Matrix model of the sQGP *with* dynamical quarks

Preliminary results,
RDP & Vladimir Skokov, arXiv:151x.xxxxx

Skokov:
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Three year postdoc,
~$10,000/year travel expenses. Japanese *or not*

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What to do in the sQGP, near T_χ?

Lattice: $T_{chiral} = T_\chi \sim 154 \pm 9$ MeV. Borsanyi et al, 1309.5258; Bazavov et al, 1407.6387

$T \leq 130$ MeV: hadron resonance gas (lattice)

$T \geq 400$ MeV: (NNLO) HTLpt, Haque et al, 1402.6907
Next-to Next-to Leading Order Hard Thermal Loop perturbation theory

What to do in the sQGP, between ~ 130 and ~ 400 MeV?

Experimentally, the region near T_χ matters most at *both* RHIC & LHC

Develop effective theory, fixed by comparing to lattice simulations *in* equilibrum.
Then use to compute transport coefficients, e.g. η/s, *near* equilibrium.
Here s=semi-QGP, in a matrix model *with* dynamical quarks.

Other models of the sQGP: quasi-particles models; e.g. Parton-String Dynamics
Polykov loop models, center domains, holography (AdS/CFT),
dyon liquids, functional renormalization group, background field method…

Anderson Localization

In a random medium, waves don't diffuse.
As a wave scatters off of *random* impurities, it gains a phase from each scattering

Let the phase for a given scattering be $e^{i\theta_j}$. In the limit of infinitely many scatterings, the total change in the wave function is

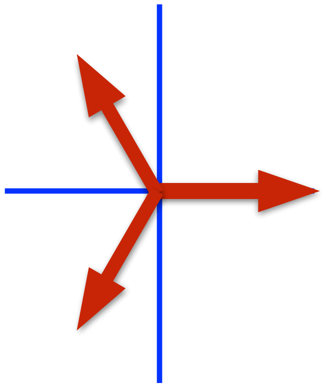
$$\sum_{j=1}^{\infty} e^{i\,\theta_j} \;\rightarrow\; \int_0^{2\pi} d\theta\; e^{i\theta} = 0$$

Probability distribution of wave function is *localized*
Not because of infinitely heavy mass: rather, *phase decoherence*.
Quantum metal-insulator transition.

Analogy: Confinement ~ Localization

't Hooft: hidden (global) Z(3) symmetry in (local) SU(3)

T=0: Quarks get Z(3) phase, $e^{2\pi i j/3}$, as they move through each random domain



$$\sum_{j=0}^{2} e^{2\pi i\, j/3} = 1 + e^{2\pi i/3} + e^{4\pi i/3} = 0$$

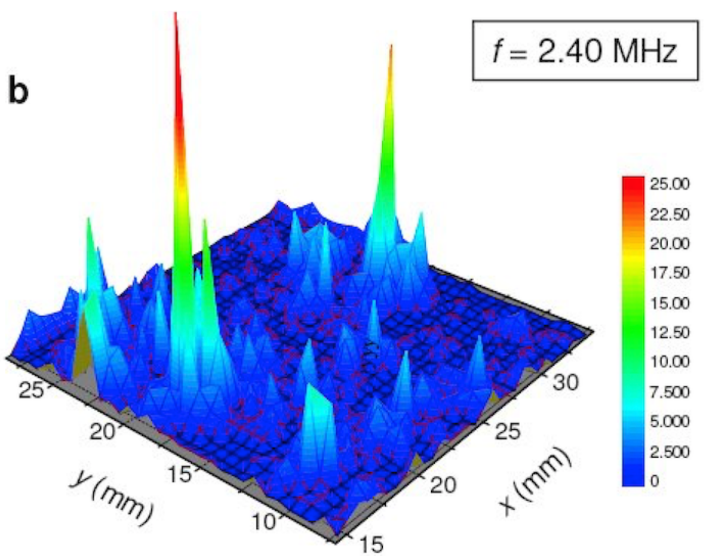
Confinement from *phase decoherence* of quark wave function (*not* ∞ heavy mass)

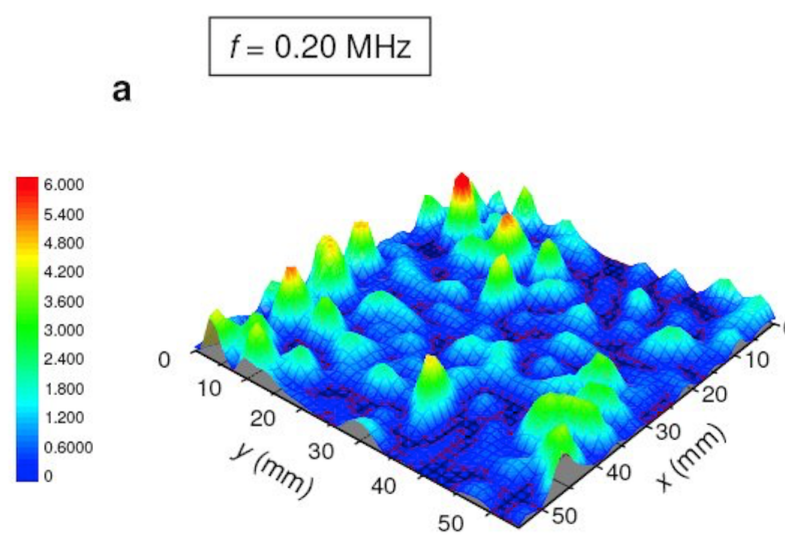
Infinite T: one big Z(3) domain ⇒ phase coherence

Hu et al, 0805.1502: use ultrasound to study brazed aluminum beads

localized = confined

delocalized = deconfined



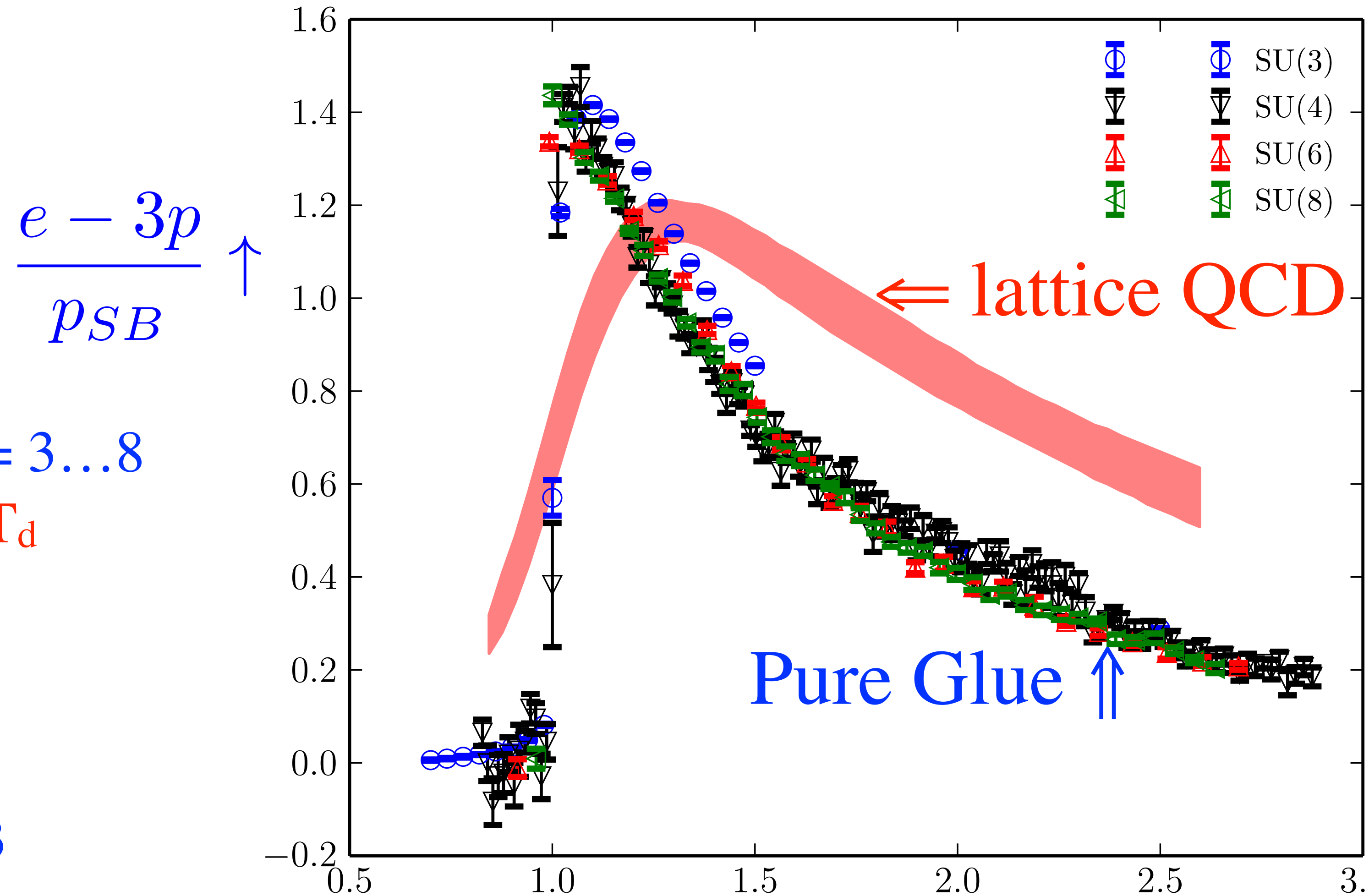


What the lattice tells us about the pressure

Consider e-3p, d*ivided by* $p_{SB} = p_{\text{Stefan-Boltzman}}$.

For pure SU(N_c) glue, e-3p/p_{SB} is ~ independent of N_c.

With quarks, e-3p/p_{SB} *changes* with N_f : “flavor independence” *not*.



Pure Glue: N_f = 0, N_c = 3…8

$T_c = T_{deconfinement} = T_d$

Panero, 0907.3719

Datta & Gupta, 1006.0938

Borsanyi et al, 1204.1684

QCD: N_f = 2+1, N_c = 3

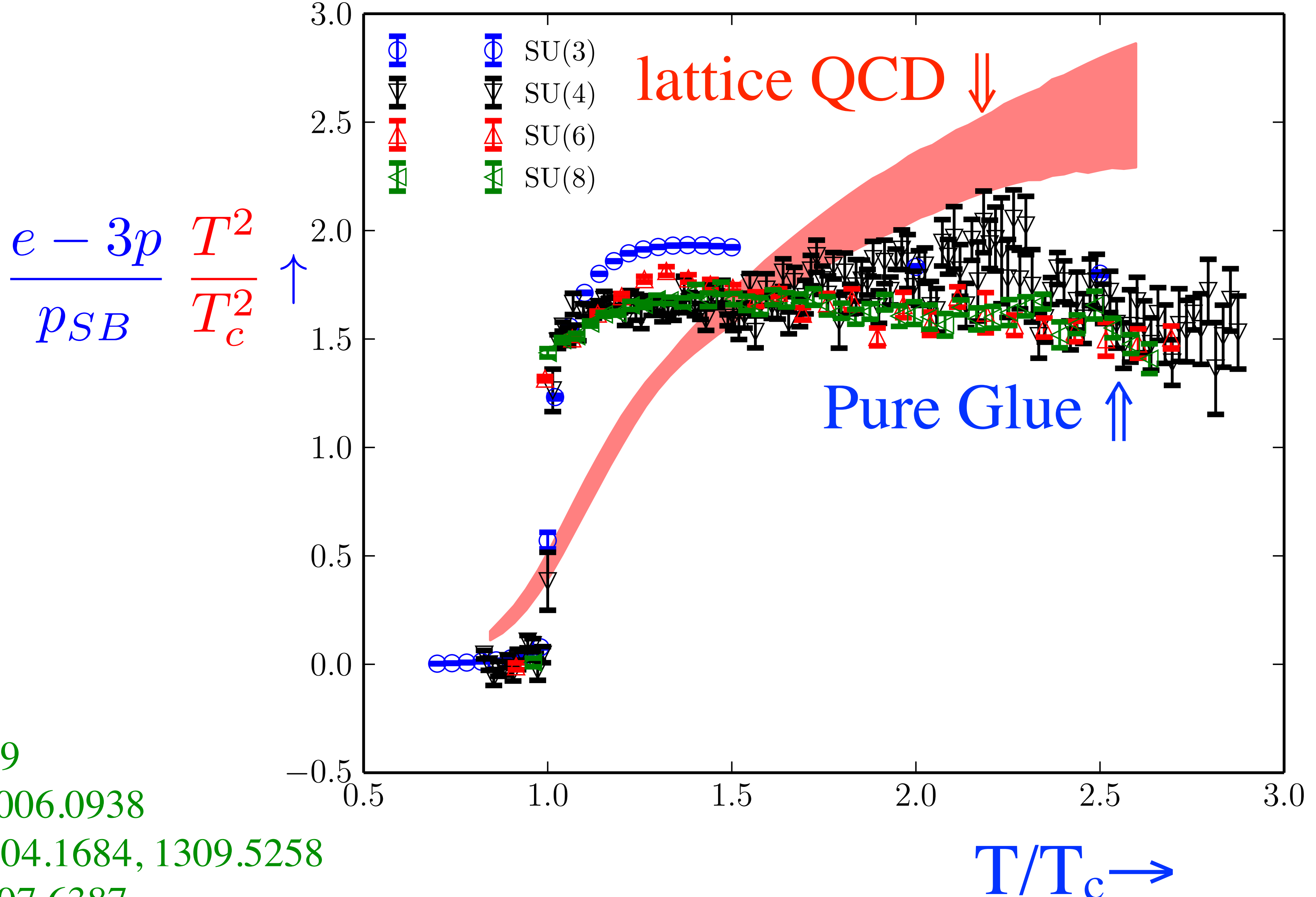
$T_c = T_{chiral} = T_\chi$

Borsanyi et al, 1309.5258; Bazavov et al, 1407.6387

With all lattice results, band is an estimate of error in continuum extrapolation

Lattice: for pure glue, T^2 term in pressure, but not in QCD

Lattice: for *pure* glue, corrections to T^4 term in pressure are nearly pure ~ T^2.
Not true with quarks; corrections to T^4 more complicated



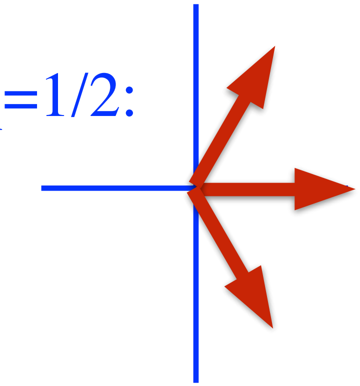
Panero, 0907.3719
Datta & Gupta, 1006.0938
Borsanyi et al, 1204.1684, 1309.5258
Bazavov et al, 1407.6387

Matrix model of pure glue in the semi-QGP

Take simplest ansatz, constant diagonal background A_0 field. Polyakov loop:

$$\ell = \frac{1}{3}\,\mathrm{tr}\,\mathcal{P}\,\mathrm{e}^{ig\int_0^{1/T} d\tau\, A_0} = \frac{1}{3}\,\mathrm{tr}\begin{pmatrix} \mathrm{e}^{2\pi i q/3} & 0 & 0 \\ 0 & \mathrm{e}^{-2\pi i q/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

q=1/2:



At $T = \infty$, complete deconfinement, $q = 0$. At $T < T_d$, confinement, with $q = 1$.

In between, $\infty > T > T_d$, is the "semi"-QGP, with $0 < q < 1$

To one loop order, perturbative potential for q:

$$V^{glue}_{pert}(q) = \frac{4\pi^2}{3}\,T^4\left(-\frac{1}{20} + q^2(1-q)^2\right)$$

By hand we add a *non*-perturbative potential for q

$$V^{glue}_{non-pert}(q) = \frac{4\pi^2}{3}\,T^2\,T^2_{deconf}\left(-\frac{c_1}{5}q(1-q) - c_2 q^2(1-q)^2 + \frac{c_3}{15}\right)$$

Dumitru, Guo, Hidaka, Korthals-Altes, RDP , 1011.3820, 1205.0137.

$T_{deconfinement} = 260$ *MeV*; $c_1 = 0.32$; $c_2 = 0.83$; $c_3 = .87$

Matrix model with quarks

Couple scalar field Φ, invariant under flavor $SU(3)_L \times SU(3)_R \times U(1)_A$, to quarks:

$$\mathcal{L}^{quark} = \overline{\psi}\left(\not{D} + y\left(\Phi P_L + \Phi^\dagger P_R\right)\right) \quad ; \quad P_{L,R} = (1 \pm \gamma_5)/2$$

Φ: $J^P = 0^-$: π , K , η , η'. $J^P = 0^+$: a_0, ϰ , σ_8, σ_0 . Yukawa coupling y between
Integrate quarks to one loop order. Gives quark potential for q, couples q to Φ…
Also need non-perturbative potential for Φ:

$$V^{quark}_{non-pert} = m^2\,\mathrm{tr}(\Phi^\dagger\Phi) - c_A(\det\Phi + \mathrm{c.c.}) + \lambda\,\mathrm{tr}(\Phi^\dagger\Phi)^2 + \mathrm{tr}(H(\Phi + \Phi^\dagger))$$

Lenaghan, Rischke, Schaffner-Bielich, nucl-th/0004006.

Integrating over quarks gives a novel Counter Term in 4-ε dimensions:

$$\mathcal{L}^{CT} \sim \frac{y^4}{\epsilon}\,\mathrm{tr}\left((\Phi^\dagger\Phi)^2\log(\Phi^\dagger\Phi)\right)$$

As usual, $H \sim m_{quark}$.
At high T, also need to add a new term ~ m_{quark}:,
to cancel $\langle\overline{\psi}\psi\rangle \sim m_{qk}\,T^2$

$$\mathcal{L}^{T\neq0}_{m_{qk}} \sim m_{qk}\int_{m_{qk}}^{m_{dyn}} dm\;\mathrm{tr}\,\frac{1}{-D^2 + m^2}$$

In a matrix model, $T_\chi \ll T_{deconfinement}$

With dynamical quarks, *precise* definition of T_χ as $m_\pi \to 0$.
No precise definition of T_{deconf}

Keep $T_{deconf} = 260$ MeV as with pure glue
Then *tune* the Yukawa coupling y to get $T_\chi \sim 154$ MeV: *so* $T_\chi \ll T_{deconf}$

Treat 2+1 flavors: $H \sim \mathrm{diag}(m_{up}, m_{up}, m_{strange})$. $\langle\Phi\rangle = (\Sigma_{up}, \Sigma_{up}, \Sigma_{strange})$

Input: the masses of π, K, η, and η'; also, f_π. $\Sigma_{up} = f_\pi/2 = 46$ MeV.

Output (MeV): $h_{up} = (122)^3$; $h_{strange} = (384)^3$; $\Sigma_{strange} = 76$; $f_K = 122$.

T_χ *not* very sensitive to Yukawa coupling y: $T_\chi \sim 154$ MeV for $y \sim 4$ - 4.5
Masses: $\sigma_0 \sim 376$; $a_0 \sim 980$.

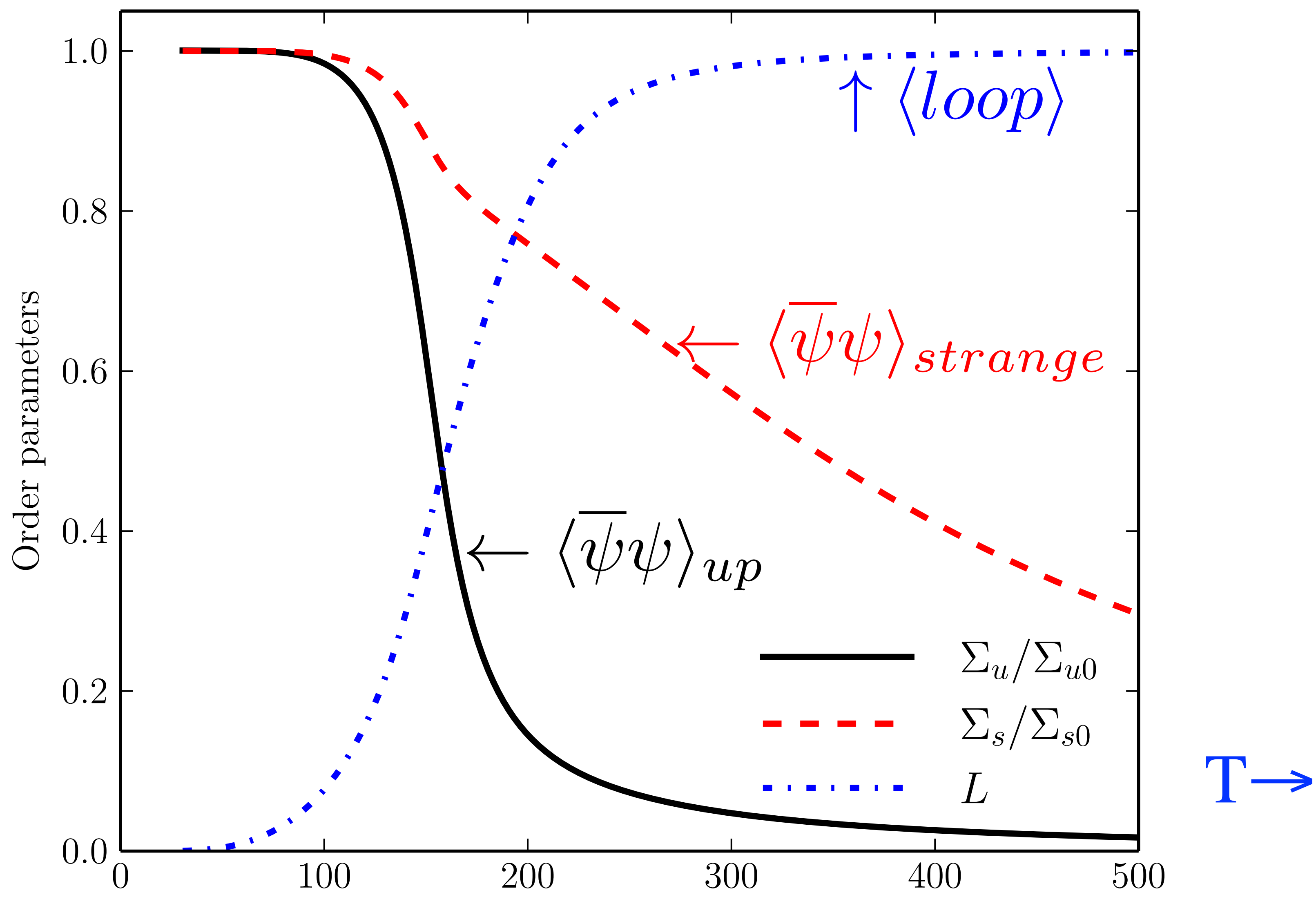
In sigma model, parameters $m = 506$; $c_A = 4560$; $\lambda \sim 28$.

Matrix model: order parameters, chiral and deconfining

Following: use mean field for Φ, neglect *any* fluctuations in Φ

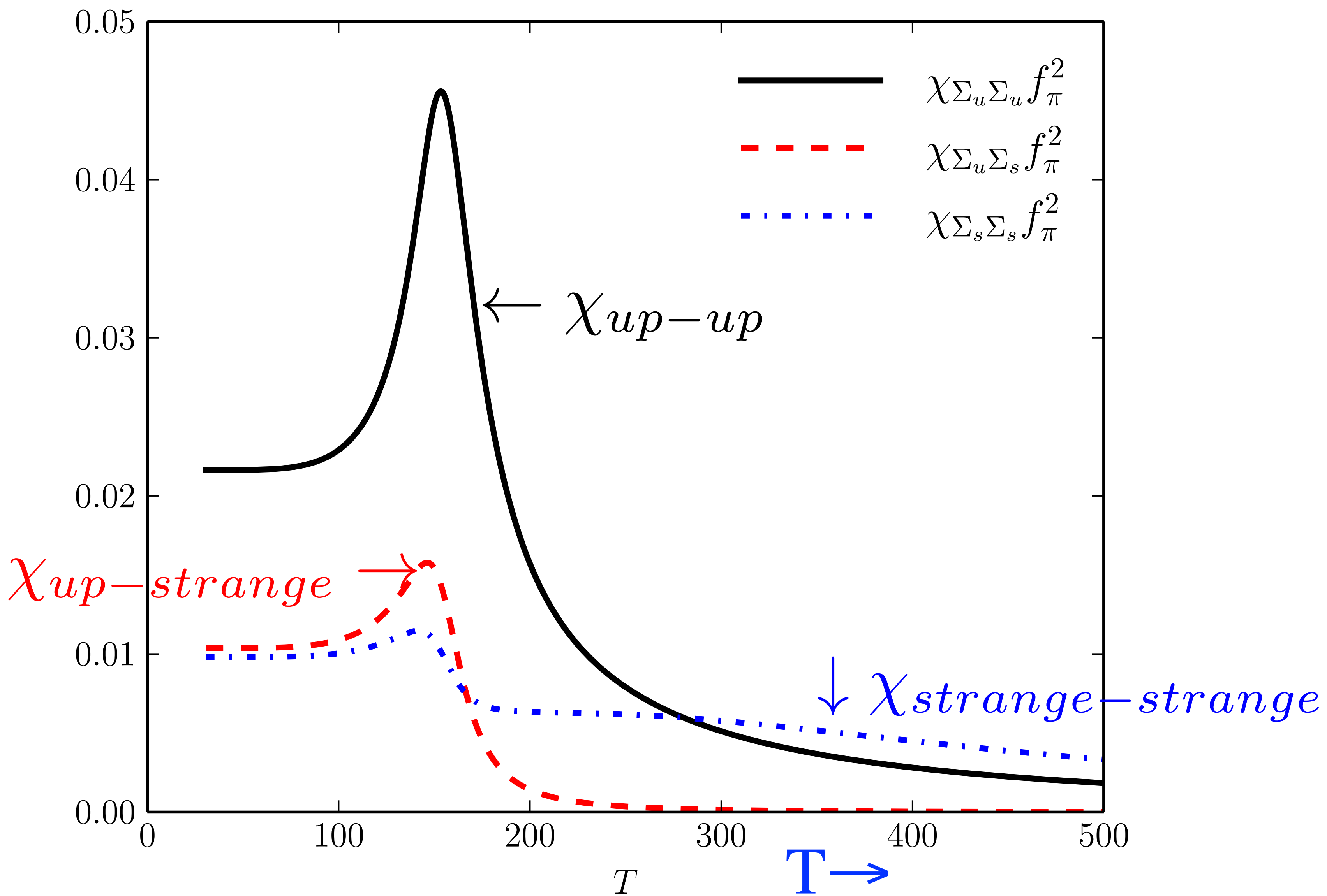
Below: ratio of chiral condensates, (T≠0)/(T=0)

Polyakov loop: as for pure glue, loop in matrix model > loop from lattice: *puzzle*



Chiral susceptibilities

At T_χ, susceptibility for light-light > light-strange > strange-strange. No surprise



Chiral-loop susceptibility: divergence!

At T_χ, loop-loop susceptibility has mild peak. But loop-up has big peak!

In chiral limit: at T_χ, divergence in both chiral *and* chiral-loop susceptibilities

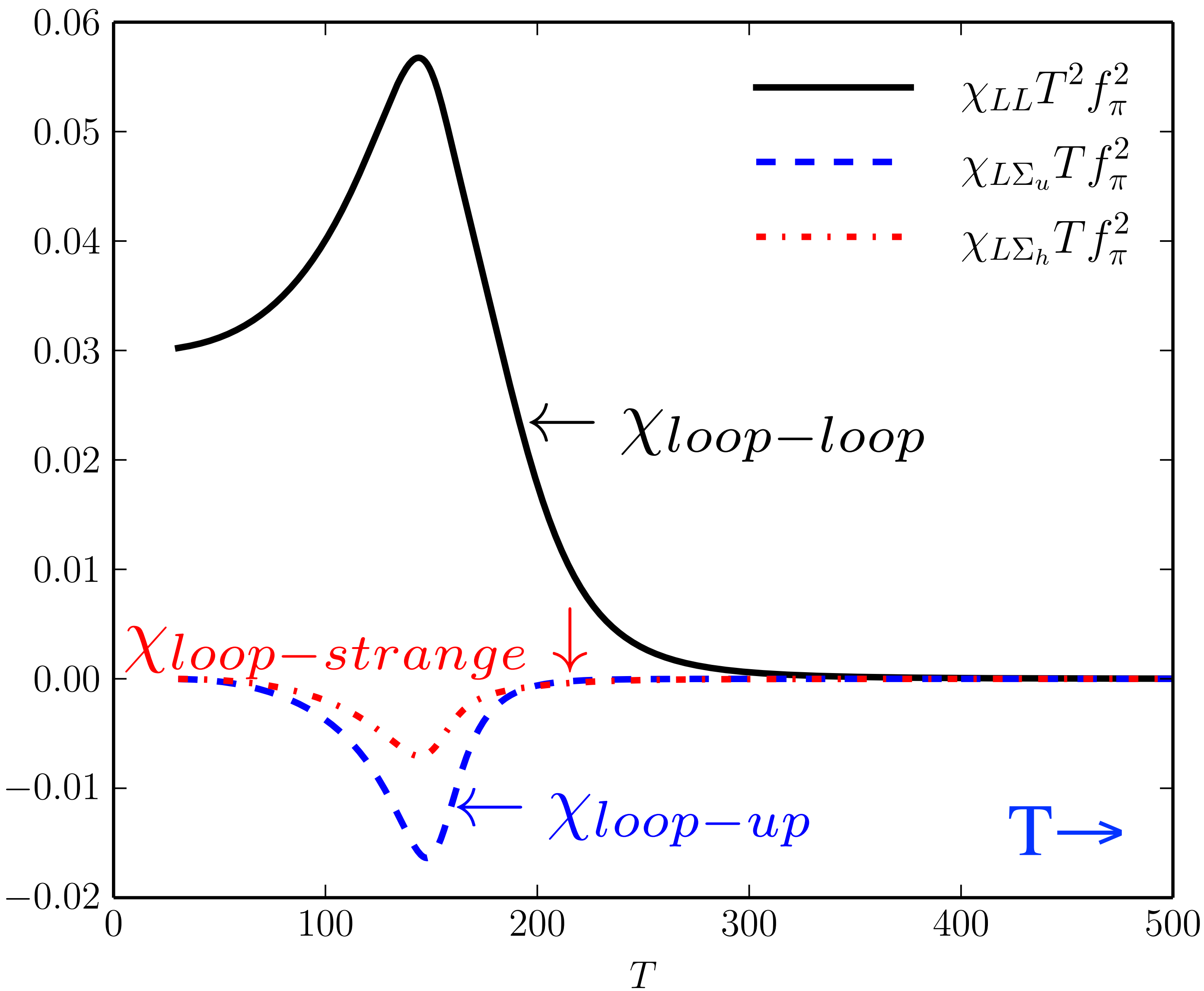
Sasaki, Friman, Redlich hep-ph/0611147

$m_\pi = 0,\ T \sim T_\chi$:

$$\chi_{up-up} \sim \frac{1}{(T - T_\chi)^1}$$

$$\chi_{up-loop} \sim \frac{1}{(T - T_\chi)^{1/2}}$$

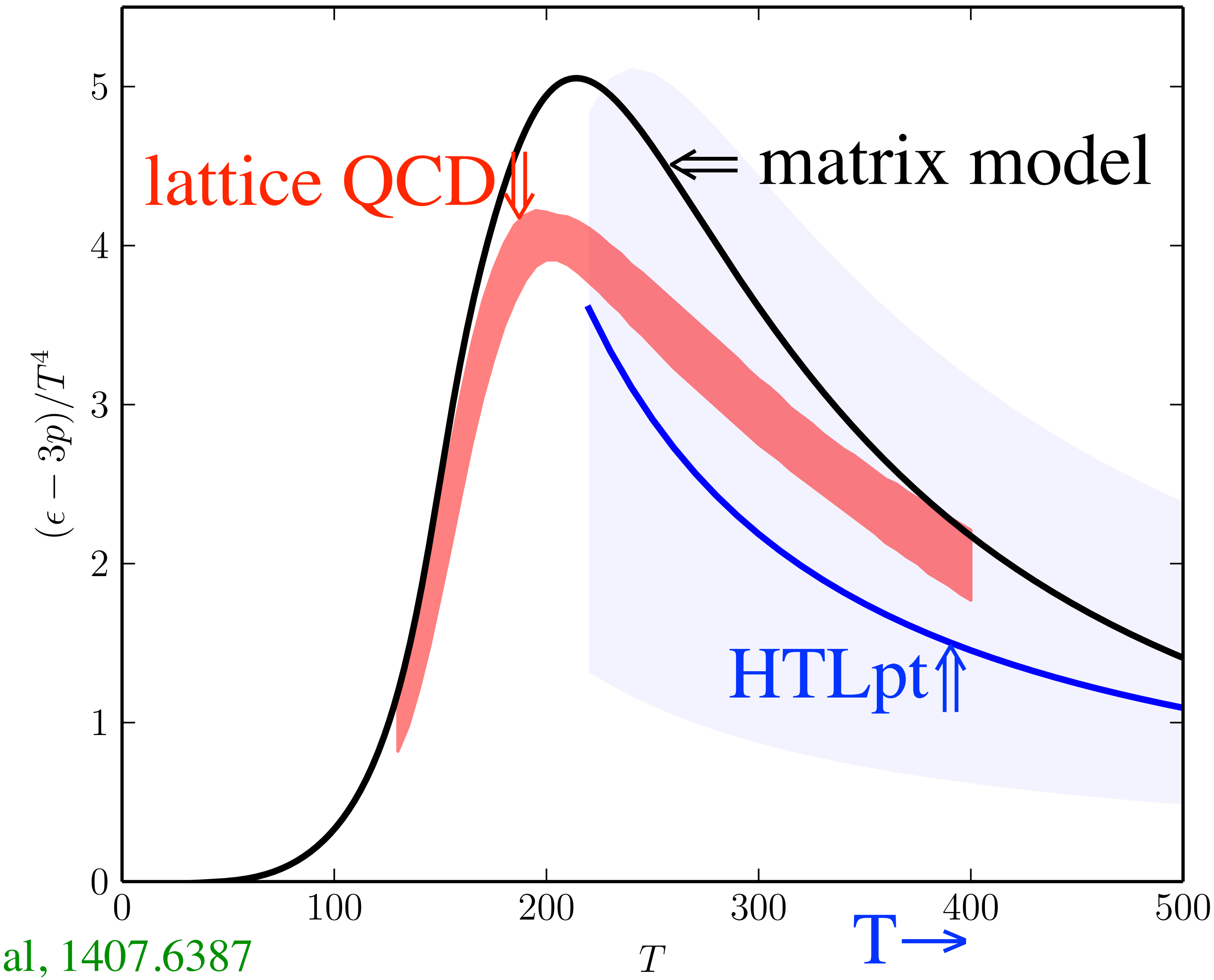
$$\chi_{loop-loop} \sim (T - T_\chi)^0$$



Results for matrix model: μ = 0, e-3p

Compare to lattice and to (NNLO) HTLpt, with quark chemical potential μ = 0
HTL pert. theory: band = changing renormalization mass scale, 2πT, by two

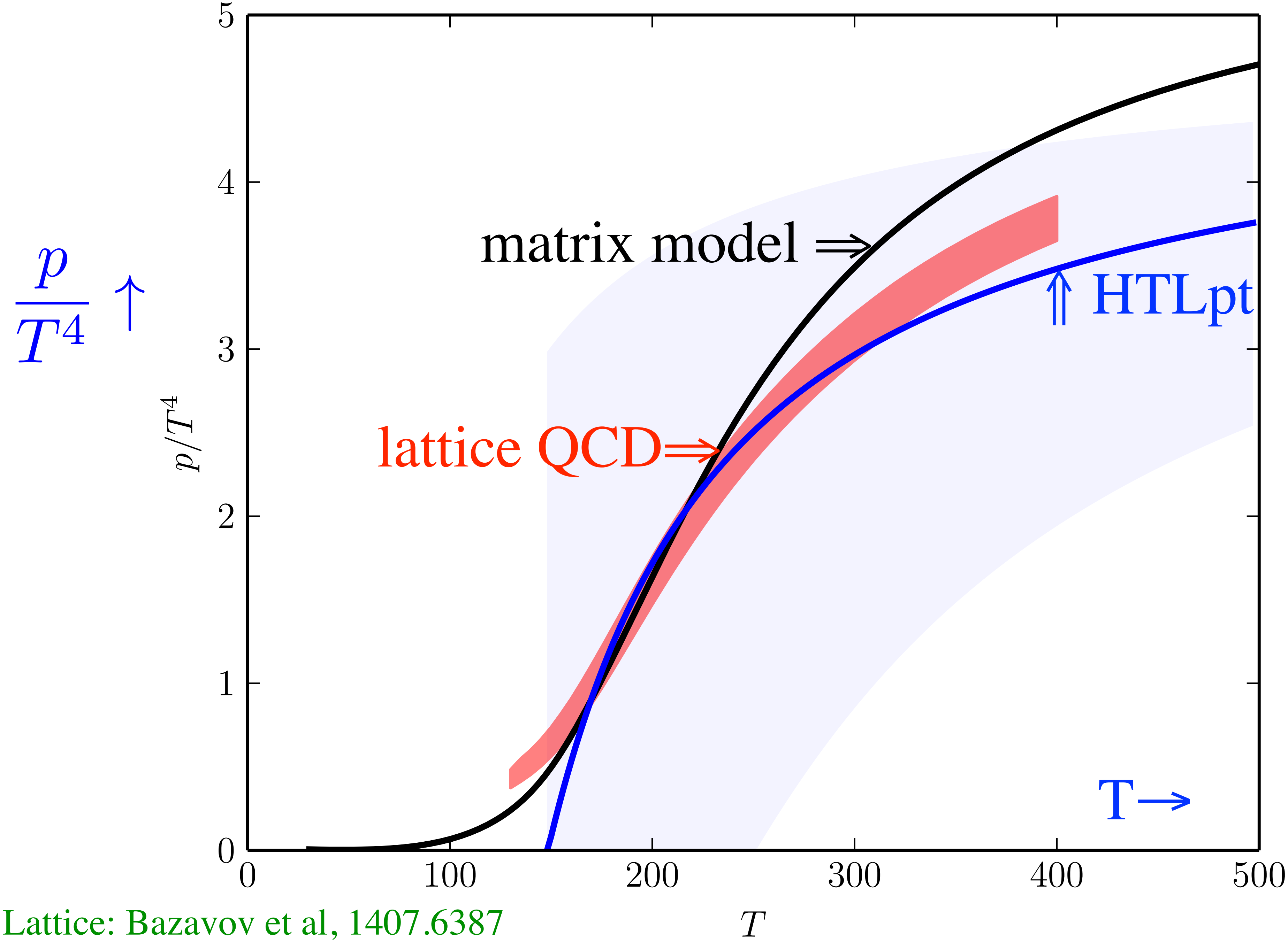
$$\frac{e-3p}{T^4}\uparrow$$



T⟶

Lattice: Bazavov et al, 1407.6387
(NNLO) HTLpt: Haque et al 1402.6907

Results for matrix model: μ = 0, pressure



Lattice: Bazavov et al, 1407.6387

HTLpt: Haque et al, 1402.6907

Generalized susceptibilities for quark chemical potentials

With quarks, several conserved flavor currents:
baryon number (B) & strangeness (S) (= light quarks (L)); electric charge (Q).

Quark chemical potential for each current: μ_B, μ_S ($\sim \mu_L$); μ_Q. Set $\mu_Q = 0$.

The pressure is a function of temperature, T, and *both* μ_B and μ_S

Instead of plotting function of three variables, useful (and computationally clean) to compute derivatives of the pressure with respect to μ_B and μ_S :

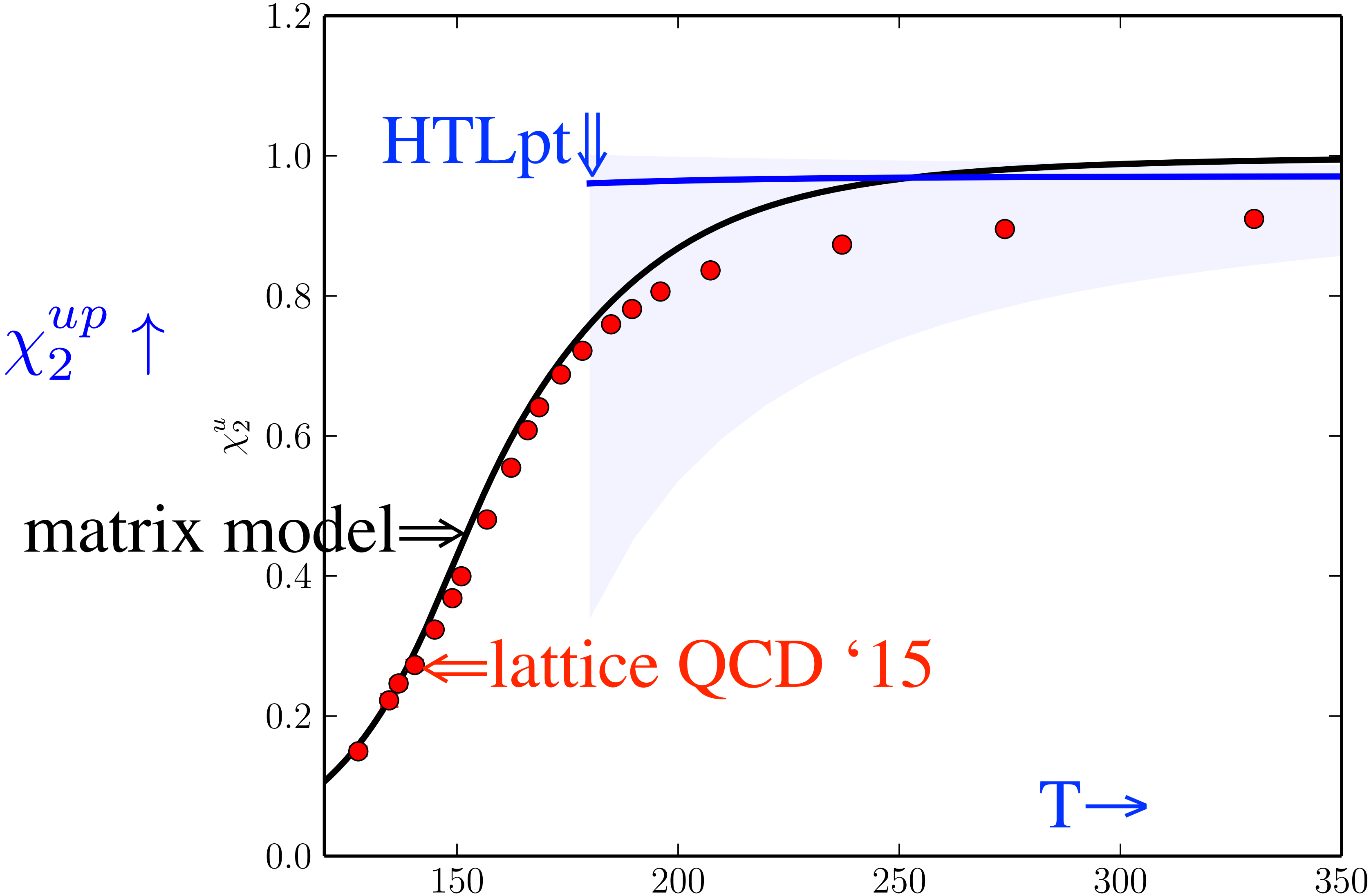
$$\chi_{ij}^{XY} = \frac{\partial^{i+j}}{\partial(\mu_X/T)^i \partial(\mu_Y/T)^j} \, p(T, \mu_X, \mu_Y) \Big|_{\mu_X = \mu_Y = 0}$$

Bazavov et al, 1304.7220

Second moment, light quarks

Simplest thing is the second moment.

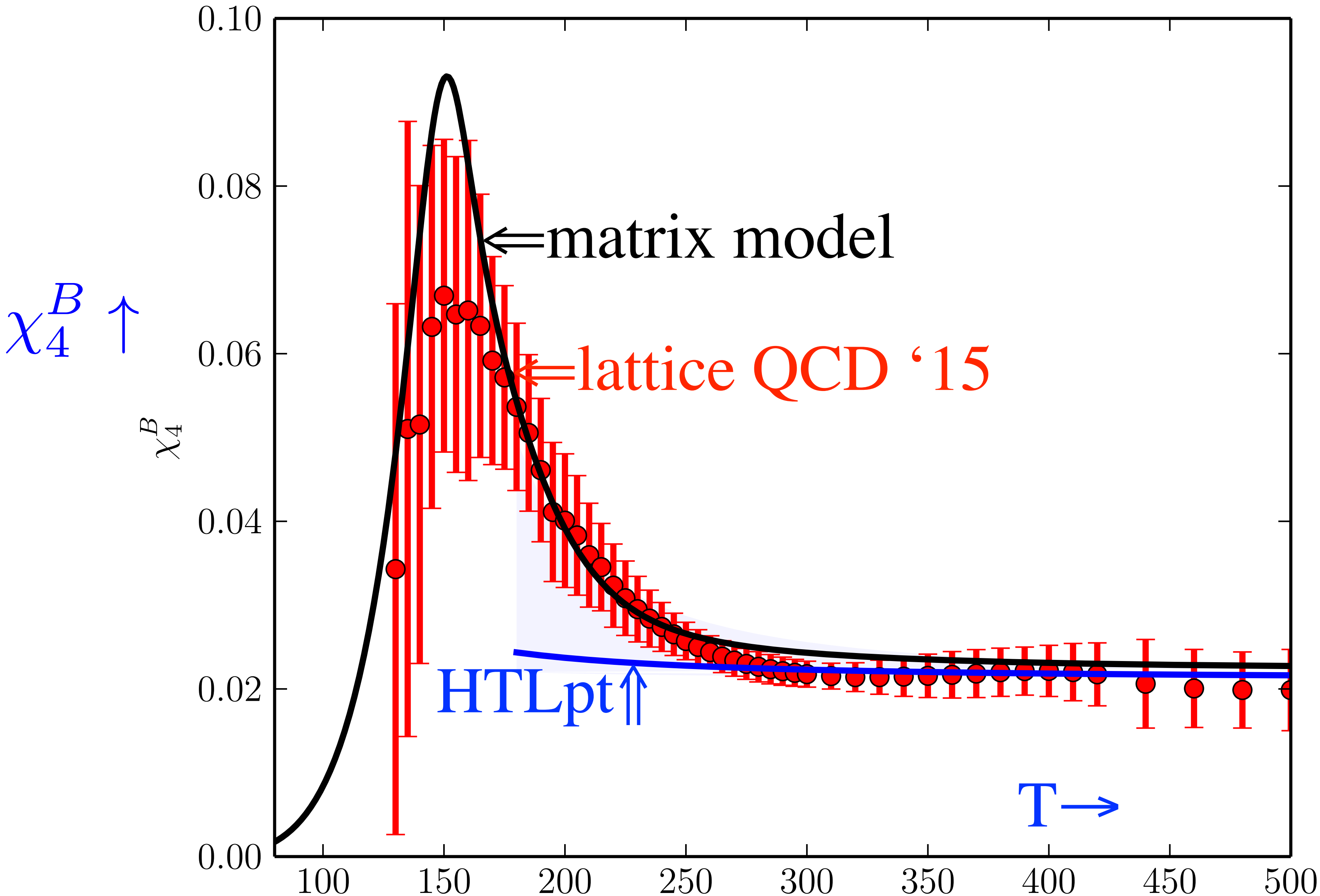
Constituent quark mass suppresses χ_2^{up} at low temperature.



Lattice: C. Schmidt, PoS(LATTICE2014)186; Bielefeld-BNL-CCNU Collaboration, in preparation

HTLpt: Haque et al, 1402.6907

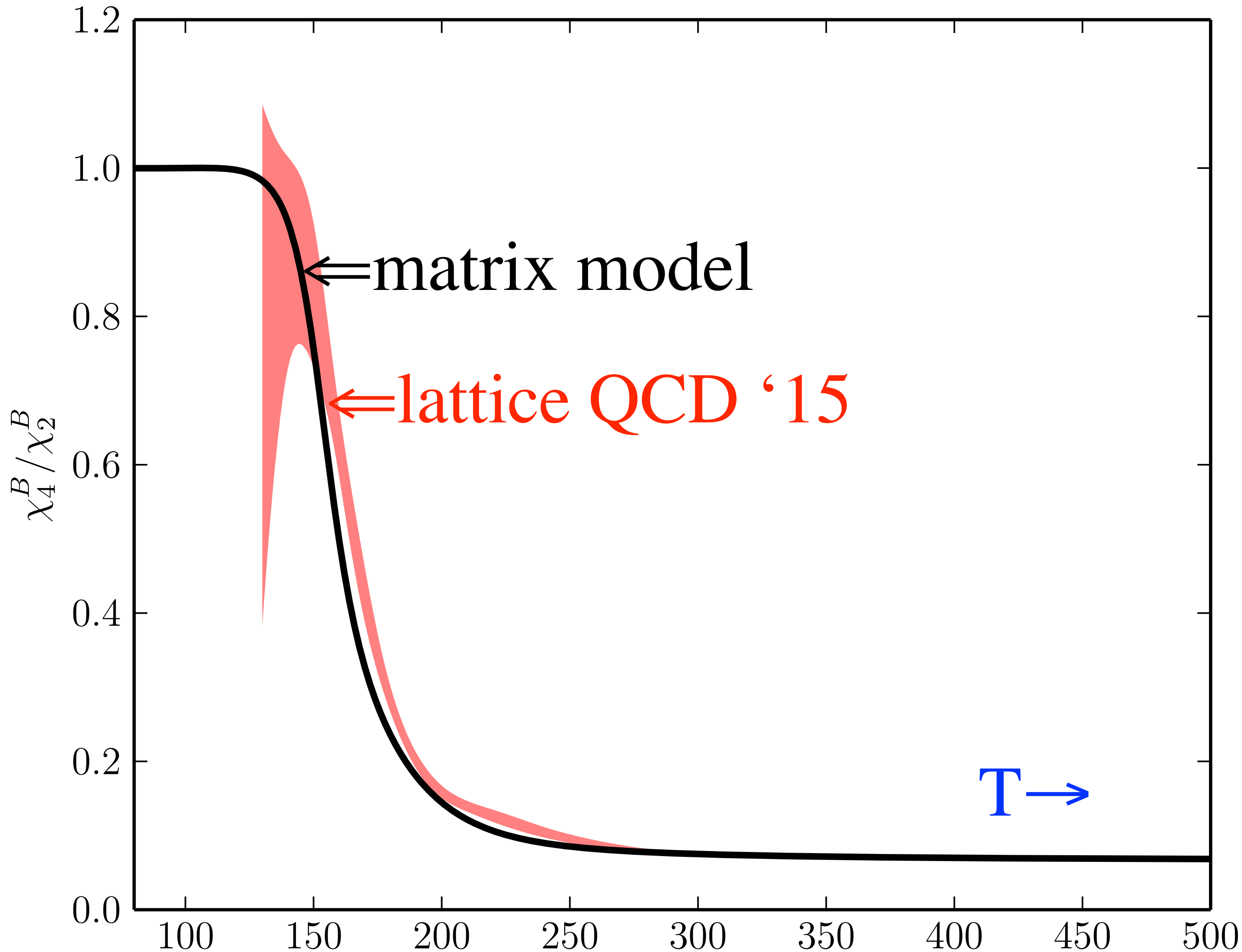
Fourth moment, baryons



Lattice: Borsanyi et al, 1305.5161; 1507.07510
HTLpt: Haque et al, 1402.6907

Ratio of fourth to second moment, baryons

$$\frac{\chi_4^B}{\chi_2^B} \uparrow$$



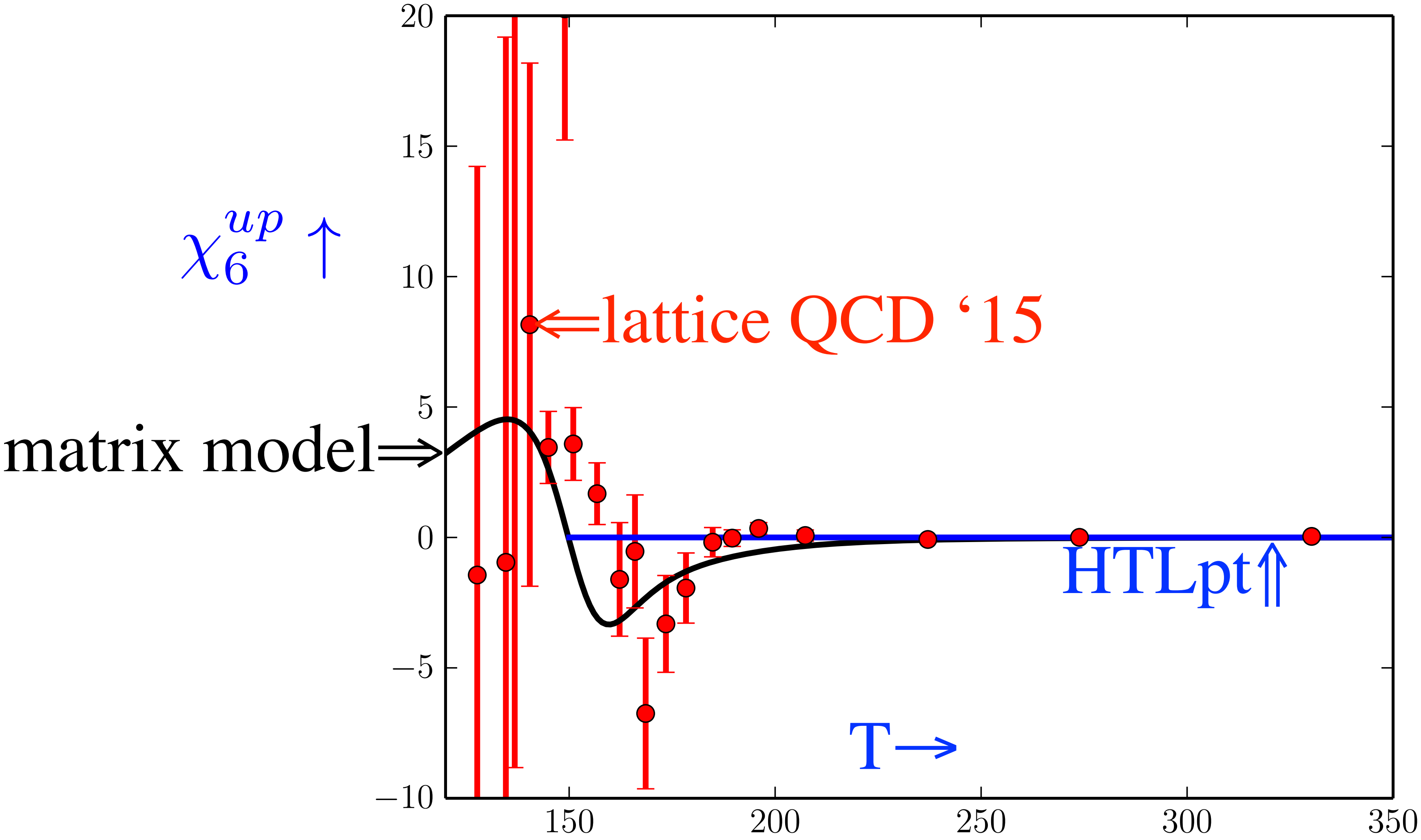
Lattice: Borsanyi et al, 1305.5161; 1507.07510

A good test: *sixth* moment, baryons

For massless quarks, order by order in pert. theory *only* terms ~μ^4 in pressure.

So HTL pert. theory gives $\chi_6 \sim d^6p/d\mu^6 \ll 1$.

Matrix model, with $m_{dynamical} \neq 0$, gives characteristic change in sign of c_6 near T_χ.



Lattice: C. Schmidt, PoS(LATTICE2014)186; Bielefeld-BNL-CCNU Collaboration, in preparation

HTLpt: Haque et al, 1402.6907

An even better test: *off*-diagonal susceptibilities

Off-diagonal susceptibilities, such as Baryon-Strange (BS), are a good test

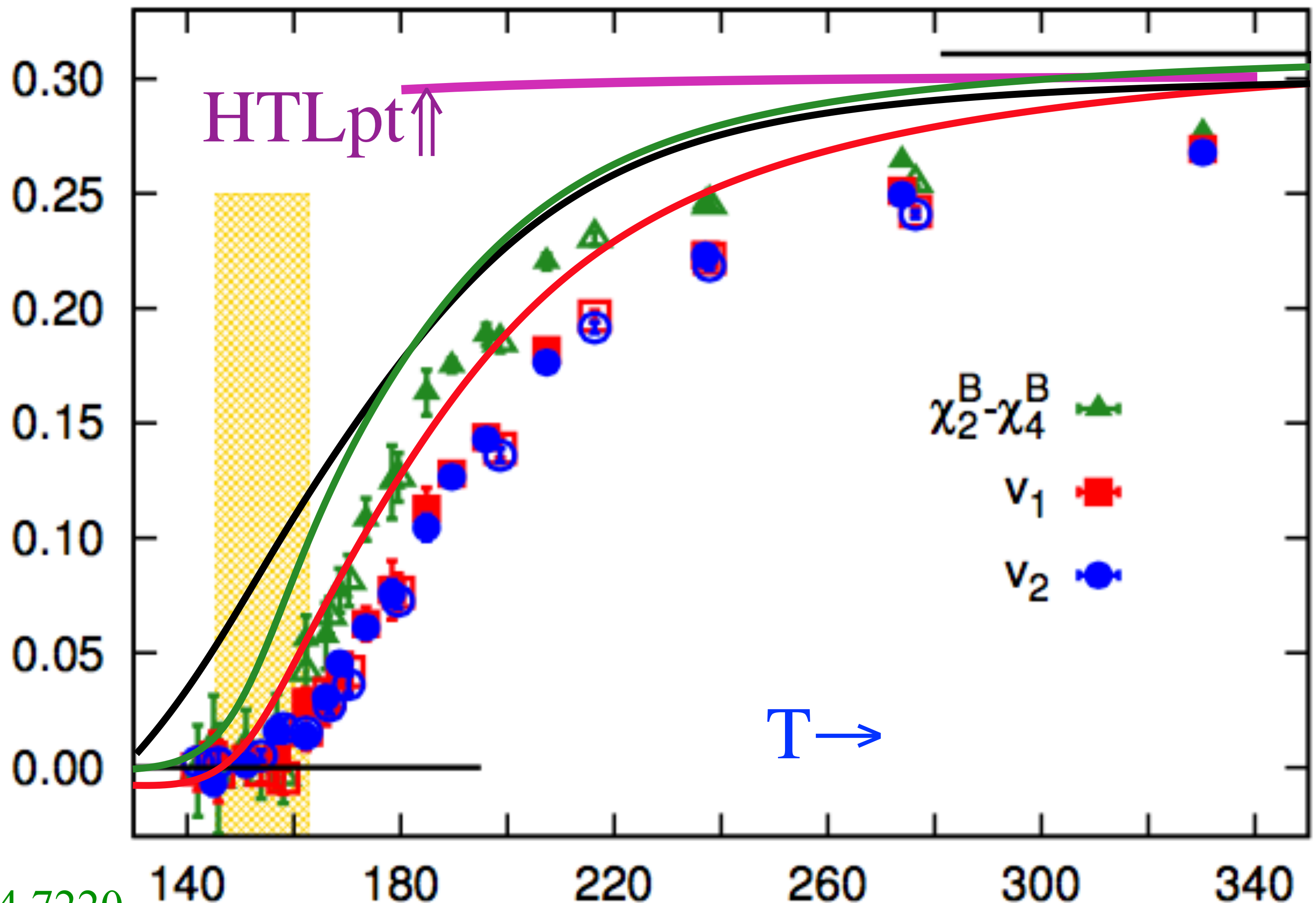
$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

Green: χ_2^B - χ_4^B
points: lattice
line: matrix model

(Black line:
Polyakov loop/3
in matrix model)

Red: v_1.
points: lattice
line: matrix model

Magenta: HTLpt



Lattice: Bazavov et al, 1304.7220
HTLpt: Haque et al, 1402.6907
To be computed, v_2:

$$v_2 = \frac{1}{3}\left(\chi^S - \chi_4^S\right) - 2\,\chi_{13}^{BS} - 4\,\chi_{22}^{BS} - 2\,\chi_{31}^{BS}$$

Summary

Took matrix model for pure glue, and included dynamical quarks by adding:

1. Linear sigma model for π's, K's…
2. Yukawa coupling *y* between quarks and π's, K's…

Determined parameters:

1. Linear sigma model: fit f_π and masses of π, K, η and η'
2. *Keep* T_{deconf} = 260 MeV, *tune y* to get T_{chiral} = 154 MeV ($T_{chiral} \ll T_{deconf}$)

Good fits to thermodynamics quantities, *especially* χ_6 *and* χ^{BS}.
Matrix model works much better in the sQGP than (NNLO) HTLpt. (duh)

To dream the impossible…: NNLO HTLpt *plus* matrix model.

Next: chiral critical end-point? Stephanov, Rajagopal, & Shuryak 9806219

More generally, phase diagram in T-μ plane, for both real and imaginary μ

Kashiwa & RDP, 1301.5344

To do

Given a matrix model with dynamical quarks, can then *directly* compute to leading (logarithmic) order:

shear viscosity

Hidaka and RDP, 0803.0453; 0906.1751; 0907.4609; 0912.0940

production of dileptons and photons

Gale, Hidaka, Jeon, Lin, Paquet, RDP, Satow, Skokov, Vujanovic, 1409.4778
Hidaka, Lin, RDP, Satow, 1504. 01770

energy loss of heavy quarks

Lin, RDP, Skokov, 1312.3340

Still need to compute: *energy loss of light quarks*