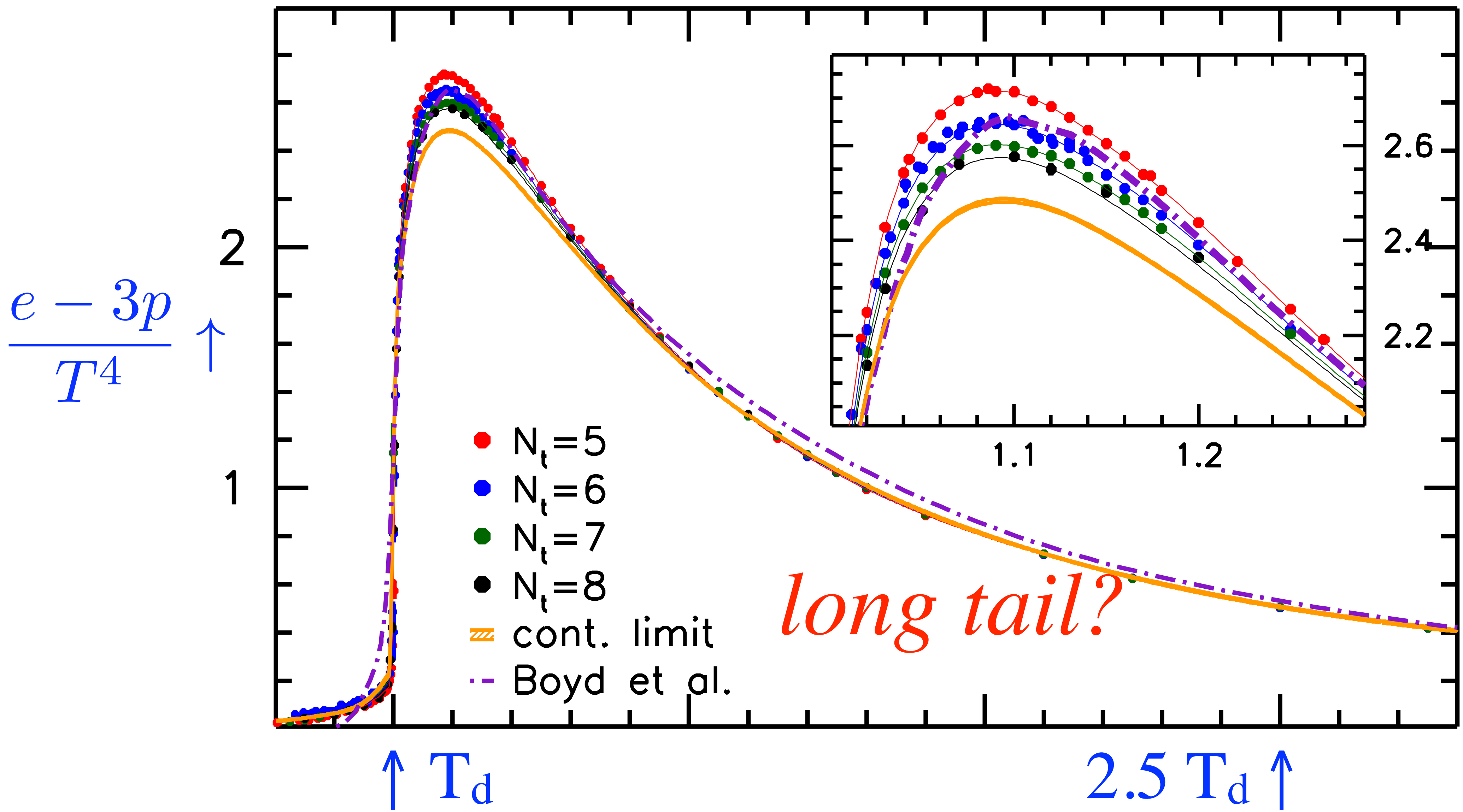
In Bjorken hydro, as $T_i \rightarrow \infty$, $\langle T \rangle \rightarrow \dfrac{3}{2} T_f$ = 215 @ RHIC; = 227 @ LHC

1. What the lattice tell us about deconfinement in pure gauge

Pure glue: leading correction to ideal gas?

"Pure" SU(3), no quarks. Peak in $(e-3p)/T^4$, just above T_d.

Borsanyi, Endrodi, Fodor, Katz, & Szabo, 1204.6184



Pure glue: *de*confined strings above T_d.

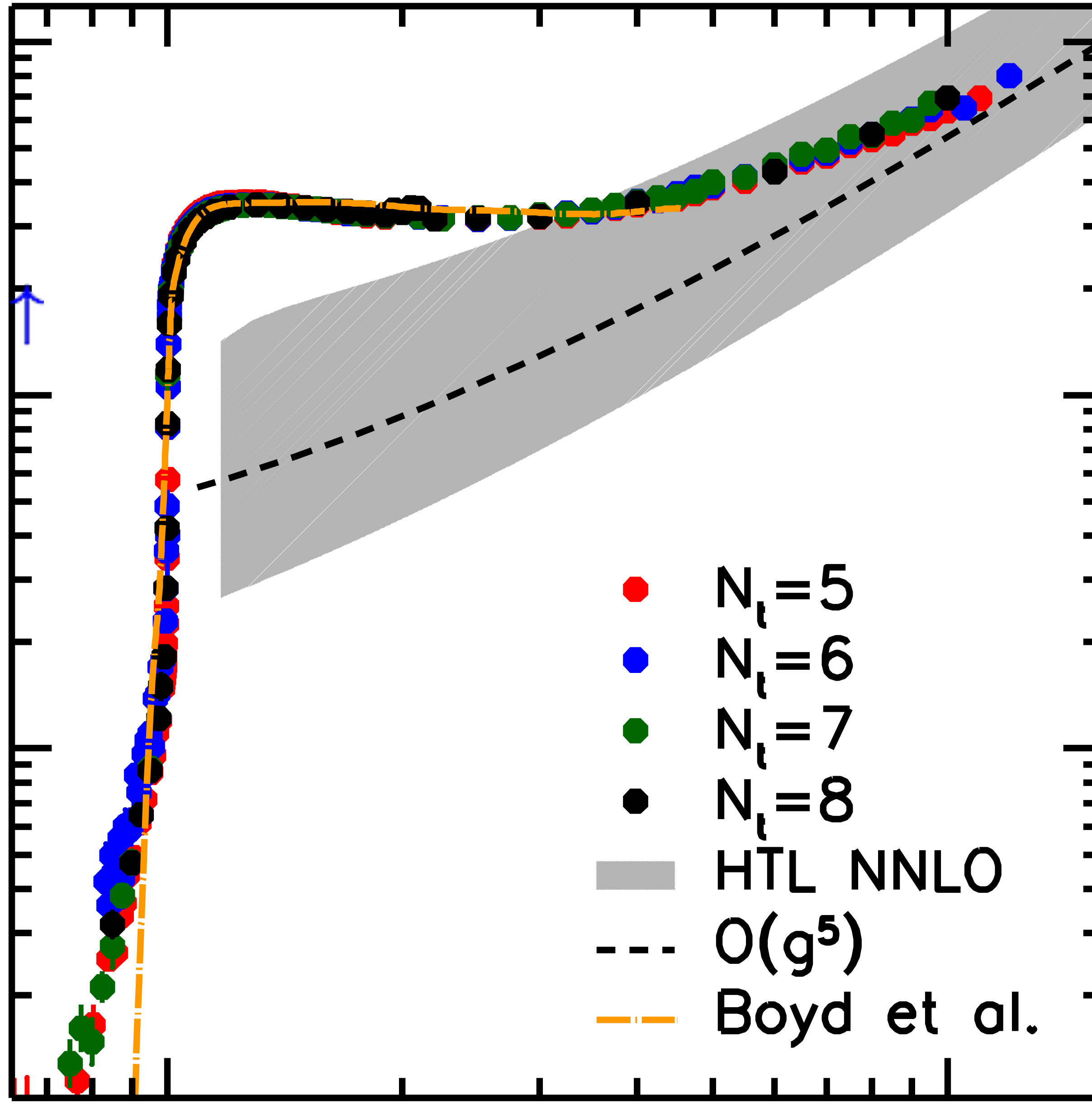
$T_d \rightarrow 4\ T_d$: for pressure, leading correction to ideal gas T^4 is *not* a bag constant, but $\sim T^2$

$$\frac{e-3p}{T^2 T_d^2}$$

For T: $1.2\ T_d \rightarrow 4\ T_d$,

$$p(T) \approx \#(T^4 - c\,T^2 T_c^2)$$

T^2 term: *de*confined strings?



$\uparrow T_c$ $\quad$ $10\ T_c \uparrow$

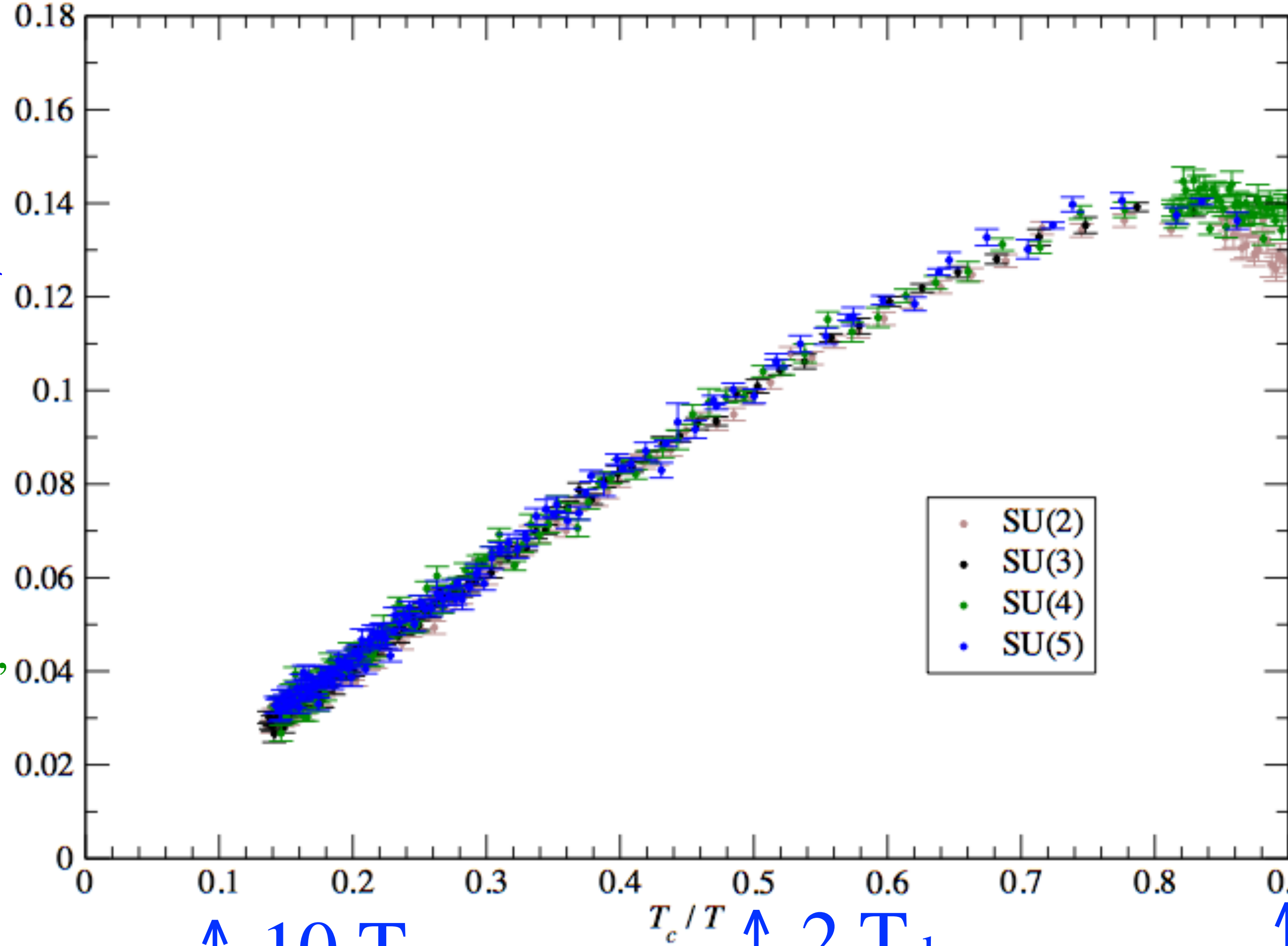
Borsanyi, Endrodi, Fodor, Katz, & Szabo, 1204.6184

Pure glue: deconfined strings in 2+1 dim.'s

In 2+ 1 dimensions, leading correction to ideal gas T^3 is *again* T^2:

$$p(T) \approx \#(T^3 - c\,T^2 T_c)\ ,\ c \approx 1$$

$$\frac{1}{N^2-1}\,\frac{e-2p}{T^3}\ \uparrow$$



T$_d$/T$\rightarrow$

$\uparrow$ 10 T$_d$ $\uparrow$ 2 T$_d$ $\uparrow$ 1.1 T$_d$

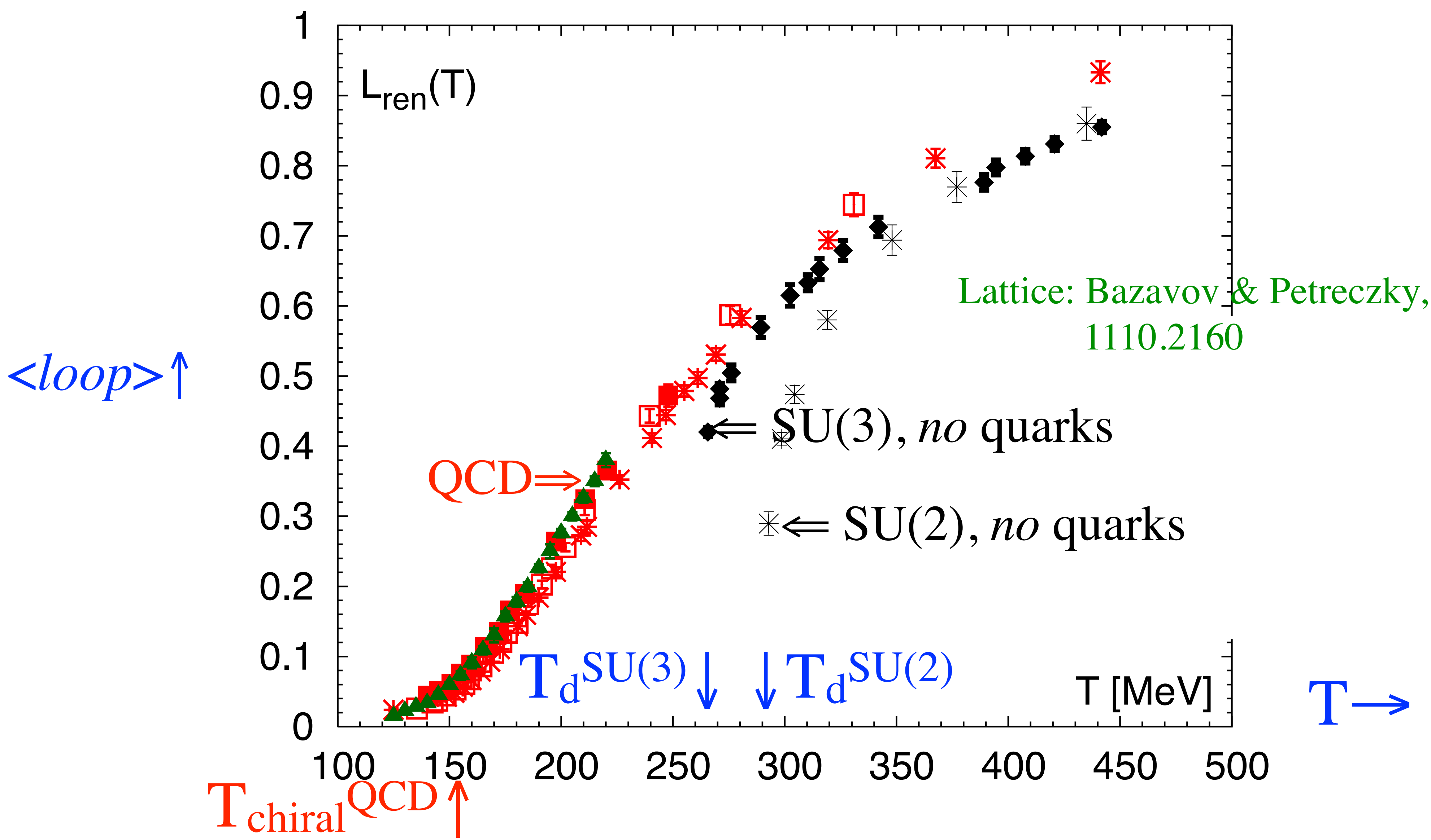
Caselle, Castagnini, Feo, Gliozzi, Gursoy, Panero, Schafer, 1111.0580

Lattice: Polyakov Loop without and *with* quarks

Without quarks: *exact* order parameter for global Z(3) = Polyakov loop

Dynamical quarks a*lways* break Z(3). But in QCD, loop *small* at T_χ , ~0.1.

Our basic assumption: in QCD, loop is an *approximate* order parameter



2. Matrix model for pure glue SU(3)

Matrix model

Polyakov Loop: $$\ell = \frac{1}{3}\,\mathrm{tr}\,\mathcal{P}\,\exp\left(i\,g\int_0^{1/T} A_0\,d\tau\right)$$

Simplest approximation to give a non-trivial loop: constant, diagonal A_0:

$$A_0^{cl} = \frac{Q}{g}\ ,\ \ Q = \frac{2\pi T}{3}\,q(T)\,\lambda_3\ \ ;\ \ \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

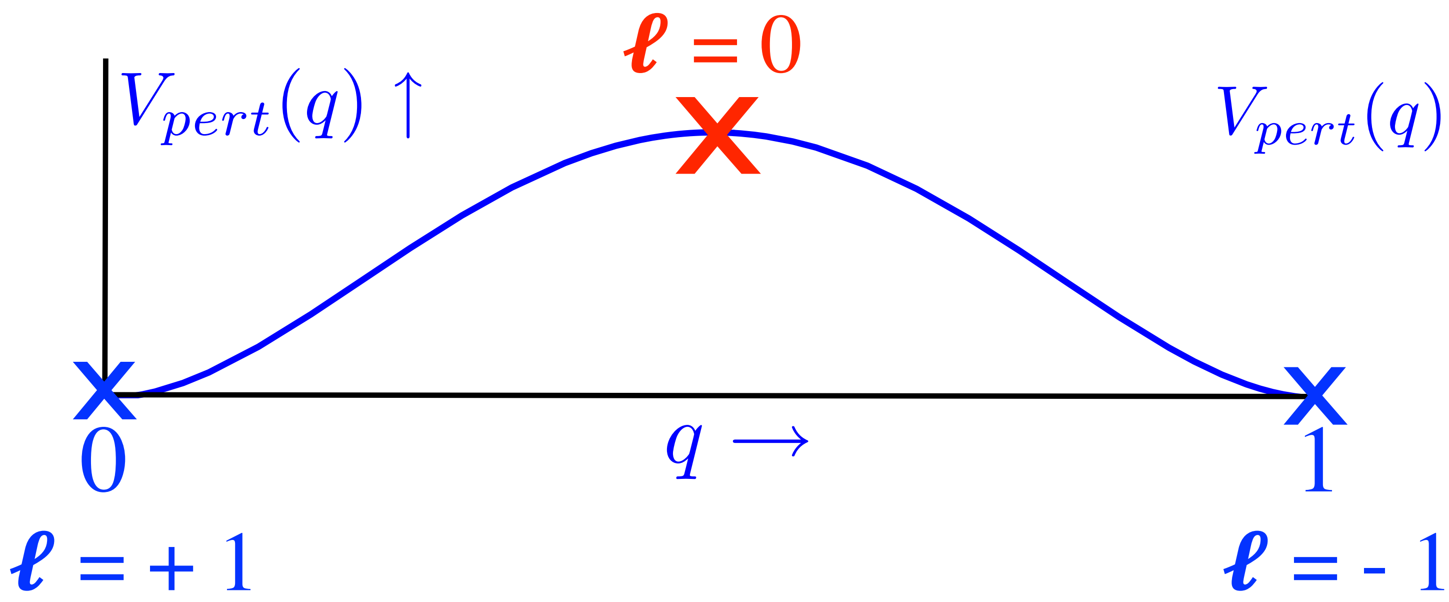
Depends upon single function, q(T), fixed from pressure(T).

Only need two parameters to fit pressure, then compute

Perturbative potential for q

Classically, no potential for q. One arises at one loop order. For two colors:

Gross, RDP, Yaffe, '81; Weiss '82



$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \, q^2 (1-q)^2$$

Use $V_{pert}(q)$ to compute 't Hooft loop:

Bhattacharya, Gocksch, Korthals-Altes, RDP, ph/9205231.

$$V_{tot}(q) = \frac{2\pi^2 T^2}{g^2} \left(\frac{dq}{dz}\right)^2 + V_{pert}(q) \qquad \Rightarrow \sigma = \frac{4\pi^2}{3\sqrt{6}} \frac{T^2}{\sqrt{g^2}}$$

Non-perturbative potential for q

To model deconfinement, add - *by hand* - a non-perturbative potential for q:
Dumitru, Guo, Hidaka, Korthals-Altes & RDP +…

$$V_{non}(q) = \frac{4\pi^2}{3} T^2 T_d^2 \left(-\frac{c_1}{5}\, q(1-q) - c_2\, q^2 (1-q)^2 + \frac{c_3}{15} \right)$$

The last term ~ c_3 is the pressure of deconfined strings. c_2 is like the perturbative. c_1 is added to ensure there isn't a second transition.

Now just like any other mean field theory.

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q)$$

$\langle q \rangle$ given by minimum of V_{eff}:

$$\frac{d}{dq} V_{eff}(q) \Big|_{q = \langle q \rangle} = 0$$

$\langle q \rangle$ depends nontrivially on temperature.

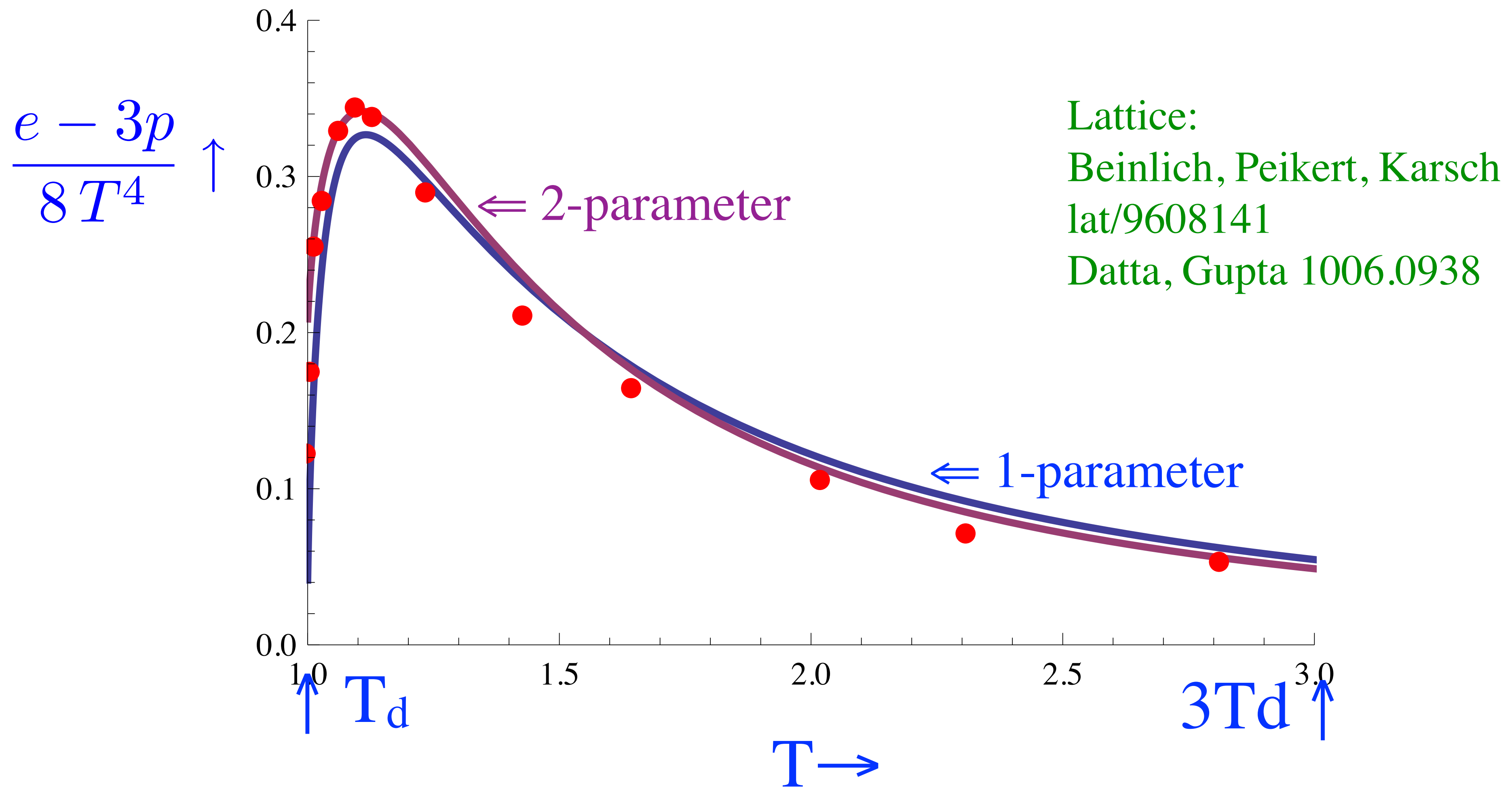
Pressure value of potential at minimum:

$$p(T) = -V_{eff}(\langle q \rangle)$$

Latent heat, and a 2-parameter model, three colors

Start with three parameters. Require transition occurs at T_d, and $p(T_d) \sim 0$.
Leave one free parameter, adjust to agree with $(e-3p)/T^4$.

$$T_d = 270 \text{ MeV} \; , \; c_1 = 0.315 \; , \; c_2 = 0.83 \; , \; c_3 = 1.13$$

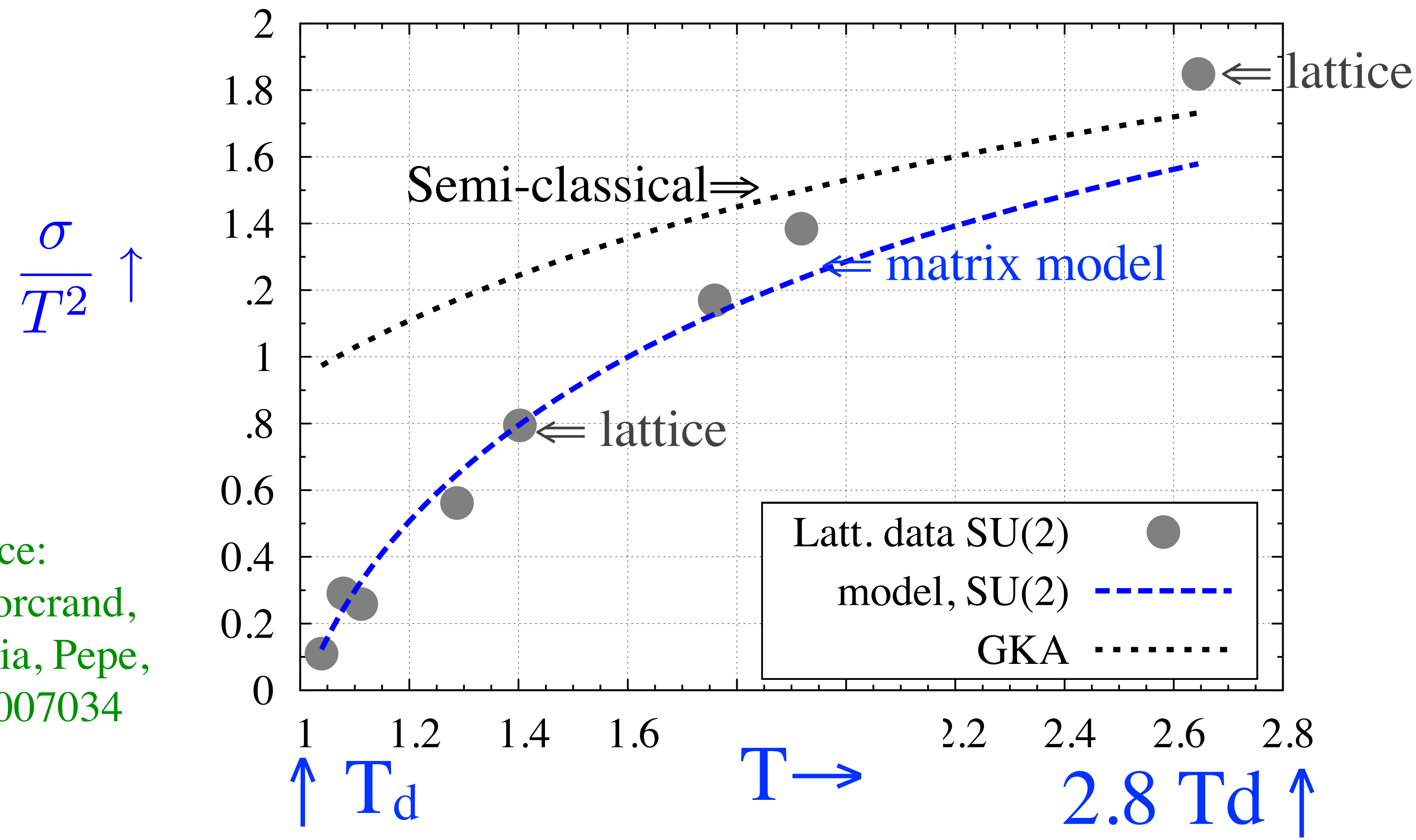


’t Hooft loop for two colors

For pure gauge, ’t Hooft loop = $Z(N_c)$ interface tension.

Can extend computation in semi-classical regime to near T_d.

With loop as in matrix model, *excellent* agreement with lattice data.



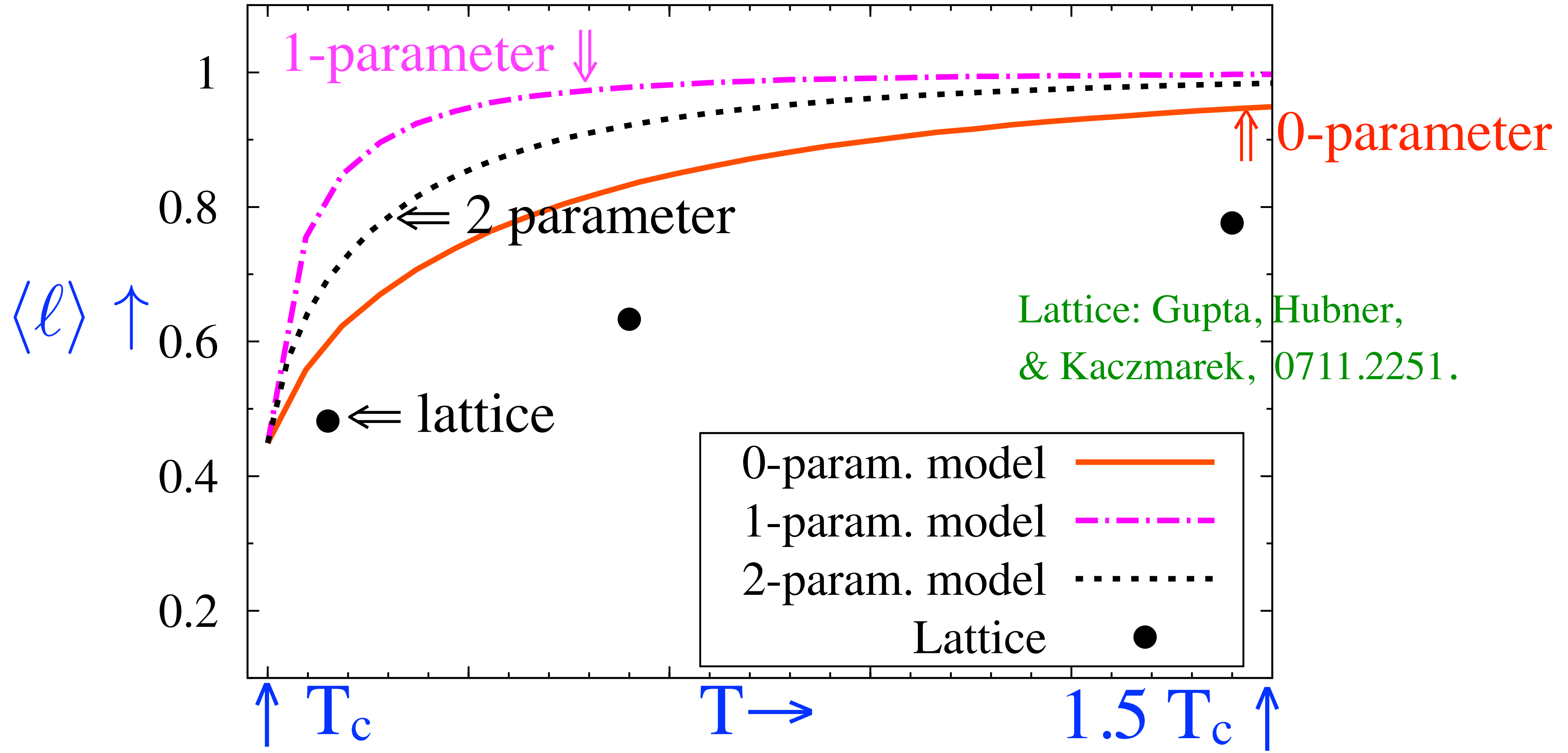
Lattice:
de Forcrand,
D’Elia, Pepe,
lat/0007034

Polyakov loop: model vs lattice?

Polyakov loop *much* smaller than the matrix model

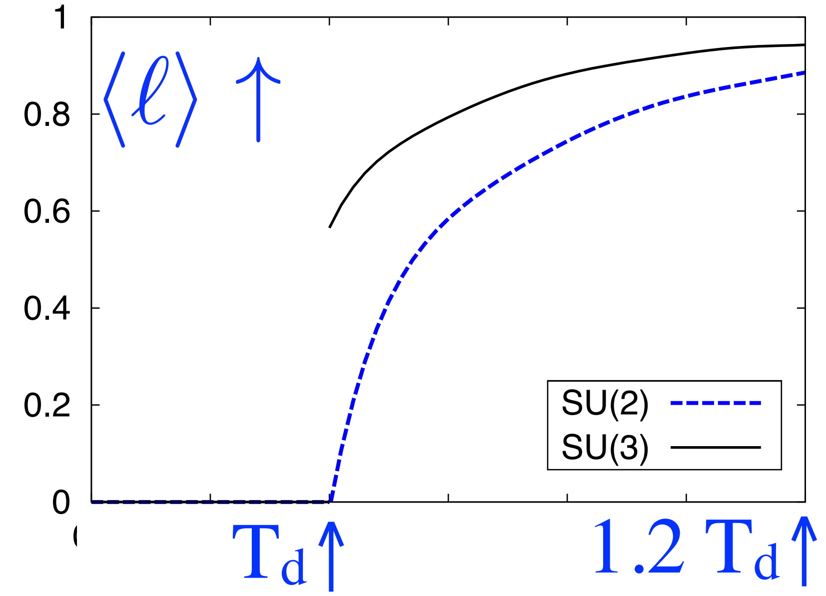
Transition region: matrix model *narrow*, to ~ 1.2 T_d.   Lattice *wide*, to ~ 4.0 T_d.

But: if one fits to lattice loop, ’t Hooft loop is *much* too small.



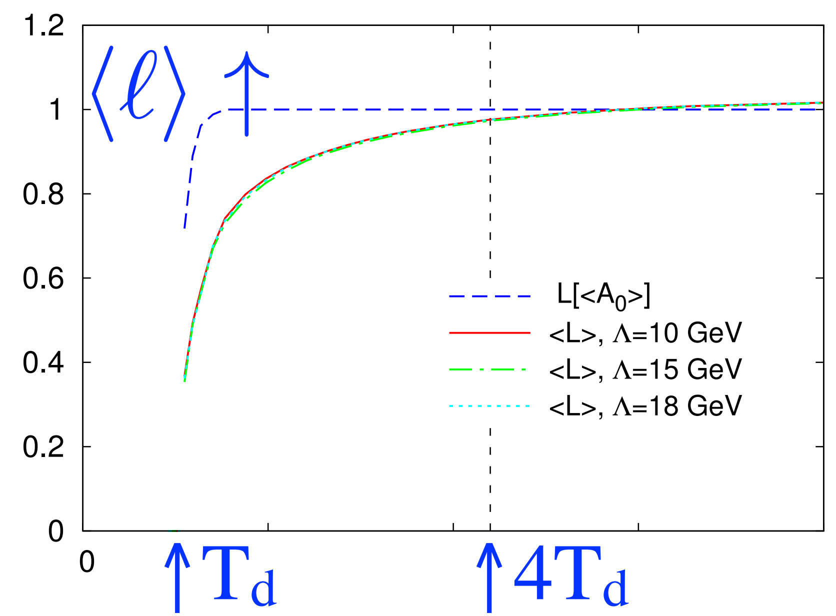
How *wide* is the transition to deconfinement?

For the Polyakov loop:
matrix model gives *narrow* transition, $T_d \to 1.2\, T_d$
Lattice: *wide* transition, $T_d \to 4\, T_d$.



Narrow transition found for *many* other models.
Functional Renormalization Group (FRG) =>
Polyakov treated in mean field
Braun, Gies, Pawlowski 0708.2413

Later, FRG including fluctuations:
Herbst, Luecker, Pawlowski 1510.03830
Polyakov loop agrees with lattice.



But what about the 't Hooft loop?

Here: assume some unknown subtlety
in going from lattice to continuum
for (non-local) Polyakov loop

2. Chiral matrix model for QCD

Chiral symmetry

For 3 flavors of massless quarks coupled to a gauge field,

$$\mathcal{L}^{qk} = \overline{q}\, \not{D}\, q = \overline{q}_L \not{D}\, q_L + \overline{q}_R \not{D}\, q_R \quad , \quad q_{L,R} = \frac{1 \pm \gamma_5}{2}\, q$$

Classically, global flavor symmetry of $SU(3)_L$ x $SU(3)_R$ x $U(1)_A$,

$$q_L \to \mathrm{e}^{-i\alpha/2}\, U_L\, q_L \quad , \quad q_R \to \mathrm{e}^{+i\alpha/2}\, U_R\, q_R$$

Simplest order parameter for χ symmetry breaking (χSB'g):
a,b… = flavor. A,B… = color

$$\Phi^{ab} = \overline{q}_L^{\,bA}\; q_R^{aA}$$

$$\Phi \to \mathrm{e}^{+i\alpha}\, U_R\, \Phi\, U_L^\dagger$$

Quantum mechanically, axial $U(1)_A$ is broken by instantons +…. to $Z(3)_A$ at T=0
't Hooft instanton vertex is invariant under $Z(3)_A$:

$$\det \Phi \to \mathrm{e}^{3i\alpha}\, \det \Phi$$

As $T \to \infty$, $U(1)_A$ approximately restored as $1/T^{7 \to 9}$.

Effective Lagrangians for chiral symmetry

Standard linear sigma model for Φ:

$$\mathcal{V}_{\Phi} = m^2 \, \mathrm{tr}\left(\Phi^{\dagger}\Phi\right) - c_A \left(\det \Phi + \mathrm{c.c.}\right) + \lambda \, \mathrm{tr}\left(\Phi^{\dagger}\Phi\right)^2$$

Drop 2nd quartic term, $(\mathrm{tr}\ \Phi^{+}\Phi)^2$.
Mass and quartic terms are invariant under $U(1)_A$, det Φ under $Z(3)_A$.

For light but massive quarks, need to add

$$\mathcal{V}_H^0 = - \, \mathrm{tr}\left(H\left(\Phi^{\dagger} + \Phi\right)\right)$$

Quarks generate potential in "q", so *must* couple Φ to quarks: $P_{L,R} = (1 \pm \gamma_5)/2$

$$\mathcal{L}_{\Phi}^{qk} = \overline{q}\left(\not{D} + \mu\, \gamma^0 + \, y\left(\Phi\, \mathcal{P}_L + \Phi^{\dagger}\, \mathcal{P}_R\right)\right) q$$

Use non-perturbative potential from pure glue theory, with *same* $T_d = 270$.
But with quarks, T_d is *just* a parameter in a potential, *not* deconfining T_c.

New logarithmic terms

Assume χSB'g occurs, $\langle \Phi \rangle = \phi$, so $m = y\,\phi$.

At T = 0, u.v. divergent terms in 4 - ε dim.s:
M = renormalization mass scale

$$\frac{3\,m^4}{16\pi^2}\left(\frac{1}{\epsilon}+\log\left(\frac{M^2}{m^2}\right)\right)$$

Need to add new logarithmic term in Φ:

$$\mathcal{V}_\Phi^{\log} = \kappa\,\mathrm{tr}\,\left[\left(\Phi^\dagger\Phi\right)^2\log\left(\frac{M^2}{\Phi^\dagger\Phi}\right)\right]$$

To 1 loop order, $\varkappa = 3y^4/(16\,\pi^2)$; we keep it as a free parameter.

In practice, log term complicates the computation, but does not signficantly alter the conclusions from $\varkappa = 0$.

New symmetry breaking term

With just usual symmetry breaking term,
at high T,

$$\mathcal{V}^{\text{eff}} \approx -h\,\phi + \frac{1}{12}\,y^2\,T^2\,\phi^2 + \ldots \;,\; T \to \infty$$

The first is SB'g, the second from fermion fluctuations.
But then there is no symmetry breaking at high T,

$$\phi \sim \frac{12h}{y^2 T^2} \;,\; m_{qk} \sim y\phi \sim \frac{1}{T^2}$$

Solve by adding a new temperature dependent term *by hand*

$$\mathcal{V}^{\text{eff}} \approx -h\,\phi - \frac{y}{6}\,m_0\,T^2\phi + \frac{1}{12}\,y^2\,T^2\,\phi^2 + \ldots$$

So $\phi \sim m_0/y$ at high T, $m_{qk} \sim m_0$. In QCD, need to be bit more clever,

$$\mathcal{V}_h^T = -\frac{m_{qk}}{V}\left(\;\mathrm{tr}\;\frac{1}{\not{D} + \mu\,\gamma^0 + \;y\,\Phi_{ii}}\bigg|_{T\neq 0} - (T=0)\right)\;.$$

Solution at T = 0

Consider first the SU(3) symmetric case, $h_u = h_d = h_s$.
Spectrum. 0^-: singlet η' & octet π. 0^+: singlet σ and octet a_0.
Satisfy a 't Hooft relation:

$$m_{\eta'}^2 - m_\pi^2 = m_{a_0}^2 - m_\sigma^2$$

The anomaly moves η' *up* from the π, but also moves σ *down* from the a_0!

QCD: $\langle \Phi \rangle = (\Sigma_u, \Sigma_u, \Sigma_s)$. From:

$$f_\pi = 93 \; , \; m_\pi = 140 \; , \; m_K = 495 \; , \; m_\eta = 540 \; , \; m_{\eta'} = 960$$

Determine:

$$\Sigma_u = 46 \; , \; \Sigma_s = 76 \; , \; h_u = (97)^3 \; , \; h_s = (305)^3 \; , \; c_A = 4560$$

$$m^2 = (538)^2 - 121\, y^4 \; ; \; \lambda = 18 + 0.04\, y^4$$

Leaves one free parameter, Yukawa coupling "y". Determine from T_χ.

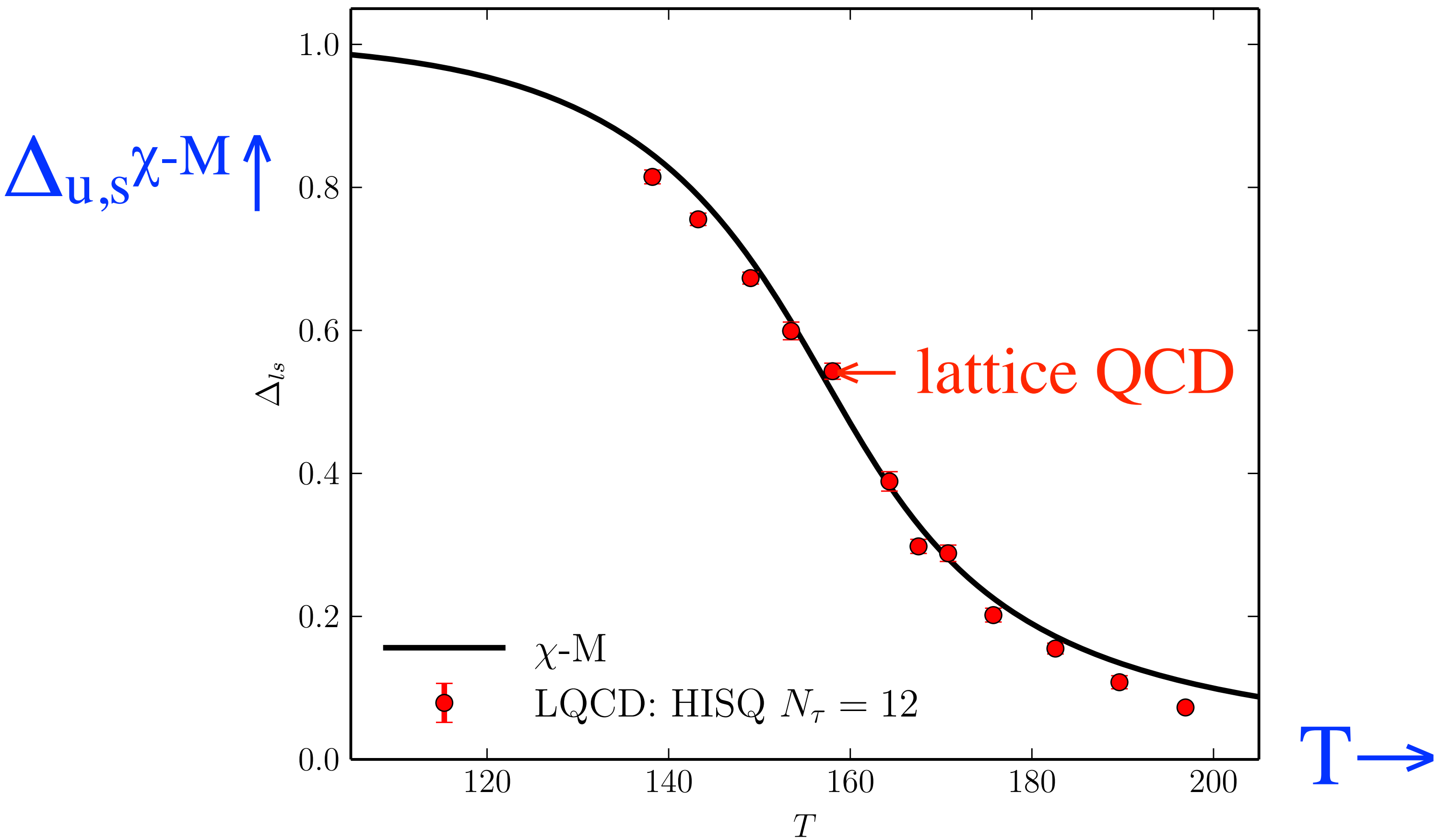
Solution at T ≠ 0

To eliminate u.v. divergences,
lattice uses substracted condensates

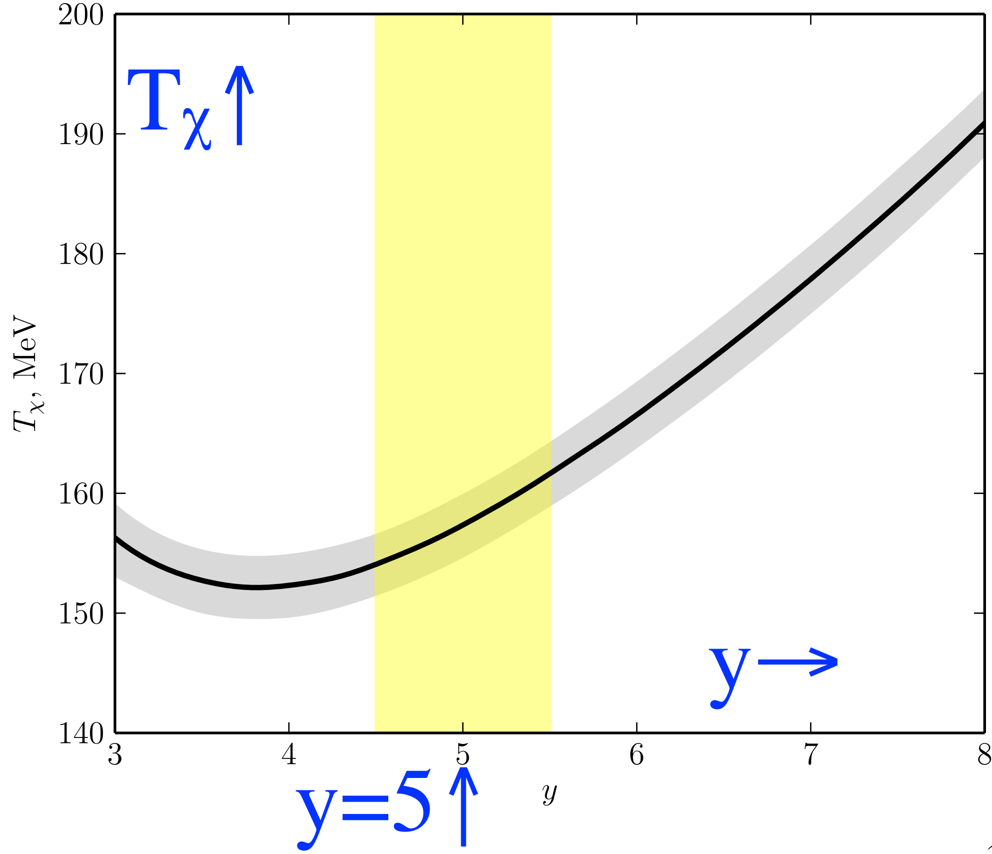
$$\Delta_{u,s}^{lattice}(T) = \frac{\langle \bar{q}q \rangle_{u,T} - (m_u/m_s)\langle \bar{q}q \rangle_{s,T}}{\langle \bar{q}q \rangle_{u,0} - (m_u/m_s)\langle \bar{q}q \rangle_{s,0}}$$

In our model we use analogous quantity
to fix y = 5.

$$\Delta_{u,s}^{\chi-M}(T) = \frac{\Sigma_u(T) - (h_u/h_s)\Sigma_s(T)}{\Sigma_u(0) - (h_u/h_s)\Sigma_s(0)}$$



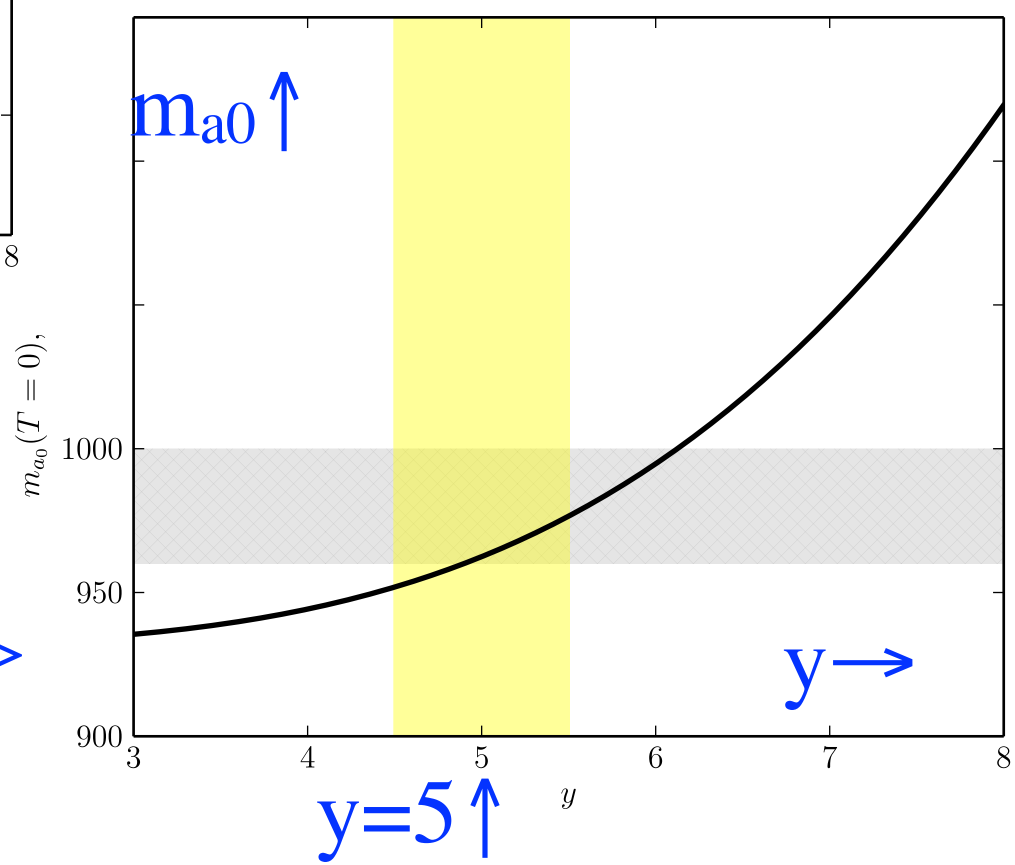
Varying the Yukawa coupling



T_χ defined from maximum in
light quark suscep., $d\Sigma_u/dT$

<= Grey band: vary T_d from 260 -> 280

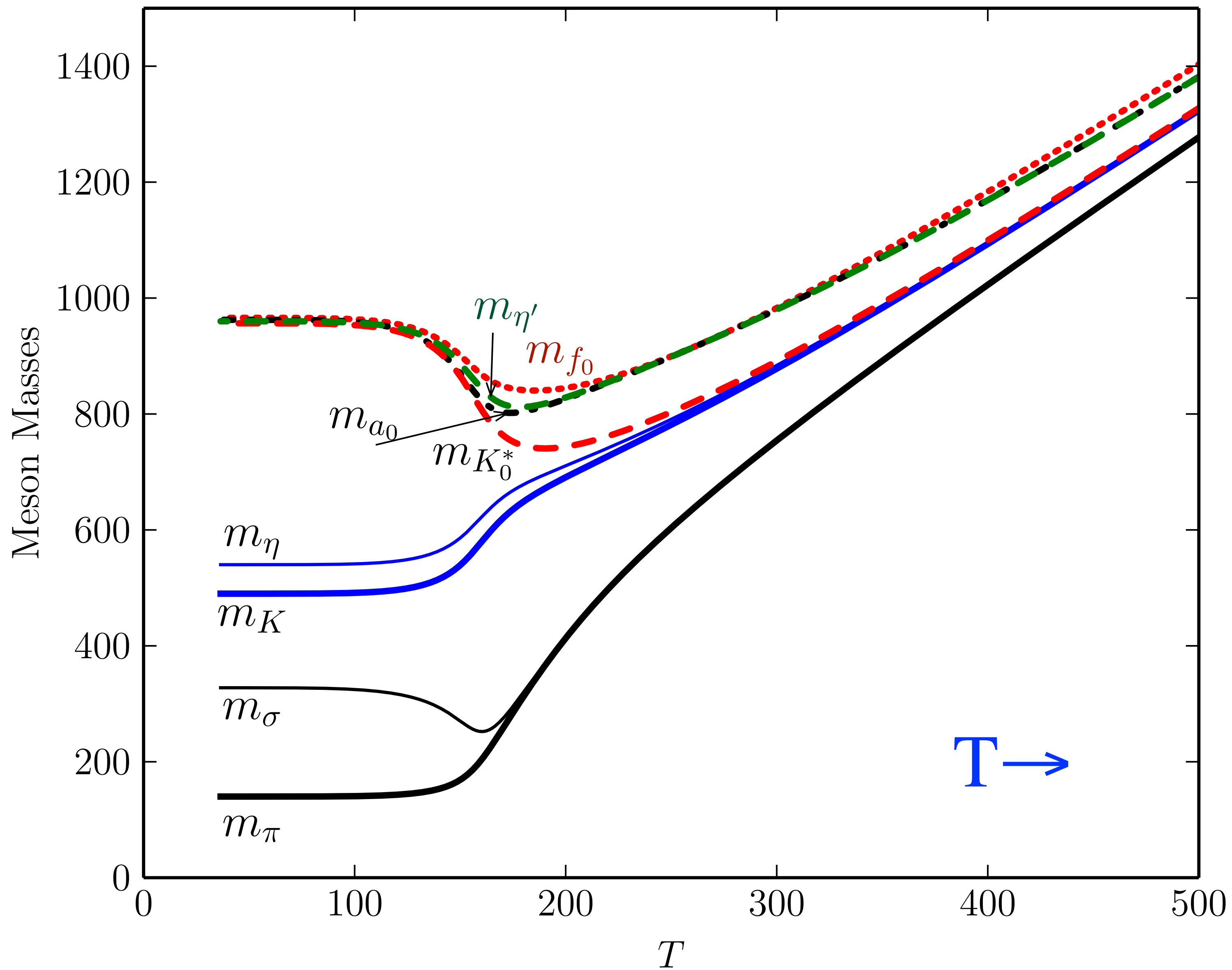
<= Yellow band = y: 4.5 -> 5.5



Grey band: experimental uncertainty
in the mass of the a_0 =>

Meson masses vs T

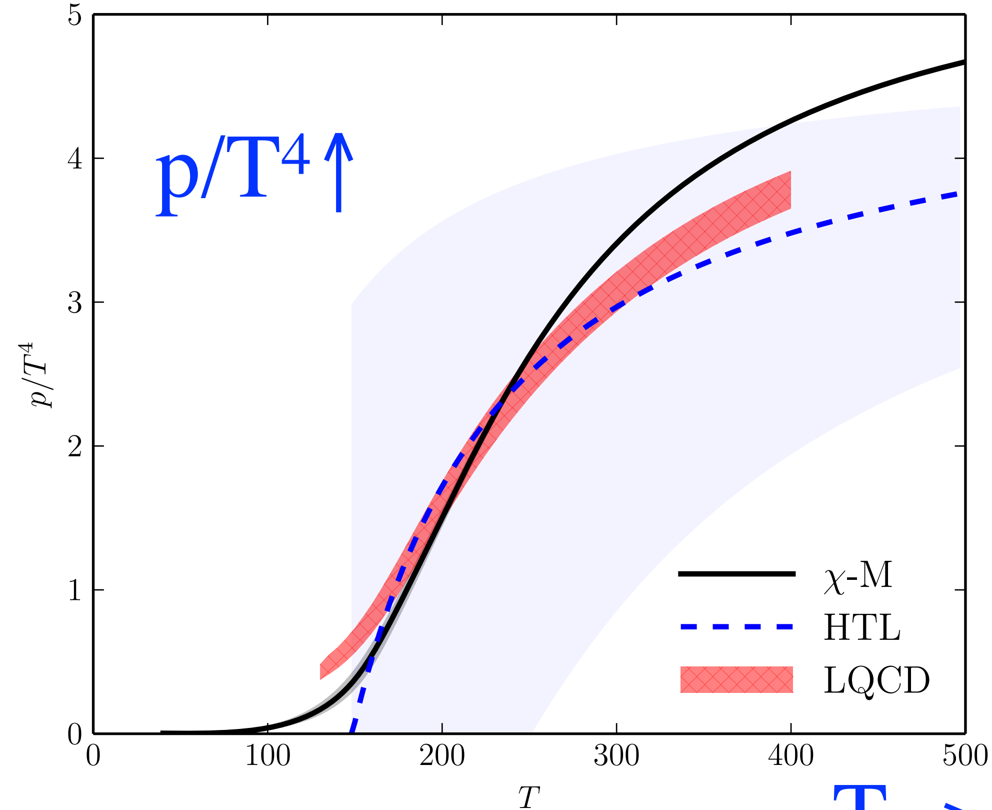
Usual pattern for $m_u = m_d \neq m_s$. y = 5.
U(1)A breaking persists to high T, unphysical.



Pressure, interaction measure vs T

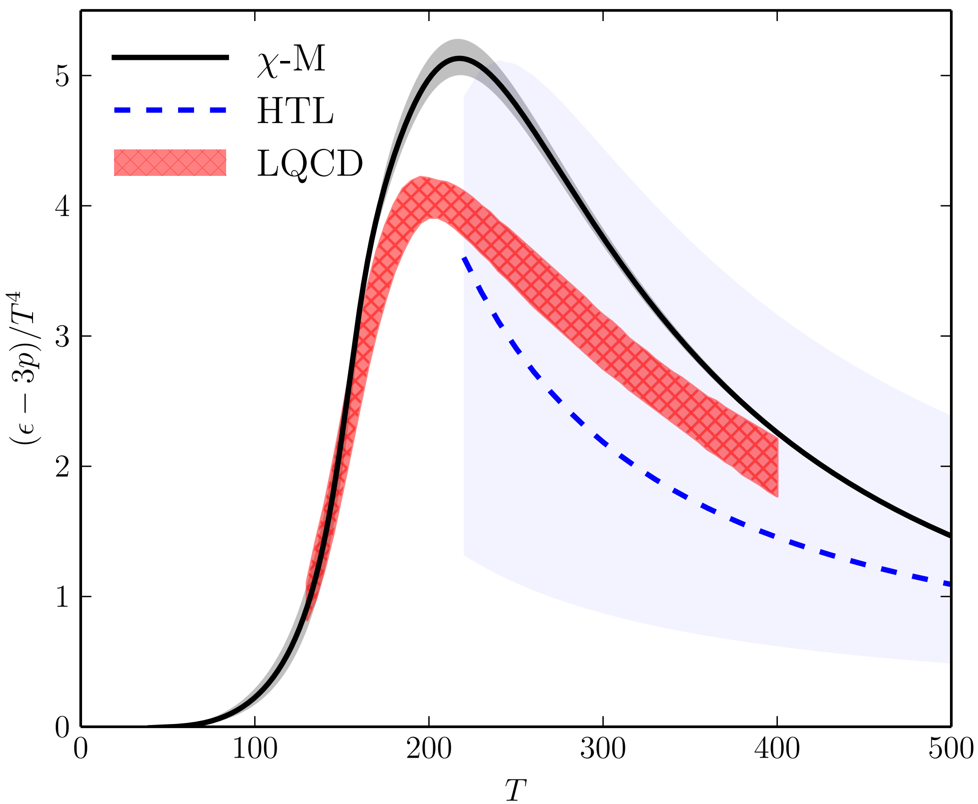
Pressure and
interaction measure, $(e-3p)/T^4$,
versus Lattice QCD
and Hard Thermal Loop (HTL)
(blue region = change ren. scale)

$p/T^4 \uparrow$



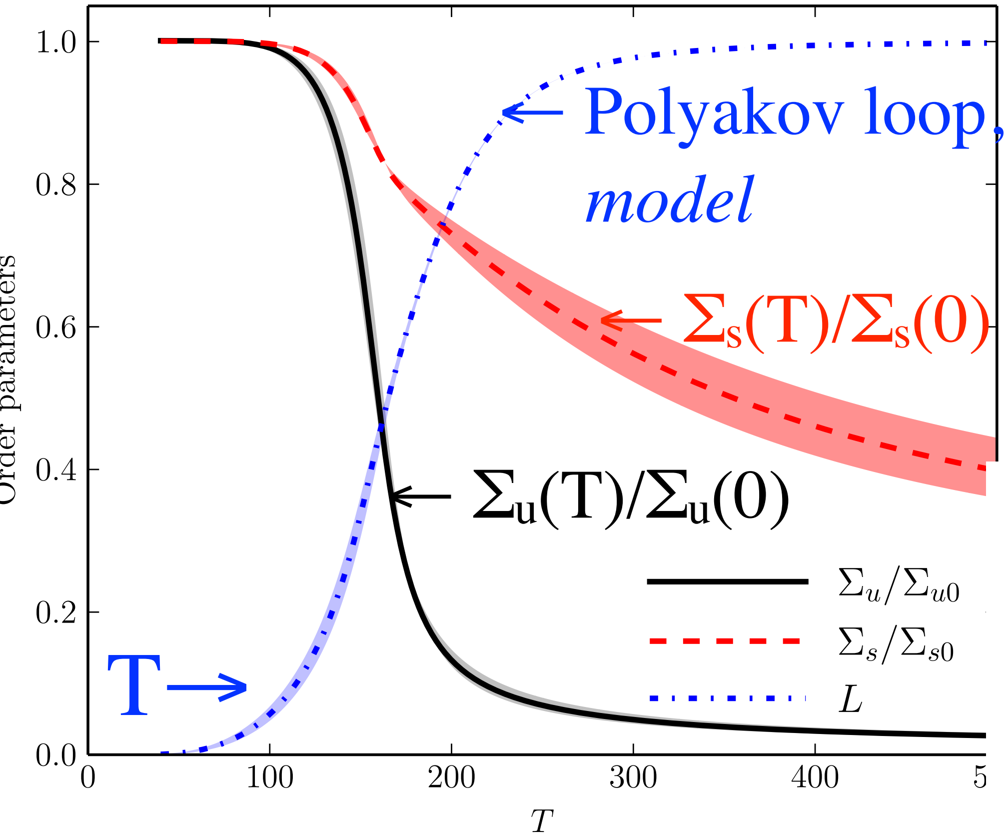
$T \rightarrow$

$(e-3p)/T^4 \uparrow$



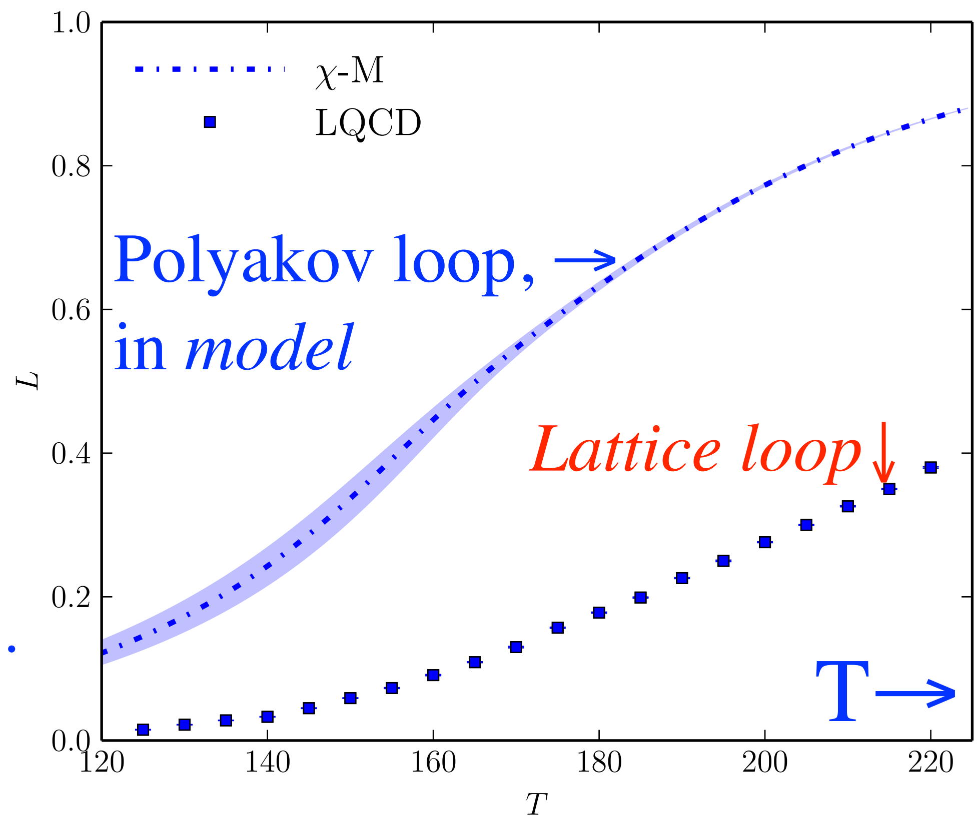
$T \rightarrow$

Order parameters, chiral and deconfining



Chiral matrix model:

Chiral and deconfining order parameters are *strongly* correlated



But Polyakov loop from lattice is *much* smaller than in model.

Persistent discrepancy, as in pure gauge.
To us: what's wrong with lattice loop?

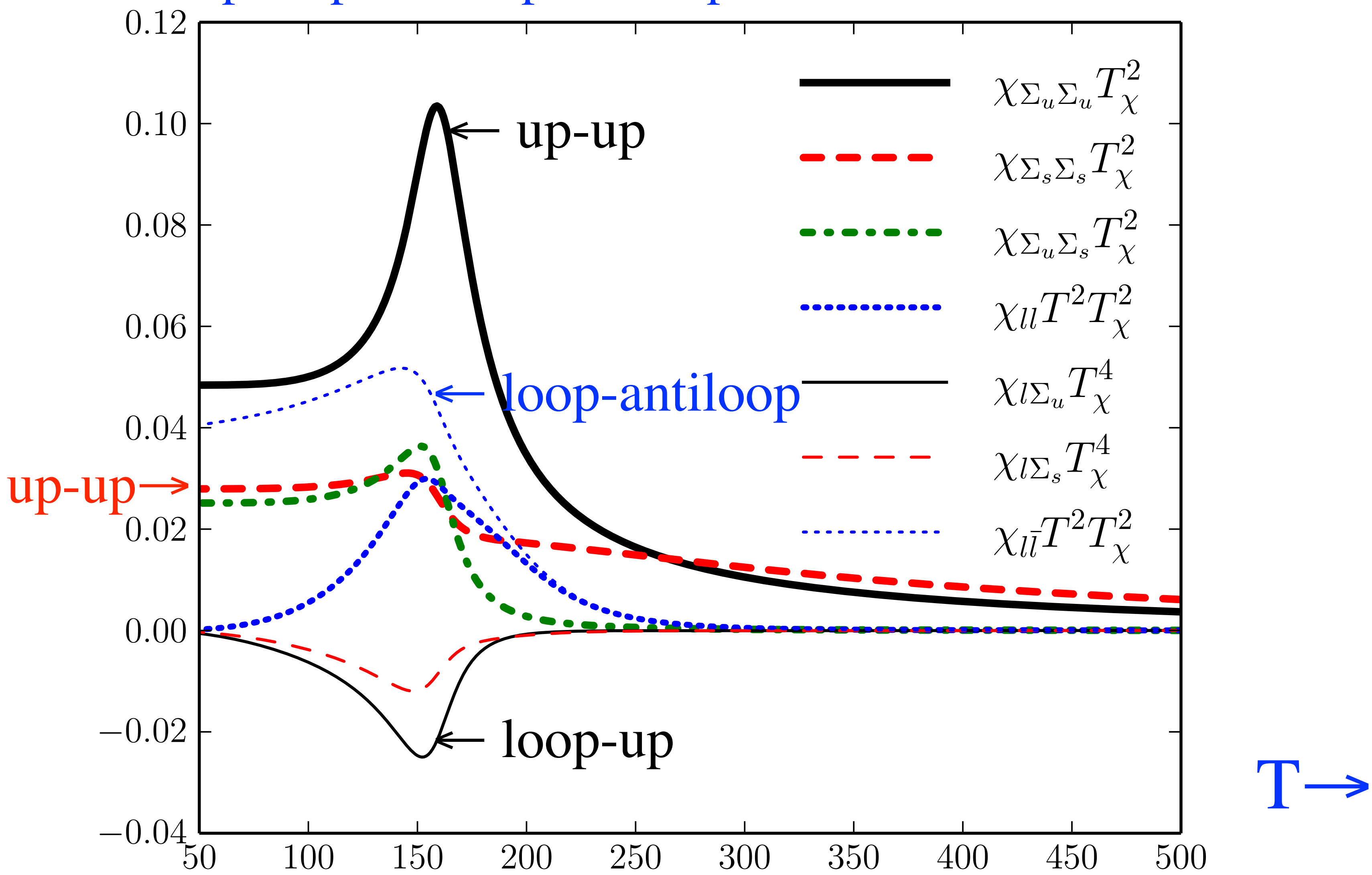
Susceptibilities, chiral and deconfining

Largest peak for up-up; strange-strange small.

In QCD, notable peaks for loop-up & loop-loop, *strongly* correlated with up-up

In chiral limit: loop-up suscep. *diverges*. (Sasaki, Friman, Redlich ph/0611147)

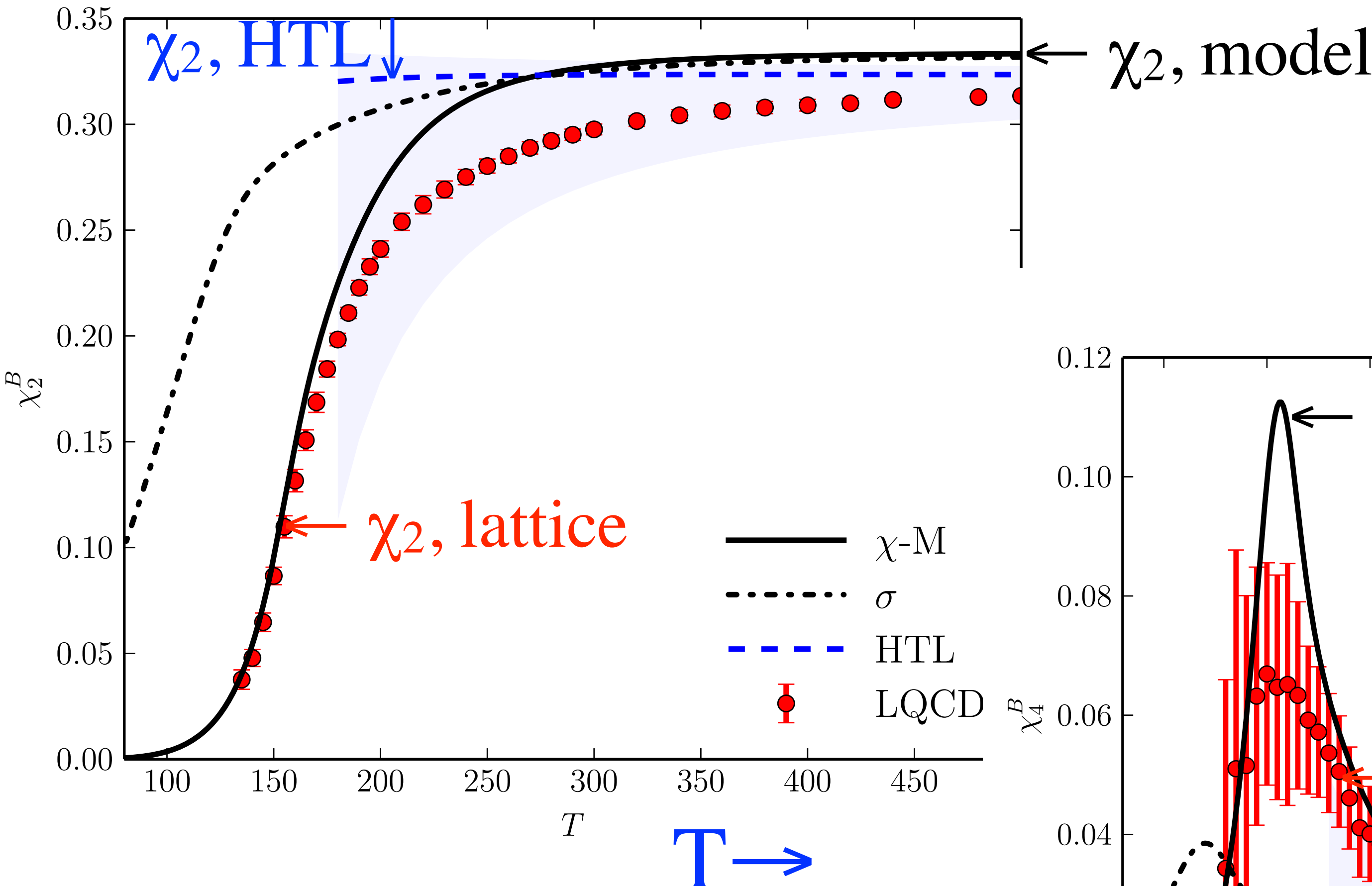
loop-loop and loop-antiloop finite

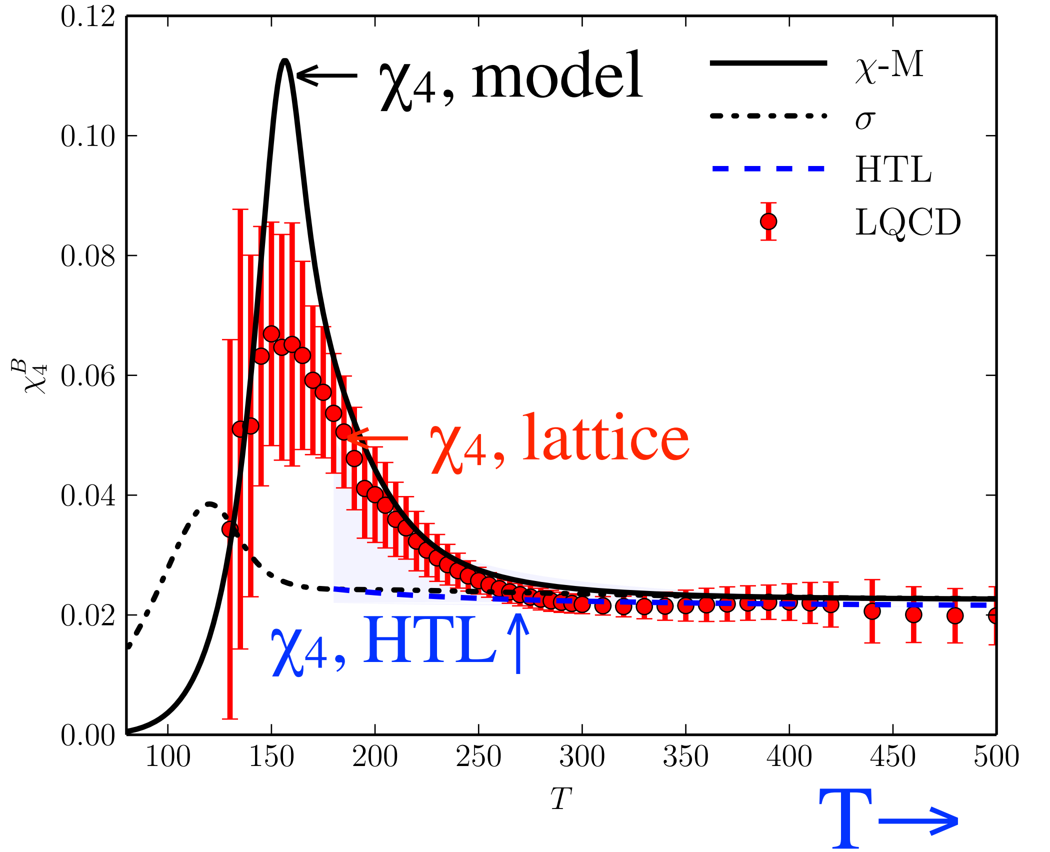


Baryon susceptibilities: 2nd & 4th

As evaluated at $\mu = 0$, lattice ok.
Baryon $\mu_B = 3\ \mu_q$.

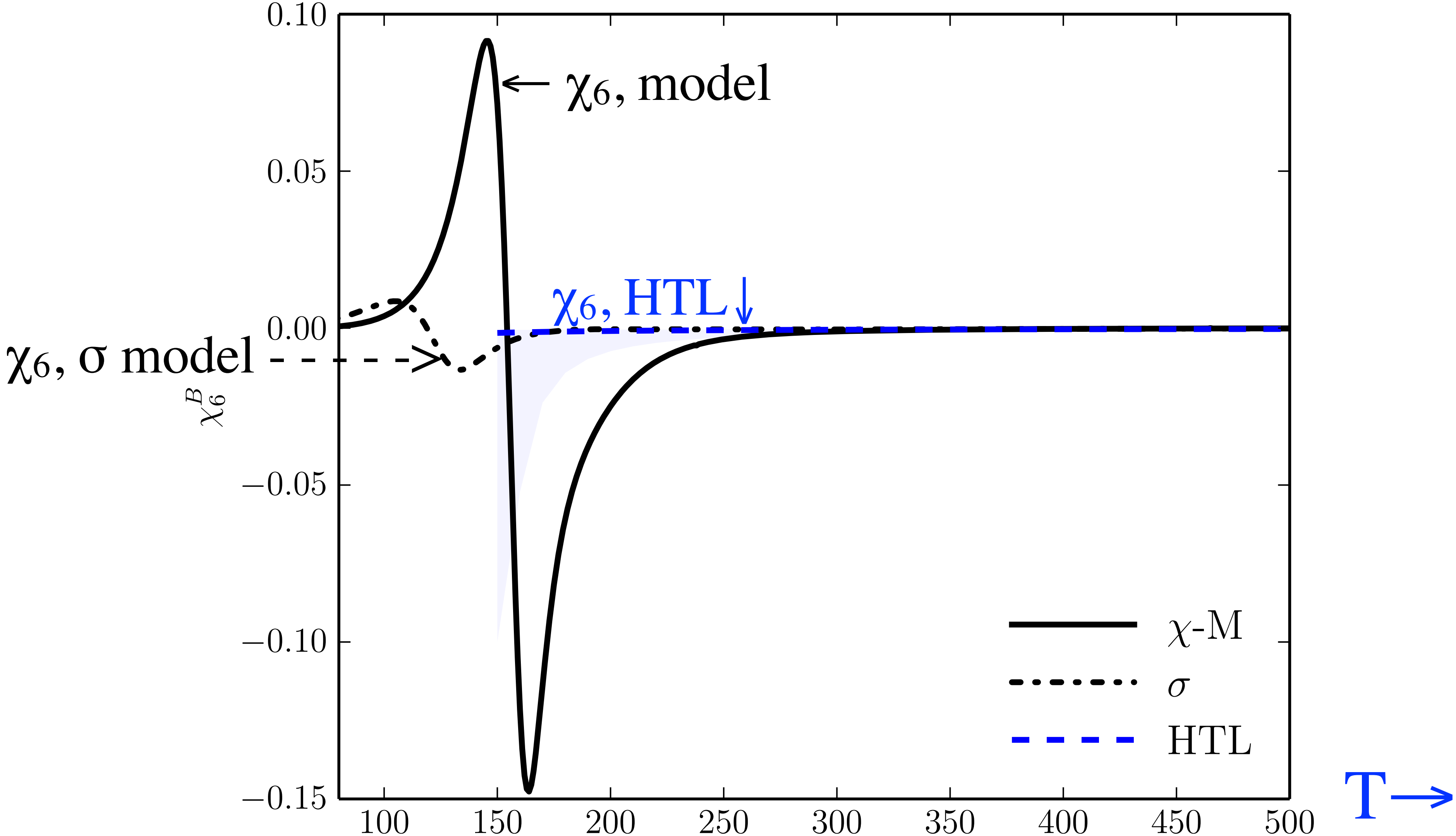
$$\chi_n^B(T) = T^{n-4}\ \frac{\partial^n}{\partial \mu_B^n}\ p(T,\mu_B)\Big|_{\mu_B=0}$$





6th order baryon susceptibility

In our model, χ_6 shows *non*-monotonic behavior near T_χ.
In HTL, χ_6 is very small (because m=0)
σ model: including change in Σ_u, but *not* in loop. Change in χ_6 *much* smaller.



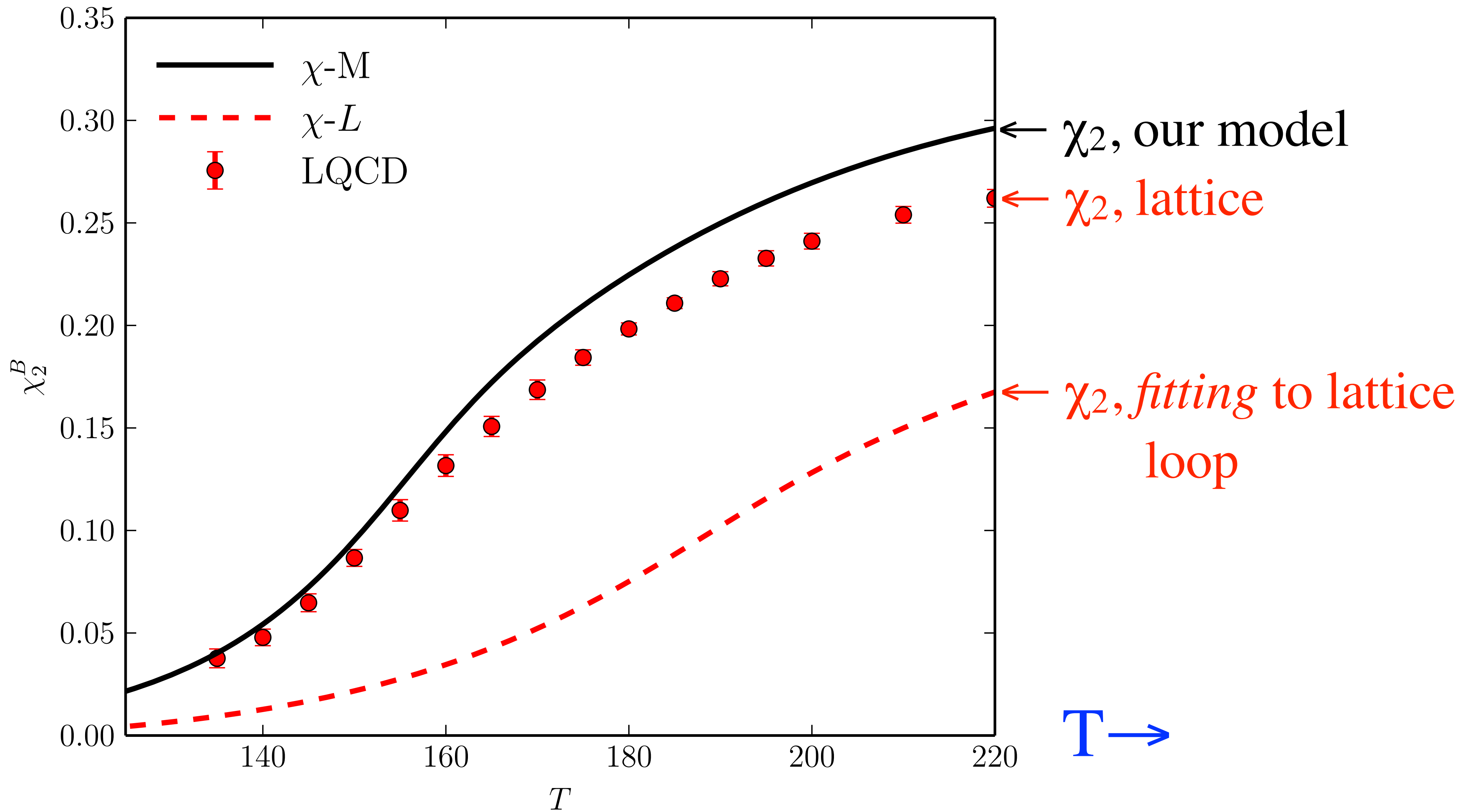
What's up with the lattice loop?

Looked at *wide* variety of possible models.

Below: χ_2 from chiral matrix model, lattice,

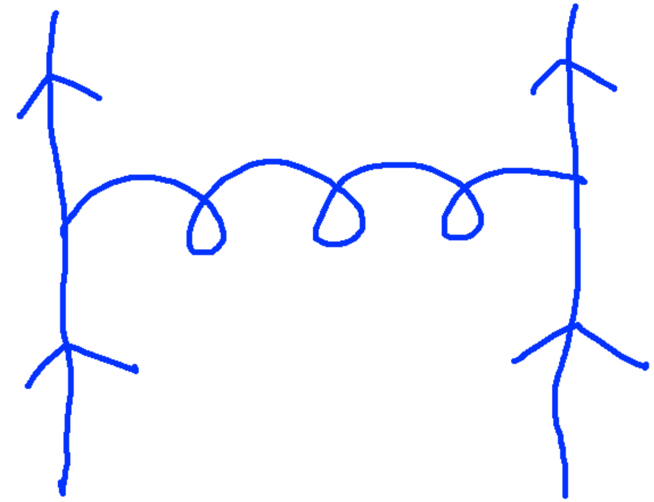
and fitting the loop to the lattice value, then computing χ_2.

If the lattice loop is right, then χ_2. is much too small.



3. Tetraquarks and perhaps a *second* chiral transition

Diquark attraction



Jaffe '79: most attractive channel for quark-quark scattering is antisymmetric in *both* flavor and color.

Color: 3 x 3 = $\underline{3}$ + 6 ; $\underline{3}$ is antisymmetric, 6 symmetric
Diquark always $\underline{3}$ in color.

Two flavors: 2 x 2 = 1 + 3 ; $\underline{1}$ is antisym., 3 symmetric
For two flavors diquark is a flavor *singlet*,

$$\chi_L^A = \epsilon^{ABC}\,\epsilon^{ab}\,(q_L^{aB})^T\,\mathcal{C}^{-1}\,q_L^{b\,C}$$

Three flavors: 3 x 3 = $\underline{3}$ + 6 . Diquark field flavor *anti-triplet*, $\underline{3}$

$$\chi_L^{aA} = \epsilon^{abc}\,\epsilon^{ABC}\,(q_L^{bB})^T\,\mathcal{C}^{-1}\,q_L^{cC}$$

Rather trivial difference in flavor structure leads to *large* difference in χSB'g

Tetraquarks for two flavors

Three combinations of (color-singlet) diquarks: LR, LL, RR

$$\zeta = (\chi_R^A)^* \, \chi_L^A \quad , \quad \zeta_L = (\chi_L^A)^* \, \chi_L^A \quad , \quad \zeta_R = (\chi_R^A)^* \, \chi_R^A$$

Only consider LR, complex ζ (ζ_L and ζ_R real, can be ignored)

ζ singlet under $SU(2)_L$ x $SU(2)_R$

$$\xi \to \mathrm{e}^{-2i\alpha} \, \xi$$

Under $U(1)_A$, Φ has charge +1, ζ charge -2

T= 0: ζ singlet under $Z(2)_A$. ζ_r = Re(ζ) has J^P = 0+, ζ_i = Im(ζ) is 0^-

Hence *arbitrary* powers of ζ_r :

$$\mathcal{V}^A_{\zeta_r} = h_r \, \zeta_r + m_r^2 \, \zeta_r^2 + \kappa_r \, \zeta_r^3 + \lambda_r \, \zeta_r^4$$

Hence ζ_r has a v.e.v. at *any* temperature where only $Z(2)_A$, not $U(1)_A$.

Potentials for 2-flavor tetraquarks

Lots of couplings between ζ and Φ! Some are invariant under $U(1)_A$:

$$\mathcal{V}_\zeta^\infty = m_\zeta^2 \, |\zeta|^2 + \lambda_\zeta \, (|\zeta|^2)^2$$

$$\mathcal{V}_{\zeta\Phi}^\infty = +\kappa_\infty \, (\zeta \, \det\Phi + \text{c.c.}) + \lambda_{\zeta\phi 1} \, |\zeta|^2 \, \text{tr}\left(\Phi^\dagger \Phi\right)$$

Some only under $Z(2)_A$:

$$\mathcal{V}_\Phi^A = \kappa_\Phi \, (\det\Phi + \text{c.c.}) + \lambda_{\Phi 3} \, (\det\Phi + \text{c.c.}) \, \text{tr}\left(\Phi^\dagger \Phi\right) + \lambda_{\Phi 4} \, (\det\Phi + \text{c.c.})^2$$

$$\mathcal{V}_{\zeta\Phi}^A = \kappa_{\zeta\Phi} \, \zeta_r \, \text{tr}\left(\Phi^\dagger \Phi\right) + \lambda_{\zeta\Phi 2} \, \zeta_r^2 \, (\det\Phi + \text{c.c.})$$

Couplings galore!

So what. ζ_r is a flavor singlet, and while it affects the v.e.v of Φ,
it shouldn't turn the chiral transition, expected to be of second order, into first.
(Also why ζ_L and ζ_R don't matter)

Tetraquarks for three flavors

Consider just the LR diquark, $\zeta^{ab} = (\chi_R^{aA})^* \, \chi_L^{bA}$

Under SU(3)$_L$ x SU(3)$_R$, ζ transforms *identically* to Φ!

$\zeta \to \mathrm{e}^{-2i\alpha} \, U_R \, \zeta \, U_L^\dagger$

Under U(1)$_A$, Φ has charge +1, ζ charge -2.

Most important coupling is *direct* mixing term, Z(3)$_A$ invariant:

$$\mathcal{V}^A_{\zeta\Phi,2} = \widetilde{m}^2 \, \mathrm{tr} \left(\zeta^\dagger \Phi + \Phi^\dagger \zeta \right)$$

Black, Fariborz, Schechter ph/9808415; 't Hooft, Isidori, Maiani, Polosa 0801.2288

An extra dozen couplings. E.g., U(1)$_A$ invariant cubic coupling

$$\mathcal{V}^\infty_{\zeta\Phi,3} = \kappa_\infty \, \epsilon^{abc} \, \epsilon^{a'b'c'} \left(\zeta^{aa'} \, \Phi^{bb'} \, \Phi^{cc'} + \mathrm{c.c.} \right)$$

"Mirror" model, T = 0

$\text{Spectrum} : \Phi = \pi, K, \eta, \eta'; a_0, \kappa, \sigma_8, \sigma_0 \; ; \; \zeta = \widetilde{\pi}, \widetilde{K}, \widetilde{\eta}, \widetilde{\eta}'; \widetilde{a}_0, \widetilde{\kappa}, \widetilde{\sigma}_8, \widetilde{\sigma}_0.$

General model has 20 couplings, just a *bit* involved:

Fariborz, Jora, & Schechter: ph/0506170; 0707.0843; 0801.2552. Pelaez, 1510.00653

Instead study "mirror" model, where Φ and ζ have *identical* couplings

$$\mathcal{V}_\Phi = m^2 \,\mathrm{tr}\left(\Phi^\dagger \Phi\right) - \kappa\left(\det \Phi + \mathrm{c.c.}\right) + \lambda \,\mathrm{tr}\left(\Phi^+ \Phi\right)^2$$

$$\mathcal{V}_\zeta = m^2 \,\mathrm{tr}\left(\zeta^\dagger \zeta\right) - \kappa\left(\det \zeta + \mathrm{c.c.}\right) + \lambda \,\mathrm{tr}\left(\zeta^+ \zeta\right)^2$$

Let the only coupling be the mass term, $\mathcal{V}^A_{\zeta\Phi,2} = \widetilde{m}^2 \,\mathrm{tr}\left(\zeta^\dagger \Phi + \Phi^\dagger \zeta\right)$

This is simple, because it *only* mixes: $\pi \leftrightarrow \widetilde{\pi}\,,\; K \leftrightarrow \widetilde{K} \ldots$

Spectrum of the mirror model

In the chiral limit, the mass eigenstates: (need to assume $\widetilde{m}^2 < 0$)

$$\pi, \widetilde{\pi} = 0\,,\; -2\,\widetilde{m}^2 \quad ; \quad \eta', \widetilde{\eta}\,' = 3\,\kappa\,\phi\,,\, 3\,\kappa\,\phi - \, 2\,\widetilde{m}^2$$

$$a_0, \widetilde{a}_0 = m^2 + \kappa\,\phi + 6\,\lambda\,\phi^2 \pm \widetilde{m}^2 \quad ; \quad \sigma_0, \widetilde{\sigma} = m^2 - 2\,\kappa\,\phi + 6\,\lambda\,\phi^2 \pm \widetilde{m}^2$$

All states are mixtures of Φ and ζ. Of course 8 Goldstone bosons.
These satisfy generalized 't Hooft relation

$$m^2_{\eta'} + m^2_{\widetilde{\eta}'} - m^2_{\pi} - m^2_{\widetilde{\pi}} = m^2_{a_0} + m^2_{\widetilde{a}_0} - m^2_{\sigma} - m^2_{\widetilde{\sigma}}$$

Since *every* multiplet is doubled, this can easily be satisfied (unlike if just one).

Even with same couplings, all masses are *split* by the mixing term.

At nonzero T, the thermal masses of the Φ and ζ can*not* be equal!

Two chiral phase transitions from tetraquarks

In chiral limit, can have have *two* chiral phase transitions. =>

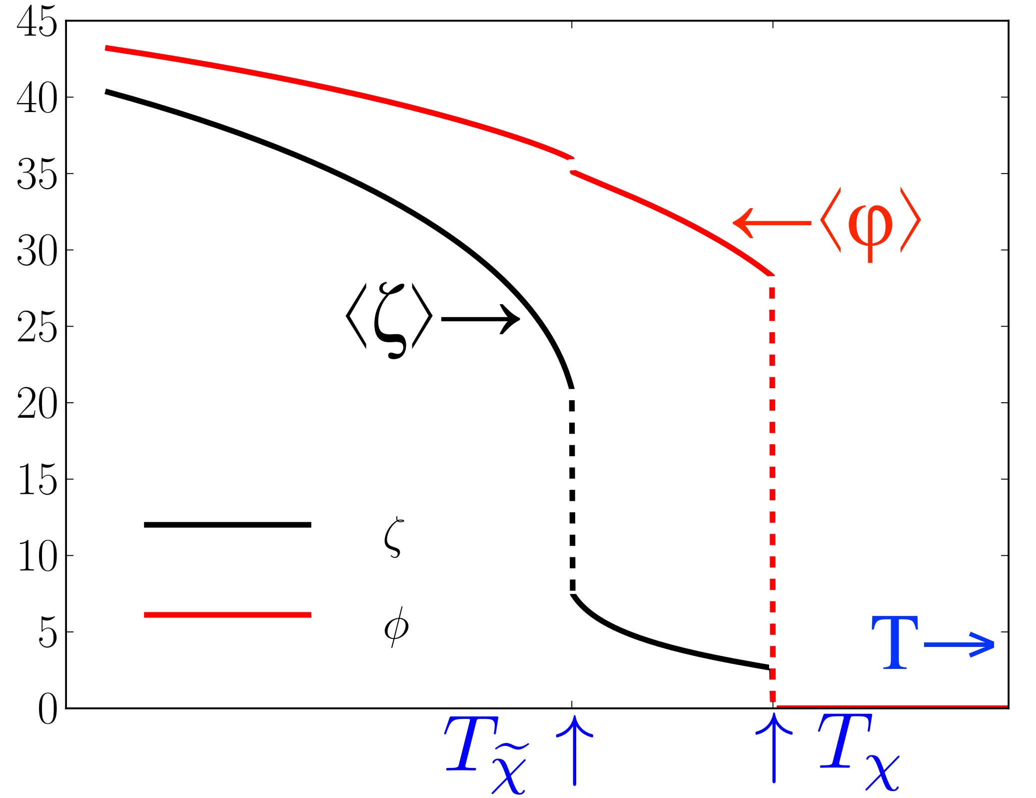
At first, both jump, remain nonzero.

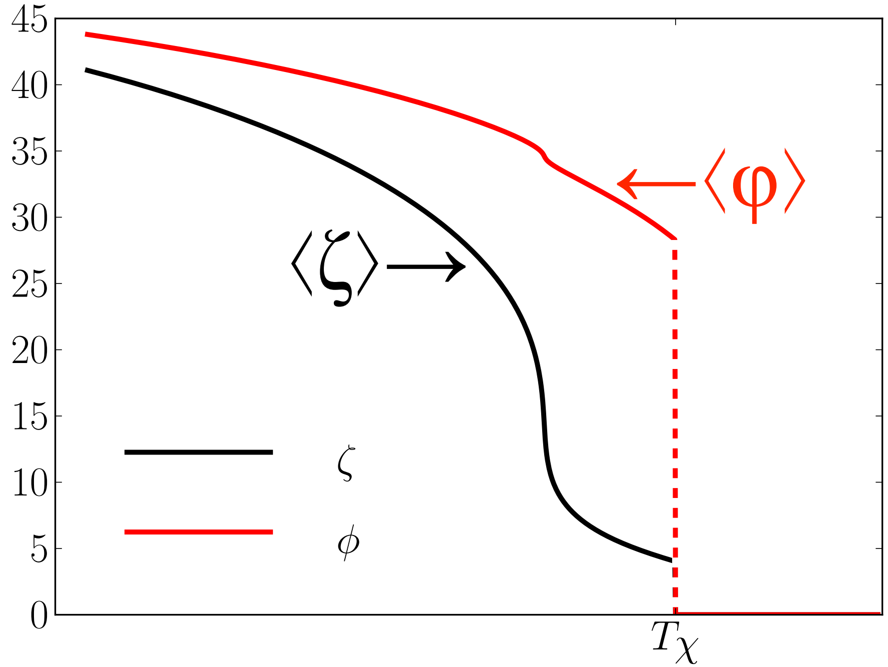
At second, both jump to zero.

$$m_\phi^2(T) = 3\,T^2\ + m^2$$

$$m_\zeta^2(T) = 5\,T^2\ + m^2$$

$$\widetilde{m}^2 = -(100)^2$$



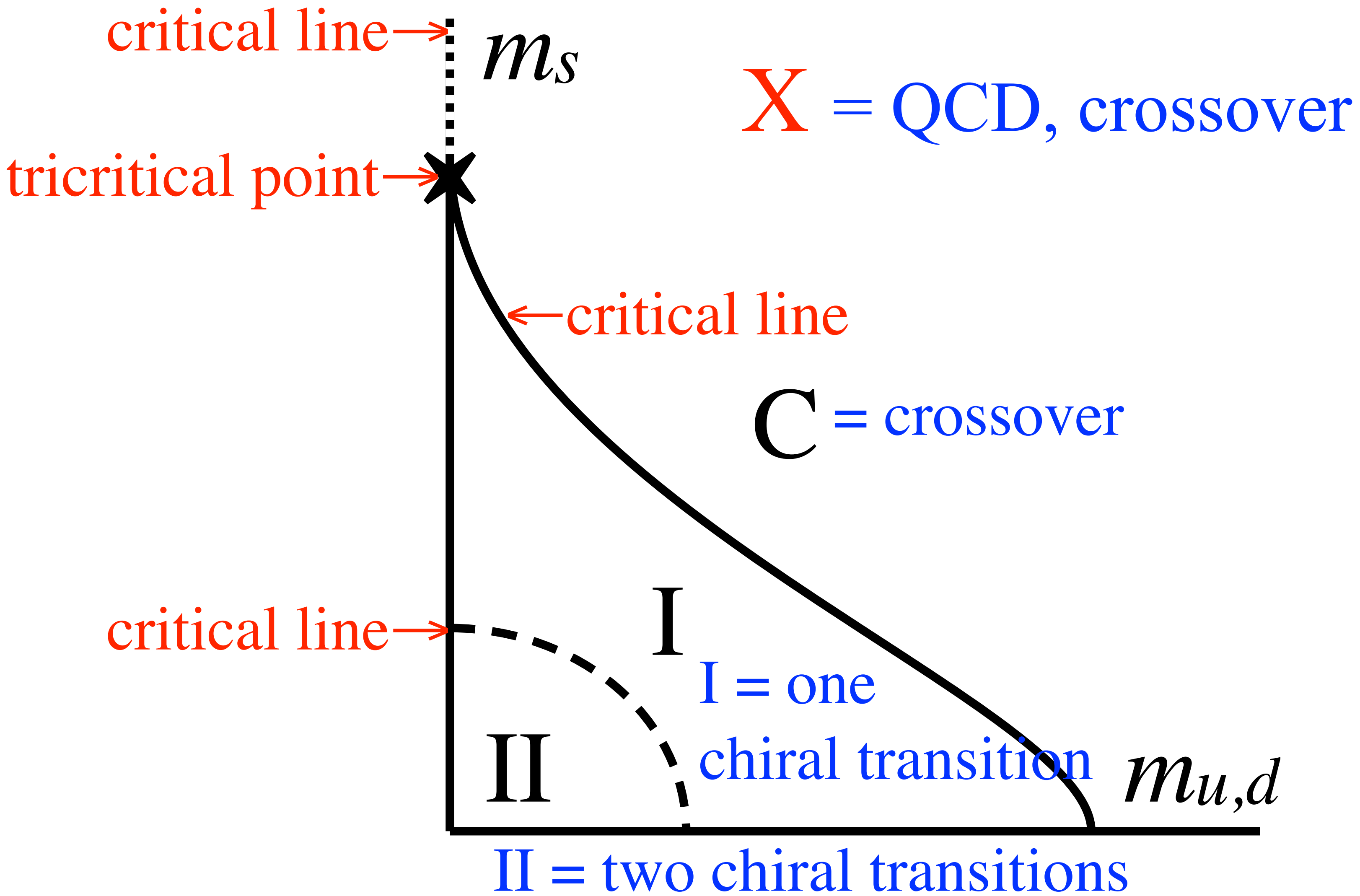


<= Also possible to have single chiral phase transition.

$$\widetilde{m}^2 = -(120)^2$$

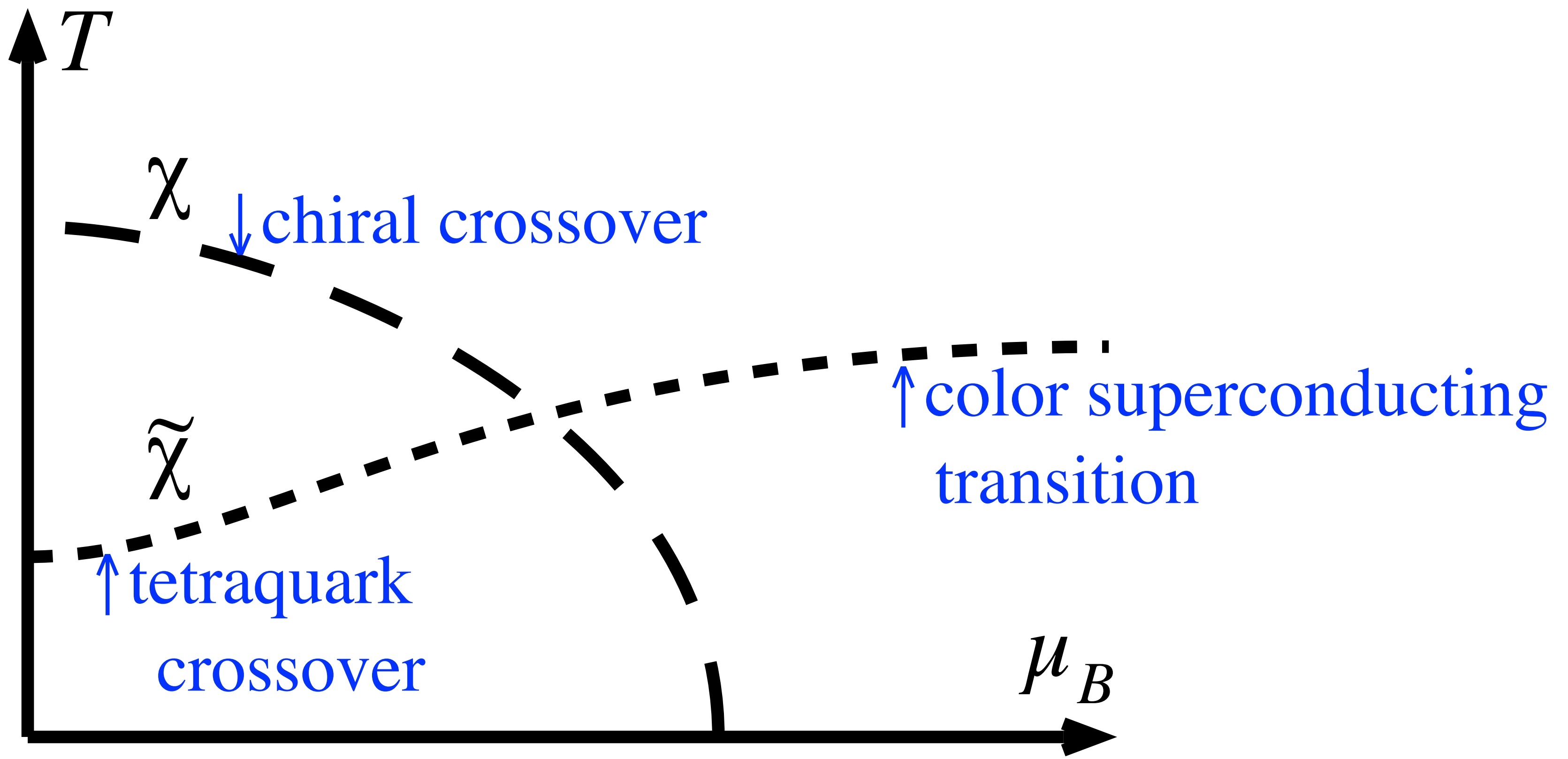
"Columbia" phase diagram for light quarks

If two chiral phase transitions for three massless flavors, will persist for nonzero range. Implies new phase diagram in the plane of $m_u = m_d$ versus m_s:



Tetraquarks and color superconductivity

Tetraquarks: most attractive channel for qq scattering.
In cold quark matter, same physics also produces color superconductivity (CS).
Tetraquark condensate is the (gauge invariant) square of CS condensate.
Perhaps:



In the plane of T and μ

Four flavors and *hexa*quarks

Four flavors, three colors: diquark is 2-index antisymmetric tensor:

$$\chi_L^{(ab)A} = \epsilon^{abcd}\,\epsilon^{ABC}\,(q_L^{cB})^T\,\mathcal{C}^{-1}\,q_L^{dC}$$

So LR tetraquark is same:

$$\zeta^{(ab),(cd)} = \left(\chi_R^{(ab)A}\right)^\dagger\,\chi_L^{(cd)A}$$

Tetraquark couples to usual Φ through cubic, quadratic terms, so what.

Instead, consider triquark field:

$$\chi_L^a = \epsilon^{abcd}\,\epsilon^{ABC}\,q_L^{bA}\,(q_L^{cB})^T\,\mathcal{C}^{-1}\,q_L^{dC}$$

Triquark is a color singlet, fundamental rep. in flavor.
Hence a LR *hexa*quark field is just like the usual Φ, and mixes *directly* with it.

$$\xi^{ab} = (\chi_R^a)^\dagger\,\chi_L^b$$

Like color superconductivity, the analysis for general numbers of flavors and colors is *not* trivial.

Hunt for the Quark Gluon Plasma

The Quark Gluon Plasma as an Unicorn.
Experimentalists are the hunters, so....“All theorists are...”