# Hard Dilepton and Photons, *near* $T_c$

arXiv.org: 1409.4778 & 1502.xxxx:

Analytic computations:

Yoshimasa Hidaka: RIKEN@Nishina (& RIKEN@BNL, ’07-’09)
Shu Lin: RIKEN@BNL
Daisuke Satow: JPS, RIKEN@BNL → ECT Trento
Vladimir Skokov : BNL NT (’11-’13) → Kalamazoo → RIKEN@BNL (’15)
RDP: BNL & RIKEN@BNL

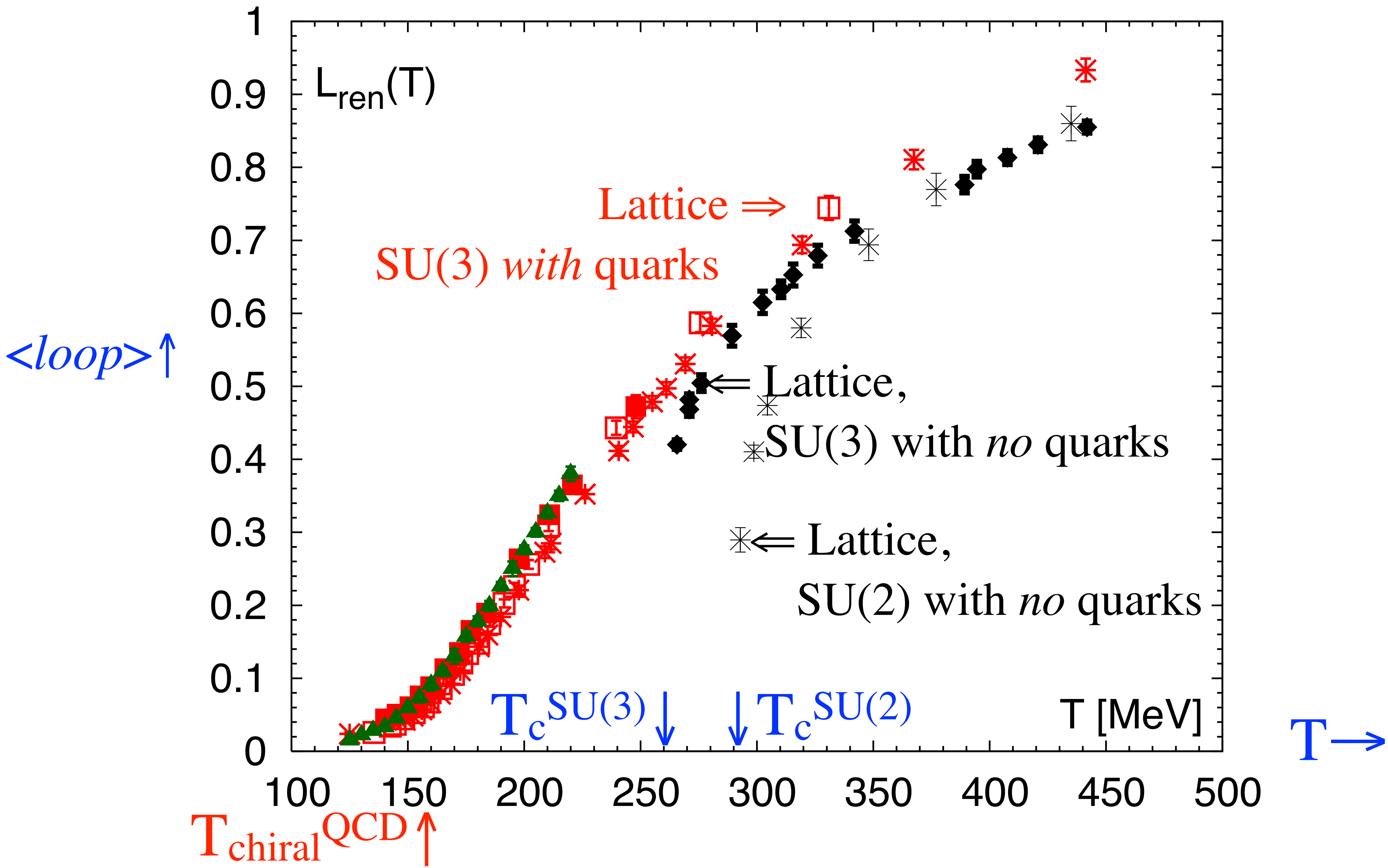
as input to 3+1 dimensional (ideal) hydrodynamics (MUSIC):

Charles Gale, McGill
Sangyong Jeon, “
Jean-Francois Paquet, “
Gojko Vujanovic, “

# Lattice: Polyakov Loop with and without quarks

Order parameter for deconfinement: Polyakov loop

Lattice: Bazavov & Petreczky, 1110.2160



# "Semi"-QGP

Polyakov Loop: $$\ell = \frac{1}{3}\,\mathrm{tr}\,\mathcal{P}\ \exp\left( i\,g\,\int_0^{1/T} A_0\, d\tau \right)$$

*Simplest* approximation to give a non-trivial loop: constant, diagonal $A_0$:

$$A_0^{cl} = \frac{Q}{g}\ ,\ \ Q = \frac{2\pi T}{3}\, q(T)\, \lambda_3\ \ ;\ \ \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Depends upon single function, q(T), fixed from pressure(T).
Only need two parameters to fit pressure, 't Hooft loop
However, for the Polyakov loop….

# 1-parameter matrix model, two colors

Dumitru, Guo, Hidaka, Korthals-Altes, RDP '10: to usual perturbative potential,

$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \left( -\frac{1}{20} + q^2(1-q)^2 \right)$$

Add - *by hand* - a non-pert. potential $V_{non} \sim T^2 T_c^2$. *Also* add a term like $V_{pert}$:

$$V_{non}(q) = \frac{4\pi^2}{3} T^2 T_c^2 \left( -\frac{c_1}{5} q(1-q) - c_2 q^2(1-q)^2 + \frac{c_3}{15} \right)$$

Now just like any other mean field theory. $\langle q \rangle$ given by minimum of $V_{eff}$:

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q) \qquad \left. \frac{d}{dq} V_{eff}(q) \right|_{q=\langle q \rangle} = 0$$

$\langle q \rangle$ depends nontrivially on temperature.

Pressure value of potential at minimum: $$p(T) = -V_{eff}(\langle q \rangle)$$

# Polyakov loop vs lattice - ?

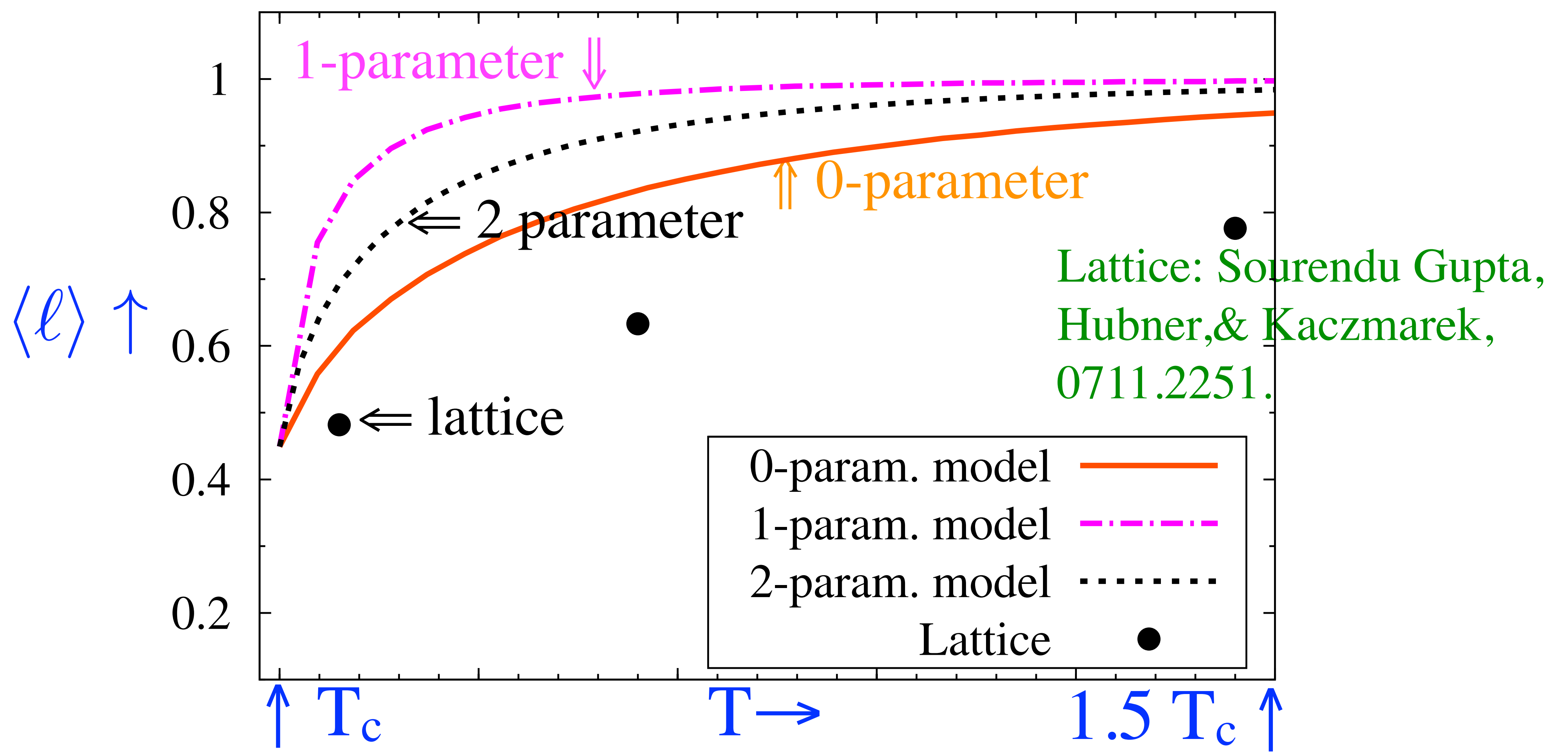
Polyakov loop from the lattice *nothing* like Matrix Model

Model: transition region *narrow*, to ~ 1.2 $T_c$. Lattice: loop *wide*, to ~ 4.0 $T_c$.

Also true for FRG, Pawlowski & Rennecke, 1403.1179 + …

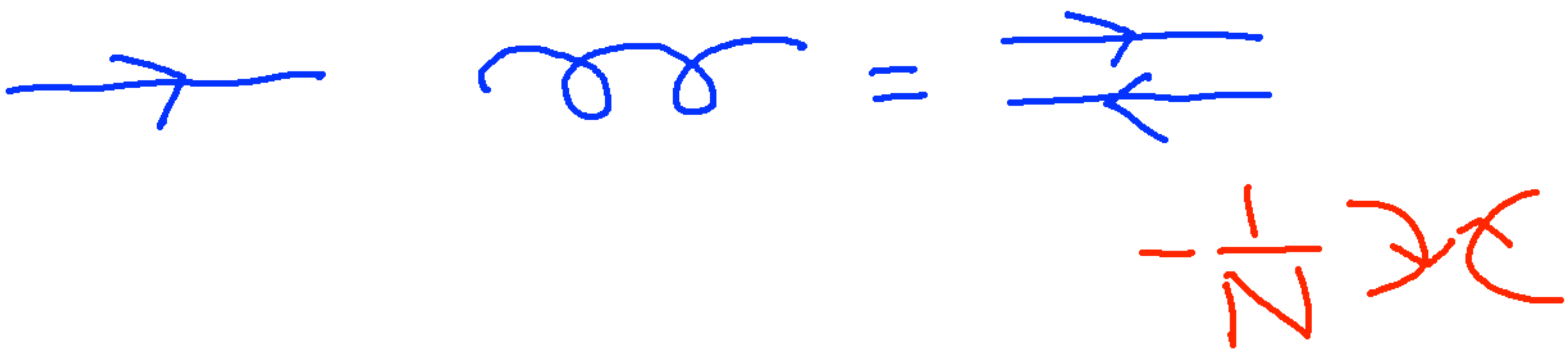
Reinosa, Serreau, Tissier & Wschebor, 1407.6469, 1412.5672

*Need…magnetic excitations?* In practice: take Q ~ T q(T) from the lattice



# Semi-QGP in imaginary time

Imaginary time: in background $A^0_{cl}$, energies carry color.
Quarks carry one line of color, gluons two:




Quarks: $$p_0 = 2\pi T(n+1/2) \rightarrow p_0 - iQ^a$$

Gluons: $$p_0 = 2\pi T n \rightarrow p_0 - i(Q^a - Q^b)$$

The background field acts like an imaginary chemical potential for color.

# Semi-QGP in real time

Statistical distribution functions those for imaginary chemical potential:

$$\widetilde{n}_a(E) = \frac{1}{e^{(E-iQ^a)/T}+1} \qquad n_{ab}(E) = \frac{1}{e^{(E-i(Q^a-Q^b))/T}-1}$$

For three colors, color chemical potential:

$$Q^a = \frac{2\pi T}{3}\, q(T)\; (1,-1,0)$$

When Q ~ T, the *only* soft gluons have $Q^a = Q^b$: *diagonal* elements.

For N colors: ~ $N^2$ off-diagonal gluons, and ~ N diagonal gluons

In the semi-QGP, soft gluons are suppressed by 1/N.

# Suppression of color near $T_c$

Consider energetic particles, $E \gg T$, Boltzmann statistics

$$\widetilde{n}_a(E) \sim e^{-(E - iQ^a)/T} \qquad\qquad n_{ab}(E) \sim e^{-(E - i(Q^a - Q^b))/T}$$

While the n(E)'s are complex, sums over color are real.
Polyakov loop:

$$\ell = \frac{1}{N} \sum_{a=1}^{N} e^{iQ^a/T}$$

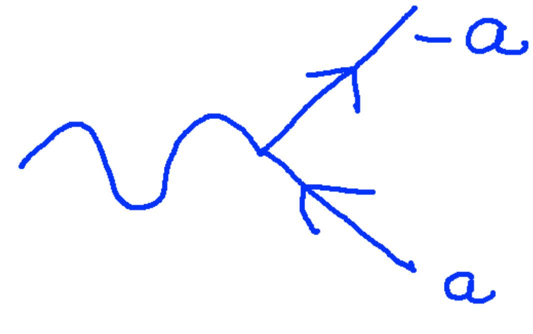
Summing over color,

$$\frac{1}{N} \sum_{a=1}^{N} \widetilde{n}_a(E) = e^{-E/T}\, \ell \qquad\qquad \frac{1}{N} \sum_{a,b=1}^{N} \widetilde{n}_{ab}(E) = e^{-E/T}\, \ell^2$$

Near $T_c$, where loop small, quarks suppressed by loop; gluons by loop *squared*.

# Hard dileptons: same!

Dileptons: off shell photon goes to quark anti-quark pair.
Consider dileptons back to back, total momentum = 0.




Diagrams same, only the distribution functions change.

$$\widetilde{n}_a(E) = \frac{1}{e^{(E-iQ^a)/T}+1} \qquad \widetilde{n}_{-a}(E) = \frac{1}{e^{(E+iQ^a)/T}+1}$$

(Imaginary) chemical potential: sign of $Q^a$ *flips* between q and q bar.
Large E: with Boltzmann statistics,

$$\sum_a \widetilde{n}_a(E)\widetilde{n}_{-a}(E) \sim e^{-(E-iQ^a)/T} e^{-(E+iQ^a)/T} = e^{-2E/T}$$

So $Q^a$'s drop out: # dileptons *identical* in deconfined and confined phases!

# Soft Dileptons: *more* in confined phase

High T: $Q^a$=0. As E $\rightarrow$ 0, # dileptons:
Fermi-Dirac dist. fnc. finite at E = 0.

$$\widetilde{n}(0)^2 \sim \frac{1}{4}$$

In the confined phase, Polyakov loop = 0, find amazing identity:

$$\frac{1}{N} \sum_{a=1}^{N} \widetilde{n}_a(E)\, \widetilde{n}_{-a}(E) \sim n(E) =_{E \rightarrow 0} \frac{T}{E}$$

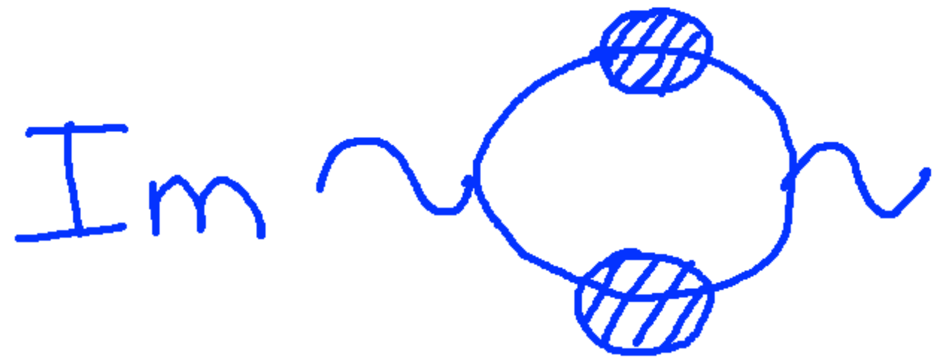
*More* dileptons in the confined phase!
Confined phase only in the pure gauge theory, but interesting point of principle.

"Statistical confinement": quark anti-quark forms "boson",
which exhibits Bose-Einstein enhancement. But *no* dynamics of confinement.

N.B.: in dynamical quasi-particle model, as T $\rightarrow$ $T_c$ quarks heavier,
but width increases, so also obtain enhanced dilepton rate.

# Dileptons

Explicitly, we computed the diagram:
Here, propagators with hatched dot are
just $p_0 \rightarrow p_0 - i\, Q^a$. *Very* straightforward




$$f_{\ell\bar{\ell}} = \#\ \text{dileptons} \left( \frac{Q \neq 0}{Q = 0} \right)$$

$$f_{\ell\bar{\ell}} = 1 - \frac{2\,T}{3\,p} \log \frac{1 + 3\,\ell\, e^{-p_-/T} + 3\,\ell\, e^{-2p_-/T} + e^{-3p_-/T}}{1 + 3\,\ell\, e^{-p_+/T} + 3\,\ell\, e^{-2p_+/T} + e^{-3p_+/T}}$$

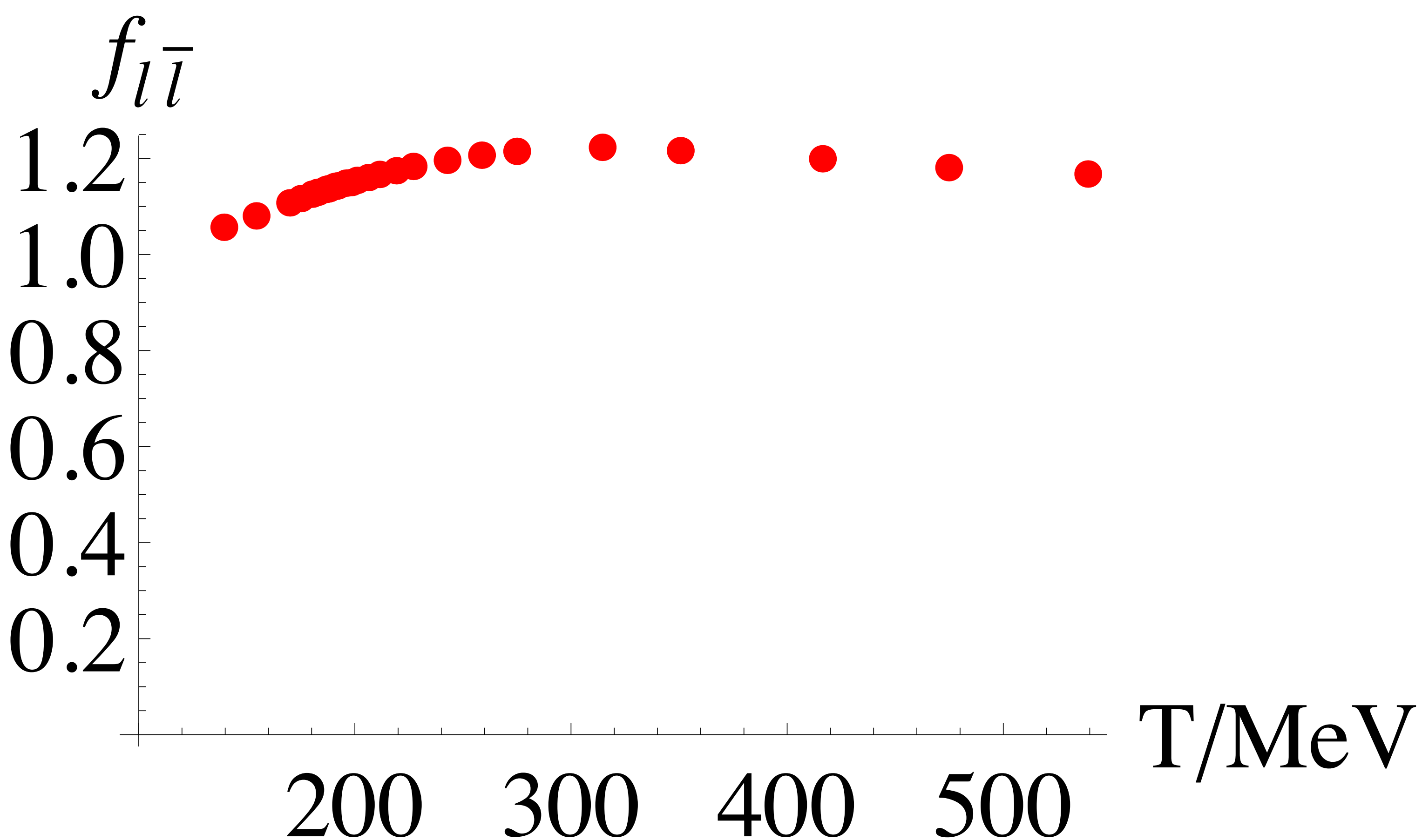
When Q = 0, # dileptons ~ $\alpha_{em}$. Photon momentum = (E,p), $E_\pm = (E \pm p)/2$.
Polyakov loop = $\ell$ : = 1 in the perturbative QGP, and = 0 in the confined phase.
Above factor analogous to PNJL model,

Abishek Atreya, Sarkar, Srivastava, 1111.3027, 1404.5697, & Das, 1406.7411

# Ratio # dileptons, vs T

Below ratio of # dileptons, vs T.  Ratio semi-QGP/perturbative QGP.
Take QCD coupling same, so only function of $Q^a$'s, taken from the lattice.
Mild enhancement of dileptons at small E.

Lee, Wirstam, Zahed, Hansson, ph/9809440:
Condensate in ~ $\langle A_0{}^2 \rangle$ ; equivalent to expanding to ~ $\langle Q^2 \rangle$.

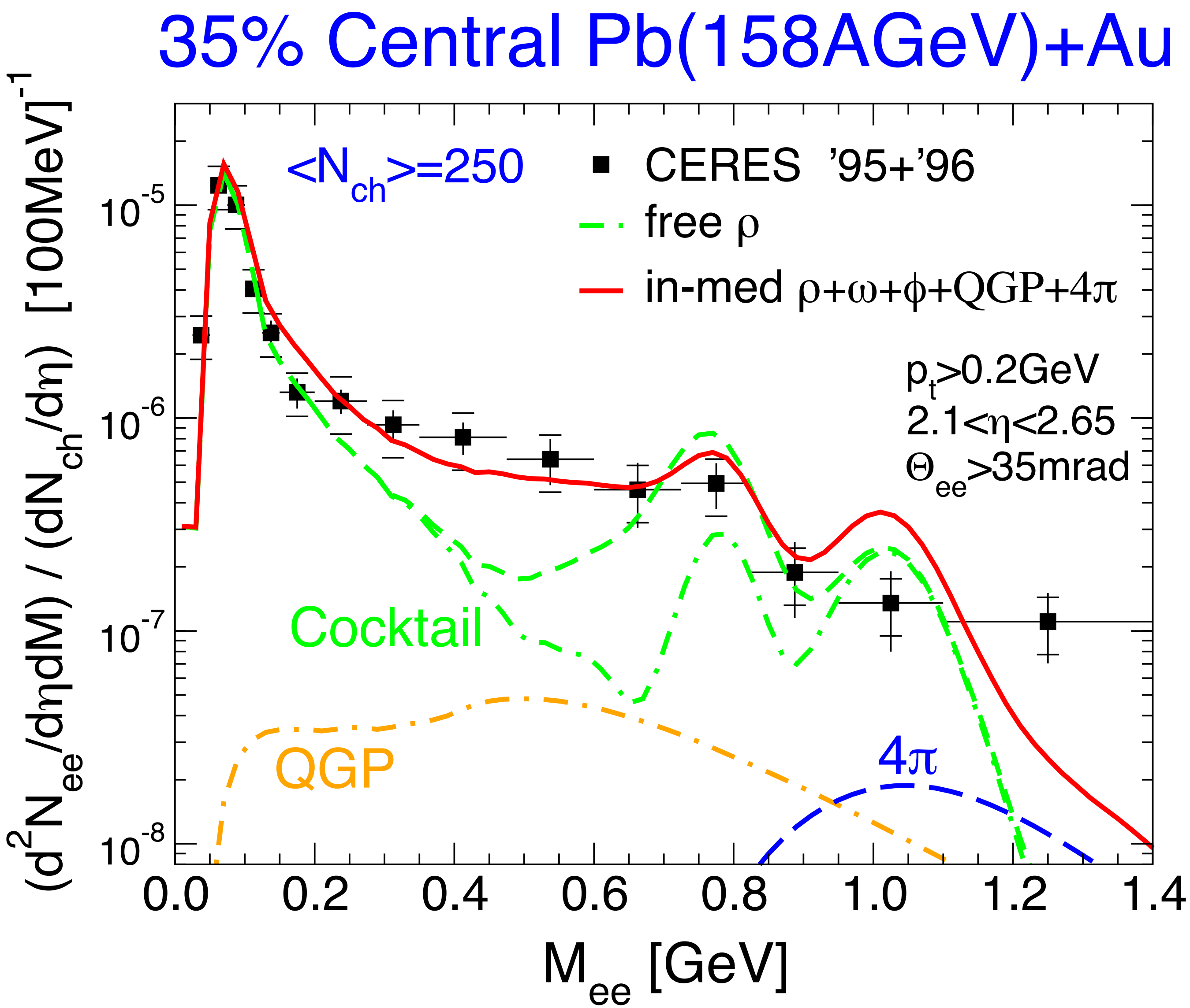


# Experiment: dilepton excess below the ϱ

CERES/NA45, $\sqrt{s}$ = 8.8 GeV/A.

Below the ϱ, QGP small, dominated by hadronic cocktail.

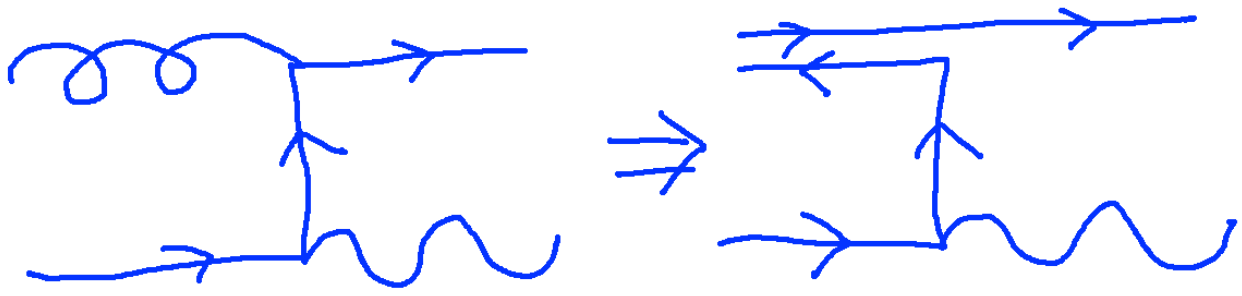
*Need* medium broadened ϱ to fit data: so *need* to fit semi-QGP to hadronic phase



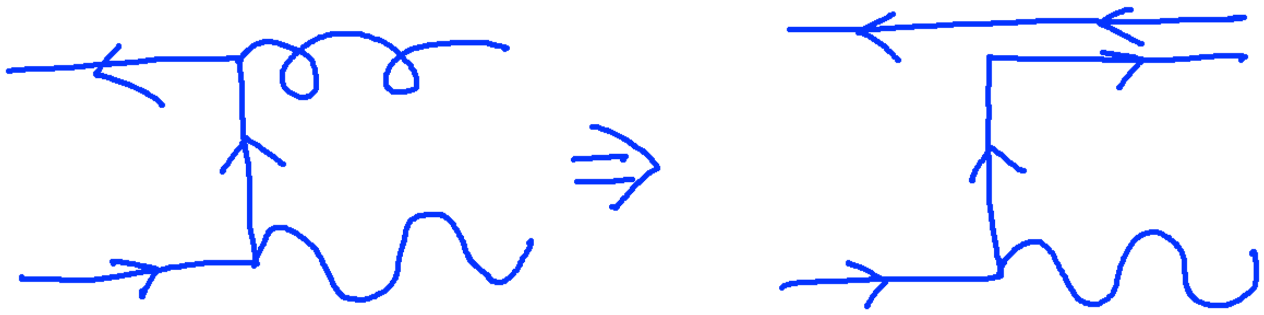

Rapp,
1306.6394

# Production of hard photons

Photon on the mass shell cannot go to quark anti-quark; must also emit a gluon
At leading order, two processes. Compton scattering:

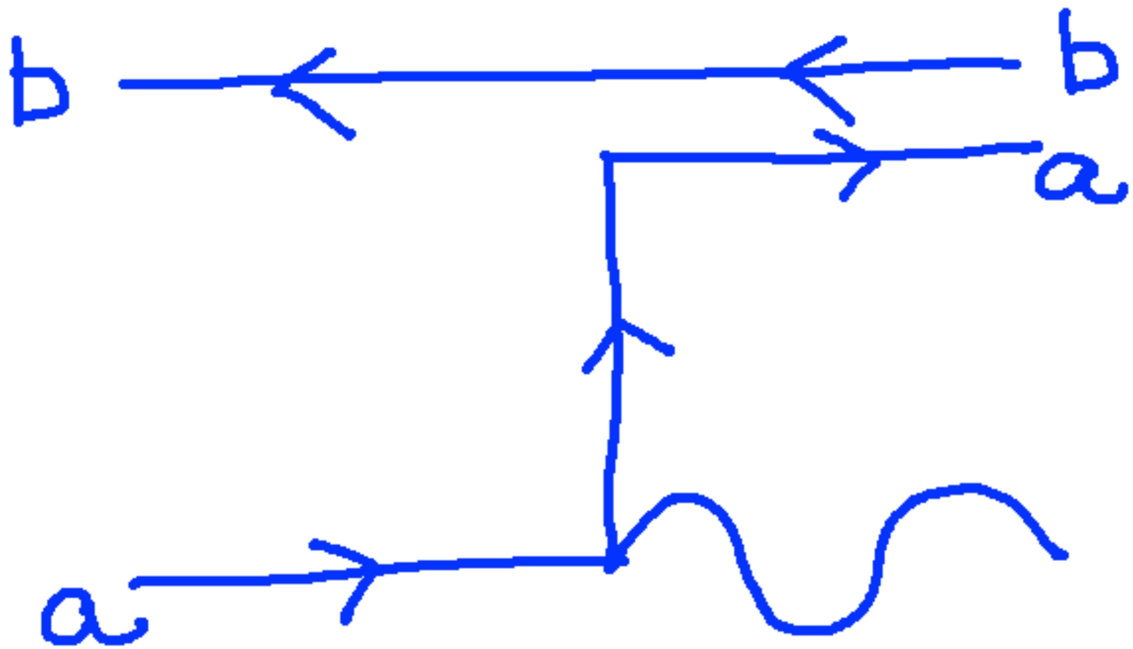


Pair annihilation:



# Suppression in confined phase by $1/N^2$

In double line notation: diagram suppressed by loop unless colors of quark and anti-quark the same, a = -b :




But if a = - b, diagonal gluon, suppression of 1/N.

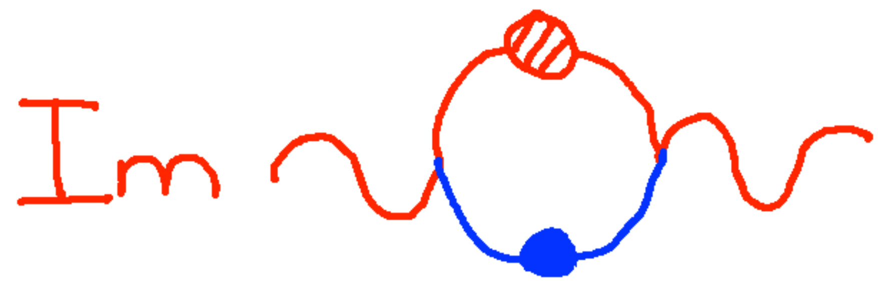
And, if a = -b, tracelessness of gluon implies extra factor of 1/N, or $1/N^2$ in all.

Similar suppression for Compton scattering.

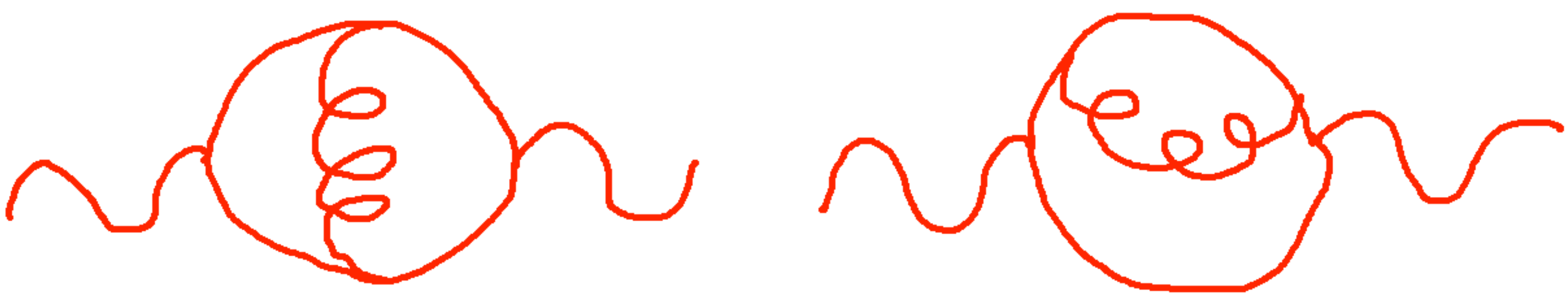
# Photon production: computation

Photon momentum "hard", $P = (E, p)$, $E = p \gg T$. Denote by red lines.
Internal lines can be soft, $E$ or $p \sim T$; denote by blue lines.

Diagrams with one soft quark line:
Hatched blob: $Q^a \neq 0$
Solid blob: HTL with $Q^a \neq 0$
Exhibits logarithmic UV divergence, when the soft quark line becomes hard.




Also two loops diagrams, in which all lines are hard.
All lines below should be hatched, with $Q^a \neq 0$.
Exhibits logarithmic IR divergence, when the gluon line becomes soft.



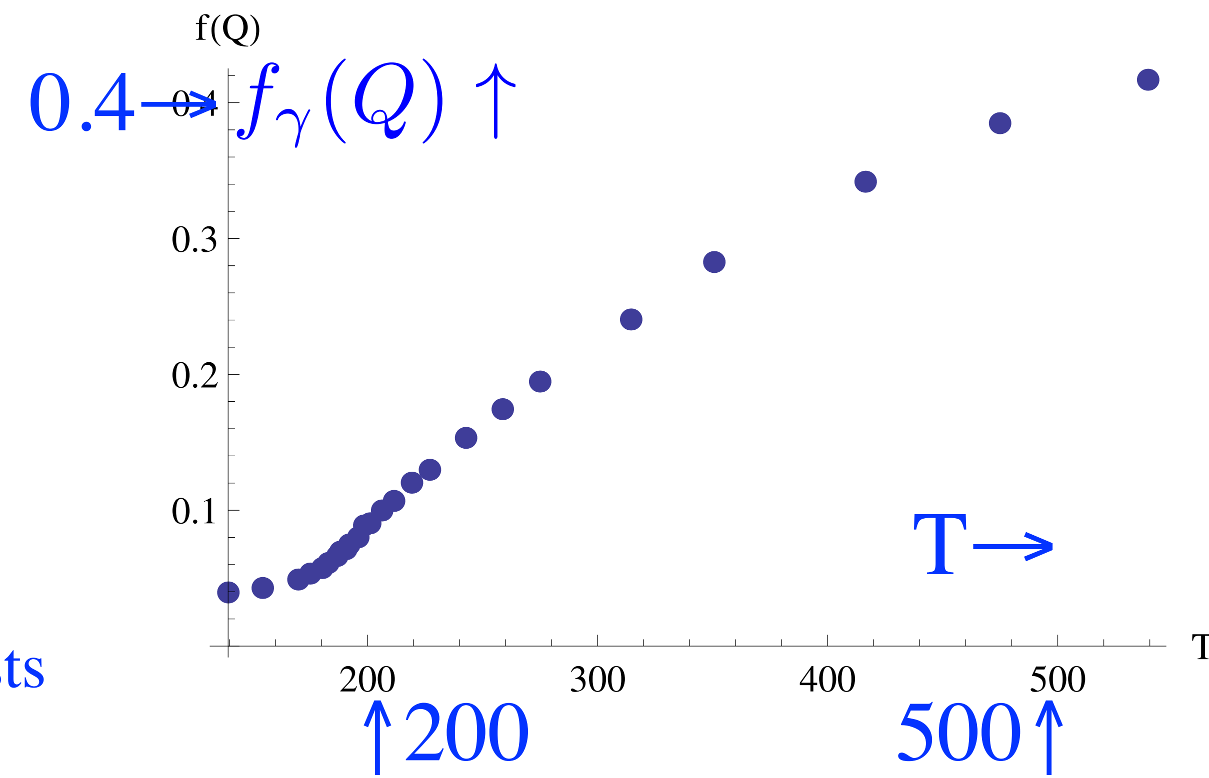
# *Strong* suppression of real photons in the confined phase

Summing soft + hard, logarithms cancel. For hard photons, very simple result:

$$f_\gamma(Q) = \#\ \text{photons}\left(\frac{Q \neq 0}{Q = 0}\right) = 1 - 4q + \frac{10}{3}q^2\ ;\ q = \frac{Q}{2\pi T}$$

In the confined phase, $q_{conf} = 1/3$, find *huge* suppression:

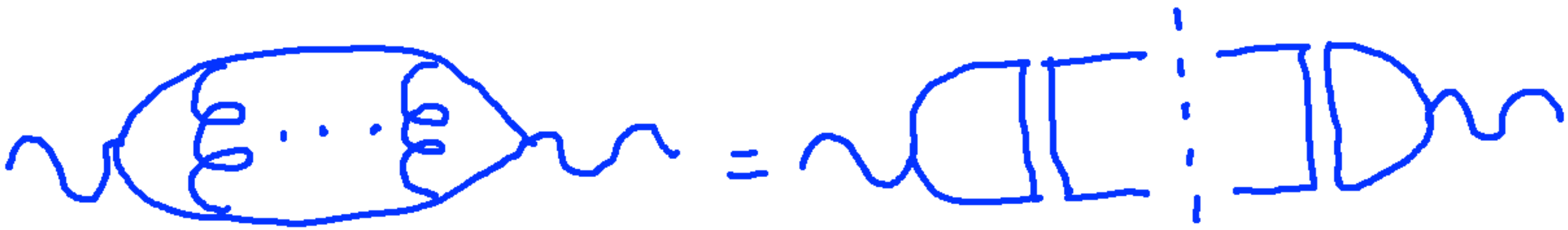
$$f_\gamma(q_{conf}) = \frac{1}{3N^2} = \frac{1}{27}$$




Suppression is so large that it persists even to T ~ 500 MeV.

# Heh, what about Landau-Pomeranchuk-Migdal!

In the perturbative QGP, even at leading order in $g^2$, LPM $\Rightarrow$ need to resum an *infinite* set of ladder diagrams: Arnold, Moore & Yaffe, ph/0111107, ph/0204343



Each new rung is down by $g^2$, but for soft gluon, $k \sim gT$, compensated by Bose-Einstein enhancement times energy denominator,

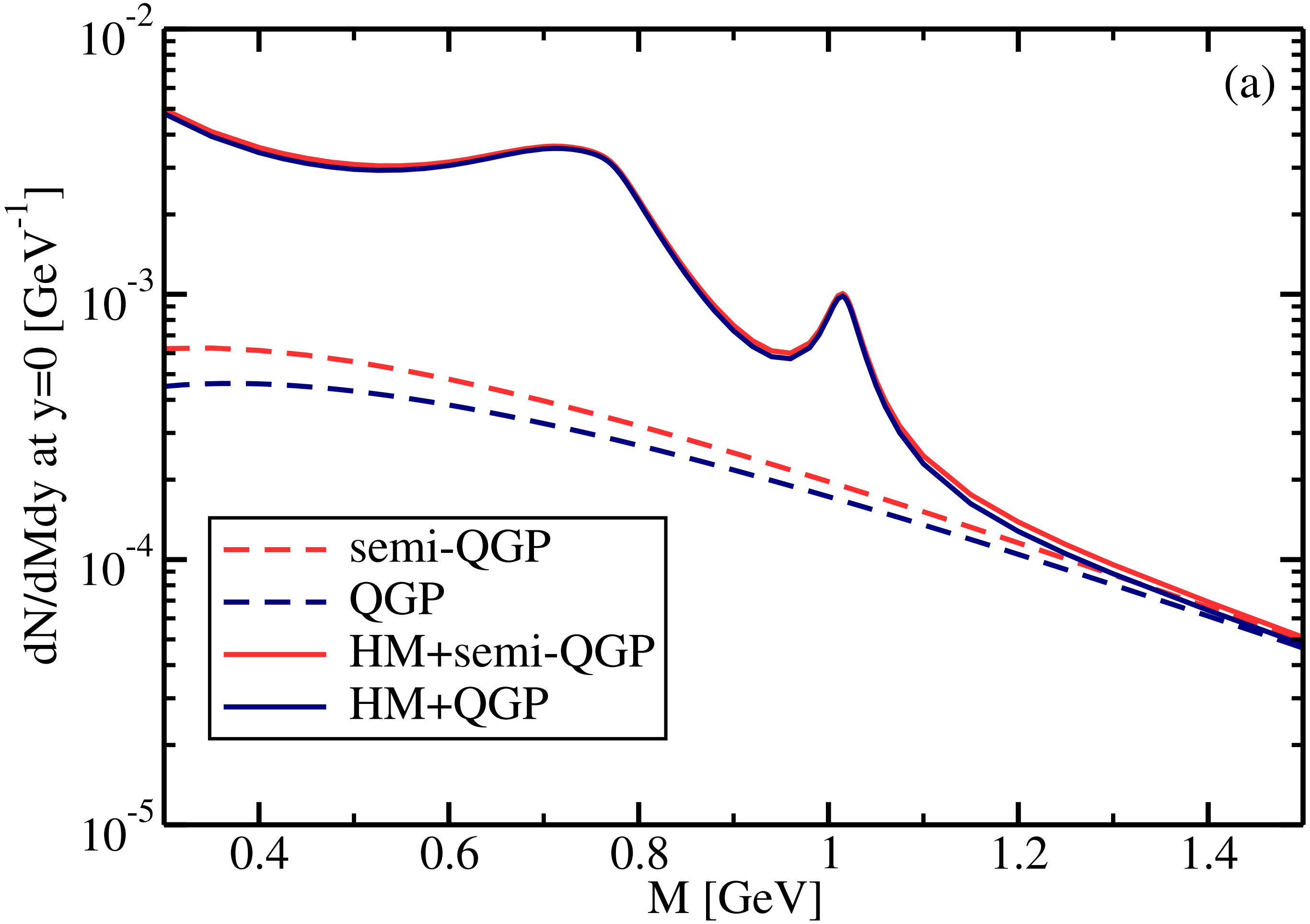
$$g^2 \; n(gT) \; \frac{T}{ip_0 - E_k + E_{p-k}} \sim g^2 \frac{T}{gT} \; \frac{T}{gT} \sim 1$$

Semi-QGP: only soft gluons are *diagonal*, so LPM is suppressed by 1/N.
What we did: only $2 \rightarrow 2$ processes, at leading logarithmic order.
Did compute LPM correction, term is large for N = 3.
Need to compute complete process, including LPM. Will do....

# Hydrodynamics: # dileptons

MUSIC: 3+1 hydro @ RHIC: $\sqrt{s}$ = 200 GeV/A, central collisions
Preliminary analysis: only ideal hydro.

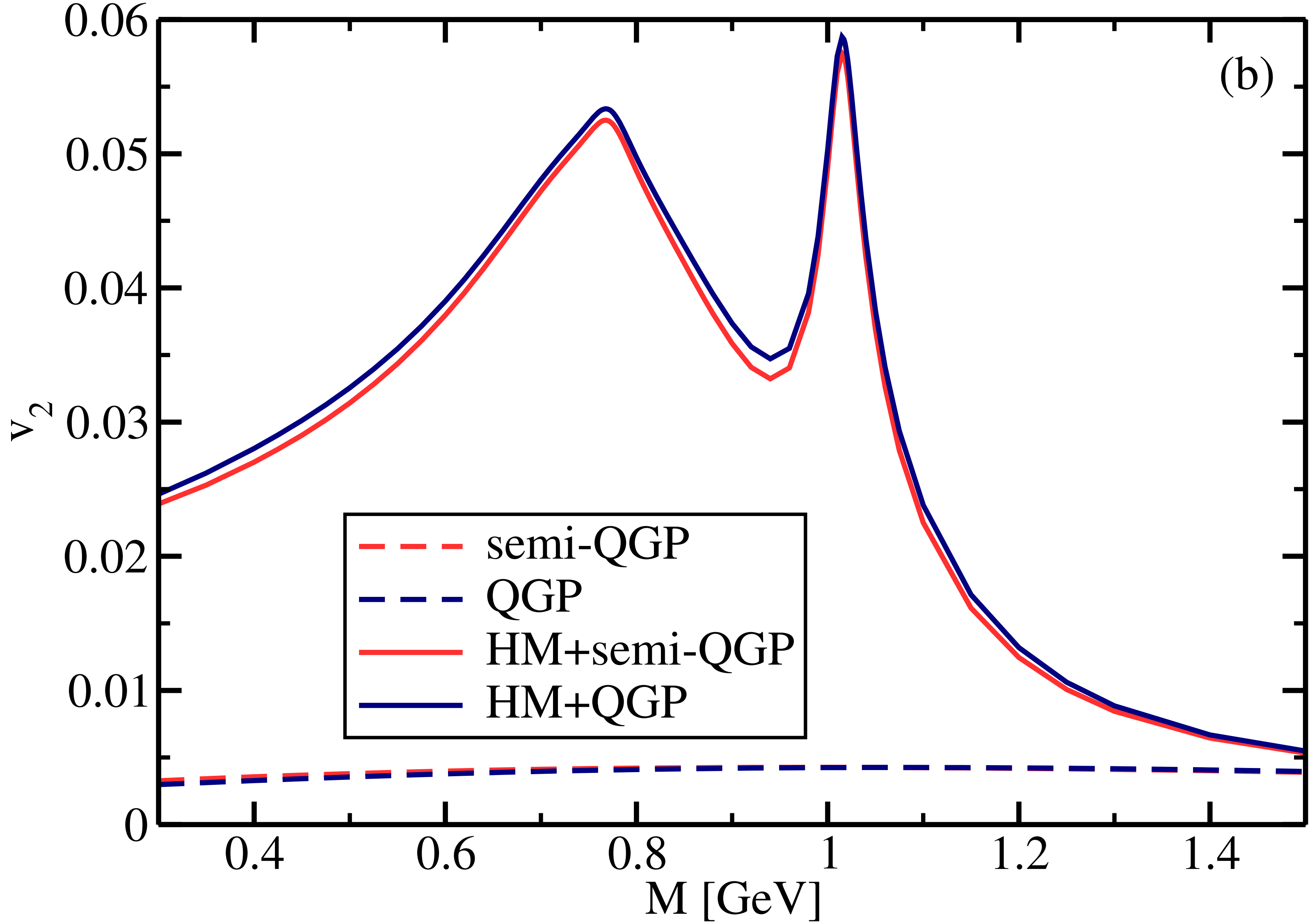
Small enhancement of dileptons in semi-QGP, swamped by hadronic phase.
*No* matching of semi-QGP to hadronic phase: clearly essential.



# Hydrodynamics: dilepton $v_2$

Since # dileptons dominated by hadrons, effect on elliptic flow, $v_2$, small.

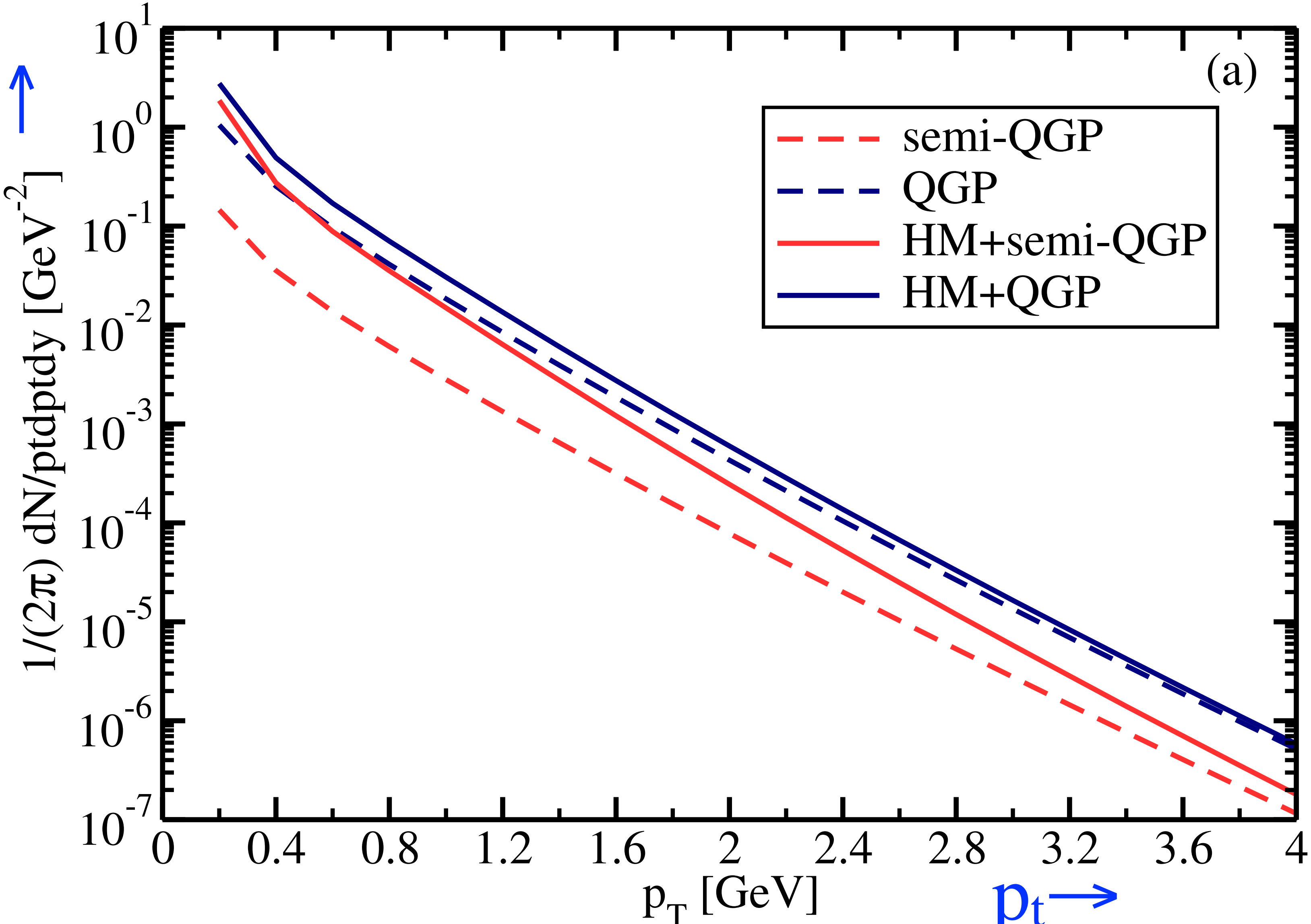
$v_2 \uparrow$



# Hydrodynamics: # photons

In semi-QGP, *far* fewer photons above $T_c$.

# photons↑



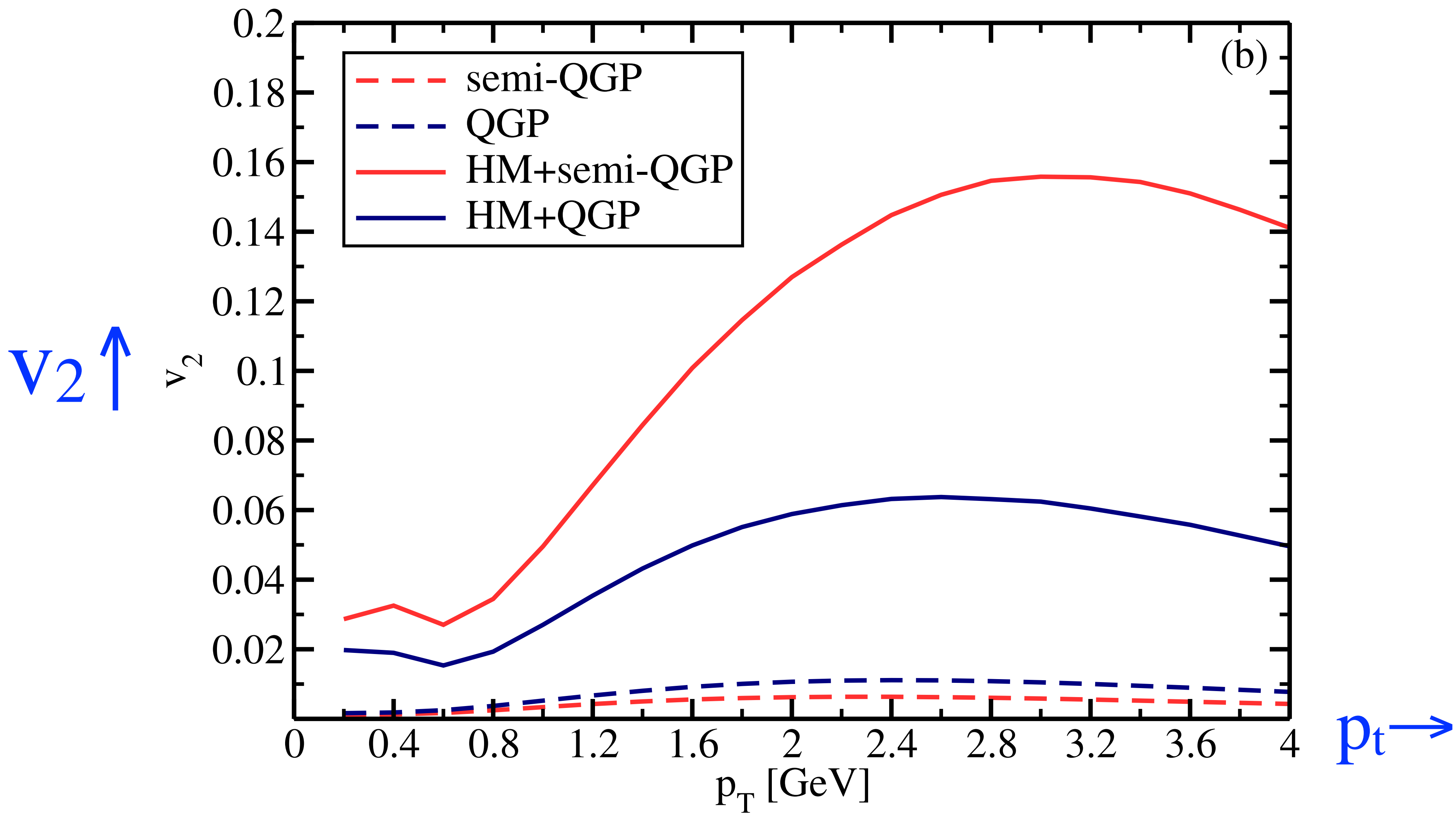

$p_t$ →

# Hydrodynamics: photon elliptic flow, $v_2$

Fewer photons near $T_c$ in semi-QGP has a big effect on the total $v_2$.

Tends to bias the total $v_2$ to that in hadronic phase.  Small "dilution" by QGP.

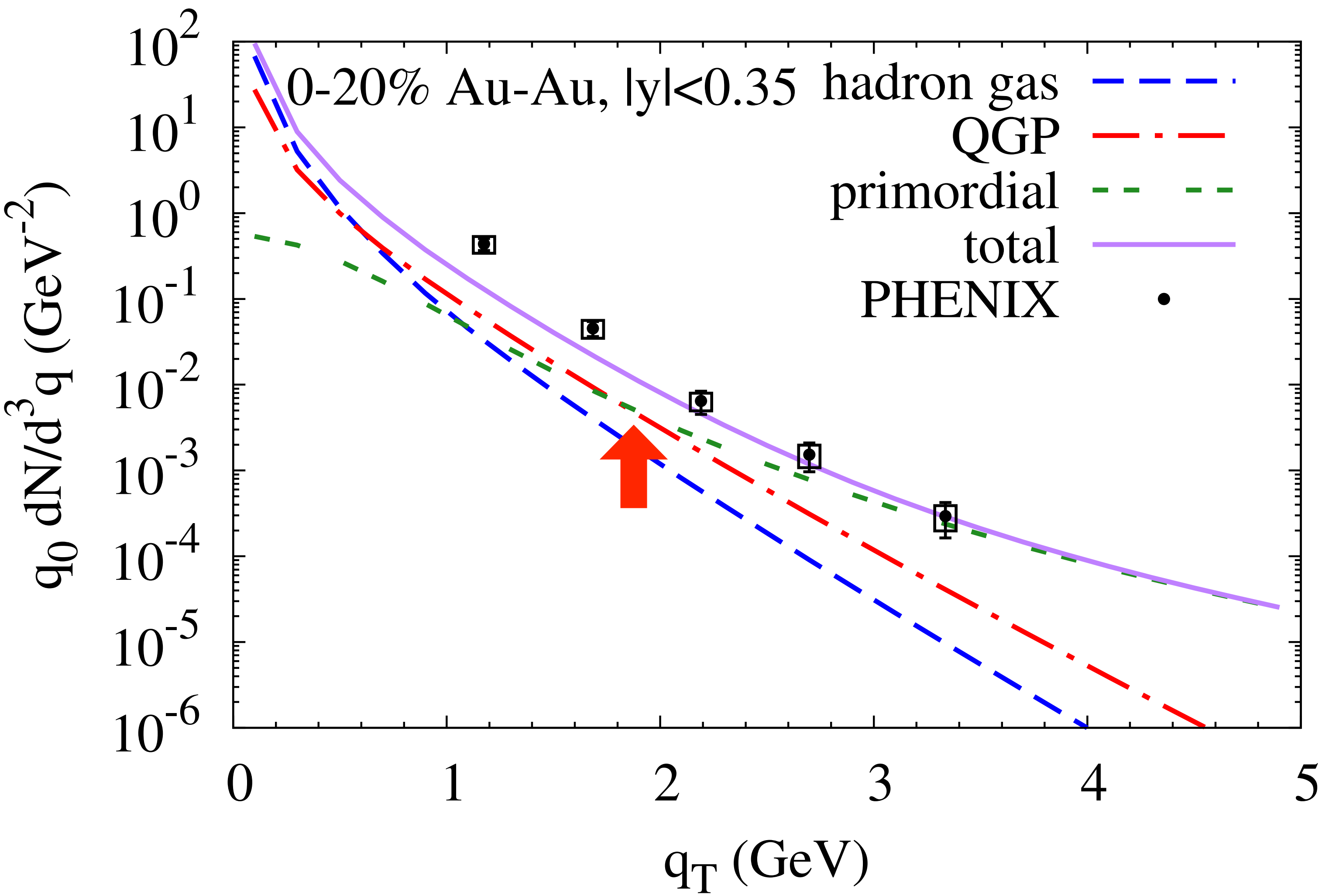
*Possible solution to experimental puzzle of "big" $v_2$ for photons?*



# PHENIX vs theory: puzzle of the "missing" photons

Sources of photons: QGP, hadron gas, "primordial" = hard initial processes
PHENIX: more photons than expected?
At RHIC: "primordial" photons appear to dominate above $p_t \sim 2$ GeV
*Experiment much larger than theory?*

van Hees, He, Rapp, 1404.2846

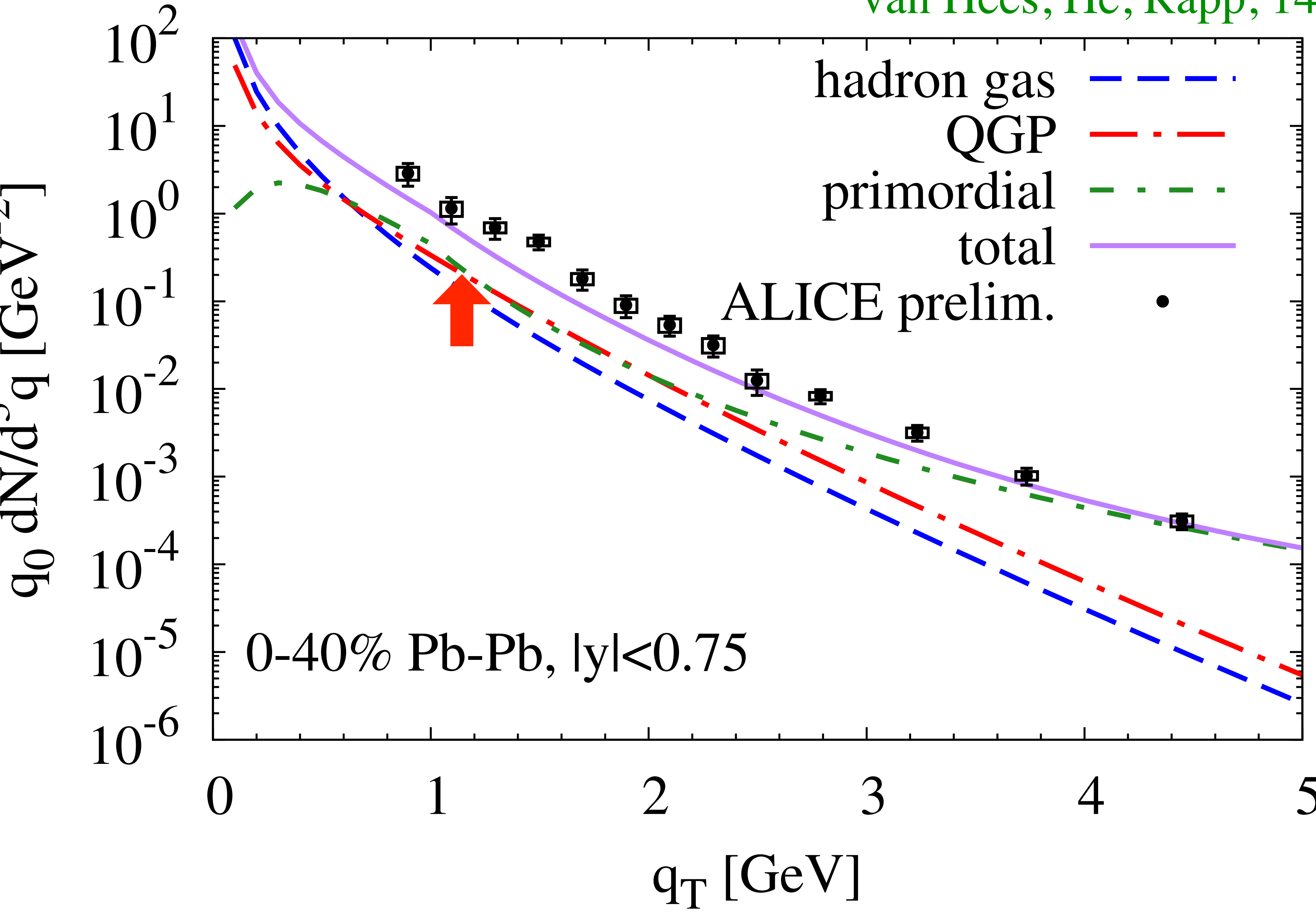


# ALICE vs theory: puzzle of the "missing" photons

At LHC, "primordial" appears to dominate above $p_t \sim 1$ GeV

*Again, experiment much larger than theory?*

van Hees, He, Rapp, 1404.2846



# Hadronic contribution to photons?

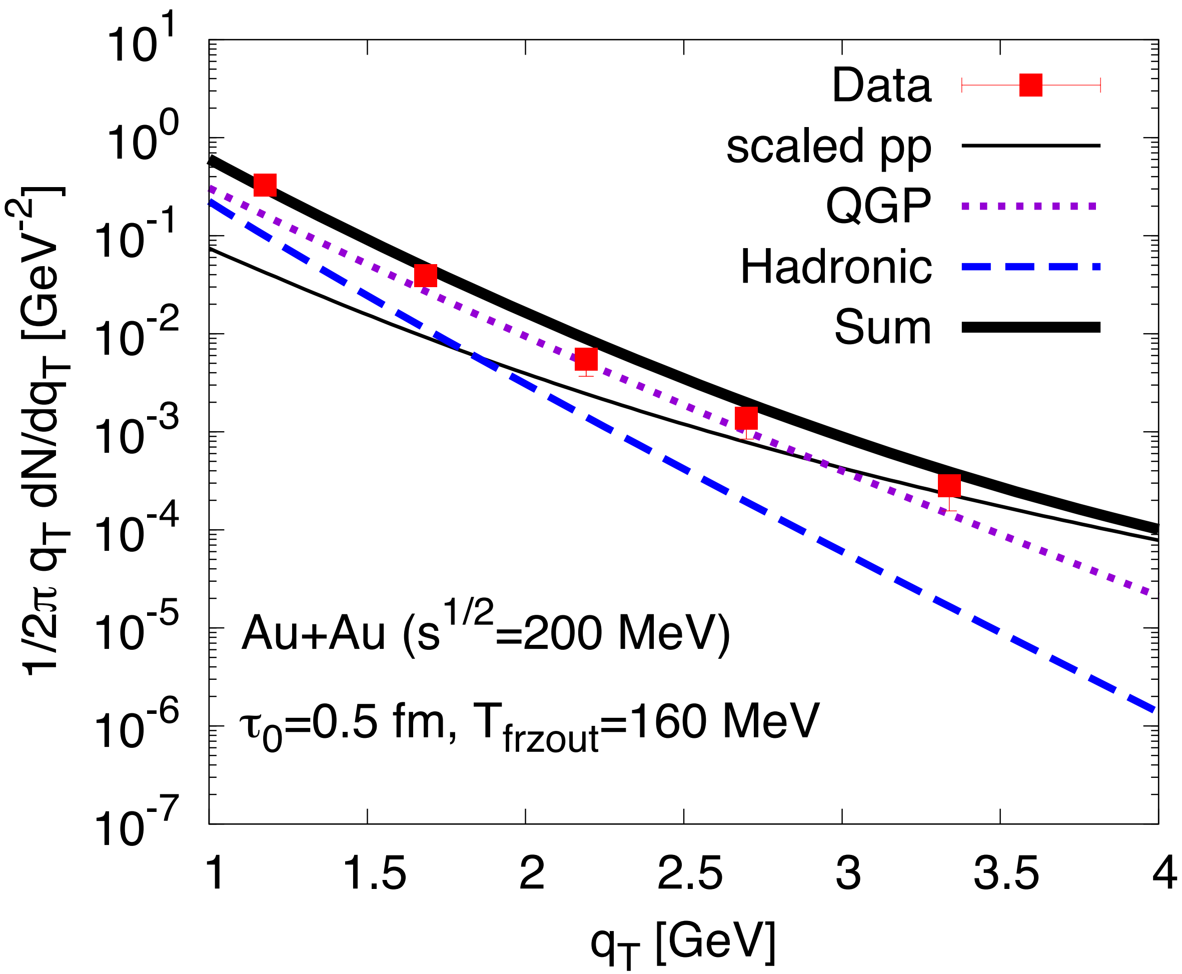
Dusling & Zahed, 0911.2426

Do virial expansion, need $\langle \pi | J_V(x) J_V(0) | \pi \rangle$ ; $\langle \pi\pi | J_V(x) J_V(0) | \pi\pi \rangle$

Use experimental input (R, τ decay) :

find hadronic contribution much larger than other analyses;

*Resolves puzzle of the "missing" photons?*



# Photon elliptic flow still too big by ~ 2!

Dusling & Zahed, unpublished,
RIKEN@BNL workshop on “Thermal Photons & Dileptons”, Aug. ’14
http://www.star.bnl.gov/~ruanlj/?dir=TPD2014/&file=Zahed.pptx