Punchline:

Effective theory for deconfinement, *near* T_c.

There's always an effective theory.

Based upon results from the lattice

Real competition for AdS/CFT

Matrix model for deconfinement

SU(N) gauge theories, with*out* quarks, at a temperature T

Lattice: in *some* ways, N = 3 is close to N = ∞.

Simple matrix model, valid in large N expansion

With 2 parameters, good fit to one function of T, pressure

Good agreement with second function of T, the 't Hooft loop (interface tension)

*Dis*agrees with third function of T, (renormalized) Polyakov loop - ?

Most unexpected: transition region *very* narrow, < 1.2 T_c, ~ *in*dependent of N

Adjoint Higgs phase, with *split* masses, in this narrow region

G(2) gauge theories: "deconfinement" without a center

Need to introduce terms to generate *maximal* eigenvalue repulsion

Generalization of Meisinger, Miller, Ogilvie (MMO), arXiv:hep-ph/0108009
Dumitru, Guo, Hidaka, Korthals-Altes, & RDP, 1011.3820 + 1205....
Also: Y. Hidaka & RDP, 0803.0453, 0906.1751, 0907.4609, 0912.0940.
RDP: ph/0608242 + ...

What the lattice tells us

Apparently: for SU(N), weak dependence on N

Lucini, Rago, & Rinaldi, 1202.6684: T_c = transition temperature, σ = string tension.

$$\frac{T_c}{\sqrt{\sigma}} = .5949(17) + \frac{0.458(18)}{N^2}$$

This ratio changes little, by 8%, from N=3, ~ 0.64 to N = ∞, ~ 0.60.

Picture more complicated: weak N dependence for some quantities, but *not* all.

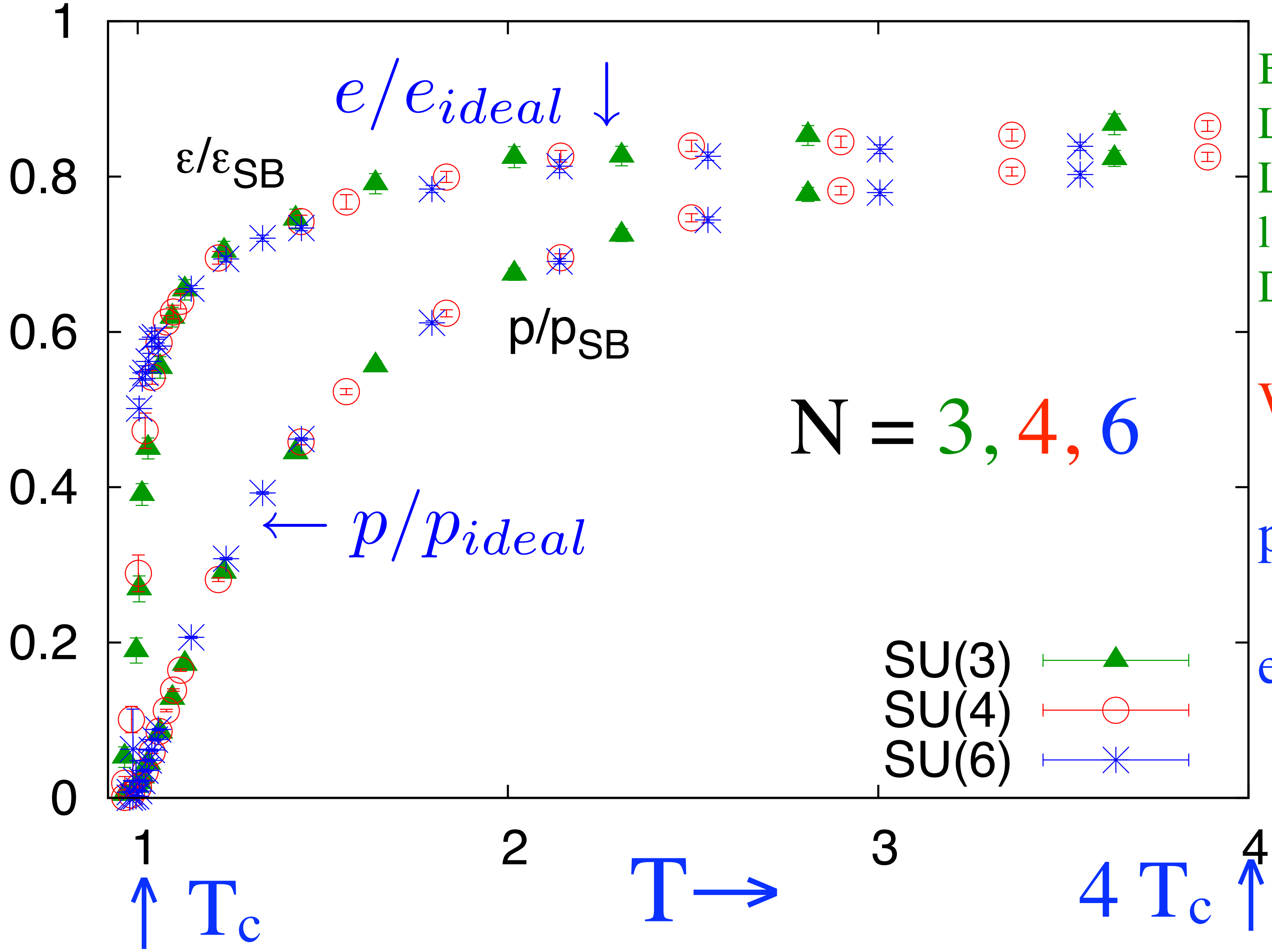
Still, use large N approximation as basic justification: *no* small masses

Lattice, SU(N): how *wide* is the transition?

SU(N) gauge theories with*out* quarks, temperature T ≠ 0

Scaled by ideal gas, energy “e” and pressure “p” *approximately* independent of N.

e and p ≈ 0 below T_c: ~ N^2 - 1 gluons above T_c, vs ~ 1 hadrons below.



Boyd, Engels, Karsch, Laermann, Legeland, Luetgemeier, Petersson, lat/9602007

Datta & Gupta, 1006.0938

Width of “semi”-QGP?

pressure: *wide* ~ 4 T_c

energy: *narrow*, ~ 1.5 T_c

Lattice: peak in conformal anomaly

For SU(N), "peak" in e-3p/T^4 just above T_c. *Approximately* uniform in N.

Not near T_c: transition *2nd* order for N = 2, *1st* order for *all* N ≥ 3

N=3: *weakly* 1st order. N = ∞: *strongly* 1st order (latent heat ~ N^2)

Datta & Gupta, 1006.0938: latent heat ~ $e(T_c^+)$.
$$\frac{e(T_c^+)}{(N^2-1)T_c^4} = 0.388 - \frac{1.61}{N^2}$$

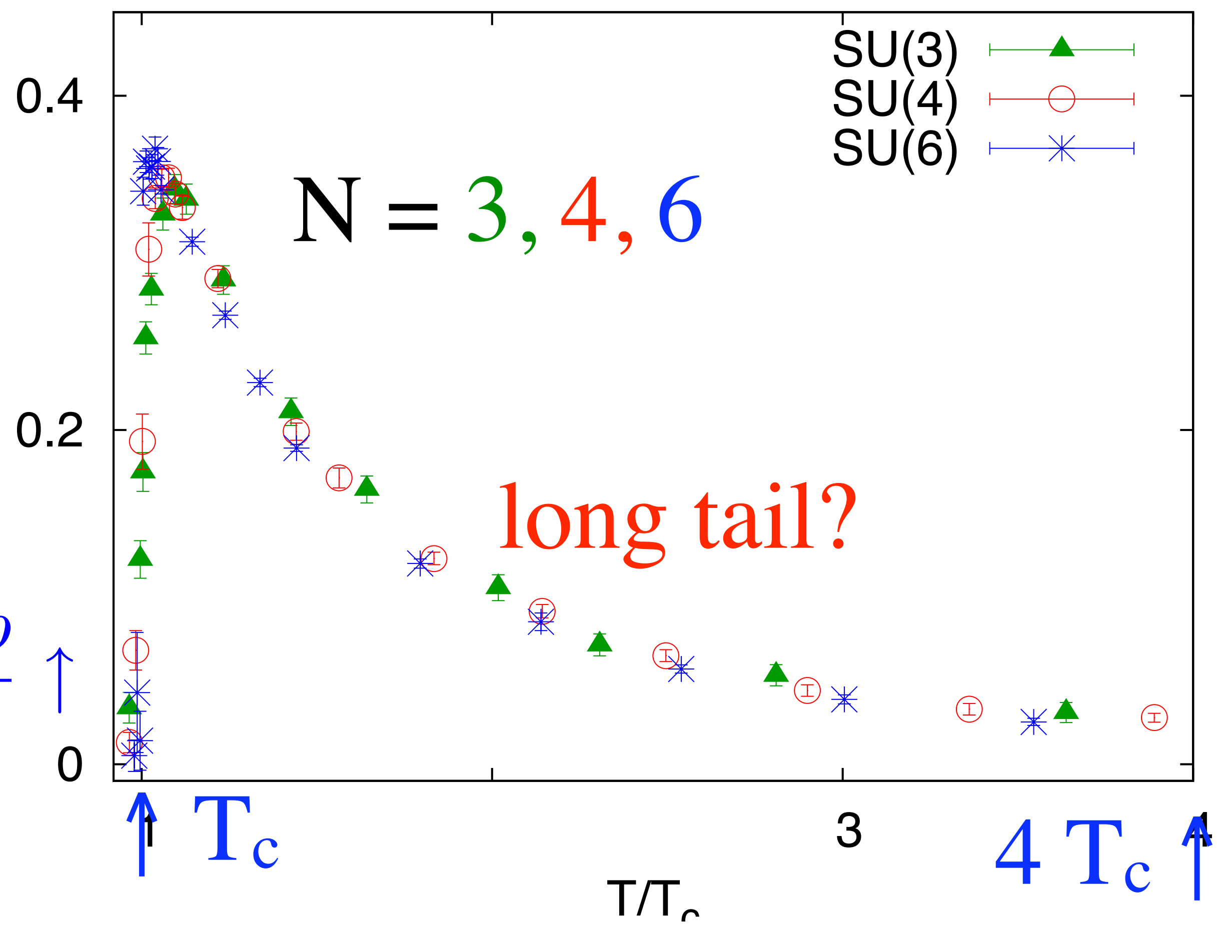
Even scaled by N^2 -1, $e(T_c)$ increases by a factor of two

from N = 3, ~ 0.21

to N = ∞, ~ 0.39

Near T_c, quantities *can* depend strongly upon N

$$\frac{1}{N^2-1}\;\frac{e-3p}{T^4}\uparrow$$

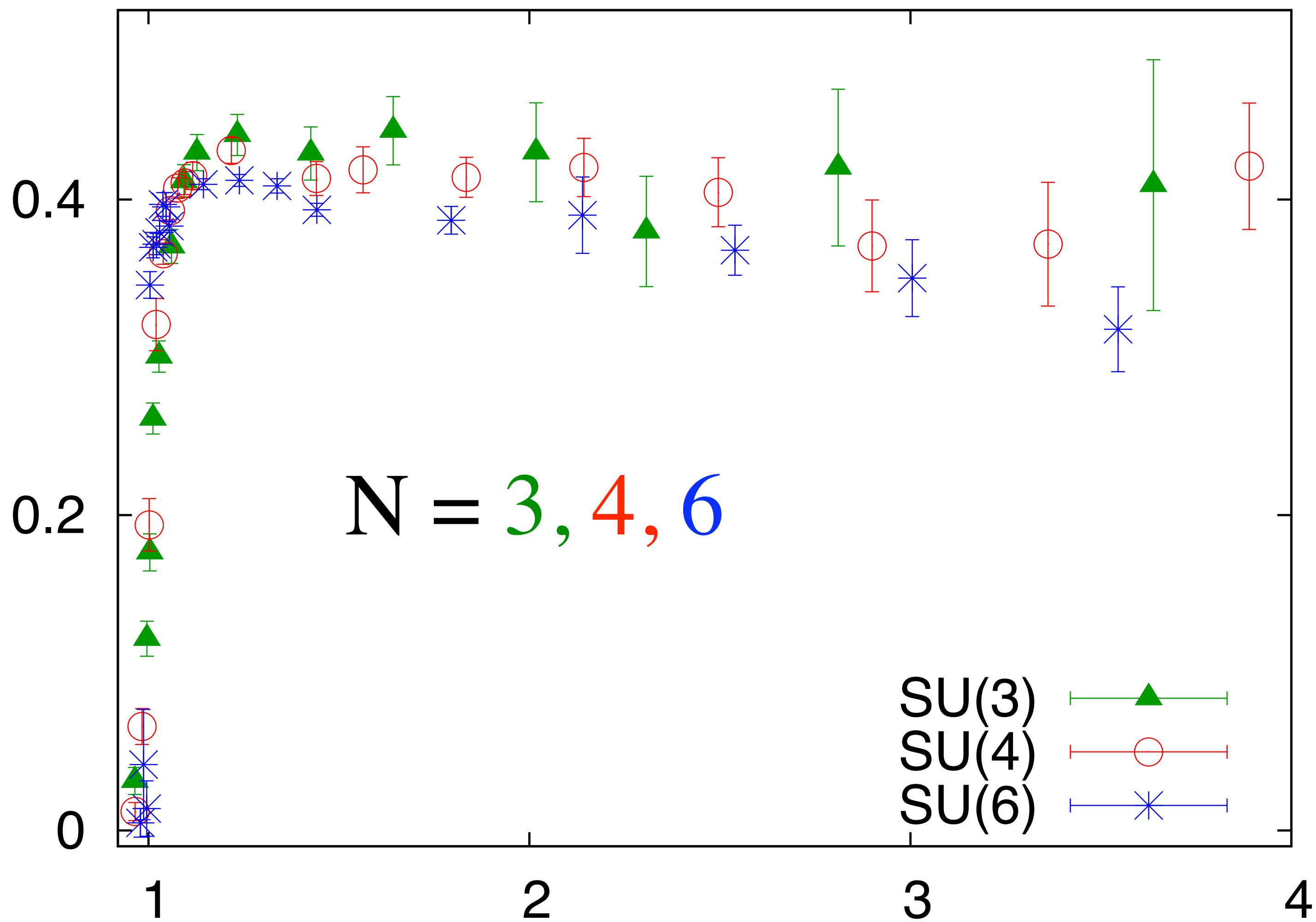


Lattice: tail in the conformal anomaly

Scaling: $(e-3p)/T^2$ *approximately* constant above 1.2 T_c: MMO '01; RDP, ph/0608242

Only true to ~ $4T_c$; eventually, $(e-3p)/T^4 \sim g^4(T)$

$$\frac{1}{N^2-1}\;\frac{e-3p}{T^2\,T_c^2}\uparrow$$



↑ T_c $\quad$ $T \rightarrow$ $\quad$ $4\,T_c$ ↑

Datta & Gupta, 1006.0938

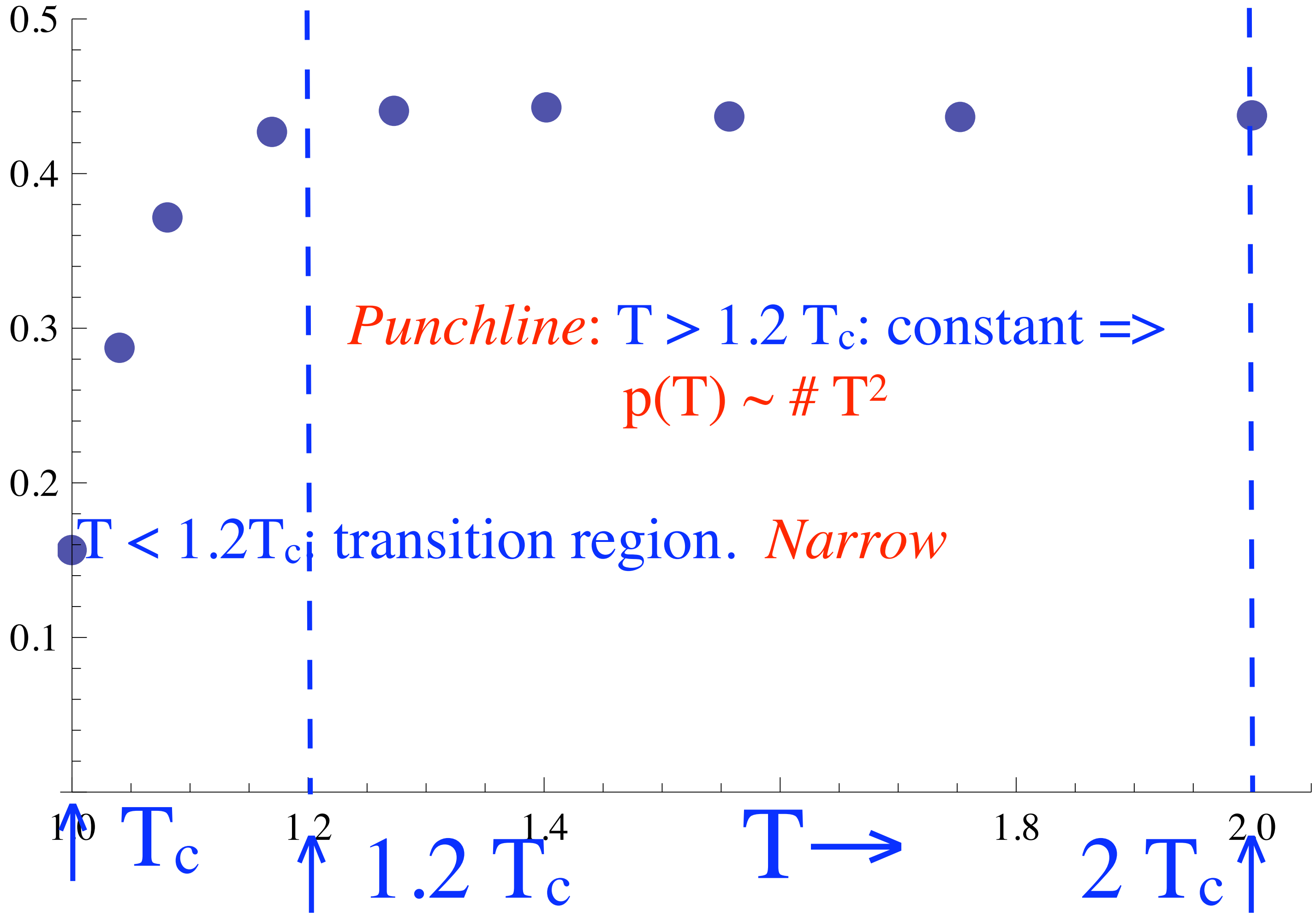
Lattice: terms ~ T^2 in pressure

Lattice: WHOT. Change # time steps at fixed lattice scale. Higher precision, ± 1%

$$T : 1.2 \to 2T_c \ : \ p(T) \approx \# \, T^2 (T^2 - c\, T_c^2) \, , \; c = 1.00 \pm .01$$

"Fuzzy bag"

$$\frac{1}{8} \, \frac{e - 3p}{T^2 \, T_c^2} \uparrow$$



WHOT: Umeda, Ejiri, Aoki, Hatsuda, Kanaya, Maezawa, Ohno, 0809.2842

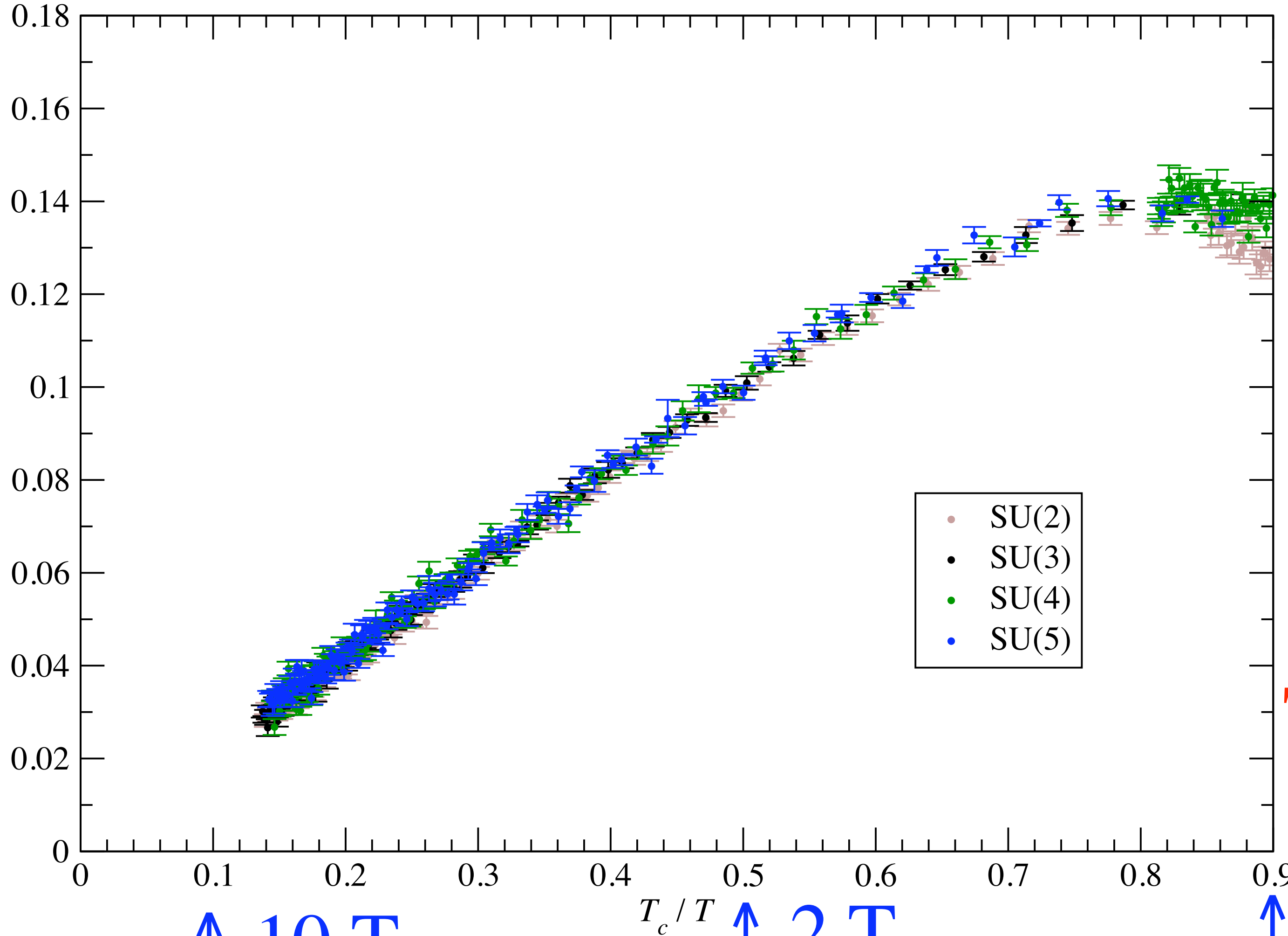
Lattice, 2+1 dim.'s: again, terms ~ T^2 in pressure

Caselle, Castagnini, Feo, Gliozzi, Gürsoy, Panero, & Schäfer, 1111.0580.

SU(N), N = 2, 3, 4, 5: again, non-perturbative terms ~ T^2 and *not* ~ T.

$$p(T) \approx \# \, T^2 (T - c\,T_c)\,,\; c \approx 1.$$

$\frac{1}{N^2-1}\,\frac{e-2p}{T^3}\,\uparrow$



$T_c/T \rightarrow$

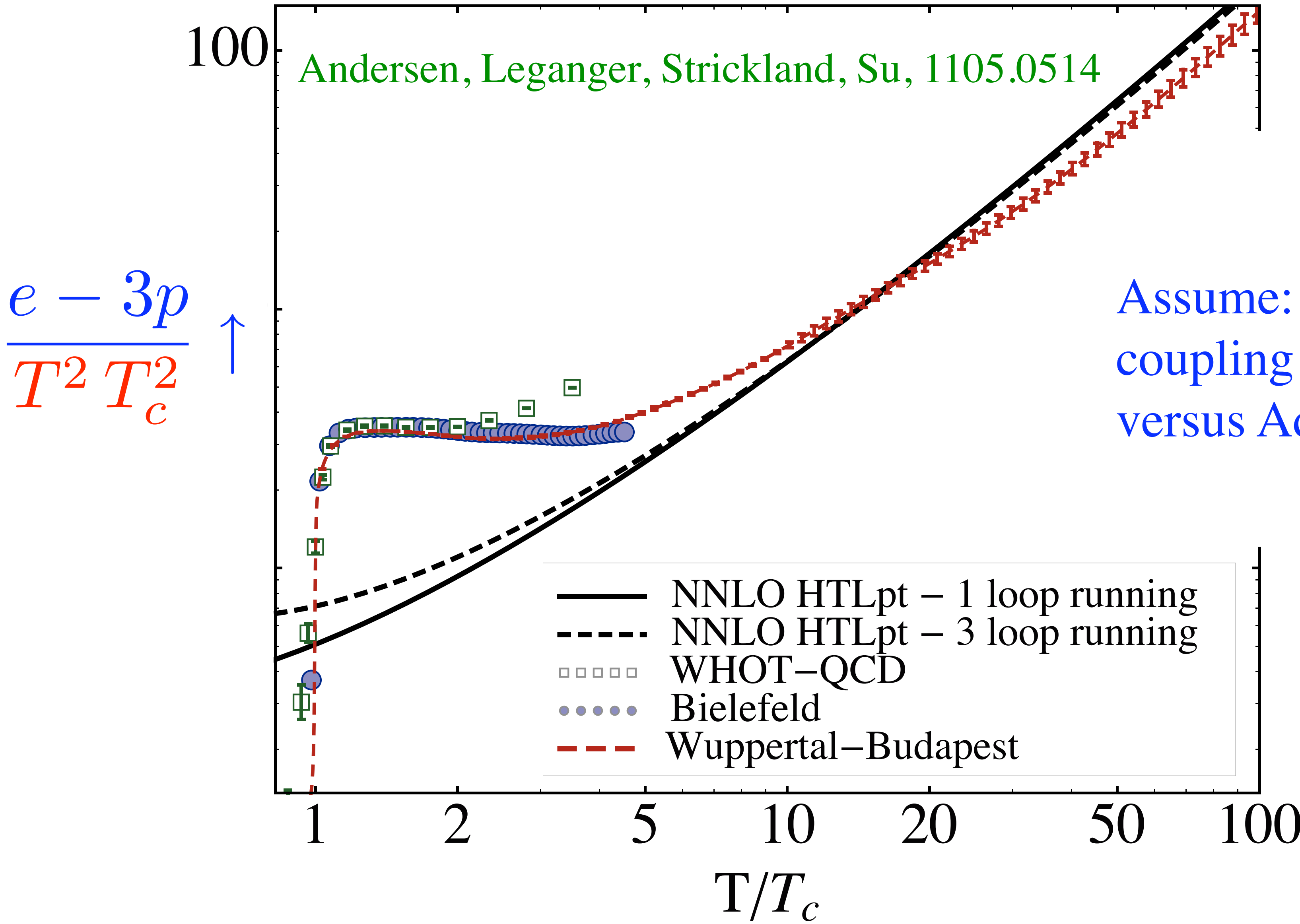
$\uparrow$ 10 T_c $\uparrow$ 2 T_c $\uparrow$ 1.1 T_c

Not strong coupling, even at T_c

QCD coupling runs like $\alpha(2\pi T)$, *intermediate* at T_c, $\alpha(2\pi T_c) \sim 0.3$

Braaten & Nieto, hep-ph/9501375, Laine & Schröder, hep-ph/0503061 & 0603048

HTL resummed perturbation theory, NNLO, good to ~ 8 T_c :



Assume: *intermediate* coupling near T_c versus AdS/CFT

Polyakov Loops and Z(N)

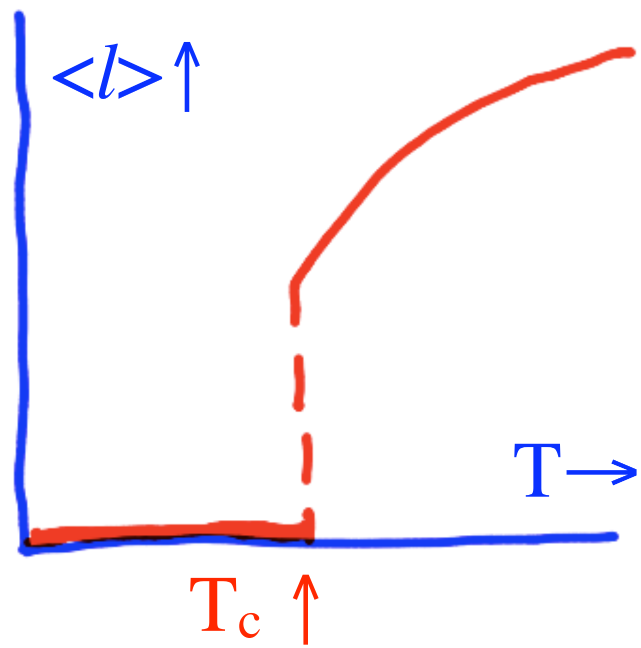
L = Wilson line. Under global Z(N) rotations: $\mathbf{L} = \mathcal{P}\, \mathrm{e}^{ig \int_0^{1/T} A_0 \, d\tau} \to e^{2\pi i/N}\, \mathbf{L}$

(Z(N) symmetry lost with dynamical quarks)

Wilson line gauge variant. Trace is invariant:
(N.B.: eigenvalues of **L** *are* gauge invariant)

$$\ell = \frac{1}{N}\, \mathrm{tr}\, \mathbf{L}$$

⟨ *loop* ⟩ measures ionization of color:
partial ionization when 0 < ⟨ *loop* ⟩ < 1 :
"semi"-QGP



Svetitsky and Yaffe '80:
SU(3) 1st order because Z(3) allows *cubic* terms:

$$\mathcal{L}_{\text{eff}} \sim \ell^3 + (\ell^*)^3$$

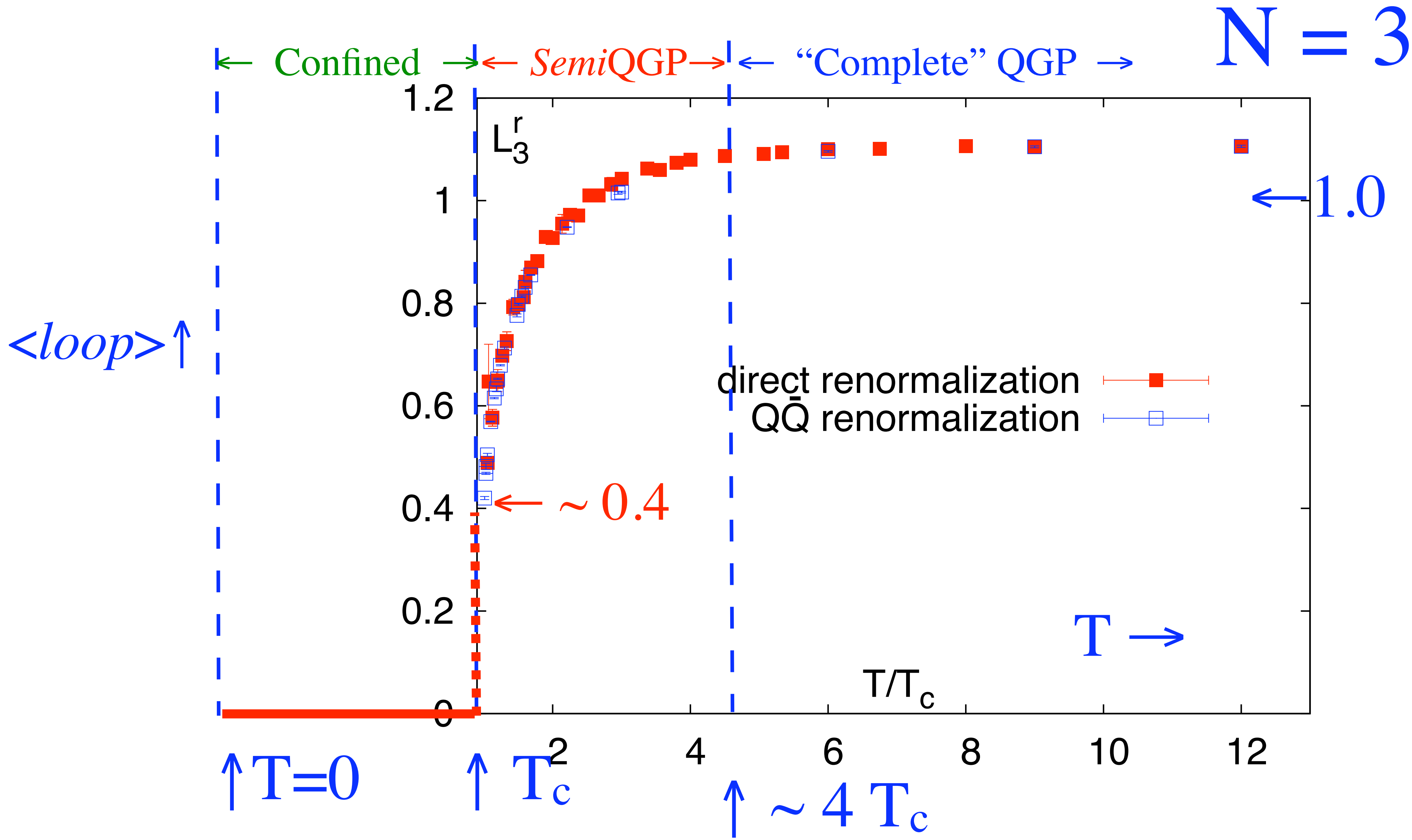
Does *not* apply for N > 3.
So why is deconfining transition 1st order for all N ≥ 3?

Lattice: Polyakov Loop, no Quarks

N=3: Gupta, Hubner, Kaczmarek, 0711.2251.

N ≥ 4: Mykkanen, M. Panero, and K. Rummukainen, 1110.3146

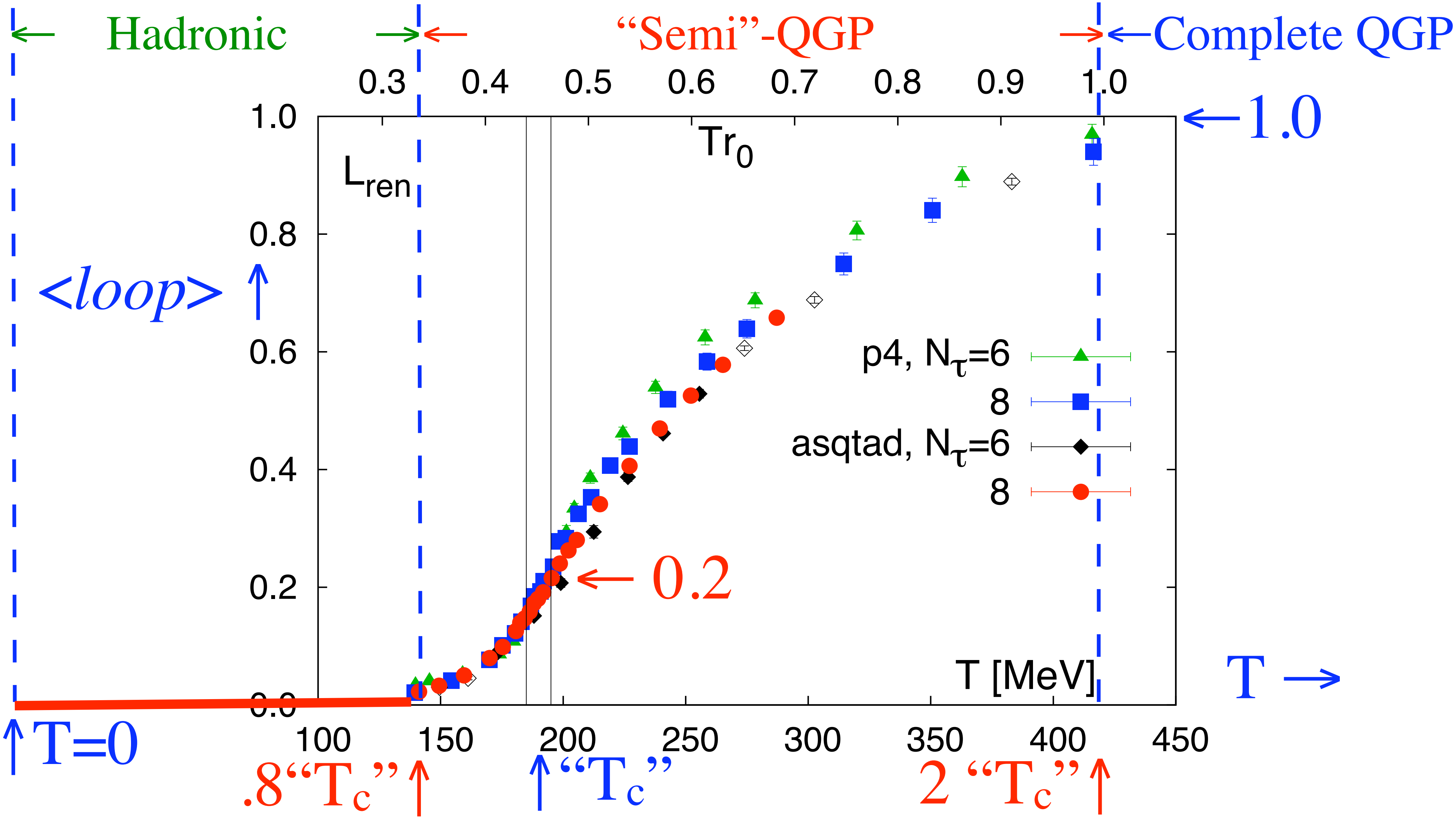
Suggests *wide* transition region, like pressure, to ~ 4 T_c.



Lattice: Polyakov Loop with Quarks, "T_c"

Quarks ~ background Z(3) field. Lattice: Bazavov et al, 0903.4379.

3 quark flavors: *weak* Z(3) field, does *not* wash out Z(3) symmetry of SU(3) glue

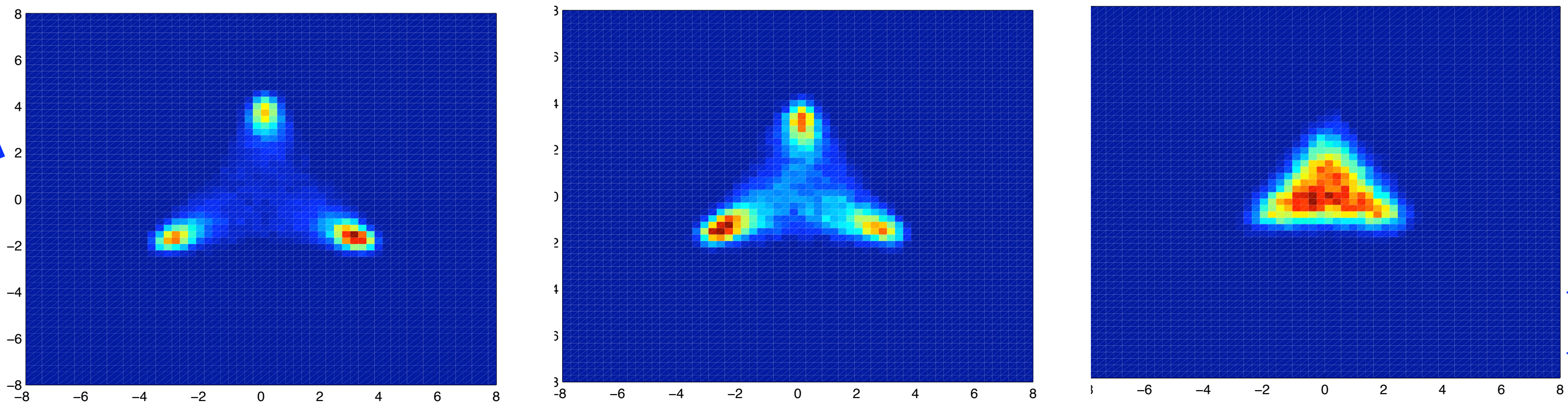


Interface tensions: order-order & order-disorder

Lattice, A. Kurkela, unpub.'d: 3 colors, loop l complex.

Distribution of loop shows Z(3) symmetry:

Im l ↑



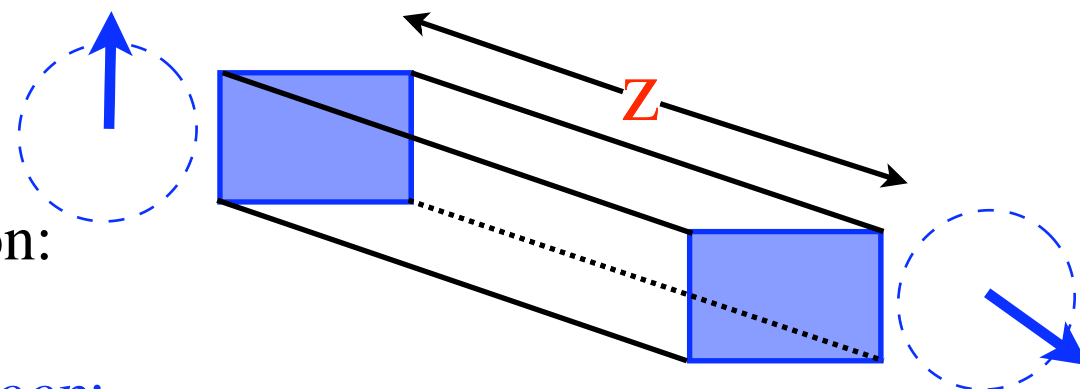
Re l →

$T \gg T_c$ $T \sim T_c$ $T < T_c$

Interface tension: box long in z.

Each end: distinct but *degenerate* vacua.

Interface forms, action ~ interface tension:



$T > T_c$: order-order interface = ‘*t Hooft loop*:

measures response to *magnetic* charge

Korthals-Altes, Kovner, & Stephanov, hep-ph/9909516

$$Z \sim e^{-\sigma_{int} V_{tr}}$$

Also: *if* trans. 1st order, order-*dis*order interface *at* T_c .

Lattice: 't Hooft loops σ

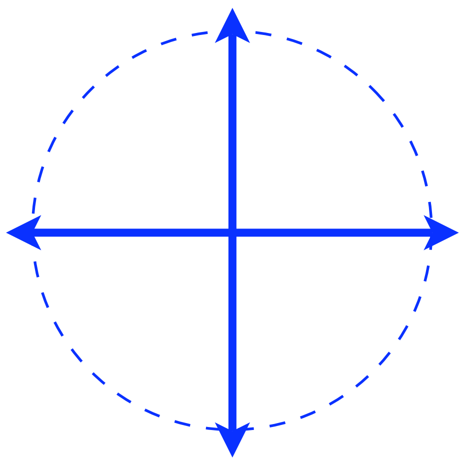
Lattice: de Forcrand & Noth, lat/0510081. σ ~ universal with N

Semi-classical σ : Giovanengelli & Korthals-Altes ph/0102022; /0212298; /0412322: *GKA '04*

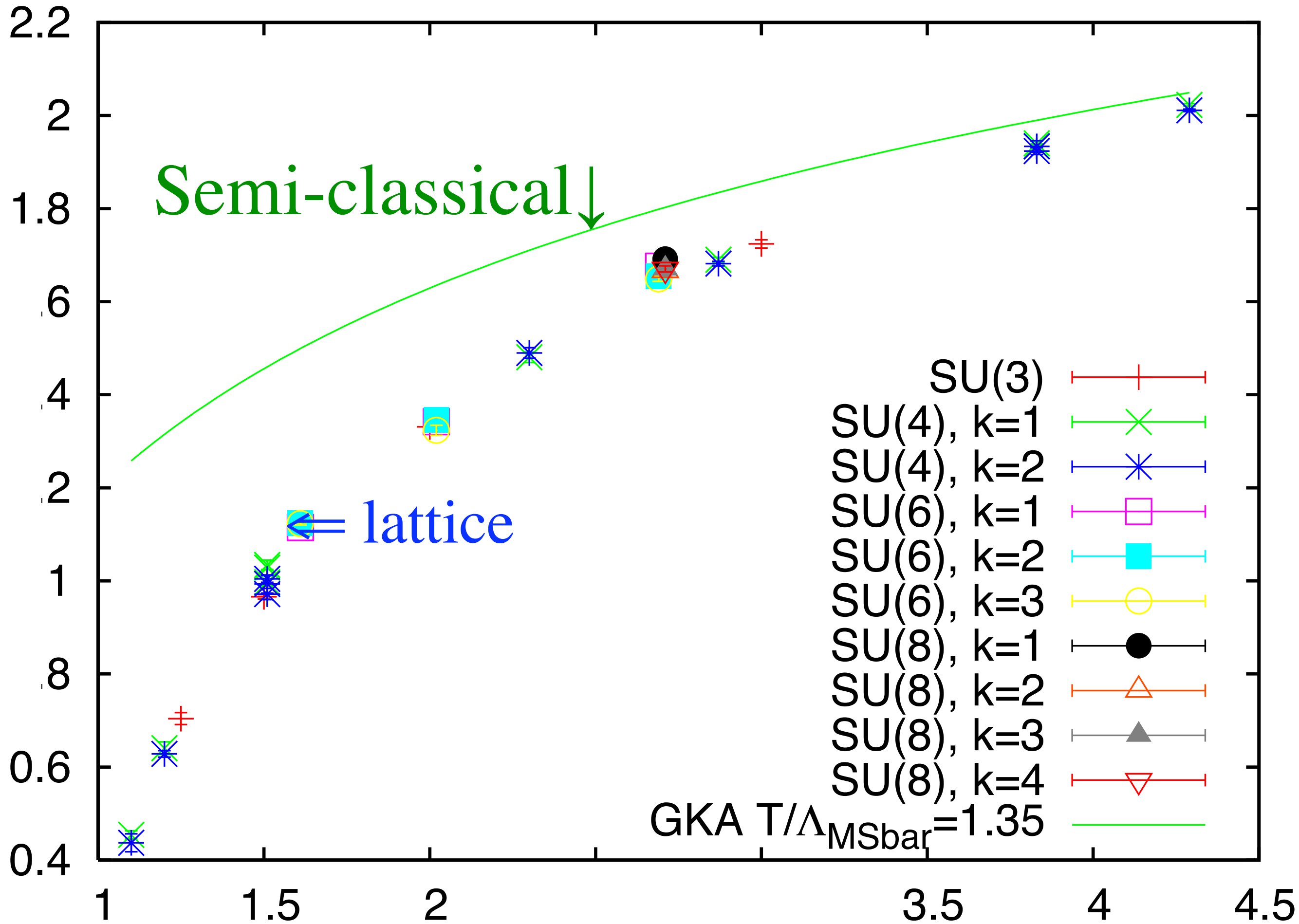
Above 4 T_c, semi-class σ ~ lattice. Below 4 T_c, lattice σ << semi-classical σ.

Even so, when N > 3, *all* tensions satisfy "Casimir scaling" at T > 1.2 T_c.

$$\frac{\sigma_k}{T^2\, k(N-k)} \uparrow$$



N = 4



↑ T_c T⟶ 4.5 T_c↑

Lattice: A_0 mass as $T \to T_c$ - *up or down*?

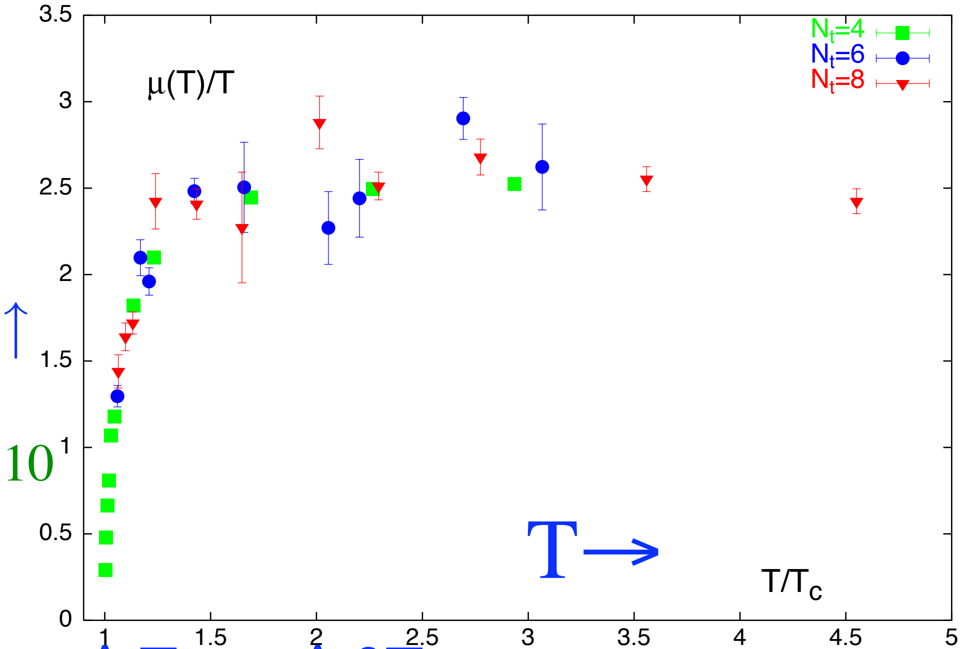
Gauge invariant: 2 pt function of loops:

$$\langle \mathrm{tr}\, \mathbf{L}^\dagger(x)\, \mathrm{tr}\, \mathbf{L}(0) \rangle \sim \mathrm{e}^{-\mu x}/x^d$$

μ/T goes *down* as $T \to T_c$

Kaczmarek, Karsch, Laermann, Lutgemeier lat/9908010

$\frac{\mu}{T} \uparrow$



$\uparrow T_c \quad \uparrow 2T_c$

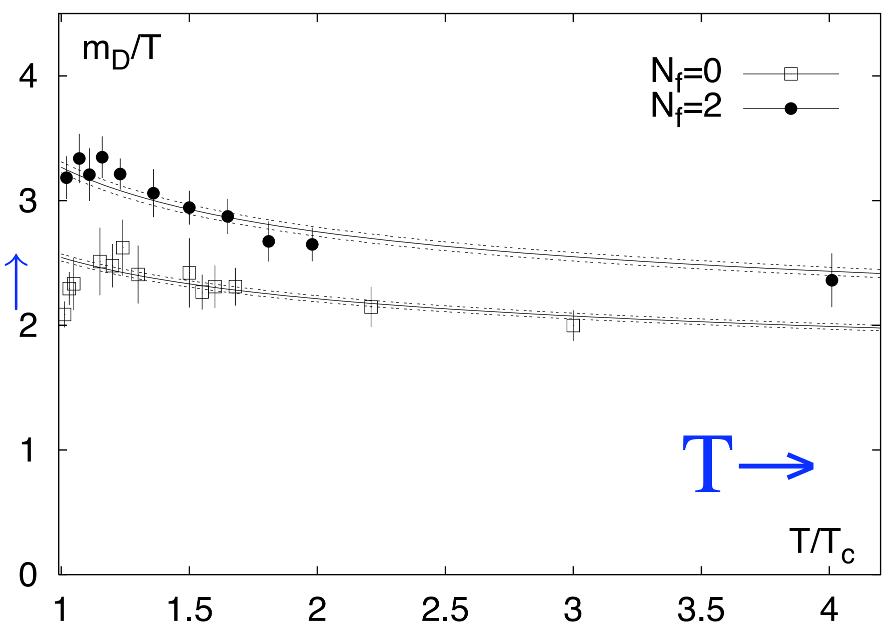
Gauge dependent: singlet potential

$$\langle \mathrm{tr}\, \left(\mathbf{L}^\dagger(x)\, \mathbf{L}(0)\right) \rangle \sim \mathrm{e}^{-m_D x}/x$$

m_D/T goes *up* as $T \to T_c$

Cucchieri, Karsch, Petreczky lat/0103009,
Kaczmarek, Zantow lat/0503017

$\frac{m_D}{T} \uparrow$



$\uparrow T_c \quad \uparrow 2T_c$

Which way do masses go as $T \to T_c$?
Both are constant above $\sim 1.5\, T_c$.

Other models

Comparable to our model with *one* free parameter

Fit only the pressure, not interface tensions.

Masses as $T \to T_c^+$:
some go up (massive gluons), some go down (Polyakov loops)

Models for the "s"QGP, T_c to 4 T_c

1. Massive gluons: Peshier, Kampfer, Pavlenko, Soff '96...Castorina, Miller, Satz 1101.1255
Castorina, Greco, Jaccarino, Zappala 1105.5902

Mass decreases pressure, so adjust m(T) to fit p(T) with 3 parameters. Gluons *very* massive near T_c.

$$p(T) = \# T^4 - m^2 T^2 + \ldots$$

2. Polyakov loops: Fukushima ph/0310121...Hell, Kashiwa, Weise 1104.0572

Effective potential of Polyakov loops.
Potential has 5 parameters...
With quarks, at $T \neq 0$, can go from $\mu = 0$ to $\mu \neq 0$

$$V_{eff}(T) \sim m^2 \ell^* \ell + T \log f(\ell^* \ell)$$

$$m^2 = T^4 \sum_{i=0}^{3} a_i (T_c/T)^i$$

3. AdS/CFT: Gubser, Nellore 0804.0434...Gursoy, Kiritsis, Mazzanti, Nitti, 0903.2859

Add potential for dilaton, φ, to fit pressure.
Only infinite N, with 2 parameters

$$V(\phi) \sim \cosh(\gamma \phi) + b\, \phi^2$$

4. Monopoles: Liao & Shuryak, 0804.0255.

Matrix model: two colors

Simple approximation

Two colors: transition 2nd order, vs 1st for $N \geq 3$

Using large N expansion at $N = 2$

Matrix model: SU(2)

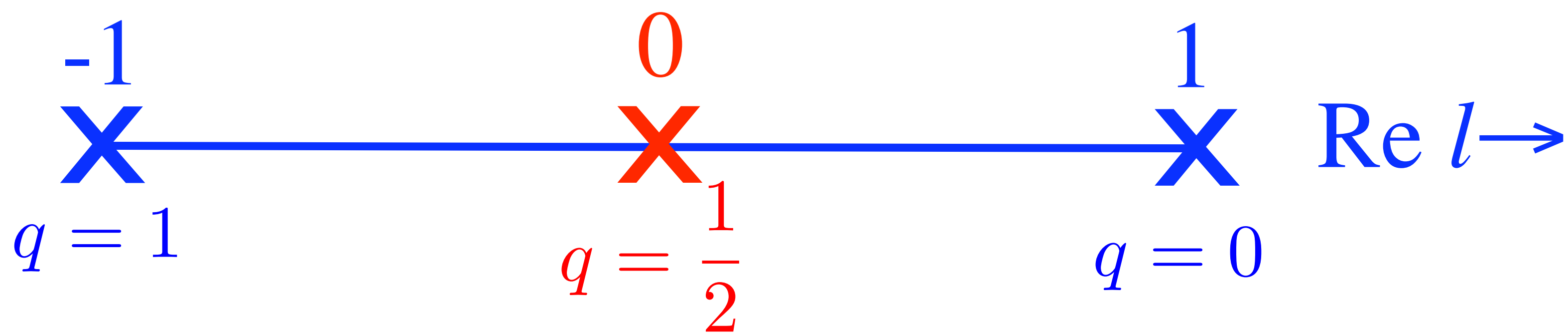
Simple approximation: *constant* $A_0 \sim \sigma_3$, *non*perturbative, $\sim 1/g$:

$$A_0^{cl} = \frac{\pi T}{g}\, q\, \sigma_3 \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \mathbf{L}(q) = \begin{pmatrix} \mathrm{e}^{i\pi q} & 0 \\ 0 & \mathrm{e}^{-i\pi q} \end{pmatrix}$$

Single dynamical field, q

Loop *l* real. Z(2) degenerate vacua q = 0 and 1:

$$\ell = \cos(\pi q)$$



Point *half*way in between: q = ½ , *l* = 0 .

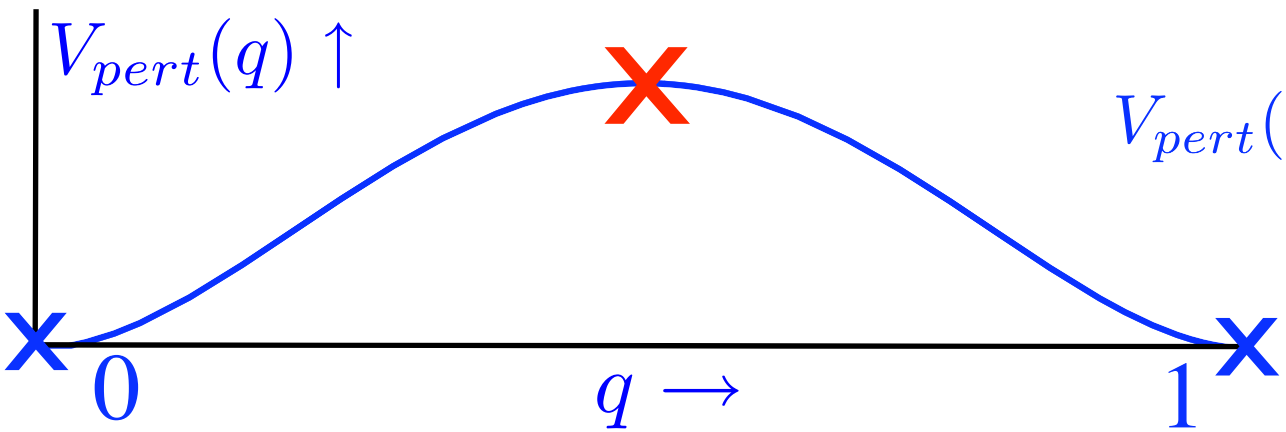
Confined vacuum, $\mathbf{L}_c$,

$$\mathbf{L}_c = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

Classically, A_0^{cl} has zero action: *no* potential for q.

Potential for q, interface tension

Computing to one loop order about A_0^{cl} gives a potential for q: Gross, RDP, Yaffe, '81



$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 q^2 (1-q)^2$$

Use $V_{pert}(q)$ to compute σ: Bhattacharya, Gocksch, Korthals-Altes, RDP, ph/9205231.

$$V_{tot}(q) = \frac{2\pi^2 T^2}{g^2} \left(\frac{dq}{dz}\right)^2 + V_{pert}(q) \qquad \Rightarrow \sigma = \frac{4\pi^2}{3\sqrt{6}} \frac{T^2}{\sqrt{g^2}}$$

Balancing $S_{cl} \sim 1/g^2$ and $V_{pert} \sim 1$ gives $\sigma \sim 1/\sqrt{g^2}$ (*not* $1/g^2$).

Width interface ~ $1/g$, justifies expansion about constant A_0^{cl}. GKA '04: $\sigma \sim \ldots + g^2$

Symmetries of the q's

Wilson line **L** *not* gauge invariant, $\mathbf{L} \to \Omega^\dagger \mathbf{L}\, \Omega$.
Its eigenvalues, $e^{\pm i \pi q}$, are.

$$\mathbf{L}(q) = \begin{pmatrix} e^{i\pi q} & 0 \\ 0 & e^{-i\pi q} \end{pmatrix}$$

O*rdering* of L's eigenvalues irrelevant.

Symmetries: $q \to q + 2$: q angular variable. Valid with quarks.

Pure glue: also, $q \to q + 1$, Z(2) transf., $\mathbf{L} \to -\mathbf{L}$

For pure glue, can restrict q: $0 \to 1$.

Then Z(2) transf. $q \to 1 - q$:
Z(2) transf., *plus* exchange of eigenvalues

$$\mathbf{L}(1-q) = - \begin{pmatrix} e^{-i\pi q} & 0 \\ 0 & e^{i\pi q} \end{pmatrix}$$

Any potential of q must be invariant under $q \to 1 - q$

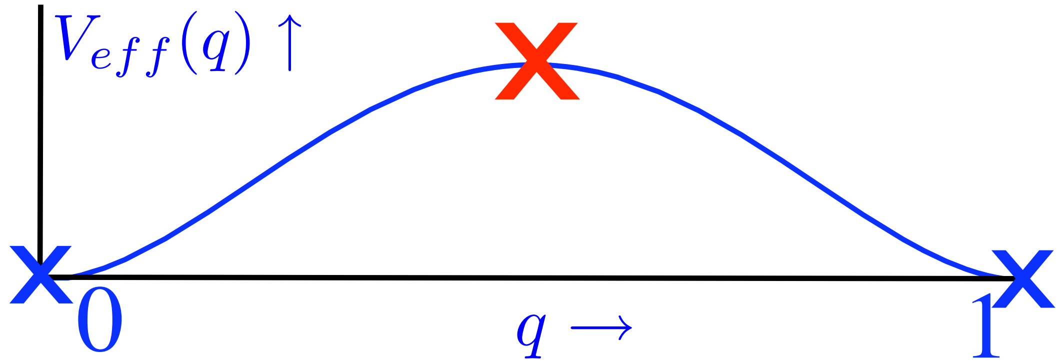
Potentials for the q's

Add *non*-perturbative terms, by *hand*, to generate <q> ≠ 0 :

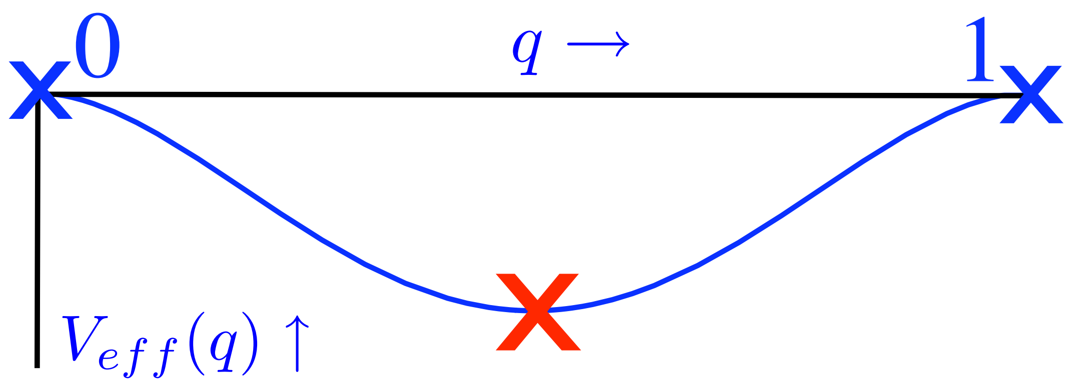
By hand? $V_{non}(q)$ from: monopoles, vortices...

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q)$$

T >> T_c: $\langle q \rangle = 0,1 \rightarrow$



T < T_c: $\langle q \rangle = ½ \rightarrow$



Three possible "phases"

Two phases are familiar:

$\langle q \rangle = 0, 1$: $\langle l \rangle = \pm 1$: "Complete" QGP: usual perturbation theory. $T >> T_c$.

$\langle q \rangle = 1/2$: $\langle l \rangle = 0$: confined phase. $T < T_c$

There is also a *third* phase, "partially" deconfined: adjoint Higgs phase

$0 < \langle q \rangle < 1/2$: $\langle l \rangle < 1$: "semi"-QGP. For some $\# T_c > T > T_c$ *What is this #?*

So *two* phase transitions are possible: from complete QGP to semi QGP,
then from semi-QGP to confined phase.

Lattice: *one* transition, to confined phase, at T_c. *No* other transition above T_c.
Still, there is an intermediate phase, the semi-QGP

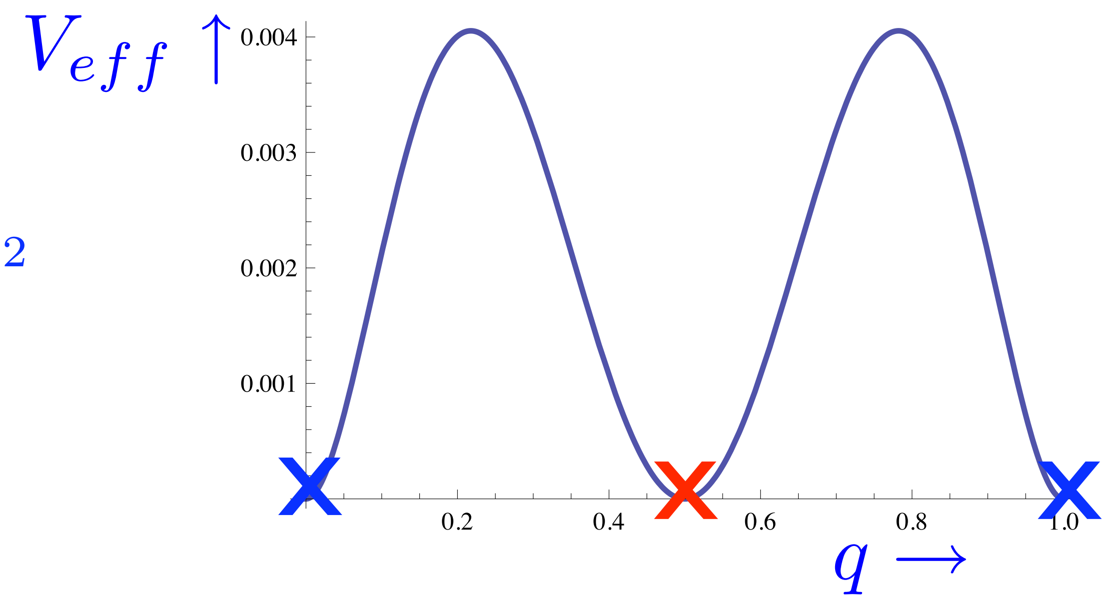
Strongly constrains possible non-perturbative terms, $V_{non}(q)$.

Getting three "phases", one transition

Simple guess: $V_{non} \sim$ loop^2,

$$V_{eff} \sim \frac{a}{\pi^2}\left(\ell^2 - 1\right) + q^2(1-q)^2$$

$$\sim q^2(1-a) - 2q^3 + \ldots$$



1s order transition *directly* from complete QGP to confined phase: no semi-QGP.

Generic if $V_{non}(q) \sim q^2$ at $q << 1$. Easy to avoid, *if* $V_{non}(q) \sim q$ for small q.
Then $\langle q \rangle \neq 0$ at all T: no complete QGP; always adjoint Higgs phase above T_c.

Imposing the symmetry of $q \leftrightarrow 1 - q$, $V_{non}(q)$ *must* include

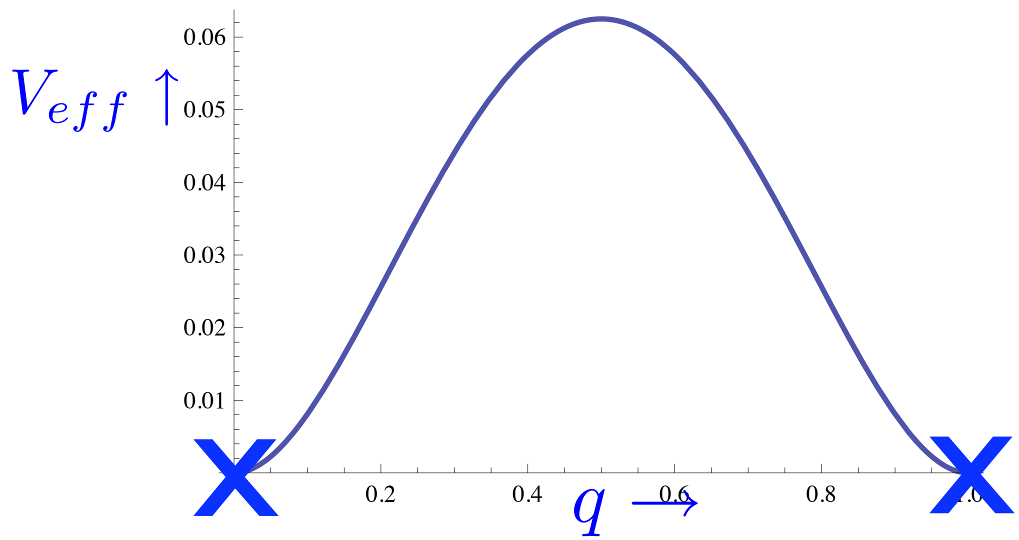
$$V_{non}(q) \sim q(1-q)$$

Cartoons of deconfinement

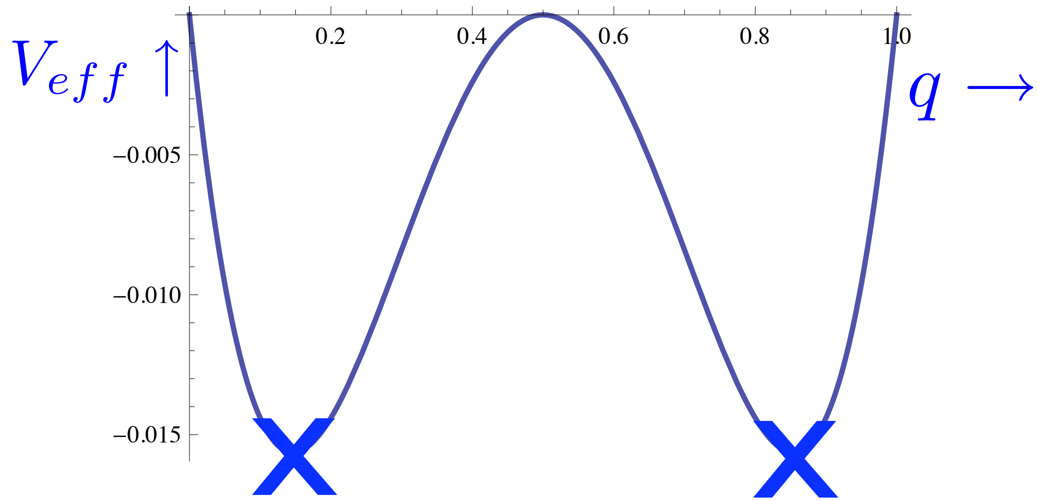
Consider:

$$V_{eff} = q^2(1-q)^2 - \, a\, q(1-q)\,, \; a \sim T_c^2/T^2$$

$\Downarrow$ a = 0: complete QGP



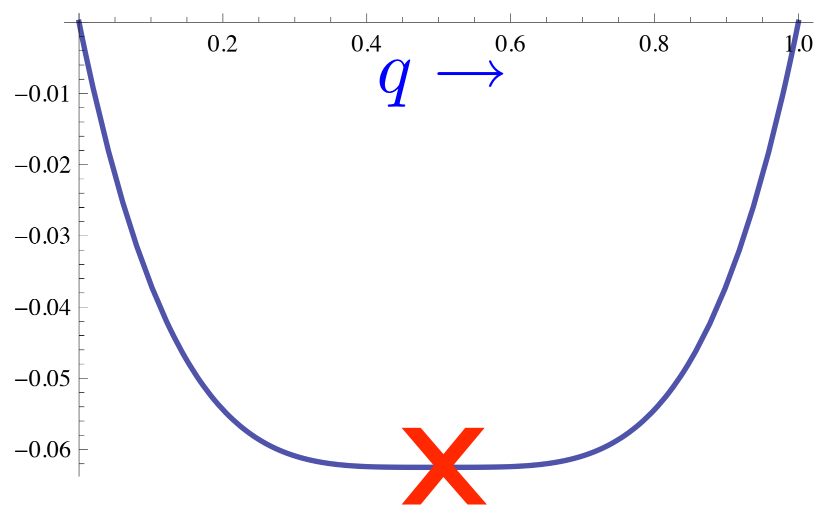
$\Downarrow$ a = ¼: semi QGP



a = ½: T_c=>

Stable vacuum at q = ½

Transition *second* order



0-parameter matrix model, N = 2

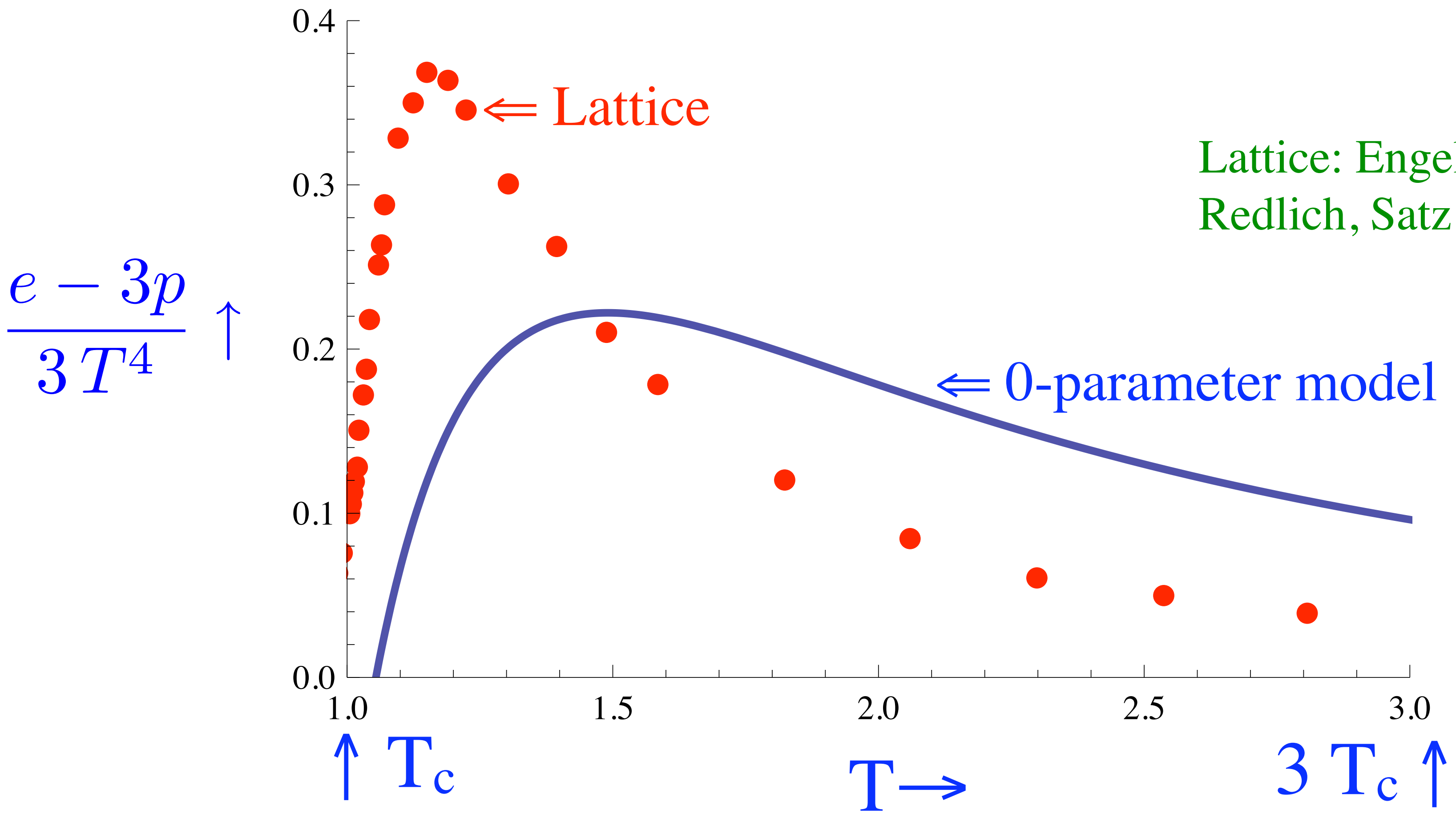
Meisinger, Miller, Ogilvie (MMO), ph/0108009:

take $V_{non} \sim T^2$

$$V_{non}(q) = \frac{4\pi^2}{3} T^2 T_c^2 \left(-\frac{c_1}{5} q(1-q) + \frac{c_3}{15} \right)$$

Two conditions: transition occurs at T_c, pressure(T_c) = 0

Fixes c_1 and c_3, *no* free parameters. *Not* close to lattice data *(from '89!)*



1-parameter matrix model, N = 2

Dumitru, Guo, Hidaka, Korthals-Altes, RDP '10: to usual perturbative potential,

$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \left(-\frac{1}{20} + q^2(1-q)^2 \right)$$

Add - *by hand* - a non-pert. potential $V_{non} \sim T^2\, T_c^2$. *Also* add a term like V_{pert}:

$$V_{non}(q) = \frac{4\pi^2}{3} T^2\, T_c^2 \left(-\frac{c_1}{5}\, q(1-q) - c_2\, q^2(1-q)^2 + \frac{c_3}{15} \right)$$

Now just like any other mean field theory. $\langle q \rangle$ given by minimum of V_{eff}:

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q) \qquad \left. \frac{d}{dq} V_{eff}(q) \right|_{q=\langle q \rangle} = 0$$

$\langle q \rangle$ depends nontrivially on temperature.

Pressure value of potential at minimum: $$p(T) = -V_{eff}(\langle q \rangle)$$

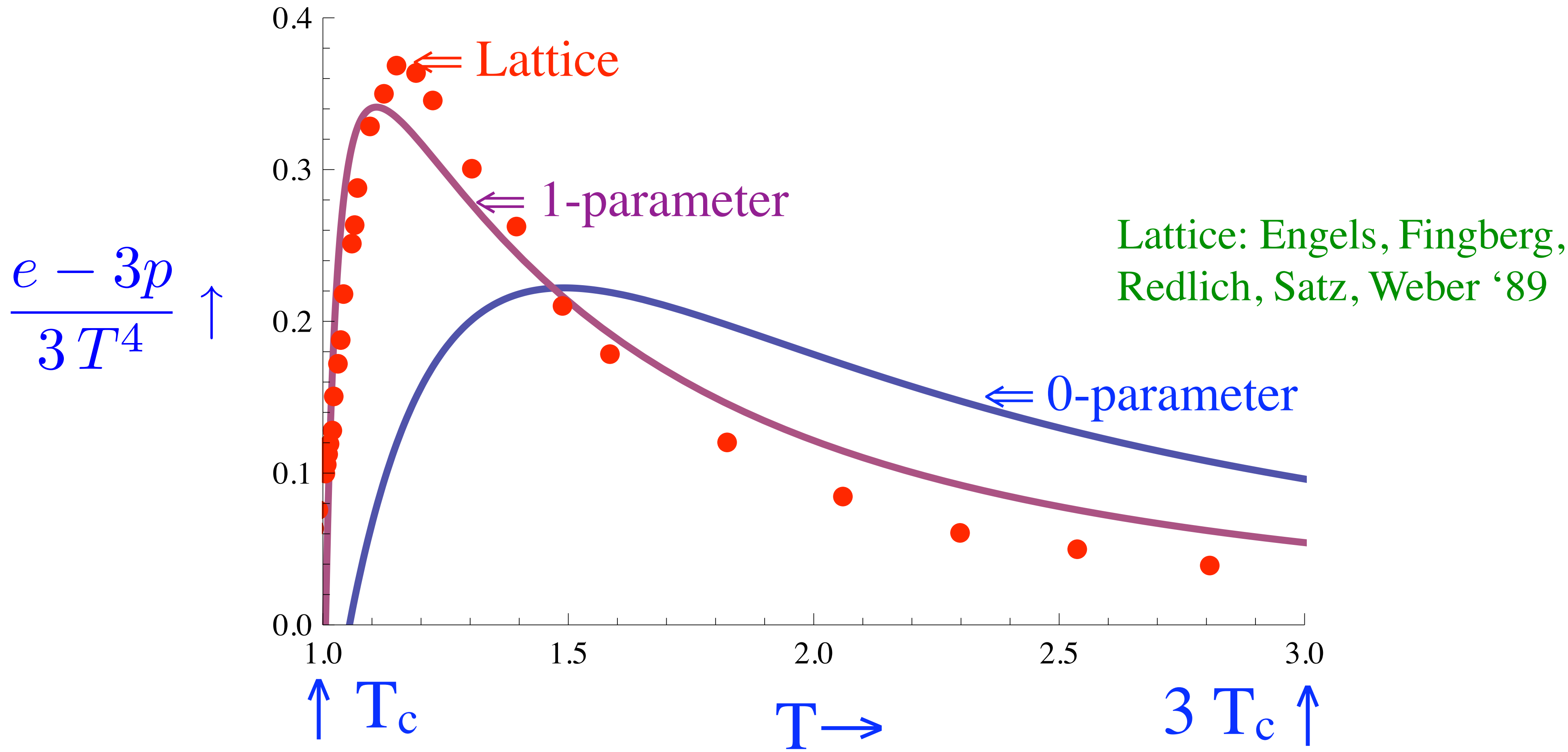
Lattice vs matrix models, N = 2

Choose c_2 to fit $e\text{-}3p/T^4$: optimal choice

$$c_1 = 0.23\,,\ c_2 = .91\,,\ c_3 = 1.11$$

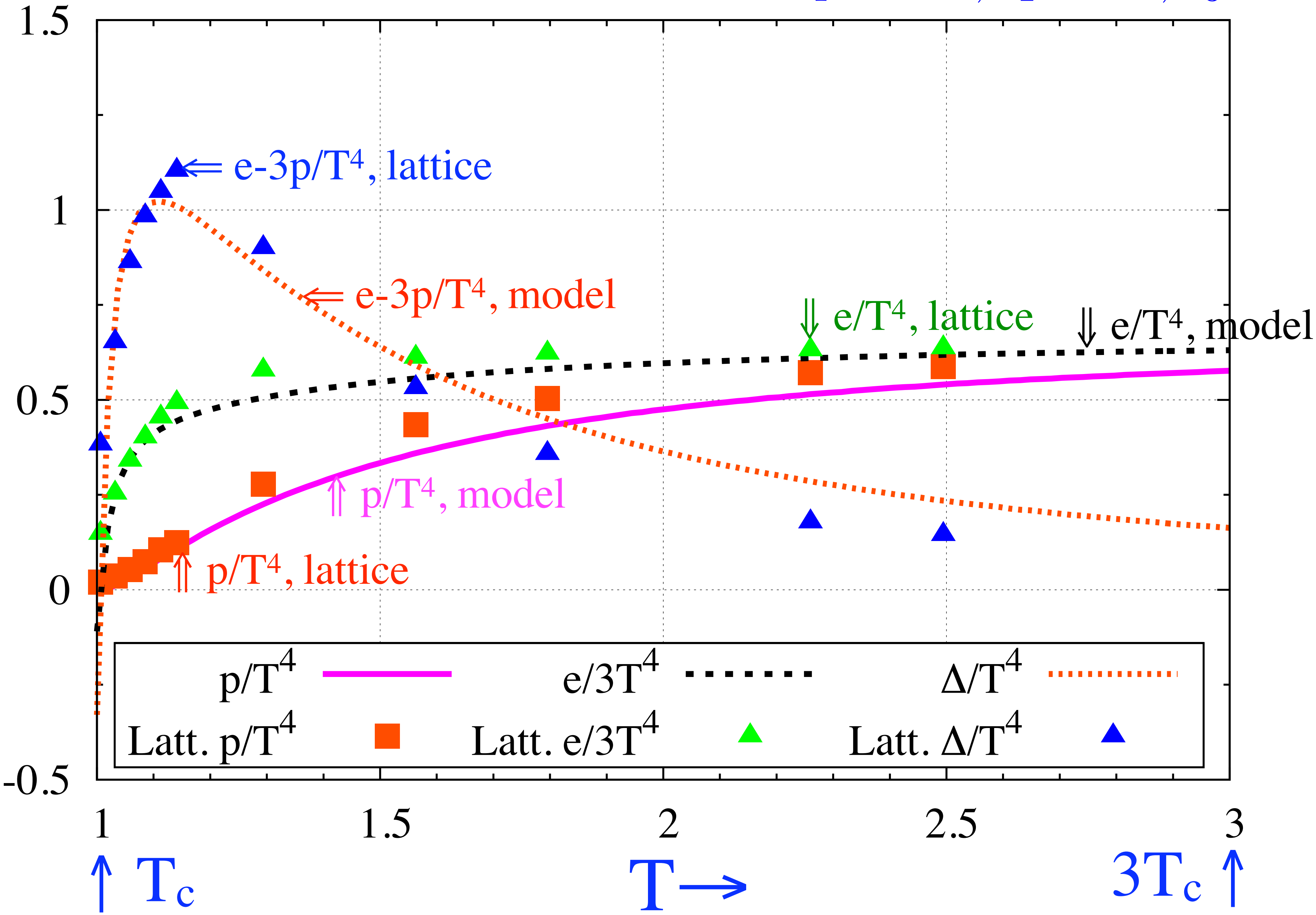
Reasonable fit to $e\text{-}3p/T^4$; also to p/T^4, e/T^4.

N.B.: $c_2 \sim 1$. At T_c, terms $\sim q^2(1\text{-}q)^2$ *almost* cancel.



Lattice vs 1-parameter model, N = 2

$c_1 = 0.23\,,\; c_2 = .91\,,\; c_3 = 1.11$



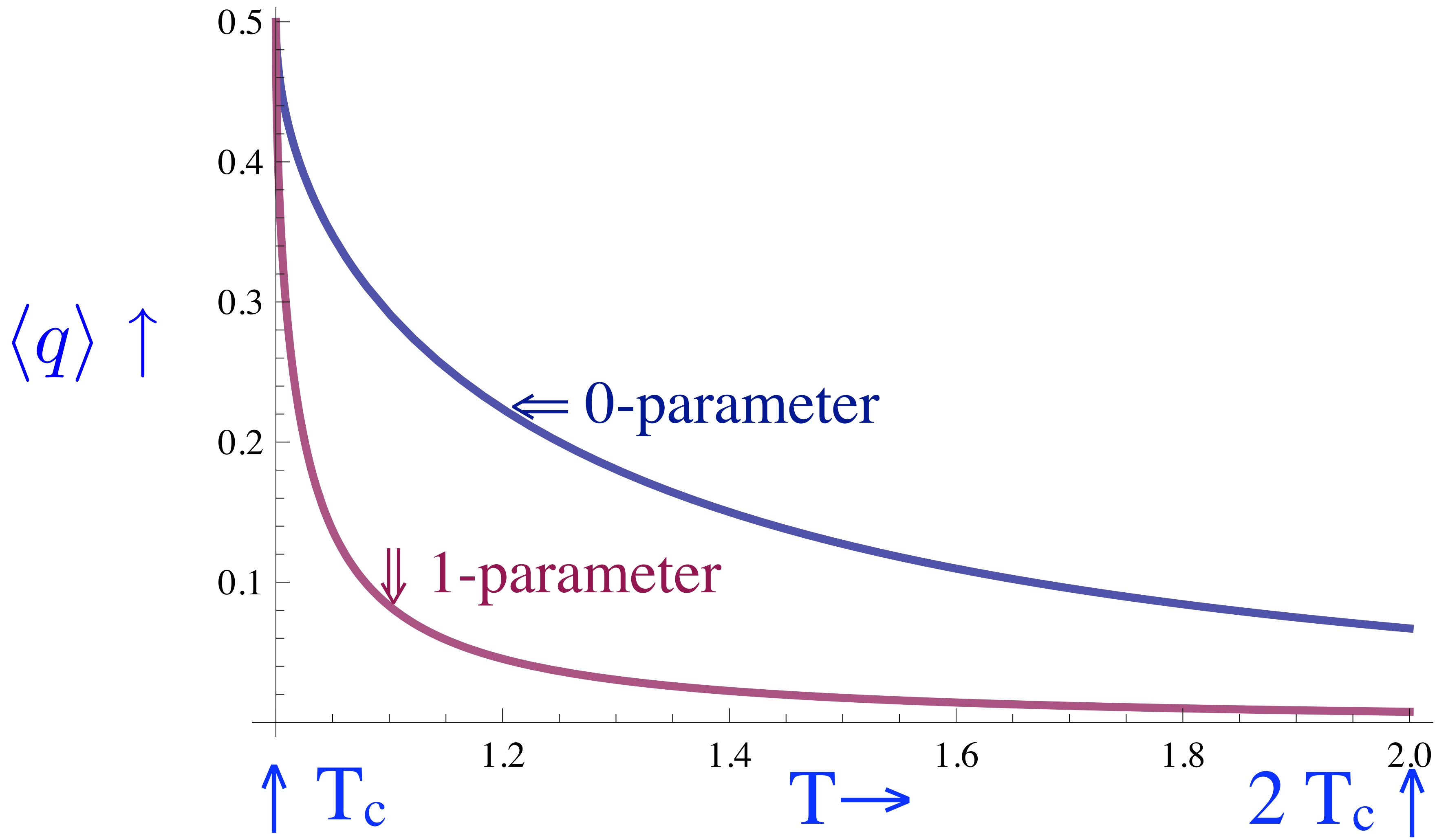
Width of transition region, 0- vs 1-parameter

1-parameter model: get sharper e-3p/T^4 because $\langle q \rangle$ -> 0 *much* quicker above T_c.

Physically: sharp e-3p/T^4 implies region where $\langle q \rangle$ is significant is *narrow*

N.B.: $\langle q \rangle \neq 0$ at all T, but numerically, *negligible* above ~ 1.2 T_c; p ~ $\langle q \rangle^2$.

Above ~1.2 T_c, the T^2 term in the pressure is due *entirely* to the *constant* term, c_3!



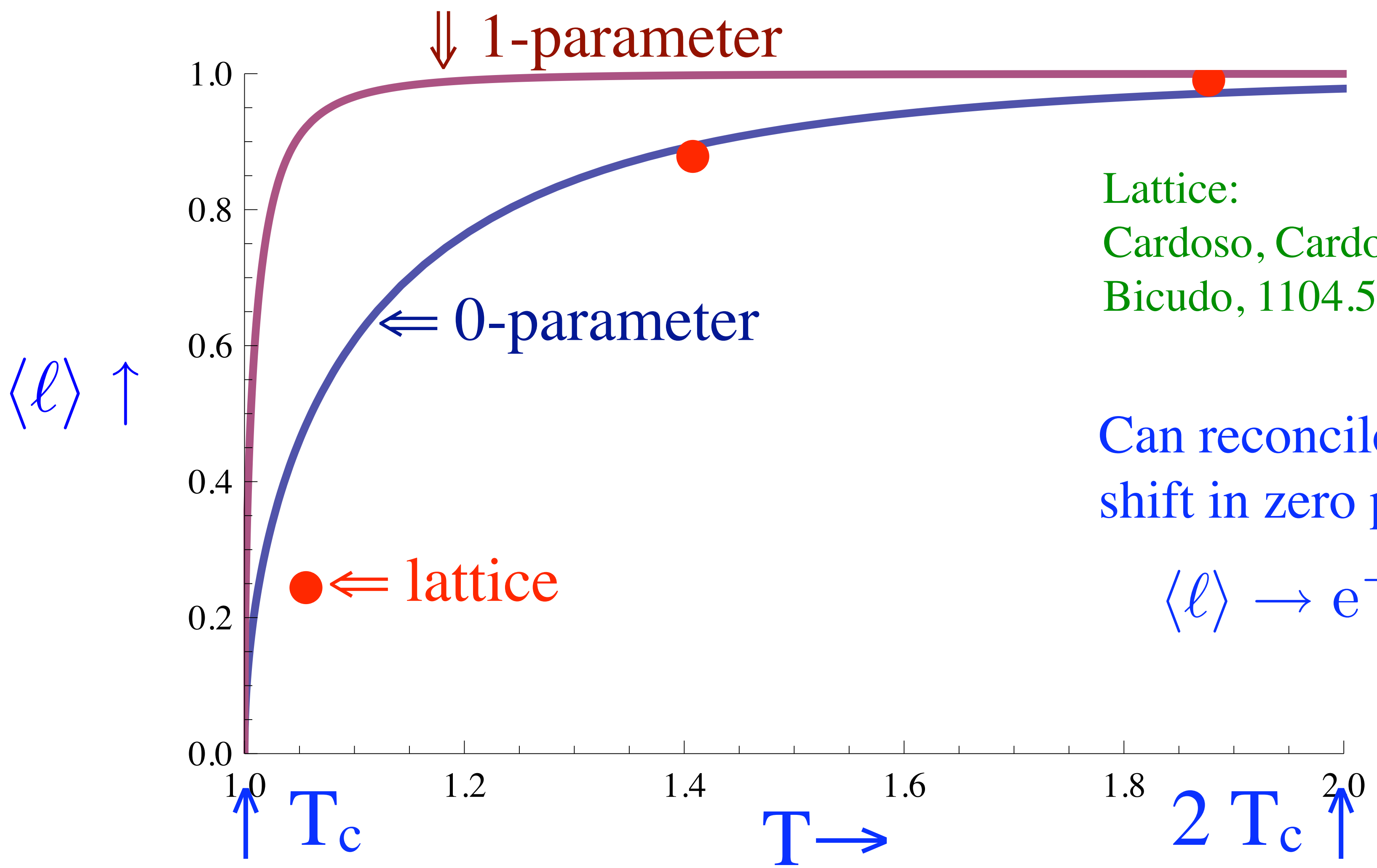
Polyakov loop: 1-parameter matrix model ≠ lattice

Lattice: *renormalized* Polyakov loop. 0-parameter model: close to lattice

1-parameter model: *sharp* disagreement. $\langle l \rangle$ rises to ~ 1 *much* faster - ?

Sharp rise also found using Functional Renormalization Group (FRG):

Braun, Gies, Pawlowski, 0708.2413; Marhauser, Pawlowski, 0812.1144



Lattice: Cardoso, Cardoso, Bicudo, 1104.5432

Can reconcile by (arbitrary) shift in zero point energy

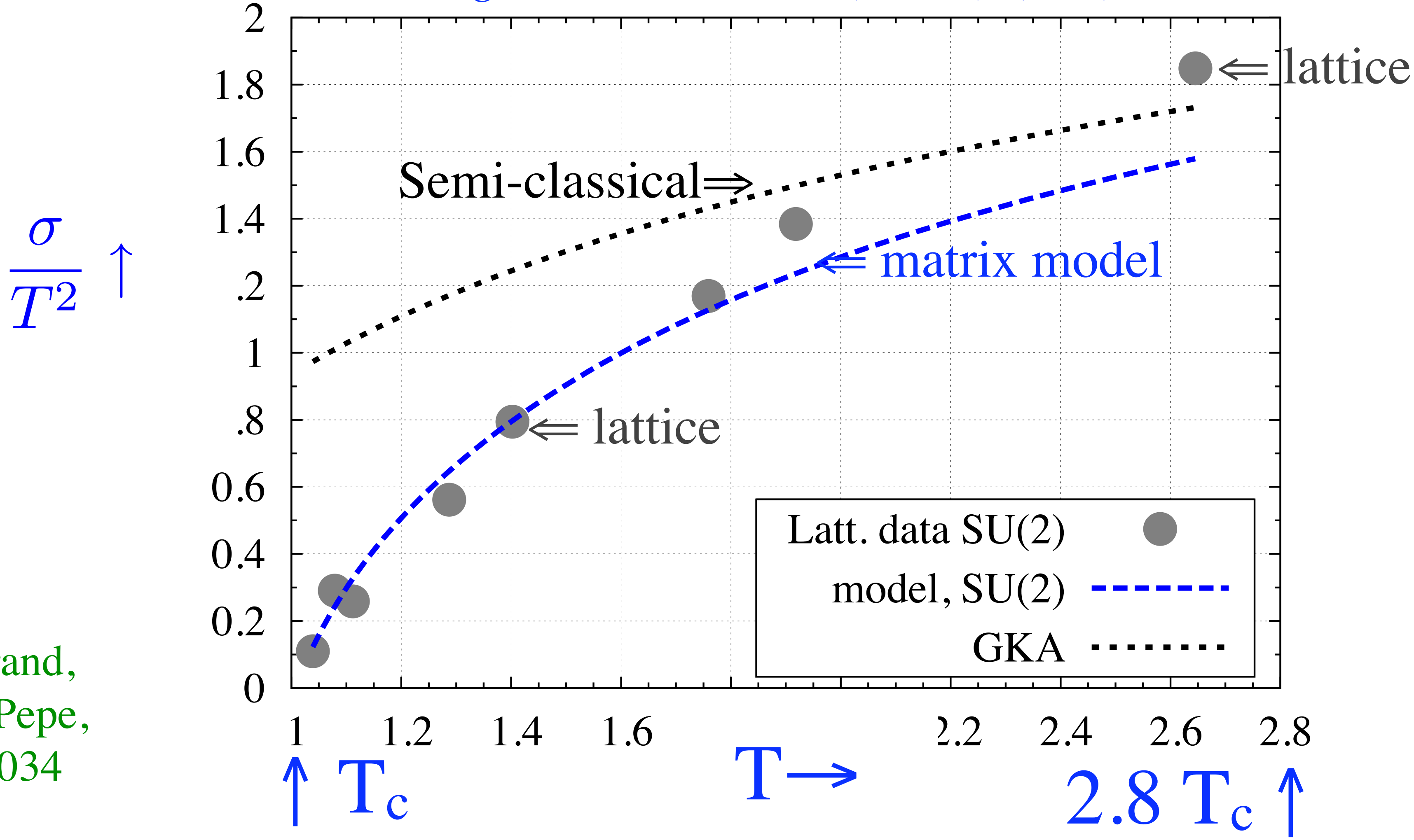
$$\langle \ell \rangle \rightarrow e^{-E_0/T} \langle \ell \rangle$$

Interface tension, N = 2

σ vanishes as $T \to T_c$, $\sigma \sim (t-1)^{2\nu}$.
Ising $2\nu \sim 1.26$; Lattice: ~ 1.32.
Matrix model: ~ 1.5: c_2 important.

$$\sigma(T) = \frac{4\pi^2 T^2}{3\sqrt{6g^2}} \frac{(t^2-1)^{3/2}}{t\,(t^2-c_2)}\,,\; t = \frac{T}{T_c}$$

Semi-class.: GKA '04. *Include* corr.'s $\sim g^2$ in matrix $\sigma(T)$ (ok when $T > 1.2\, T_c$)
N.B.: width of interface *diverges* as $T \to T_c$, $\sim \sqrt{(t^2 - c_2)/(t^2-1)}$.



Lattice:
de Forcrand,
D'Elia, Pepe,
lat/0007034

Adjoint Higgs phase, N = 2

$A_0^{cl} \sim q\,\sigma_3$, so $\langle q \rangle \neq 0$ generates an (adjoint) Higgs phase:

RDP, ph/0608242; Unsal & Yaffe, 0803.0344, Simic & Unsal, 1010.5515

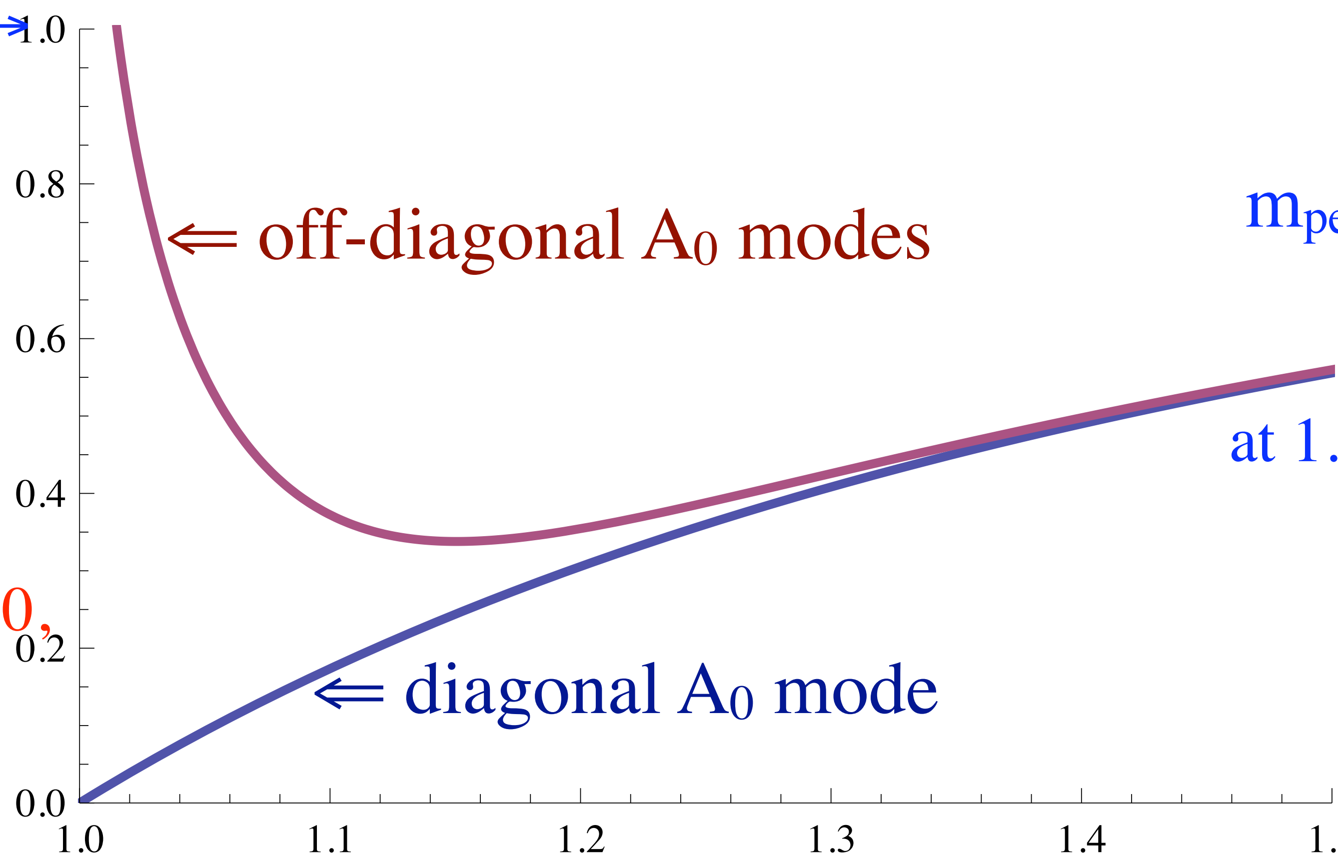
In background field, $A = A_0^{cl} + A^{qu}$: $D_0^{cl} A^{qu} = \partial_0 A^{qu} + i\, g\, [A_0^{cl}, A^{qu}]$

Fluctuations $\sim \sigma_3$ not Higgsed, $\sim \sigma_{1,2}$ Higgsed, get mass $\sim 2\,\pi\, T \langle q \rangle$

Hence when $\langle q \rangle \neq 0$, for $T < 1.2\, T_c$, *splitting* of masses:

$1 \rightarrow$

$\dfrac{m_{eff}}{m_{pert}} \uparrow$



$m_{pert} = \sqrt{2/3}\, g\, T$:

$m/m_{pert} \sim .56$ at $1.5\, T_c$, from V_{non}.

At T_c: $m_{diag} = 0$, $m_{off} \sim 2\, m_{pert}$.

$T \rightarrow$

$\uparrow T_c$ $\uparrow 1.2\, T_c$ $1.5\, T_c \uparrow$

Why the deconfining transition is of first order for *all* N $\geq$ 3

General potential for any SU(N)

Ansatz: constant, diagonal matrix
i, j = 1...N

$$A_0^{ij} = \frac{2\pi T}{g} q_i \, \delta^{ij} \qquad \mathbf{L}_{ij} = \mathrm{e}^{2\pi i \, q_j} \, \delta_{ij}$$

For SU(N), $\Sigma_{j=1...N}\, q_j = 0$. Hence N-1 independent q_j's, = # diagonal generators.

At 1-loop order, the perturbative potential for the q_j's is

$$V_{pert}(q) = \pi^2 \, T^4 \, \left(-\frac{N^2-1}{45} + \frac{2\pi^2}{3} \sum_{i,j} q_{ij}^2 (1 - q_{ij})^2 \right) \;,\; q_{ij} = |q_i - q_j|$$

As before, *assume* a non-perturbative potential ~ $T^2 \, T_c^2$:

$$V_{non}(q) = \frac{2\pi^2}{3} T^2 T_c^2 \left(-\frac{c_1}{5} \sum_{i,j} q_{ij}(1 - q_{ij}) - c_2 \sum_{i,j} q_{ij}^2 (1 - q_{ij})^2 + \frac{4}{15} c_3 \right)$$

Path to Z(3), three colors

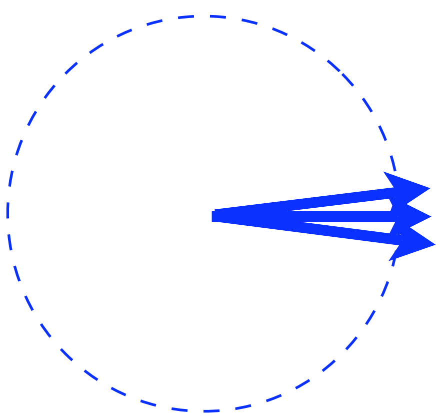
SU(3): *two* diagonal λ's, so *two* q's:

$$A_0 = \frac{2\pi T}{3\,g}\left(q_3\,\lambda_3 + q_8\,\lambda_8\right)$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \; \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

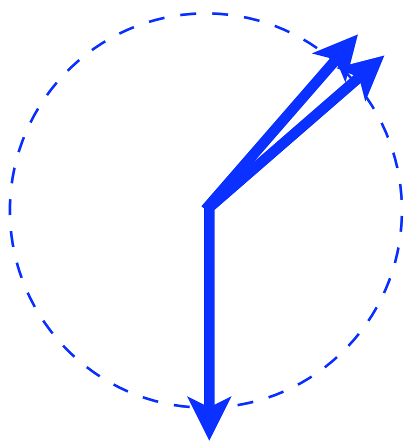
Z(3) paths: move along λ_8, not λ_3: $q_8 \neq 0$, $q_3 = 0$.

$$\mathbf{L} = \mathrm{e}^{2\pi i q_8 \lambda_8/3}$$

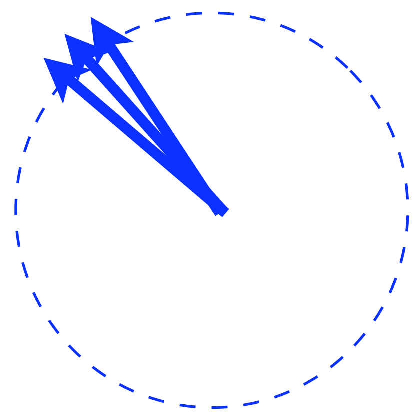


$$q_8 = 0$$

$$\mathbf{L} = \mathbf{1}$$



$$q_8 = 3/8$$



$$q_8 = 1$$

$$\mathbf{L} = \mathrm{e}^{2\pi i/3}\,\mathbf{1}$$

Path to confinement, three colors

Now move along λ_3: $$\mathbf{L} = \mathrm{e}^{2\pi i q_3 \lambda_3/3}$$

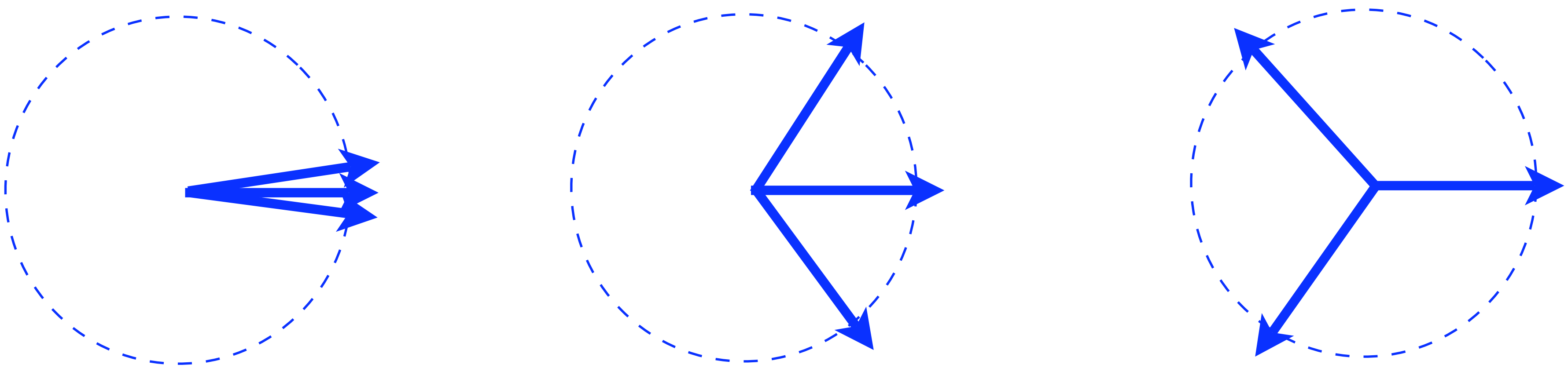
In particular, consider $q_3 = 1$:

Elements of $e^{2\pi i/3}$ $\mathbf{L}_c$ *same* as those of $\mathbf{L}_c$.

Hence tr $\mathbf{L}_c$ = tr $\mathbf{L}_c^2 = 0$: $\mathbf{L}_c$ *confining* vacuum

$$\mathbf{L}_c = \begin{pmatrix} \mathrm{e}^{2\pi i/3} & 0 & 0 \\ 0 & \mathrm{e}^{-2\pi i/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Path to confinement: along λ_3, not λ_8, $q_3 \neq 0$, $q_8 = 0$.



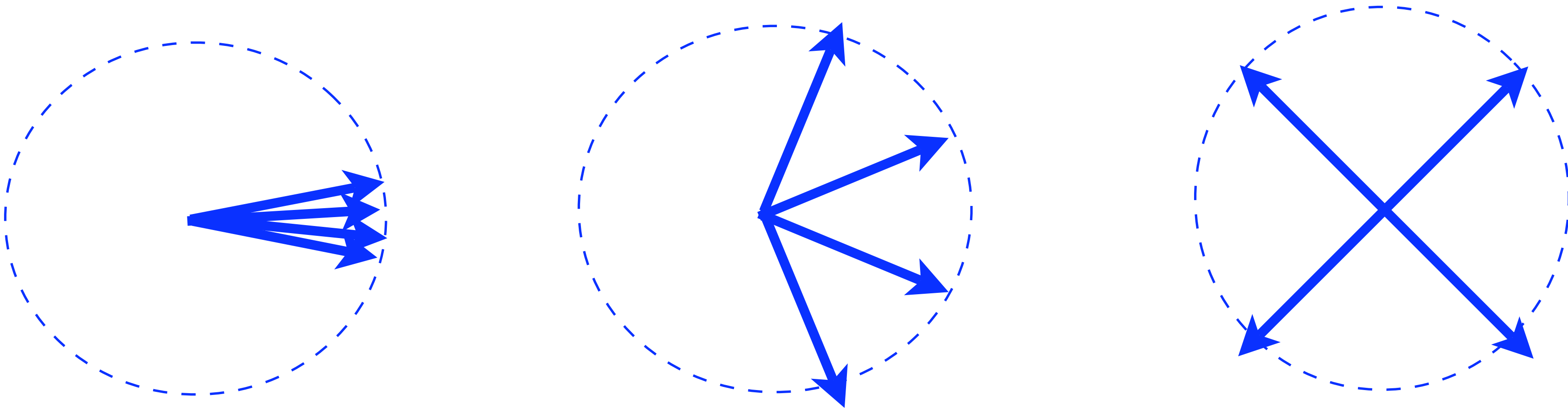
$q_3 = 0$, $\ell = 1$ | $q_3 = 3/8$, $\ell \approx .8$ | $q_3 = 1$, $\ell = 0$

Path to confinement, four colors

Ansatz: move to the confining vacuum along *one* direction, q_j^c, with uniform spacing of eigenvalues. Close to the exact solution, determined numerically.

Perturbative vacuum: $q = 0$.
Confining vacuum: $q = 1$.
Four colors:

$$q_j^c = \left(\frac{2j - N - 1}{2N}\right) q \, , \; j = 1 \ldots N$$



$q = 0$, $\ell = 1$

$q = 1/2$, $\ell \approx .65$

$q = 1$, $\ell = 0$

Why deconfinement is of first order for *all* N ≥ 3

Define $\phi = 1 - q$,
Confining point $\phi = 0$

$$V_{tot} = \frac{\pi^2(N^2-1)}{45} T_c^4 \, t^2 \, (t^2 - 1) \, \widetilde{V}(\phi, t) \, , \; t = \frac{T}{T_c}$$

$$\widetilde{V}(\phi, t) = -\, m_\phi^2 \, \phi^2 - 2 \left(\frac{N^2 - 4}{N^2} \right) \phi^3 + \left(2 - \frac{3}{N^2} \right) \phi^4$$

$$m_\phi^2 = 1 + \frac{6}{N^2} - \frac{c_1}{t^2 - c_2}$$

No term linear in ϕ: confining vacuum center of Weyl chamber.
Cubic term in ϕ for *all* N ≥ 3. Not special to particular ansatz.
Cubic terms, and so a first order transition, are *ubiquitous*.

Special to matrix model, with the q_i's elements of Lie *algebra*.

Svetitsky and Yaffe '80: V_{eff}(loop) ⇒ 1st order *only* for N=3; loop in Lie *group*

Also 1st order for N ≥ 3 with FRG: Braun, Eichhorn, Gies, Pawlowski, 1007.2619.

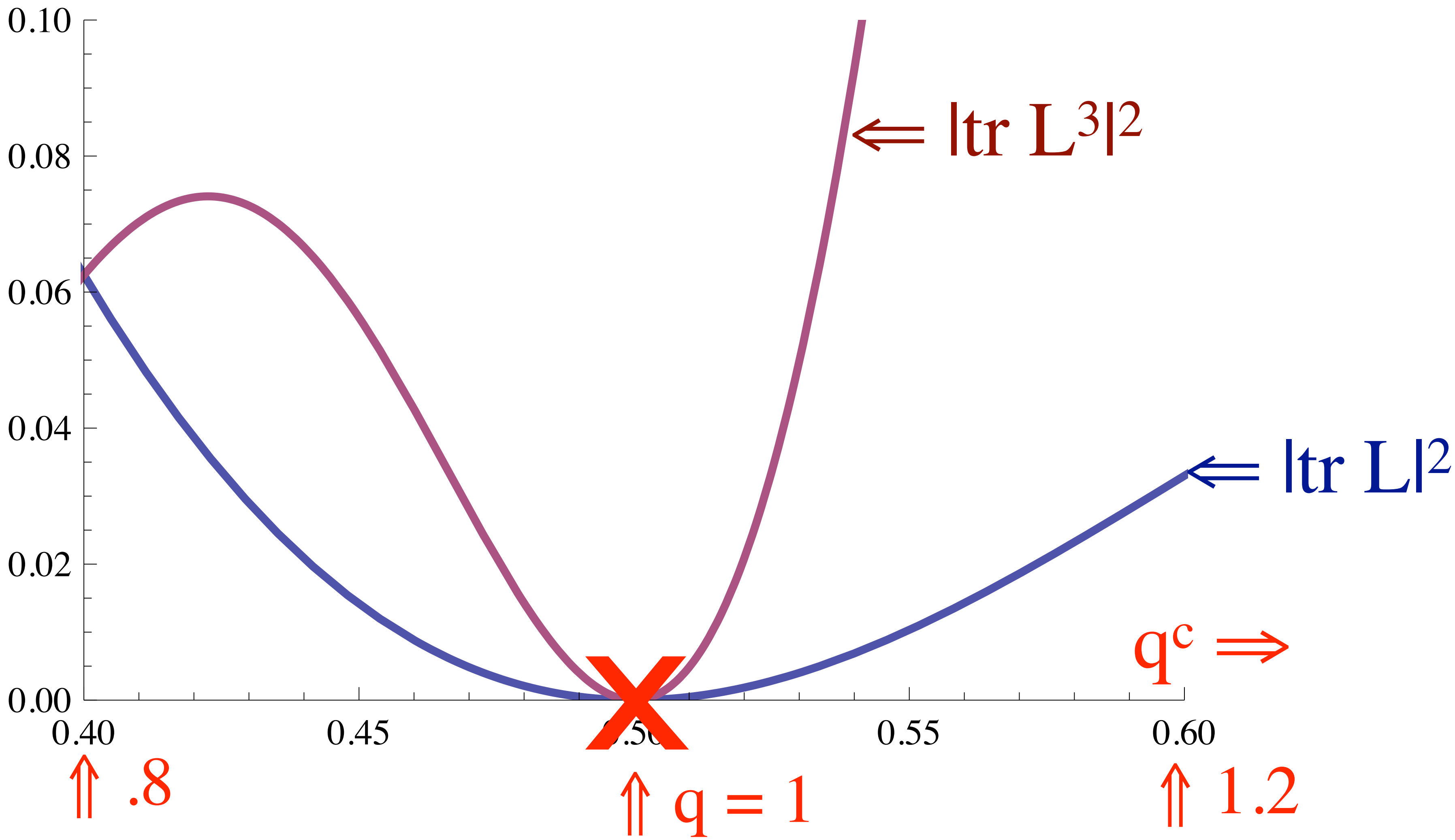
Cubic term for four colors

Construct V_{eff} either from q's, or equivalently, loops: tr $\mathbf{L}$, tr $\mathbf{L}^2$, tr $\mathbf{L}^3$....

N = 4: $|\mathrm{tr}\,\mathbf{L}|^2$ and $|\mathrm{tr}\,\mathbf{L}^3|^2$ *not* symmetric about q = 1, so cubic terms, ~ $(q - 1)^3$.

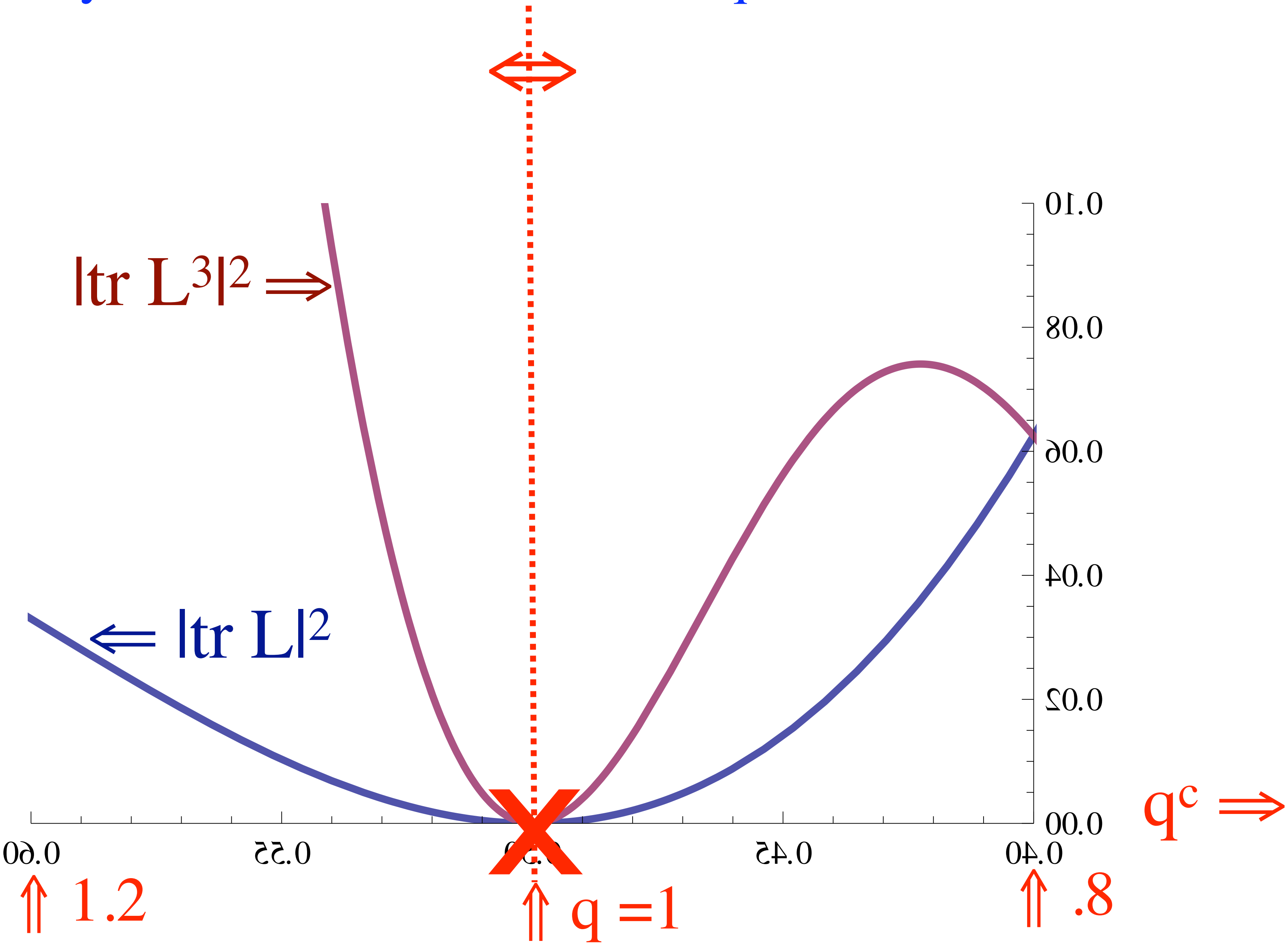
($|\mathrm{tr}\,\mathbf{L}^2|^2$ symmetric, residual Z(2) symmetry)

Cubic terms *special* to moving along q_c in a *matrix* model. *Not* true in loop model



Cubic term for four colors

Asymmetric in reflection about q = 1

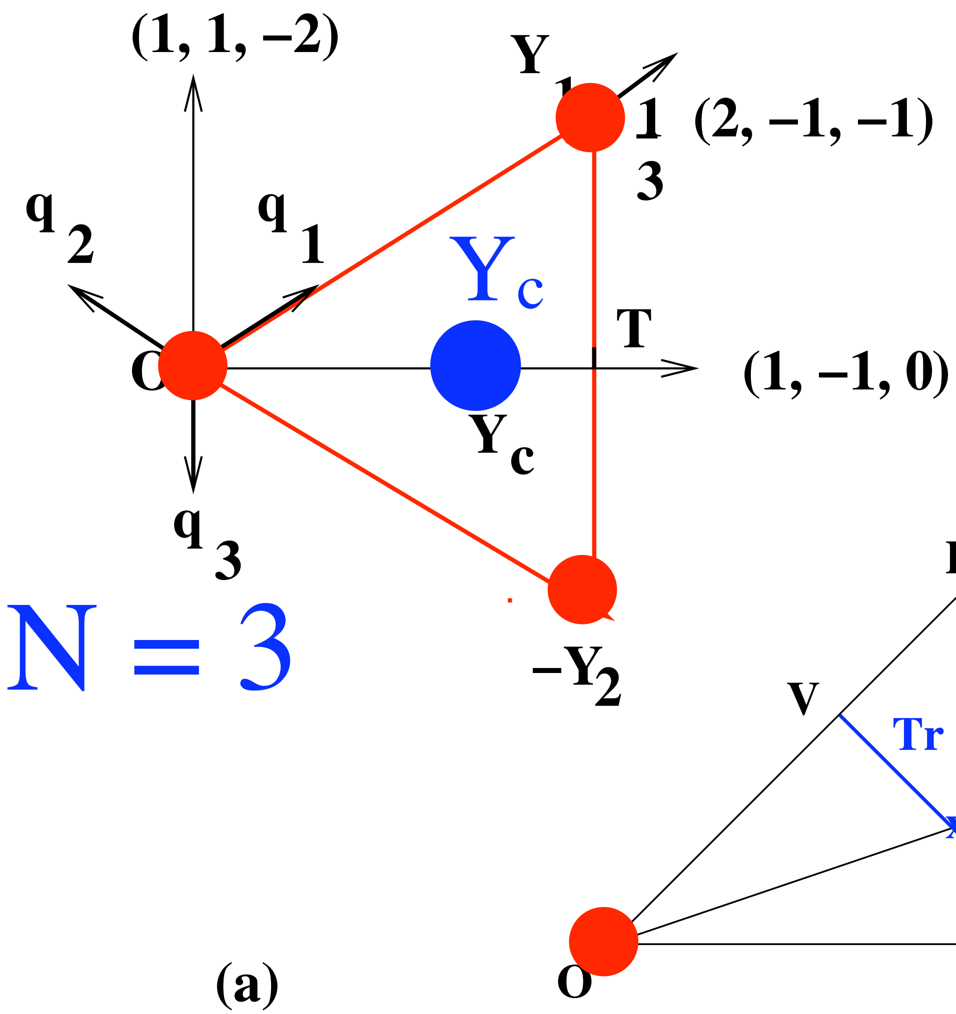


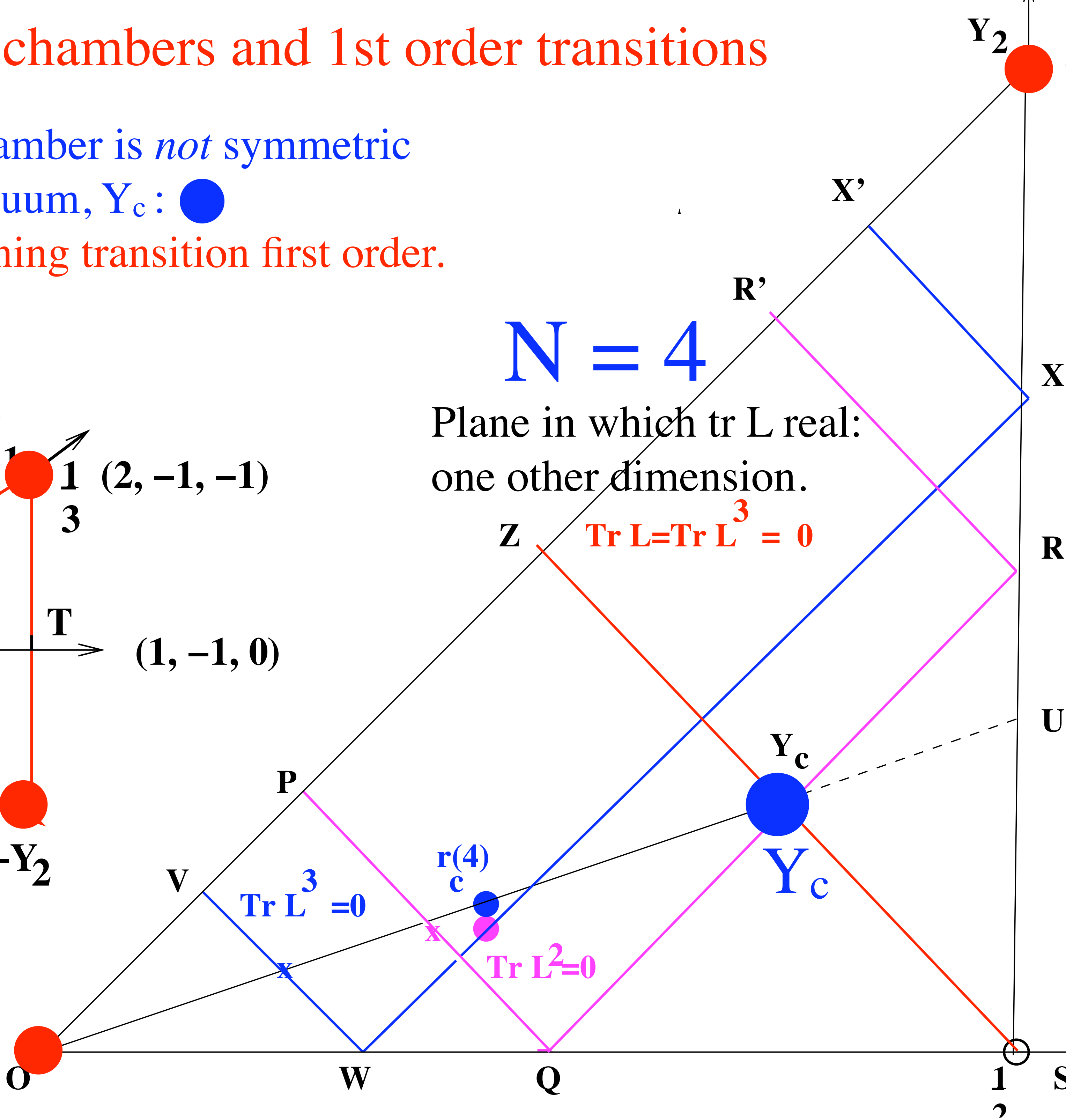
Weyl chambers and 1st order transitions

For N ≥ 3, the Weyl chamber is *not* symmetric about the confining vacuum, Y_c : ●

This drives the deconfining transition first order.

Z(N) vacua: ●





Matrix model: $N \geq 3$

One parameter model

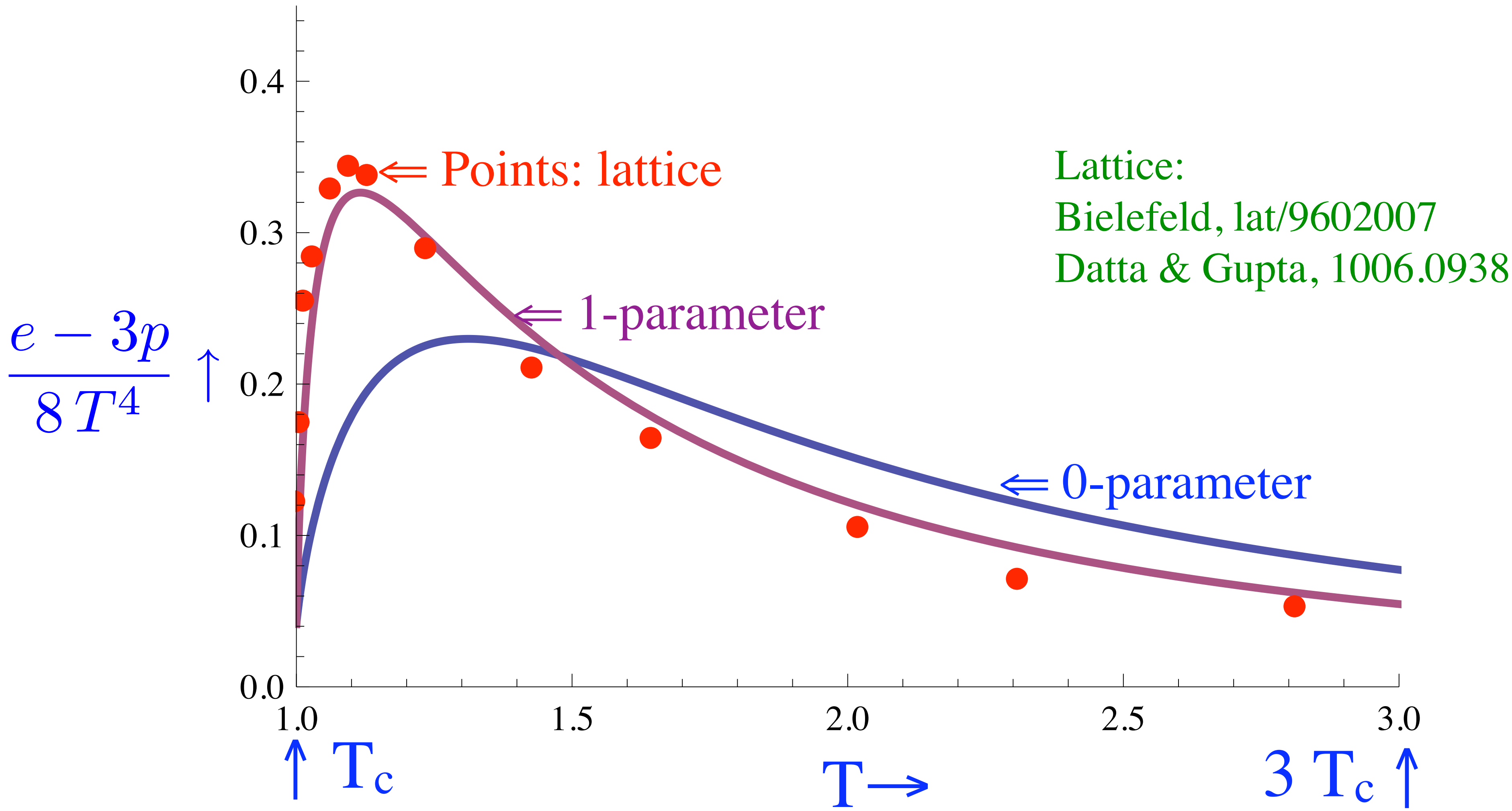
Lattice vs 0- and 1- parameter matrix models, N = 3

Results for N=3 similar to N=2.

0-parameter model way off.

Good fit e-3p/T^4 for 1-parameter model, $c_1 = 0.32\,,\ c_2 = 0.83\,,\ c_3 = 1.13$

Again, $c_2 \sim 1$, so at T_c, terms $\sim q^2(1-q)^2$ *almost* cancel.



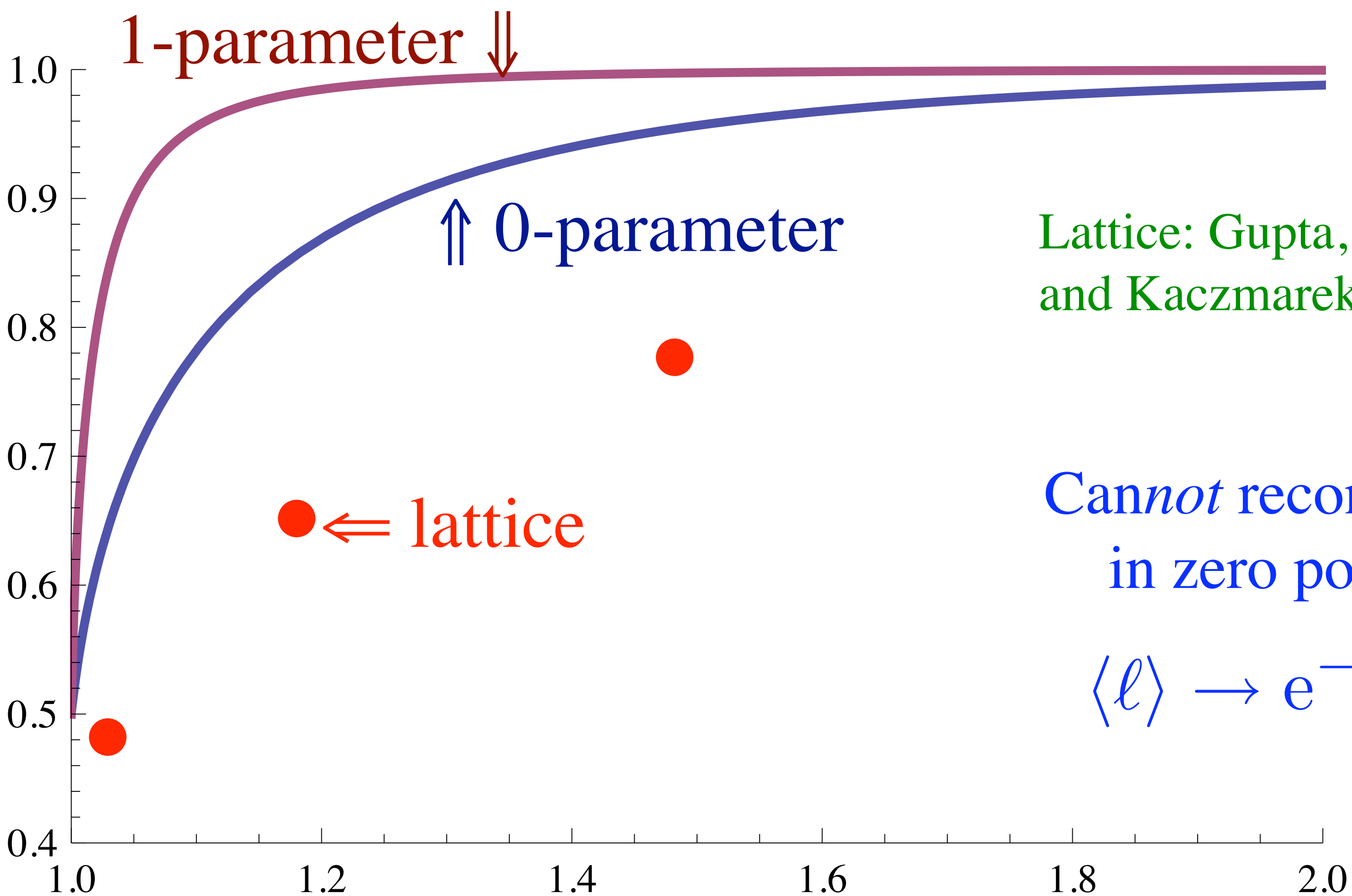
Polyakov loop: matrix models ≠ lattice

Renormalized Polyakov loop from lattice does *not* agree with *either* matrix model

$\langle l \rangle$ - 1 ~ $\langle q \rangle^2$: By 1.2 T_c, $\langle q \rangle$ ~ .05, negligible.

Again, for T > 1.2 T_c, the T^2 term in pressure due *entirely* to the *constant* term, c_3!

Rapid rise of $\langle l \rangle$ as with FRG: Braun, Gies, Pawlowski, 0708.2413



$\langle \ell \rangle$ ↑

↑ T_c T→ 2 T_c ↑

Lattice: Gupta, Hubner, and Kaczmarek, 0711.2251.

Can*not* reconcile by shift in zero point energy

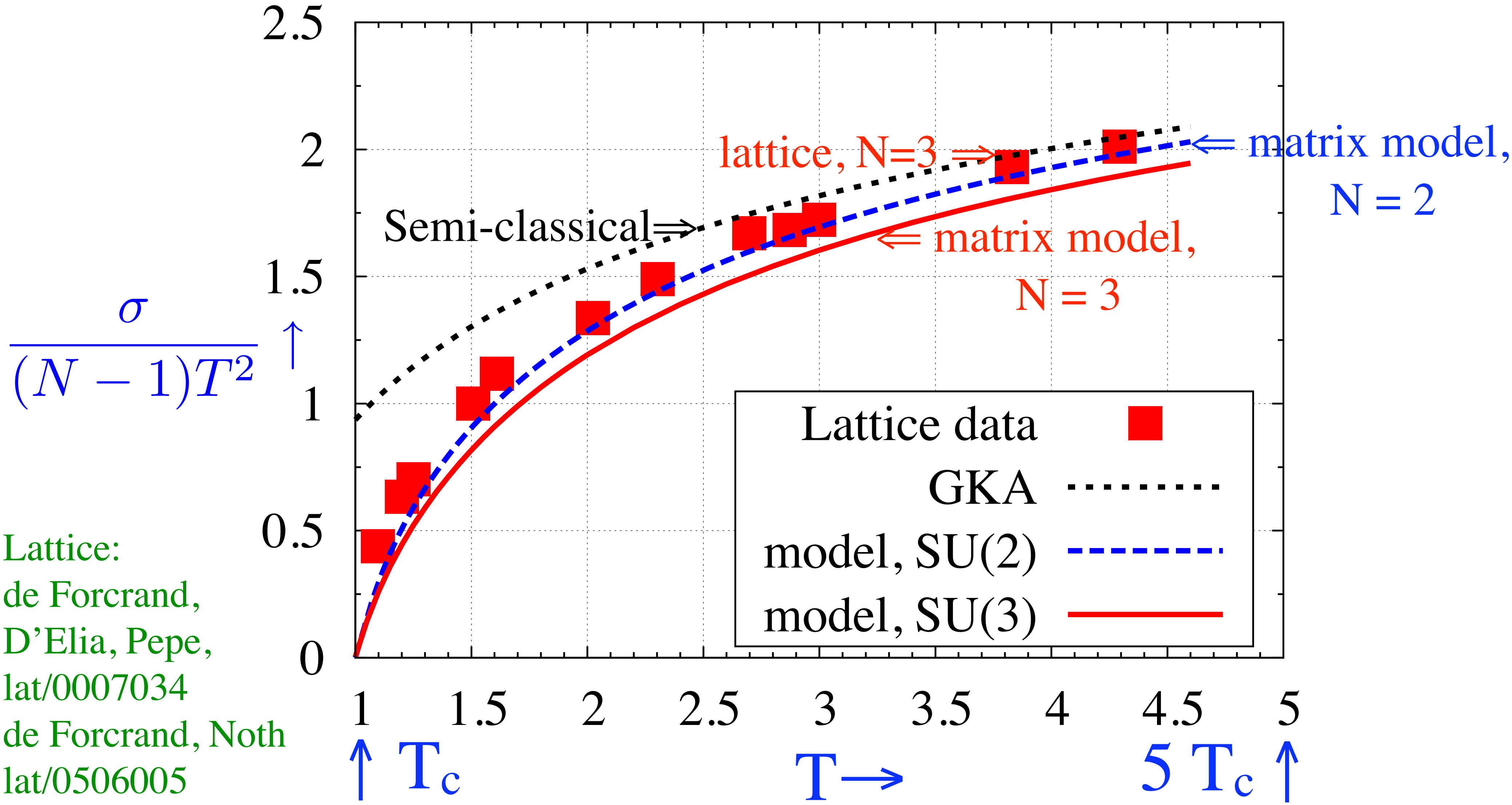
$$\langle \ell \rangle \rightarrow e^{-E_0/T} \langle \ell \rangle$$

Interface tension, N = 2 and 3

Order-order interface tension, σ, from matrix model close to lattice.

For $T > 1.2\,T_c$, path along λ_8; for $T < 1.2\,T_c$, along *both* λ_8 and λ_3.

$\sigma(T_c)/T_c^2$ nonzero but *small*, ~ .02. Results for N =2 and N = 3 similar - ?



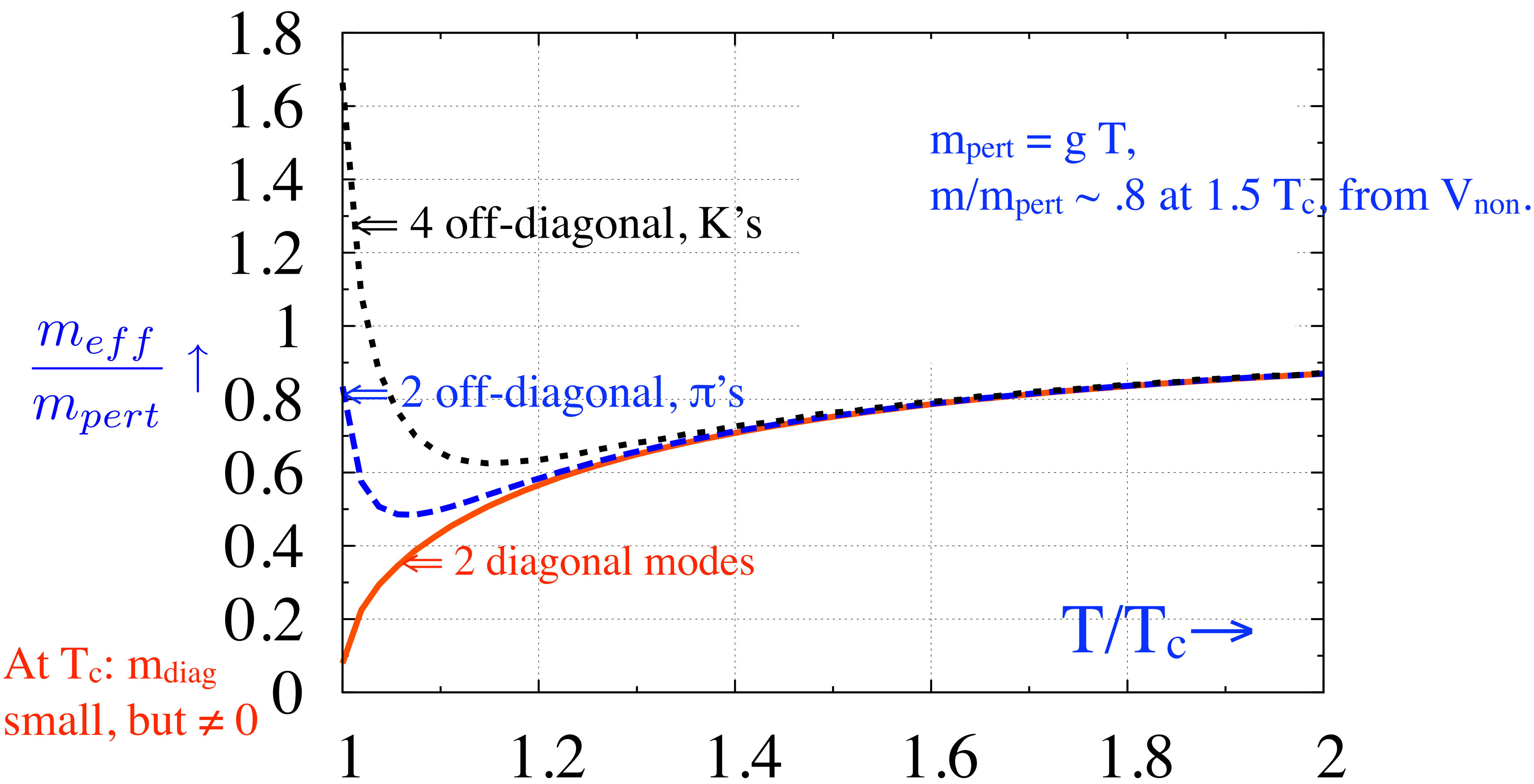
Lattice:
de Forcrand, D'Elia, Pepe, lat/0007034
de Forcrand, Noth lat/0506005

Adjoint Higgs phase, N = 3

For SU(3), deconfinement along $A_0^{cl} \sim q\,\lambda_3$. Masses $\sim [\lambda_3, \lambda_i]$: two off-diagonal.

Splitting of masses only for $T < 1.2\ T_c$:

Measureable from singlet potential, $\langle \mathrm{tr}\, L^\dagger(x)\, L(0) \rangle$, over *all* x.



Matrix model, $N = 3$

To get the latent heat right, two parameter model.

Thermodynamics, interface tensions improve

Latent heat, and a 2-parameter model

Latent heat, $e(T_c)/T_c^4$: 1-parameter model too small:

1-para.: 0.33. BPK: 1.40 ± .1; DG: 1.67 ± .1.

$$c_3(T) = c_3(\infty) + \frac{c_3(1) - c_3(\infty)}{t^2}\ ,\ t = \frac{T}{T_c}$$

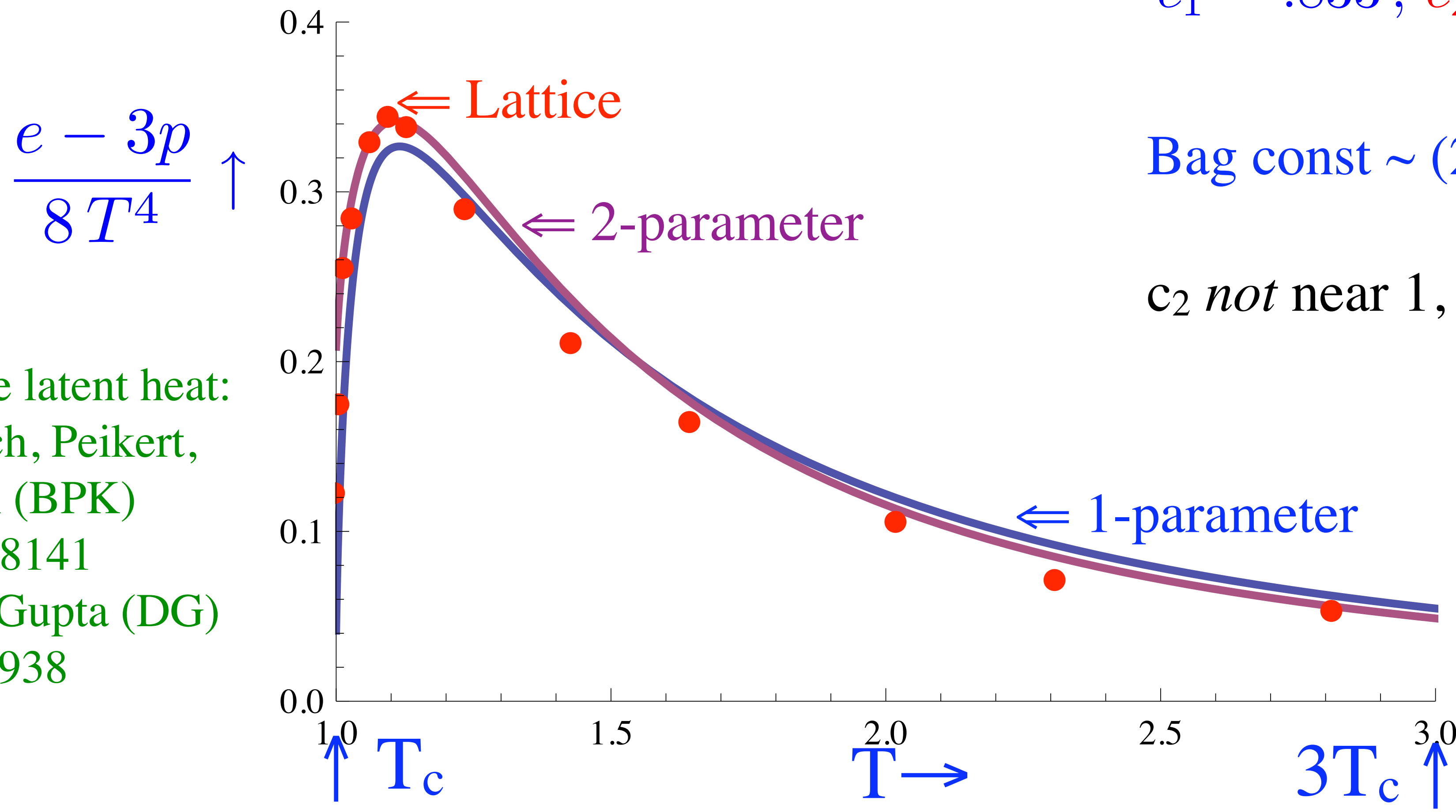
2-parameter model, $c_3(T)$. Like MIT bag constant

WHOT: $c_3(\infty) \sim 1$. *Fit* $c_3(1)$ to DG latent heat

Fits lattice for $T < 1.2\ T_c$, overshoots above.

$$c_3(1) = 1.33\ ,\ c_3(\infty) = .95$$

$$c_1 = .833\ ,\ c_2 = .552$$

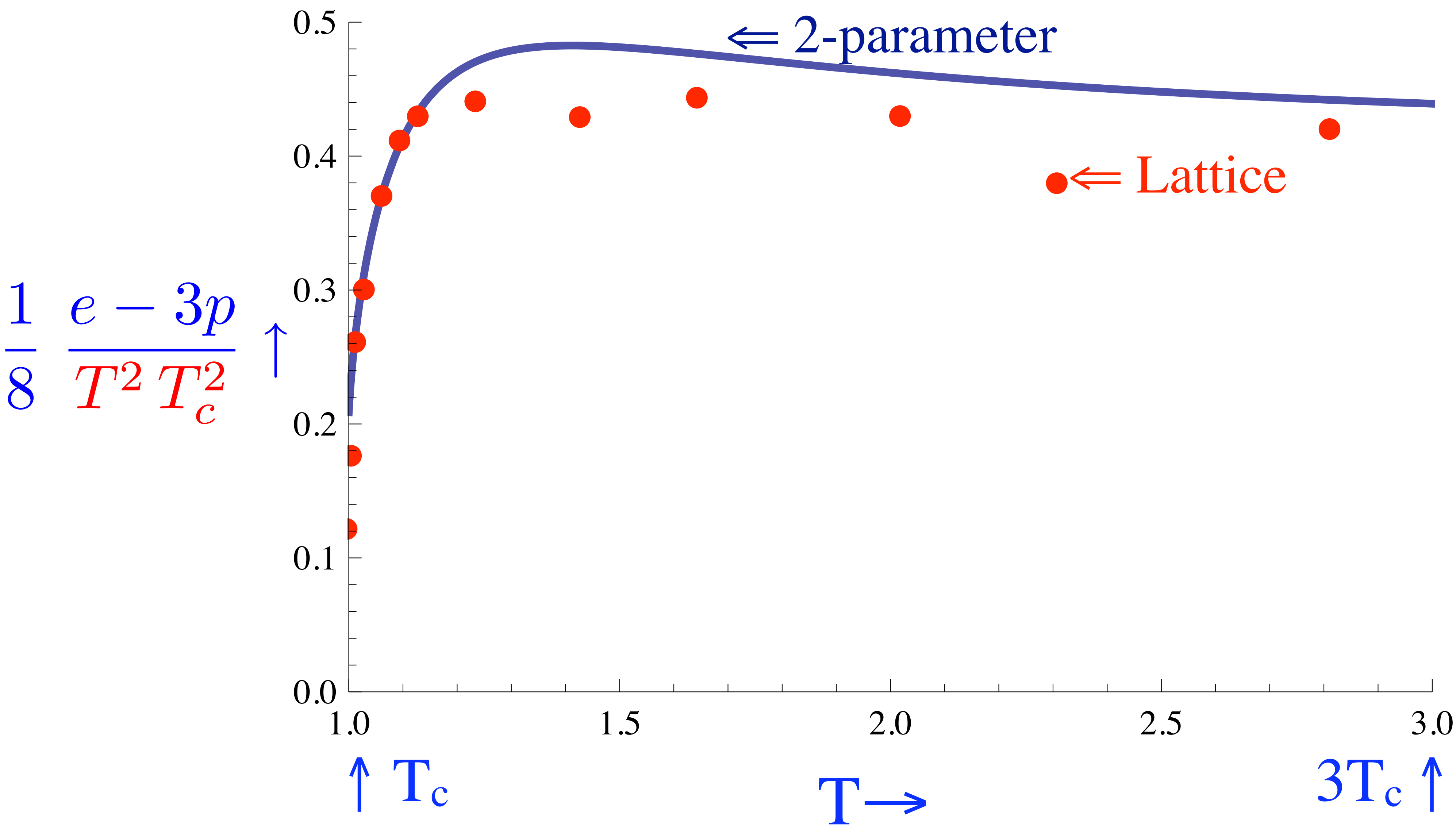


Bag const ~ (262 MeV)^4

c_2 *not* near 1, vs 1-para.

Latttice latent heat:
Beinlich, Peikert, Karsch (BPK)
lat/9608141
Datta, Gupta (DG)
1006.0938

Anomaly times T²: 2-parameter model vs lattice



G(2) and “deconfinement”

“Confinement” from eigenvalue repulsion

G(2) group: confinement without a center

Holland, Minkowski, Pepe, & Wiese, lat/0302023...

Exceptional group G(2) obtained from SO(7).

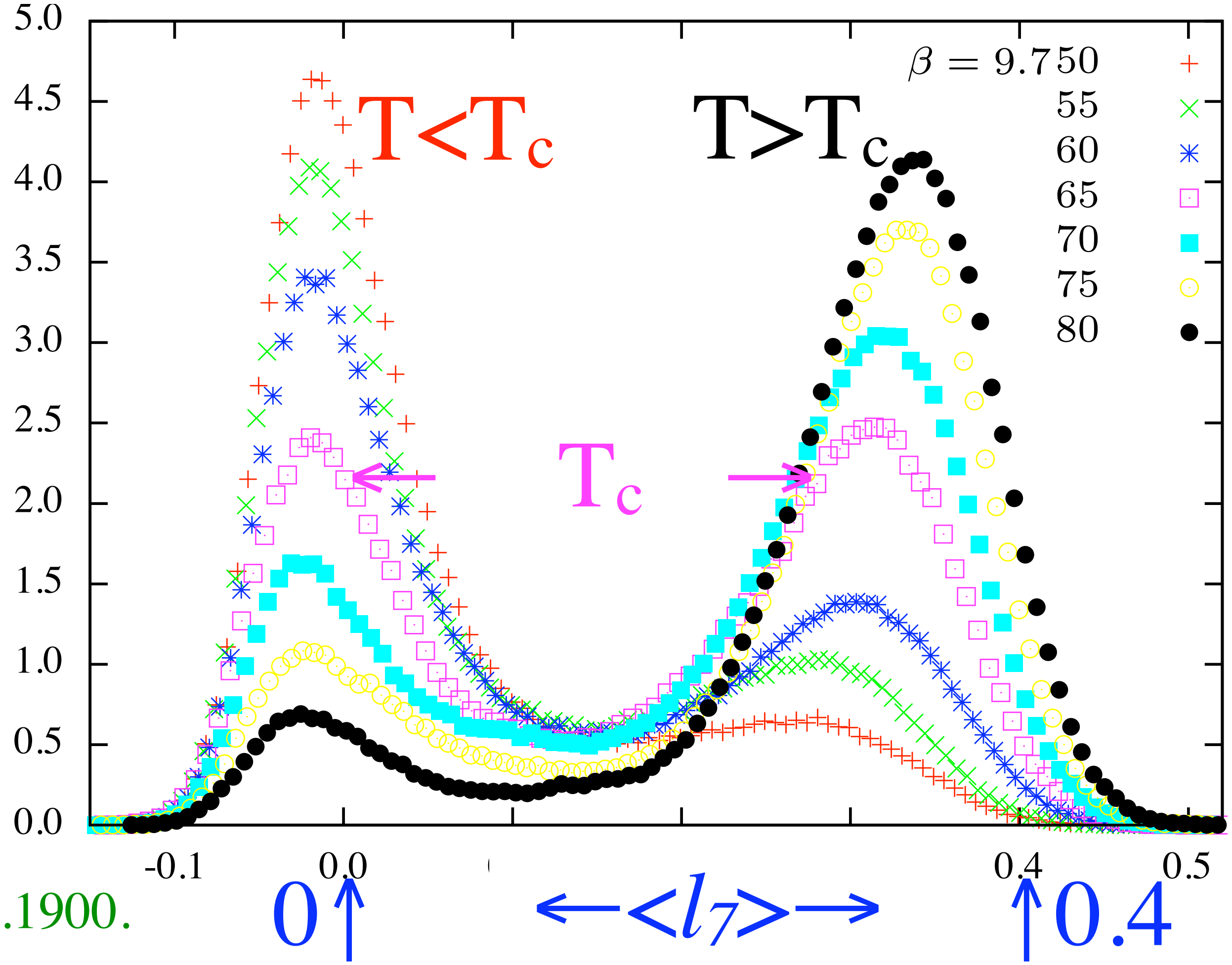
No center, so in *principle*, *no* “deconfinement”

Fund. = **7**. Adjoint = **14**. Fund. screened by adj’s: **7 × 14 × 14 × 14 = 1 + ...**

With no center, $\langle loop_7 \rangle$ can be *nonzero* at *any* T > 0.

But: lattice finds 1st order transition, $\langle l_7 \rangle \sim 0$ for $T < T_c$, $\langle l_7 \rangle \neq 0$ for $T > T_c$!

There *is* a “deconfining” transition!



Welleghausen, Wipf, & Wozar 1102.1900.

G(2): perturbative potential

Natural SU(3) embedding: fundamental **7 = 1 + 3 + 3***

$$\mathbf{L}_7 = \mathrm{e}^{2\pi i \mathbf{q}_{G(2)}} \;,\; \mathbf{q}_{G(2)} = (0, q_1, q_2, -q_1 - q_2, -q_1, -q_2, q_1 + q_2))$$

Only two parameters, q_1 and q_2.

Adjoint = **14 = 3 + 3* + 8**. Perturbative potential: $V_2(q) = q^2(1-q)^2$

$$V_2^{G2} = V_2(q_1) + V_2(q_2) + V_2(q_1 + q_2)$$

$$+ V_2(q_1 - q_2) + V_2(2q_1 + q_2) + V_2(q_1 + 2q_2)$$

Looks like a SU(3) gluon potential *plus* fundamental fields.

Hard to get confinement with G(2) potential: **3** and **3***'s give *non*-zero loop

$q_{G(2)}$ looks like a special case of SU(7):

$$\mathbf{q}_{SU(7)} = (0, q_1, q_2, q_3, -q_1, -q_2, -q_3)$$

By taking $q_3 = q_1 + q_2$ and permuting the order of the eigenvalues.

G(2): non-perturbative potentials

To one loop order, perturbative V: $$V_{pert}(q) = \pi^2\, T^4 \left(-\frac{14}{45} + V_2^{G2}(q_1, q_2) \right)$$

Without Z(N) symmetry, have *many* possible terms:

$$V_{non}(q) = T^2\, T_c^2\, \left(-c_1^{G2} V_1^{G2} - c_2^{G2} V_2^{G2} - c_1^{SU7} V_1^{SU7} - c_2^{SU7} V_2^{SU7} - d\, \ell_7 \right)$$

All V's = V(q_1,q_2). Generally, $$V_n(q_1, q_2) = \sum_{q_1, q_2} |q|^n (1 - |q|)^n$$

The V^{G2}'s have the symmetry of the perturbative G(2) term.

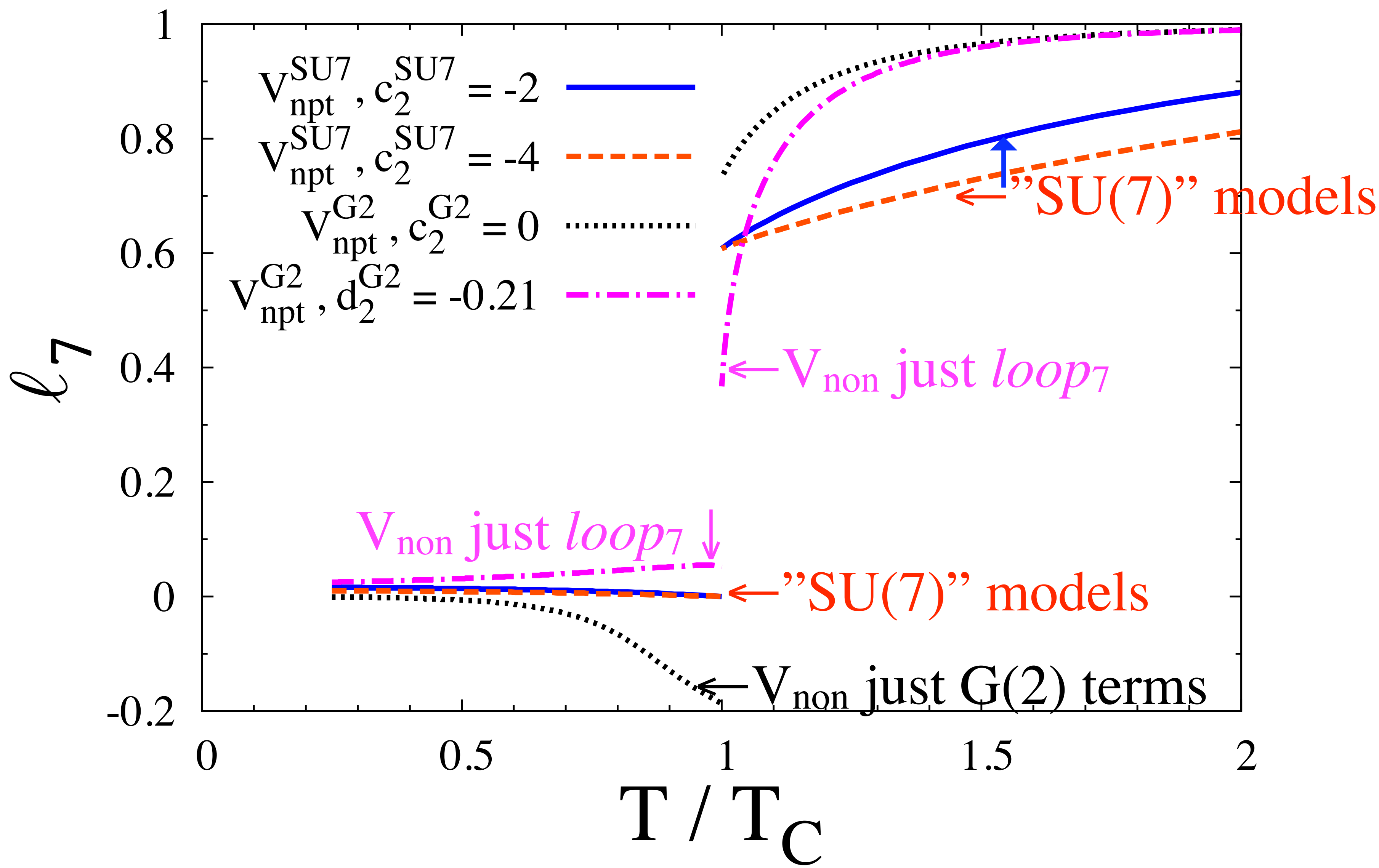
The V^{SU7}'s have the symmetry of the SU(7) theory, fixing $q_3 = q_1 + q_2$.

Note a term ~ *loop₇* is allowed in the potential, unlike for SU(N). No center!

Both V^{SU7} and *loop₇* generate eigenvalue *repulsion*, and so *small* ⟨*loop₇*⟩.

Predictions for Polyakov loop in G(2)

Generically, *easy* to find 1st order transitions. *Most* have $\langle l_7 \rangle$ *non*zero below T_c.
To obtain zero (or small) $\langle l_7 \rangle$ below T_c, *must* have either V^{SU7} and/or l_7 ,
and adjust terms to cancel pert. V_2^{G2} at T_c.

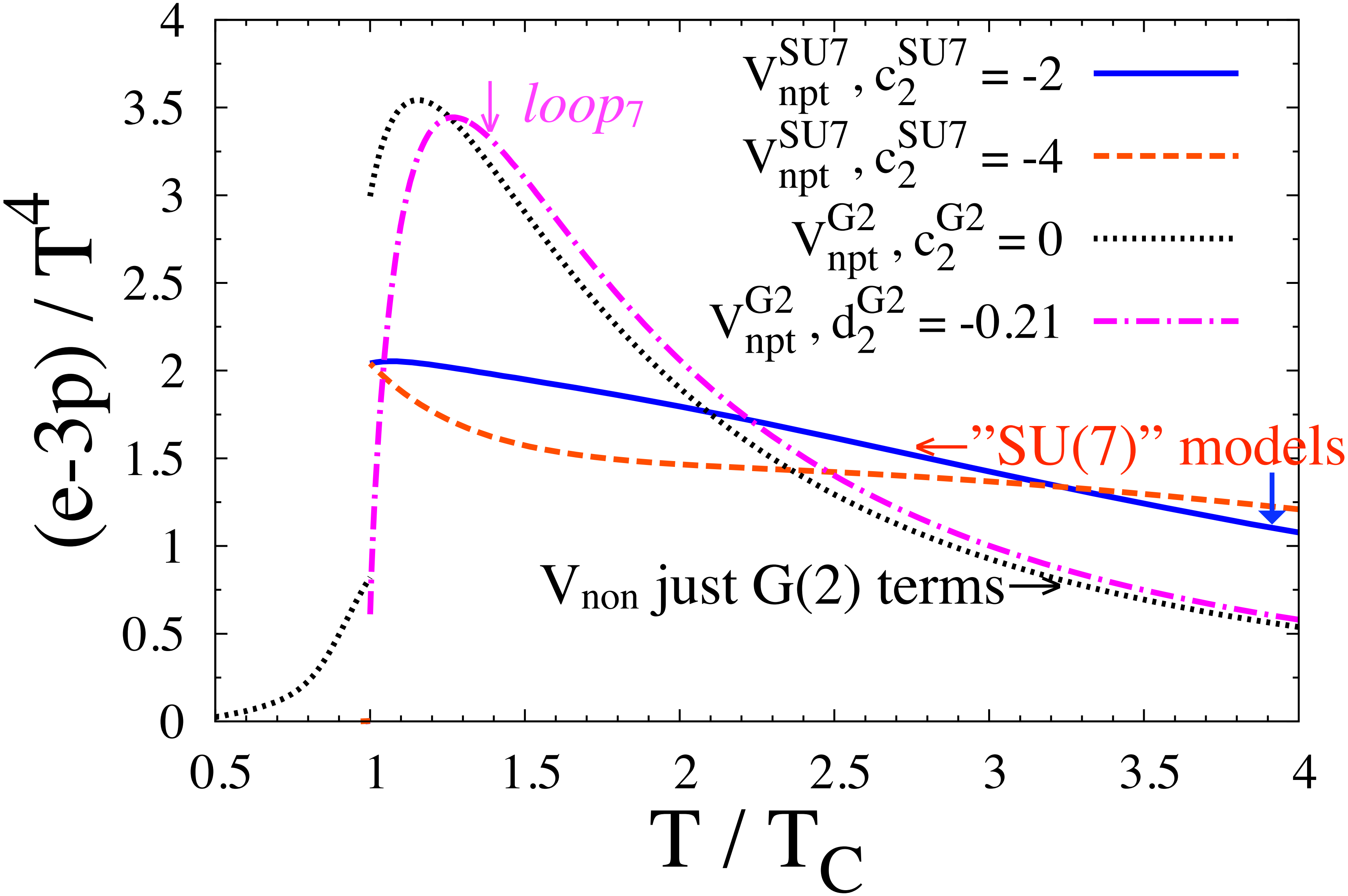


Predictions for interaction measure of G(2)

Appear to have a potential with 5 - 2 = 3 parameters.

Simply requiring $\langle l_7 \rangle$ small below Tc *greatly* restricts the possible parameters.

Yields *dramatic* differences in the behavior of (e-3p)/T^4.



Summary

For SU(N), transition region *narrow*: for pressure, T_c to ~1.2 T_c!

Special to pressure: for interface tensions, T_c to ~4 T_c...

Above 1.2 T_c, pressure dominated by *constant* term ~ T^2 .

What does this come from? Free energy of massless fields in 2 dimensions?
Strings? But *above* T_c.

G(2) gauge group crucial test of model. Lattice simulations in progress.

Need to include quarks.

Can then compute temperature dependence of:

shear viscosity, energy loss of light quarks, damping of quarkonia...

With quarks, is there a single "T_c"?