Technicalities of the QCD phase diagram: $T \neq 0, \mu = 0$

1. Z(N) interface tension for pure SU(N)

2. Deconfinement at *zero* coupling: SU(∞) on a small sphere
   (Sundborg '99, Aharony et al '03, '05)

3. Renormalization of Polyakov loops
   Representation(s); zero point energy of ren.'d loops
   One point vs. two point renormalization

4. Lattice data on renormalized loops
   Bare loops, adjoint loops, Casimir scaling

5. Goal: complete effective theory near T_c

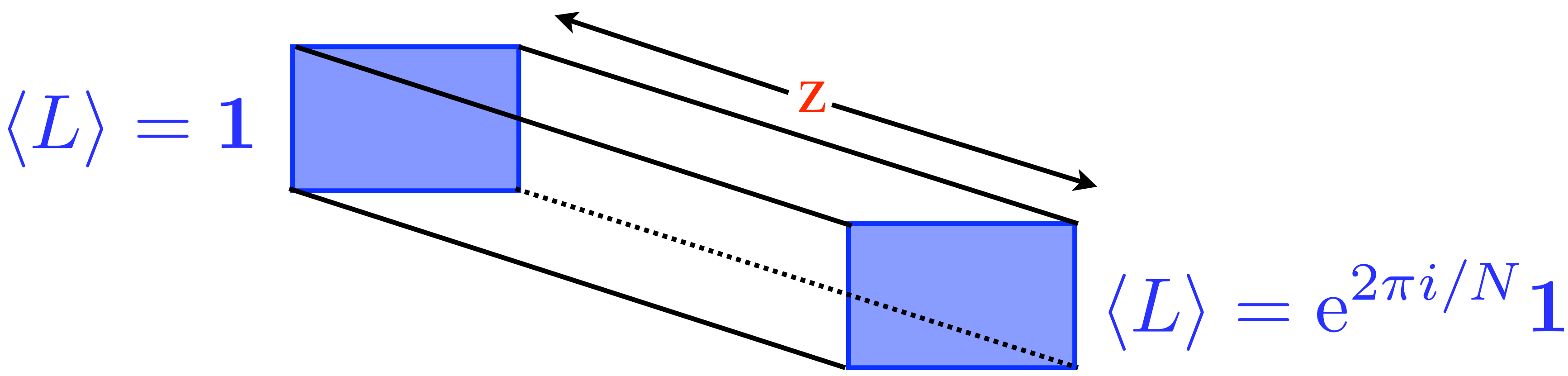
6. Effective theory in Euclidean spacetime

7. Effective theory in Minkowski spacetime
   Hard Thermal Loops, shear viscosity

1. Z(N) interface tension (‘t Hooft loop) for pure SU(N)

Z(N) interface: boundary conditions

In pure SU(N) (no quarks), consider a box which is long in one (spatial) direction, and which differ by a Z(N) rotation at the two ends:



Each end represents an allowed vacuum. Since they differ, an interface forms between the two ends. Look for a solution which tunnels between the two vacua:

$$A_0^{cl}(z) = \frac{2\pi T}{g}\, q(z)\, t^{NN} \;\; , \;\; t^{NN} = \frac{1}{N}\begin{pmatrix} \mathbf{1}_{N-1} & 0 \\ 0 & -(N-1) \end{pmatrix}$$

With $\mathbf{L} = \mathcal{P}\, e^{ig \int A_0 \, d\tau}$, the boundary conditions are that q(0) = 0 at one end of the box, and q(L) = 1 at the other end.

Z(N) interface: classically

With this ansatz, the classical action only receives a contribution from the electric field:

$$S^{cl}(A^{cl}) = \int d^4x \, \frac{1}{2} \mathrm{tr}(G_{\mu\nu})^2 = V_{tr} \, \frac{4\pi^2 T^2}{g^2 N} \, (N-1) \int dz \, \left(\frac{dq}{dz} \right)^2$$

But then the equation of motion is trivial,

$$\frac{d^2 q}{dz^2} = 0$$

As is the solution for the interface:

$$q(z) = \frac{z}{L}$$

The action is $\int$dz (dq/dz)^2 ~ L (1/L)^2 ~ 1/L: the action *vanishes* as L → ∞. There is really no interface, the theory smoothly rolls from one vacuum to another. This is possible classically, where any q is valid. This degeneracy is lifted by quantum mechanical effects.

Z(N) interface: quantum effects

Thus compute the one loop action, in the presence of this background field. Only the ladder generators for t^{NN}, t^{aN} and t^{Na}, feel the background field.

Ladder generators obey $[t^{NN}, t^{aN}] \sim t^{aN}$, so reduces to an *Abelian* problem

$$D_0^{cl}\, t^{aN} = (\partial_0 - ig[A_0^{cl},])\, t^{aN} = i(2\pi T)(n+q)\, t^{aN}$$

Background field just shifts the "energy" from an integer (n) * 2 π T, to a fractional number, (n+q), * 2 π T. *Some*thing like fractional statistics...

$$S^{qu}(A^{cl}) = 2\,(N-1)\ \mathrm{tr}\log\left((p_0^+)^2 + \vec{p}^{\,2}\right)$$

The N-1 is from the number of ladder generators t^{jN}. This is independent of the gauge fixing parameter. Now treat "q" as *constant*. Then this is easy!

$$\frac{\partial S^{qu}}{\partial q} = 4(N-1)(2\pi T)\ \mathrm{tr}\left(\frac{p_0^+}{(p_0^+)^2 + \vec{p}^{\,2}}\right)$$

Justify, after the fact, why ok to take constant "q".

Z(N) interface: quantum effects, cont.'d

Do the integral by integrating over spatial momenta *first*, *then* summing over "n" for p_0:

$$V_{tr}\, L\, T \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \left(\frac{p_0^+}{(p_0^+)^2 + \vec{p}^{\,2}} \right) = -V_{tr} L \pi T^3 \sum_{n=-\infty}^{+\infty} (n+q)|n+q|$$

The sum over "n" is a type of zeta-function:

$$\mathrm{tr} \left(\frac{p_0^+}{(p_0^+)^2 + \vec{p}^{\,2}} \right) = -V_{tr} L\, \pi T^3 \left(\zeta(-2,q) - \zeta(-2, 1-q) \right)$$

$$\zeta(r,q) = \sum_{n=0}^{\infty} \frac{1}{(n+q)^r} \;\; ; \;\; \zeta(-2,q) = -\frac{1}{12} \frac{d}{dq} \left(q^2 (1-q)^2 \right)$$

Putting all the constants together, we obtain a *quantum* potential for "q":

$$S^{qu}(A^{cl}) = V_{tr} \frac{4\pi T^4}{3} (N-1) \int dz\; q^2 (1-q)^2$$

This is valid for $0 < q < 1$, and periodic outside of that domain.

Only the Z(N) vacua, $q = 0$ and 1, are minima of the potential.

Z(N) interface at leading order

Notice the classical action is ~ $1/g^2$, while the one loop term is ~ 1.
Thus if we introduce a rescaled coordinate, $\tilde{z} = \sqrt{N/3}\, gT\, z$
then the sum of the classical and one loop actions is

$$S^{cl} + S^{qu} = V_{tr}\, \frac{4\pi^2(N-1)}{\sqrt{3N}}\, \frac{T^3}{\sqrt{g^2}} \int d\tilde{z} \left(\left(\frac{dq}{d\tilde{z}} \right)^2 + q^2(1-q)^2 \right)$$

It is now easy solving for the interface! By the boundary conditions, the "energy" of the solution, $\mathcal{E} = (dq/d\tilde{z})^2 - q^2(1-q)^2$ vanishes. Thus one doesn't even need the explicit solution to find the action:

$$\int d\tilde{z} \left(\left(\frac{dq}{d\tilde{z}} \right)^2 + q^2(1-q)^2 \right) = 2 \int_0^1 dq\, q(1-q) = \frac{1}{3}$$

The interface tension is the coefficient of the transverse volume, $V_{tr}\, T$:

$$\tilde{\sigma} = \frac{4\pi^2(N-1)}{3\sqrt{3N}}\, \frac{T^3}{\sqrt{g^2}}$$

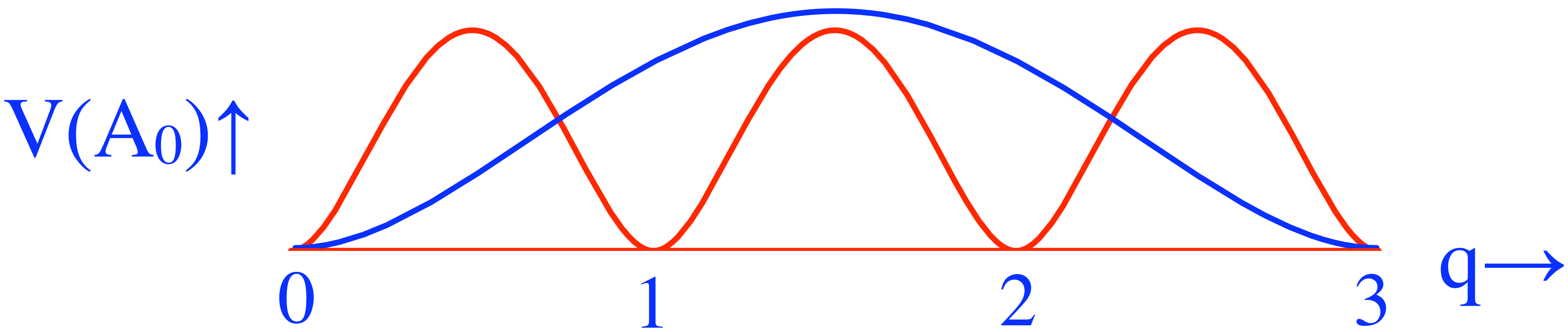
Z(N) interfaces, with quarks

Why constant A_0 ok? In terms of $\tilde{z} = \sqrt{N/3}\, gT\, z$, interface width is ~1. So in terms of z, it is large (for small g), ~ 1/g.

With "fat" interface, can systematically expand about constant A_0.

Without quarks, Z(N) interface equivalent to a (spatial) 't Hooft loop:
area behavior in deconfined phase (converse to Wilson loop).

With quarks, Z(N) degeneracy lifted. For N=3:



Bhattacharya, Gocksch, Korthals-Altes & RDP, hep-ph/9205231

Z(N) interface as 't Hooft loop: Korthals-Altes, Kovner & Stephanov, hep-ph/9909516

Corrections ~ g^3: Giovannangeli & Korthals-Altes hep-ph/0412322

~ g^4: Korthals-Altes, Laine, Romatschke 09...

SUSY interfaces: Armoni, Kumar, & Ridgeway 0812.0773, Korthals-Altes 09...

Lattice: Z(N) interfaces = ‘t Hooft loop

Can enforce Z(N) interface by boundary conditions, and so measurable on lattice. Find semi-classical result works well to ~ 10 T_c, see below

For N ≥ 4, also interface tension for kth interface: tunneling $\mathbf{L} : \mathbf{1} \rightarrow e^{2\pi i k/N}\,\mathbf{1}$

Satisfies semi-classical relation, right down to T_c:

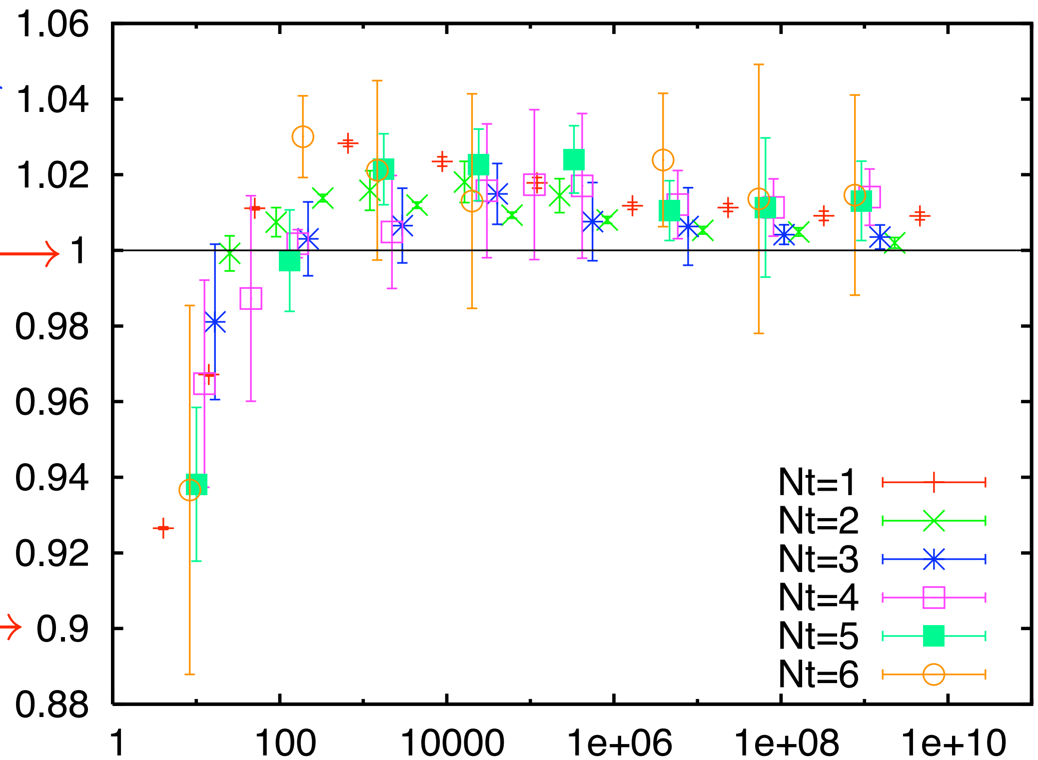
$$\widetilde{\sigma}_k = \frac{k(N-k)}{N-1}\,\widetilde{\sigma}_1$$

Bursa & Teper, hep-lat/0505025

$\dfrac{\widetilde{\sigma}_{\rm lattice}}{\widetilde{\sigma}_{\rm semi-class.}} \uparrow$

1.0 $\longrightarrow$

0.9 $\longrightarrow$



$T/\Lambda_{\overline{MS}} \;\rightarrow$

←de Forcrand & Noth, hep-lat/0506005.

2. Deconfinement at *zero* coupling: SU(∞) on a small sphere

SU(∞) on a small sphere: Hagedorn temperature

Sundborg, hep-th/9908001
AMMPV: Aharony, Marsano, Minwalla, Papadodimas, & Van Raamsdonk,
hep-th/0310285 & 0502149

Consider SU(N) on a *very* small sphere: radius R, with $g^2(R) << 1$.
(Sphere because constant modes simple, spherically symmetric)

At $N = \infty$, can have a phase transition even in a *finite* volume.

When $g^2 = 0$: by counting gauge *singlets*, find a Hagedorn temperature, T_H :

$$\rho(E) \sim \exp(E/T_H) \quad , \quad E \to \infty$$

At $N = \infty$, Hagedorn temperature is *precisely* defined. When $g^2 = 0$,

$$T_H = \frac{1}{\log(2+\sqrt{3})} \, \frac{1}{R} \quad , \quad g^2 = 0.$$

SU(∞) on a small sphere: effective theory

Construct effective theory for low energy (constant) modes,
by integrating out high energy modes, with momenta ~ 1/R:

Consider (thermal) Wilson line:

$$\mathbf{L} = \mathcal{P}\ \exp\left(ig \int_0^{1/T} A_0\ d\tau \right)$$

L is gauge dependent matrix,

$$\mathbf{L} \to \Omega(1/T)^\dagger\, \mathbf{L}\, \Omega(0)$$

Traces of L are gauge invariant,

$$\ell_j = \frac{1}{N}\,\mathrm{tr}\,\mathbf{L}^j \quad , \quad j = 1 \ldots (N-1)$$

Effective theory for l_j: compute free energy in *constant* background A_0 field:
Q = diagonal matrix. Calc's like Z(N) interface,
but for arbitrary background.

$$A_0 = \frac{T}{g}\, Q \quad , \quad \mathbf{L} = \mathrm{e}^{iQ}$$

SU(∞) on a small sphere & the Polyakov loop

When $g^2 = 0$:

$$\mathcal{V}_{eff} = N^2 \left(m^2 \, \ell_1^2 + \mathcal{V}_{\rm Vdm} + \ldots \right) \quad ; \quad m^2 \sim T_H^2 - T^2$$

At the Hagedorn temperature, T_H, *only* the first mode, l_1, is unstable;

all other modes are stable. Concentrate on that mode, $l \equiv l_1$.

Vandermonde determinant in measure for constant mode gives "Vdm potential":

(N.B.: special to *small* volume; measure terms regularization dependent in infinite volume)

$$\mathcal{V}_{\rm Vdm} = + \, \ell^2 \quad , \quad \ell < \frac{1}{2}$$

$$\mathcal{V}_{\rm Vdm} = - \, \frac{1}{2} \, \log \left(2 \left(1 - \ell \right) \right) + \frac{1}{4} \quad , \quad \ell \geq \frac{1}{2}$$

Vdm potential has discontinuity of *third* order at $l = 1/2$.

Gross & Witten '81; Kogut, Snow & Stone '82.... Sundborg, '99....AMMPV '03 & '05

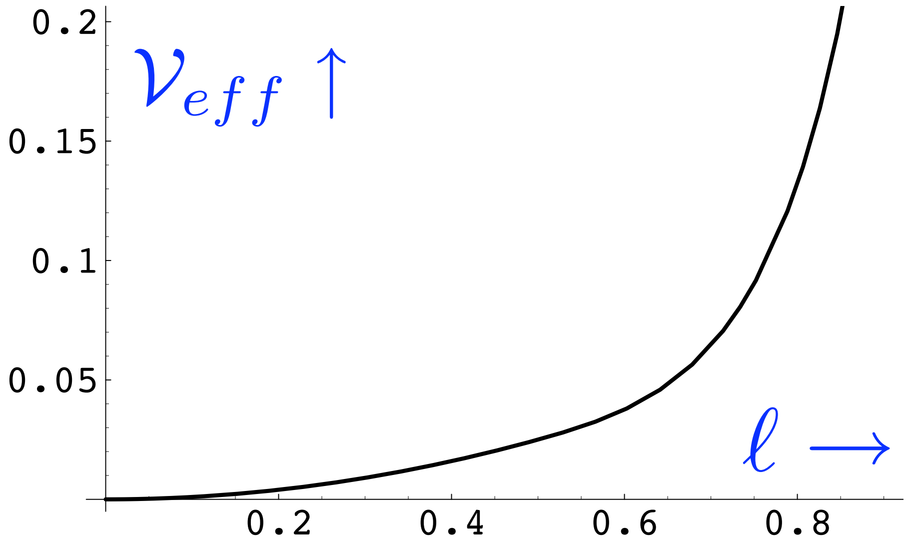
Dumitru, Hatta, Lenaghan, Orginos & RDP, hep-th/0311223 = DHLOP.

Dumitru, Lenaghan & RDP, hep-ph/0410294 = DLP.

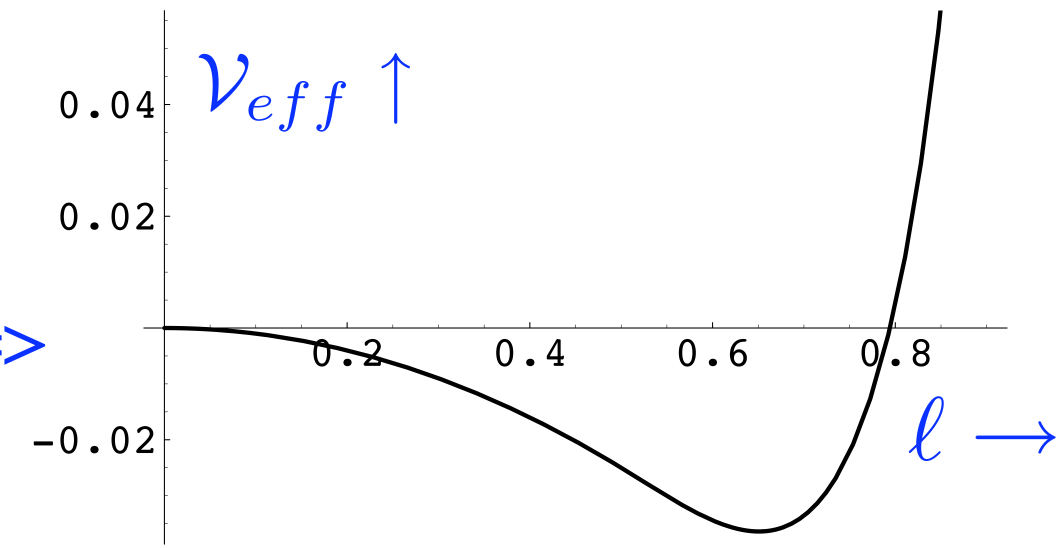
Deconfinement on a small sphere

Deconfining phase transition when $m^2 = 0$: *first* order, <*loop*> = 1/2 at $T_c = T_H$.

Obvious from potentials above and below T_c:



<= $m^2 = +.1$, confined phase,
<*loop*> = 0



$m^2 = -.1$, deconfined phase =>
<*loop*> $\neq 0$

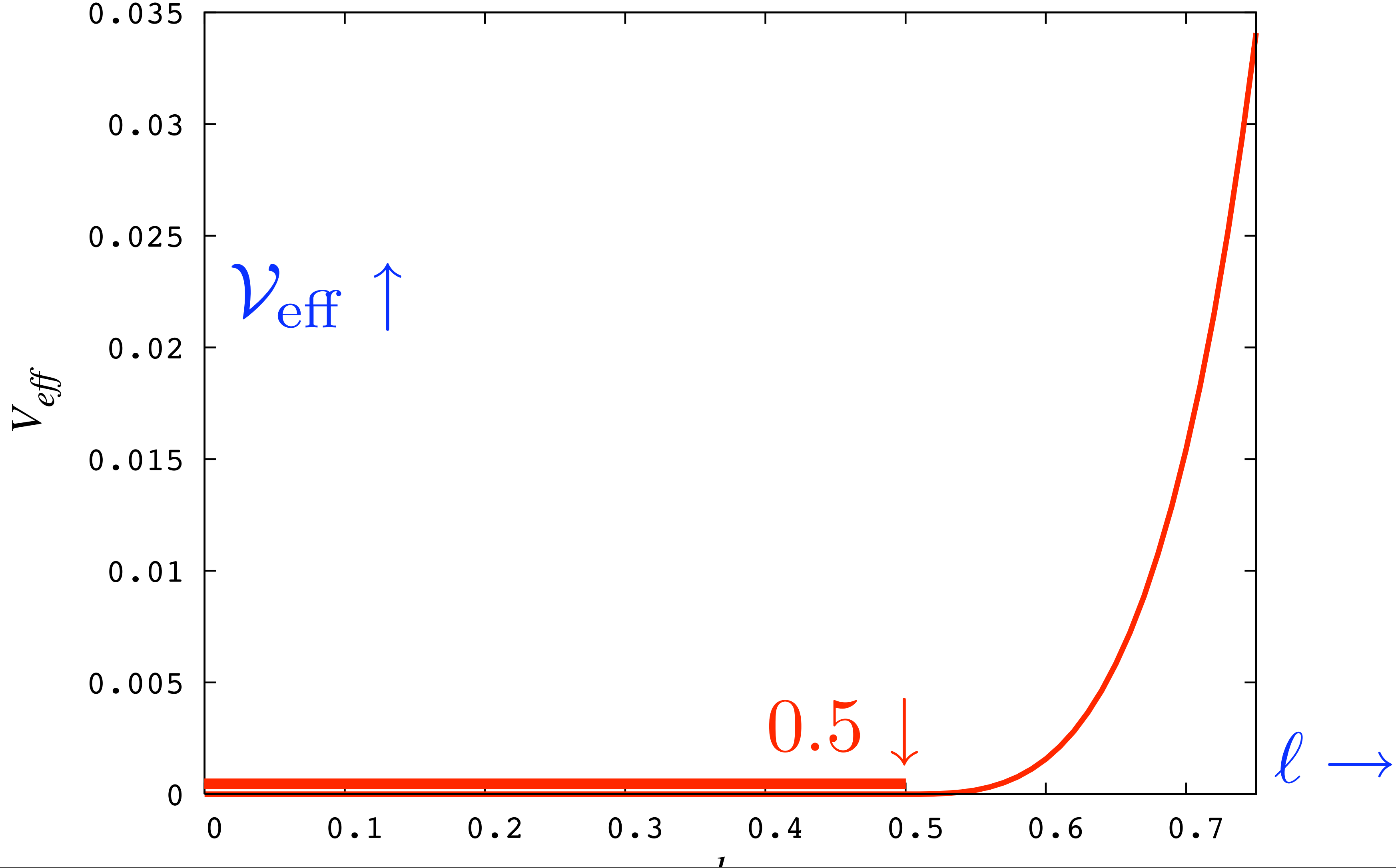
Gross-Witten point

At transition, order parameter <*loop*> jumps from 0 to 1/2. Latent heat nonzero.

DLP: masses vanish, asymmetrically: "critical" 1st order transition: "GW point".

At $m^2 = 0$, <*loop*> jumps because of 3rd order discontinuity in Vdm potential

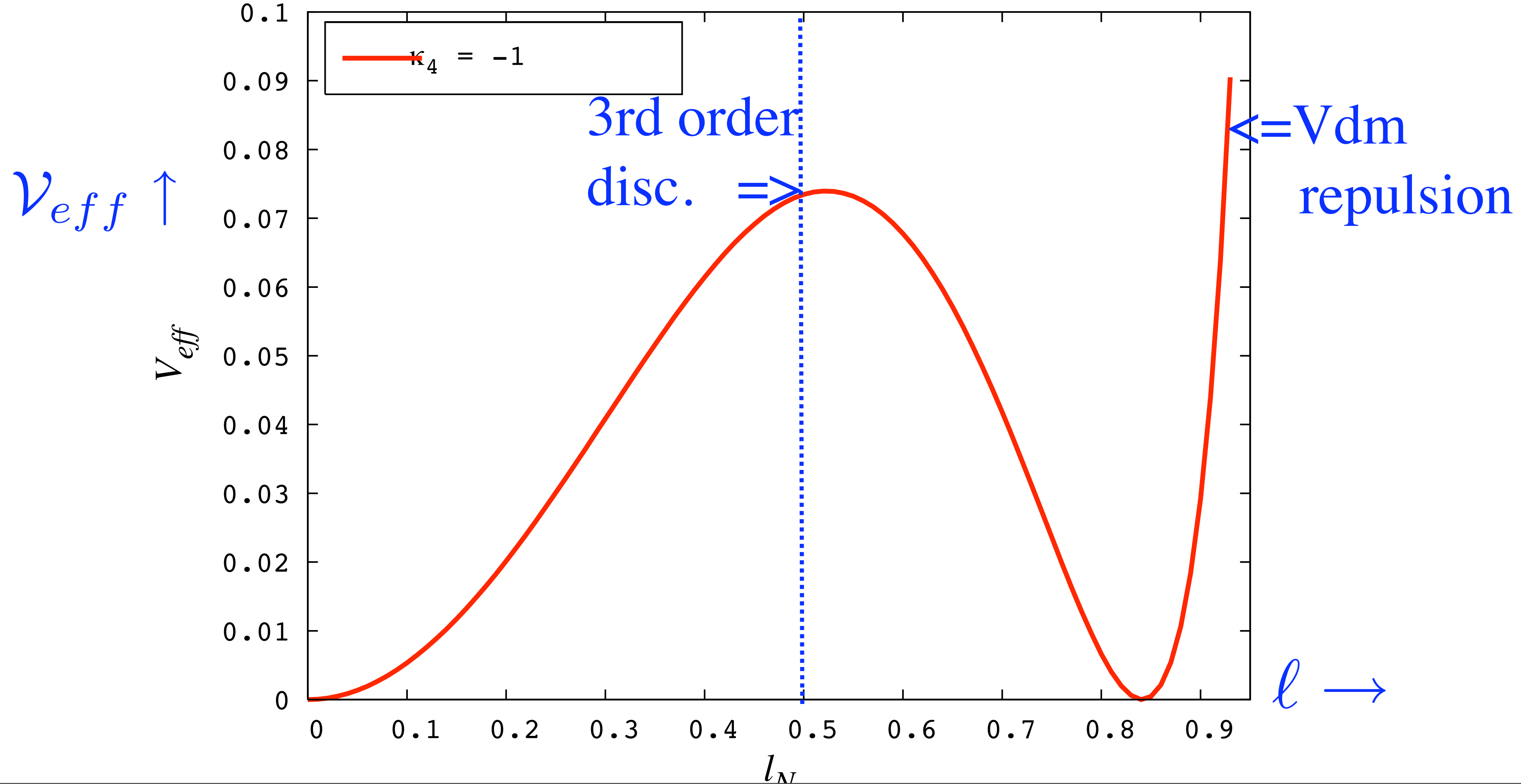
GW point like tricritical point in extended phase diagram.



Away from the GW point

Add negative quartic coupling: $\mathcal{V}/N^2 = m^2|\ell|^2 - (|\ell|^2)^2$

Typical strongly 1st order transition: masses nonzero at transition (below)
New minimum ≠ 1/2. So 3rd order discontinuity at 1/2 is no big deal.



GW = "ultra"-critical point

Phase diagram: tri-critical => Gross-Witten point.

$$\mathcal{V}_{eff}/N^2 = \widetilde{m}^2|\ell|^2 + \kappa_4(|\ell|^2)^2 + \kappa_6(|\ell|^2)^3 + \ldots \qquad \ell < 1/2$$

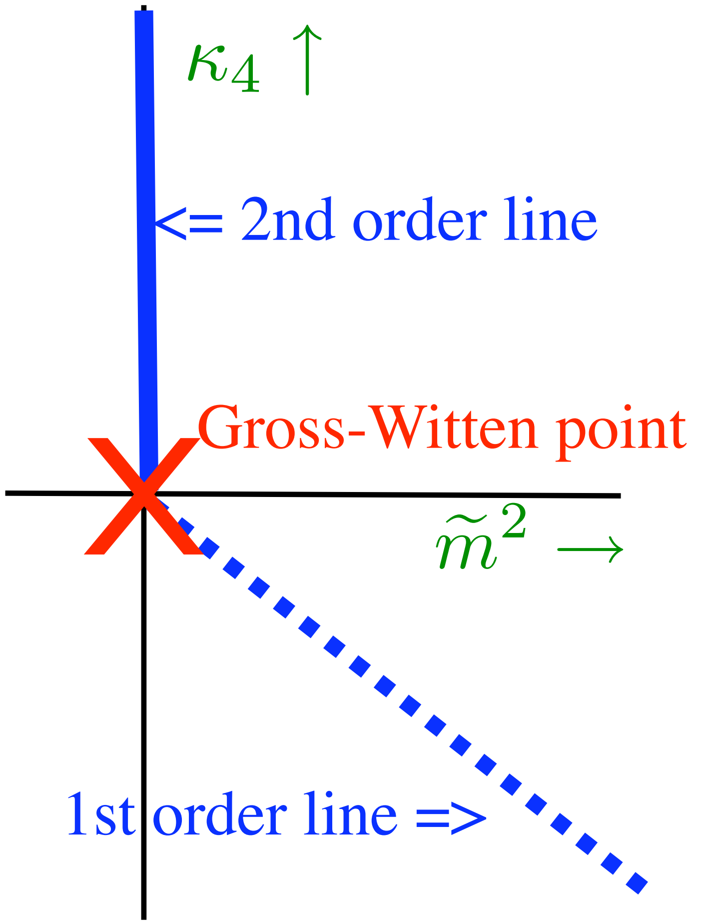
Away from GW point,
ordinary 1st or 2nd order transitions.

Only at GW point:

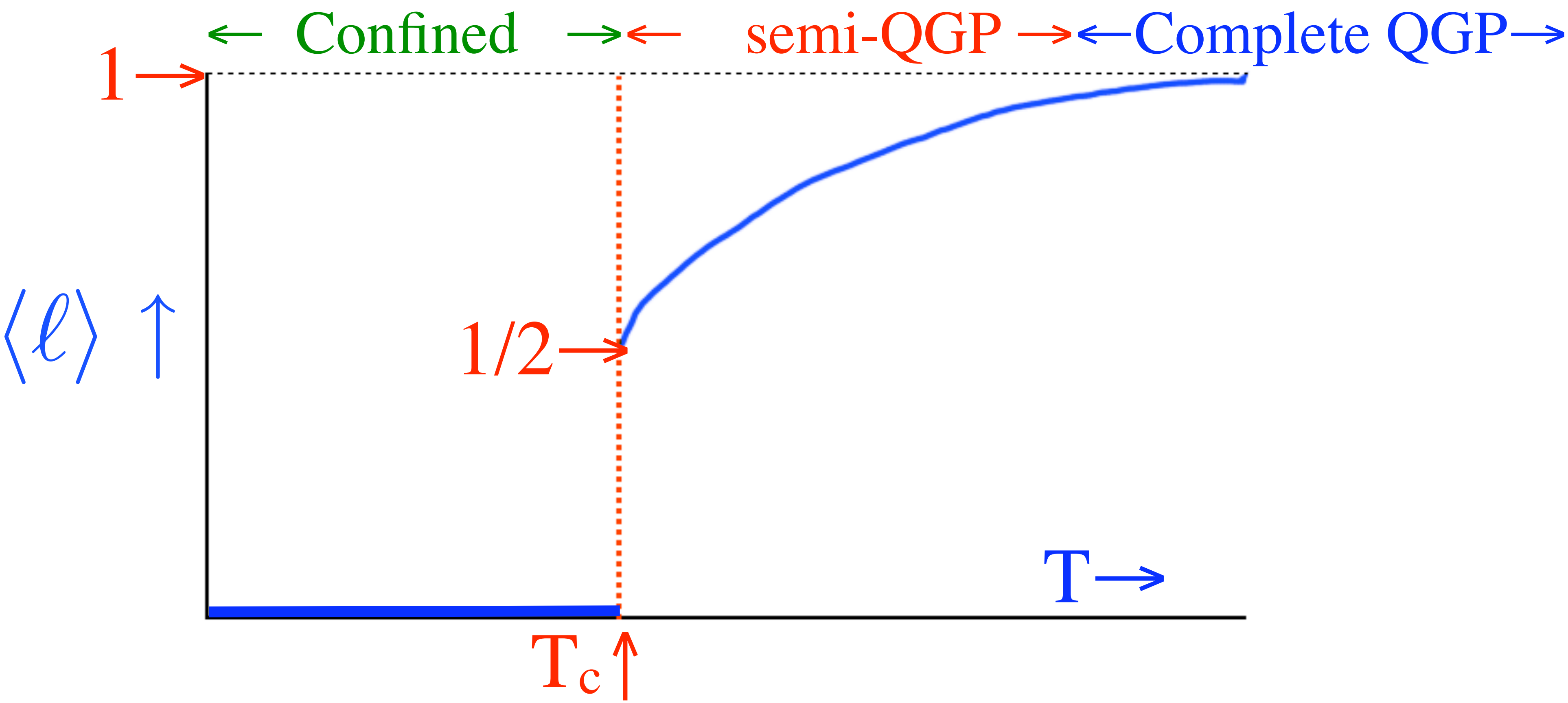
*Non*zero latent heat, jump in order parameter
and masses vanish

"Ultra"-critical as infinite # couplings vanish
Non-analytic behavior only possible at N = ∞

AMMPR '03, DLP



Semi-QGP on a small sphere



Boundary between complete & semi-QGP *not* precise; <*loop*> → 1 by T ~ # T_c?

AMMPV '05: calculate free energy with $Q \neq 0$ to *two* loop order at small R

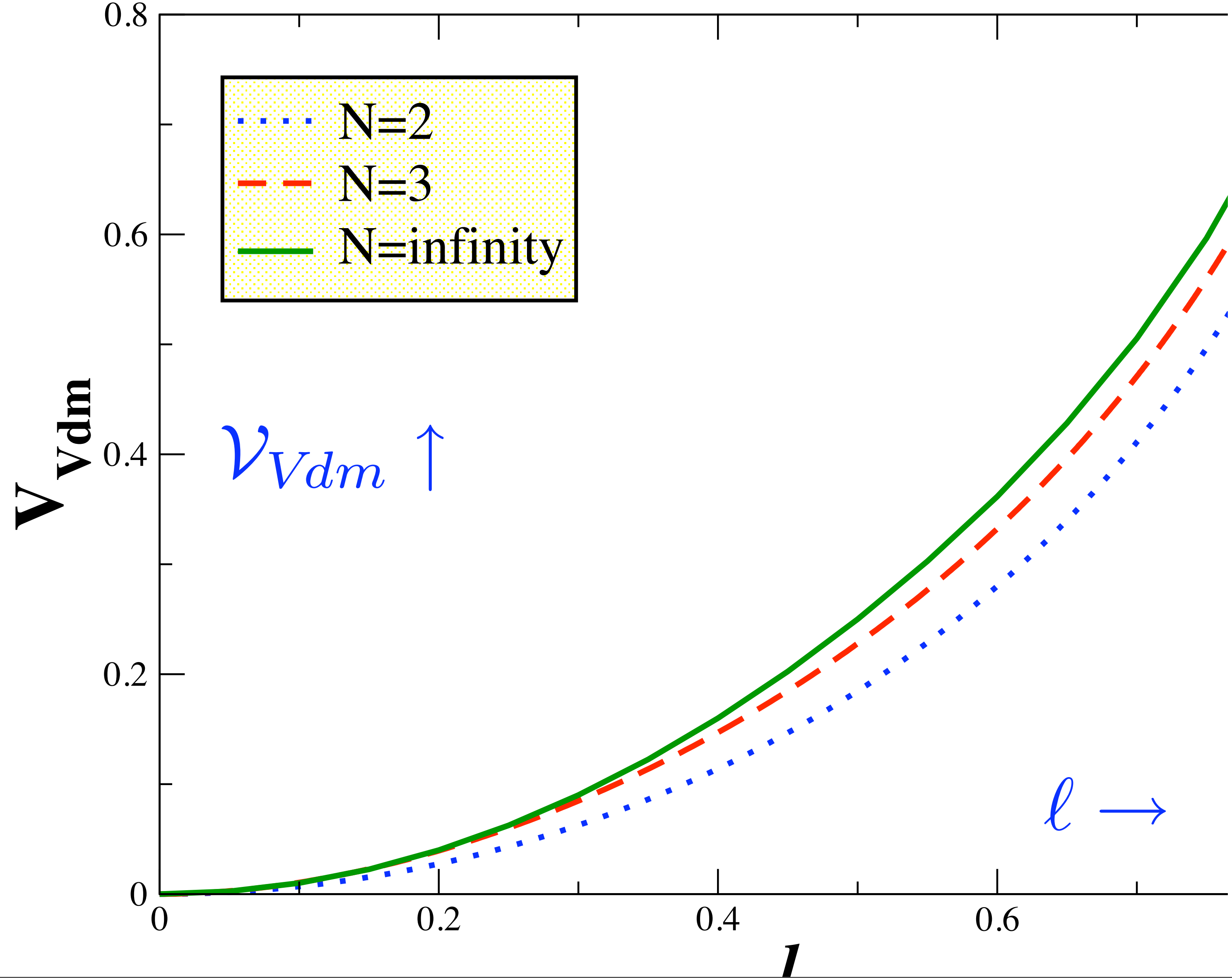
$$\mathcal{V}_{eff} = \mathcal{V}_{eff}(g^2 = 0) - c_3 \, g^4 \left(\ell^2\right)^2 \quad c_3 > 0.$$

$c_3 > 0 \Rightarrow T_c = T_H - O(g^4)$. Deconfinement first order, *below* T_H

Finite N: Vandermonde potential

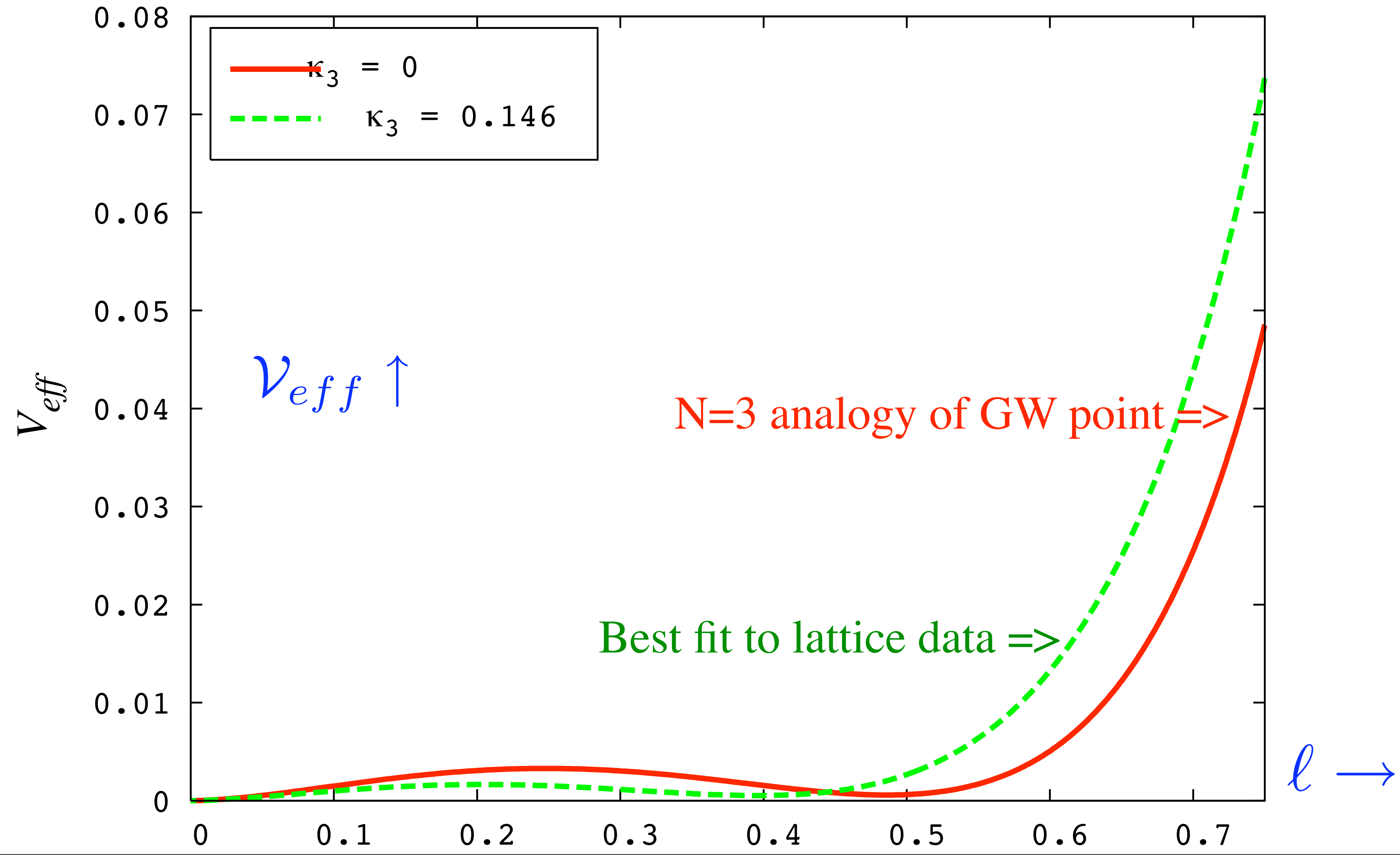
Infinite N: discontinuity of 3rd order at 1/2. Continuous at finite N.

Numerically, N=2 and 3 close to infinite N. DLP ‘04



Lattice: N = 3 close to GW point

Take ren'd loops from lattice data.
Fit matrix model, with $m^2 \sim T_d - T$ Only need small cubic term. DLP '04



3. Renormalized loops

Representation(s)

Lattice: bare loops

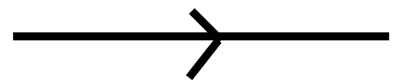
Renormalization of Wilson loops

Zero point energy of ren’d Wilson loops

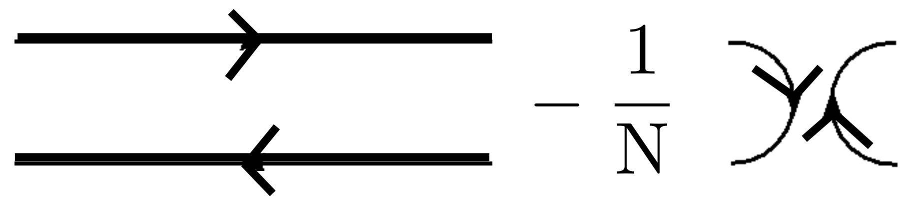
Representations of Polyakov loops

Wilson lines, and so Polyakov loops, are classified by irreducible representations.
Birdtracks for the simplest representations are:

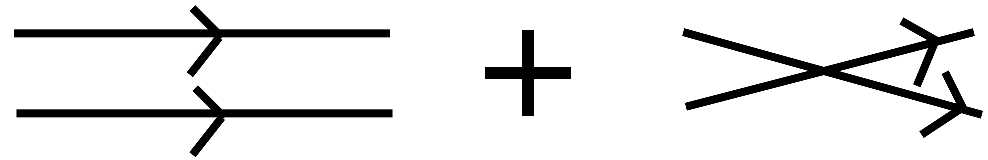
Fundamental
dim. = N
= 3 , N = 3



Adjoint: fund. + anti-fund.
dim. = N^2 - 1
= 8 , N = 3



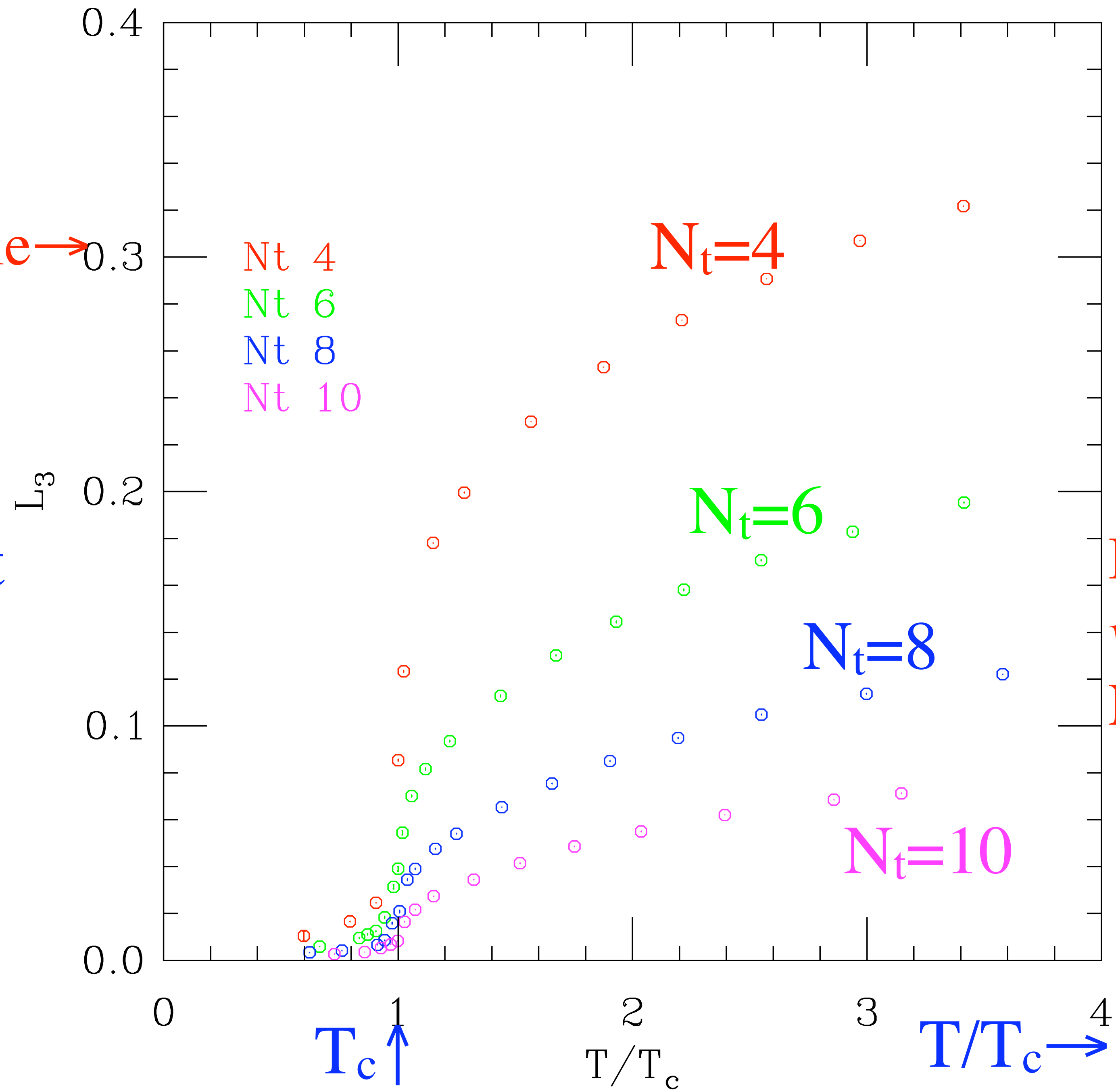
Symmetric 2 index tensor,
dim. = (N^2 + N)/2
= 6, N=3



Lattice: bare triplet loop vs T and N_t

Note scale→ ~ .3 !

Triplet loop↑



N_t = # time steps.

Bare loop *vanishes* as $N_t \rightarrow \infty$

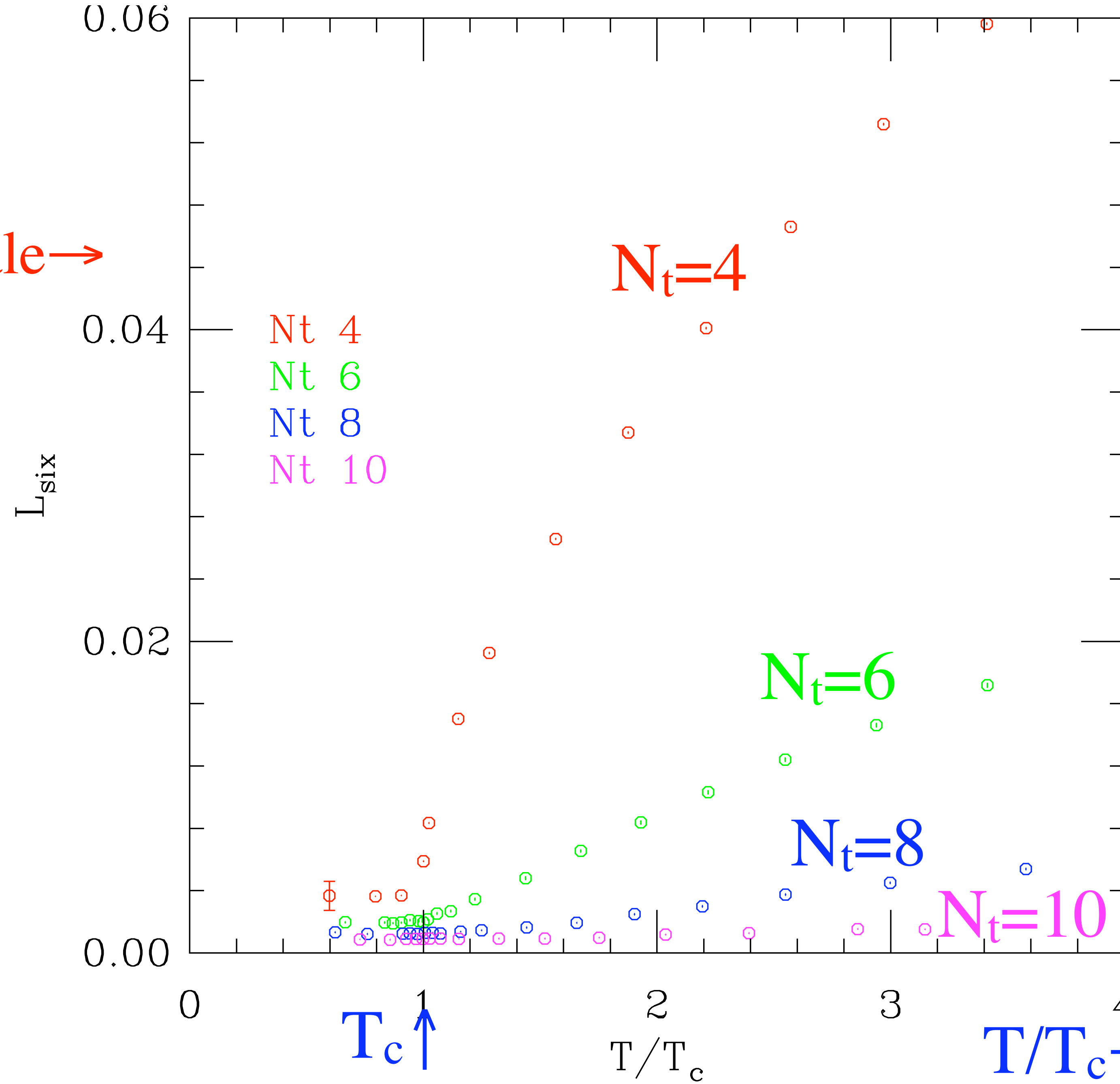
T_c↑

T/T_c→

Lattice: bare sextet loop vs T and N_t

Note scale→ ~ .04 !

Sextet loop↑



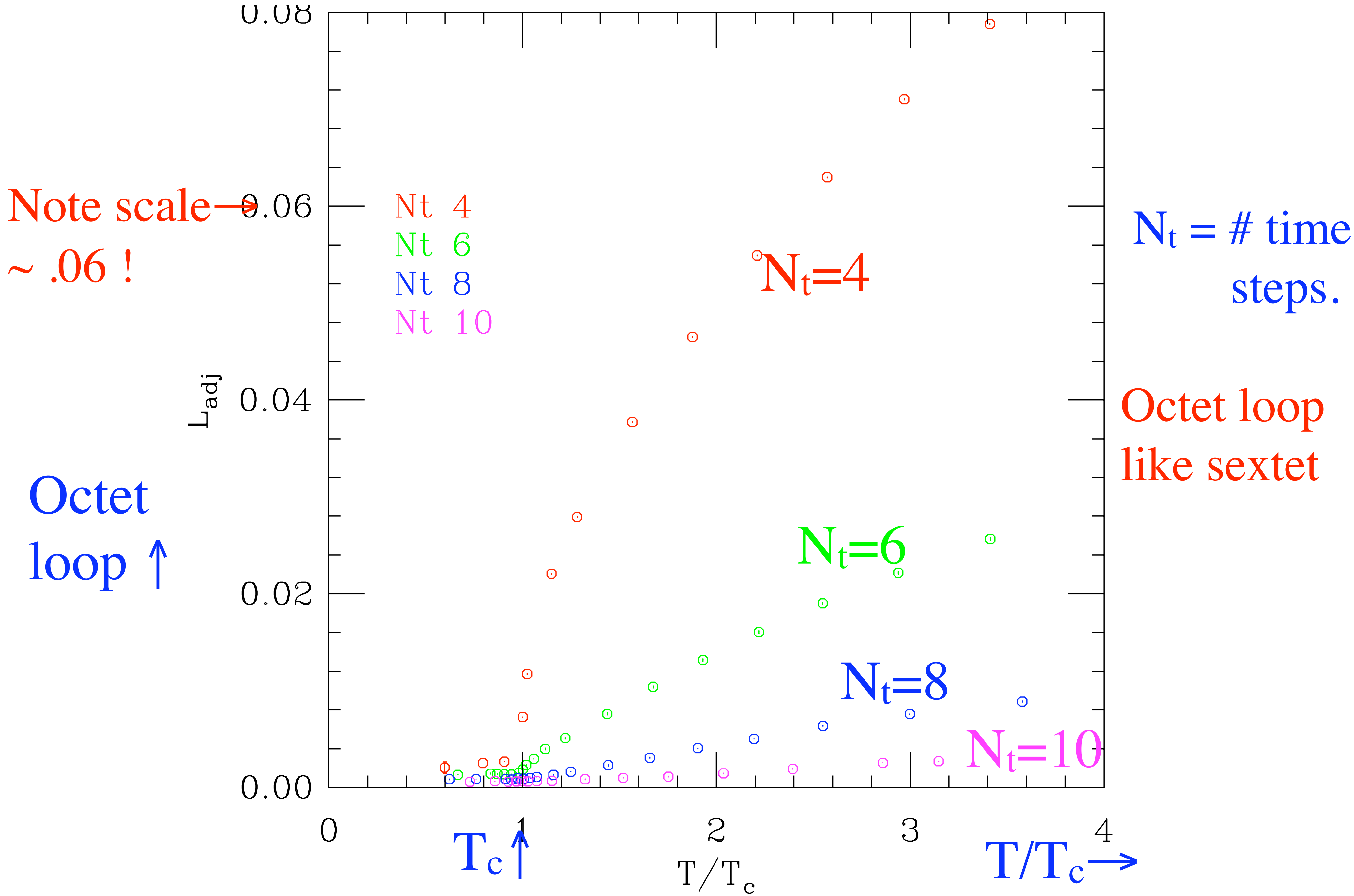
T_c↑

T/T_c→

N_t = # time steps.

Bare sextet loop vanishes *more* quickly as $N_t \to \infty$

Lattice: bare octet loop vs T, at different N_t

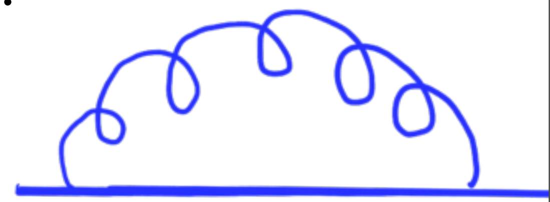


Renormalization of Wilson loops

Old story: Gervais & Neveu ‘80. Polyakov ‘80. Dotsenko & Vergeles ‘80....
Kaczmarek, Karsch, Petreczky & Zantow: hep-lat/0207002, hep-lat/0406036,
DHLOP ’03... Petreczky & Petrov hep-lat/0405009, Cheng et al. hep-lat/0608013.
Gupta, Hubner & Kaczmarek 0711.2251 = GHK

Usual interest: loops with cusps. Here: term special to lattice:

$N_t = 1/(a\, T)$ = # time steps, “a” = lattice spacing



$$< \ell_R > -1 = (-)\, \frac{\mathcal{C}_R\, g^2}{T} \int^{1/a} \frac{d^3 k}{k^2} + \ldots = (-)\, f_R(g^2)\, N_t$$

Vanishes with dimensional regularization, hence ignored previously.
Renormalize (non-local) operator by multiplicative renormalization:

$$< \ell_R^{\mathrm{bare}} >= \mathcal{Z}_R \; < \ell_R^{\mathrm{ren}} > \; ; \quad \mathcal{Z}_R = \mathrm{e}^{-f_R(g^2)\, N_t}$$

N.B.: the function $f_R(g^2)$ is *not* determined perturbatively, but numerically from lattice simulations. $N_t \rightarrow \infty$ in continuum limit: *all* loops vanish! ($f_R > 0$)

Ambiguities in renormalized loops

Renormalization valid for *arbitrary* Wilson loops:

$$\mathcal{W} = \mathrm{tr}\, \mathcal{P}\, e^{ig \oint A_\mu dx^\mu} \quad ; \quad \mathcal{W}_{\rm bare} = \mathcal{Z}_{\rm div}\, \mathcal{W}_{\rm ren}$$

Two ambiguities:

$$\mathcal{Z}_{\rm div} = \mathrm{e}^{E_0 L}\, \mathcal{Z}_0\, \mathcal{Z}(g^2 \ldots)^{L/a} \quad ; \quad \mathcal{W}_{\rm ren} \to \mathrm{e}^{-E_0 L}\, \mathcal{Z}_0^{-1}\, \mathcal{W}_{\rm ren}$$

Overall scale trivial: $Z_0 = 1$ by requiring <*loop*> → 1 as T →∞.

E_0? At T = 0 in a pure gauge theory, for a rectangular loop R x t in size,

$$< \mathcal{W} >= \mathrm{e}^{-V(R)t} \, , \; V(R \to \infty) \sim \sigma R + E_0 - \frac{\pi}{12\, R} + \ldots$$

σ = string tension, term ~1/R = Luscher term, universal.

In quantum mechanics, E_0 is unphysical, just overall phase in wave function.

Claim: for a *renormalized* loop, E_0 is physical.

Zero point energy, perturbative & non-pert.

$E_0^{\rm pert} = 0$ order by order in perturbation theory: obvious with dimensional reg. Also Pauli-Villars, higher derivatives, which eliminate power-law divergences.

Dim.'y: $E_0^{\rm pert} \sim \Lambda_{\overline{MS}}$. But then *not* renormalization group invariant.

Could have $E_0 \sim \Lambda_{\overline{MS}}\, e^{-\#/g^2}$ but this is a *non*-perturbative zero point energy.

Generally, $E_0^{\rm non-pert} = \#\sqrt{\sigma}$. Consider $E_0^{\text{non-pert}}$ in (effective) string models.

Nambu model: $E_0^{\rm non-pert} = 0$ Corrections to Nambu ~$1/R^5$!

Arvis '83, Luscher & Weisz hep-th/0406205,
Drummond hep-th/0411017

$$V_{\rm Nambu}(R) = \sigma R\,\sqrt{1 - \frac{\pi}{12\sigma R^2}}$$

Generally, though, *non*-zero. For "smooth" string:

$$E_0^{\rm non-pert} = -\frac{\kappa}{4}\sqrt{\sigma}$$

where $\varkappa$ is the coupling of the extrinsic curvature.

Braaten, RDP, Tse '87

$E_0^{\text{non-pert}}$ provides *sensitive* test of corrections to Nambu string model!

4. Lattice: renormalized Polyakov loops

Two methods for renormalizing loops

Casimir scaling of ren. constant

(Approx.) Casimir scaling of ren’d loops

“Semi”-QGP

Adjoint loops

(Non-perturbative) renormalization of Polyakov loops

Ren.'d loops unique after imposing $E_0^{pert} = 0$.

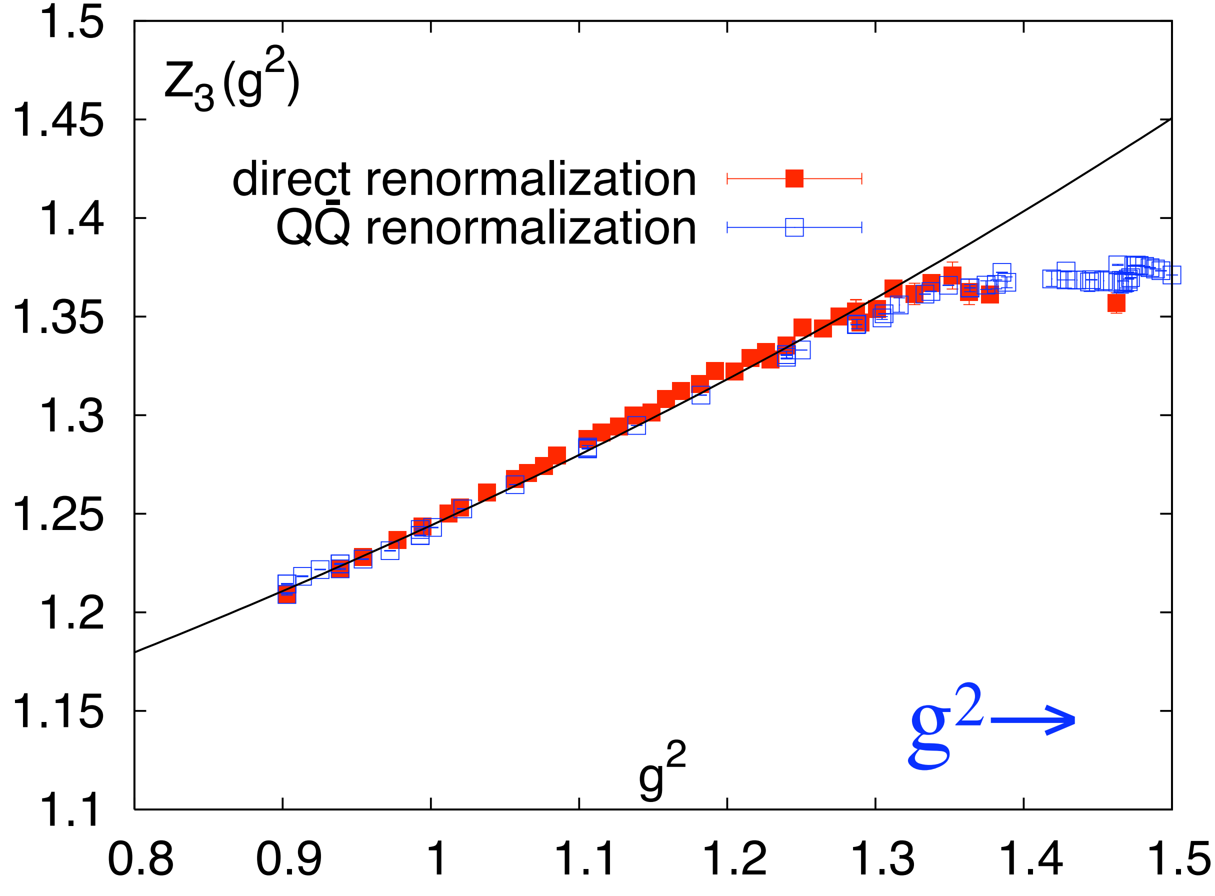
How to determine $Z(g^2)$ without computing it in perturbation theory?

Bielefeld: compare to perturbation theory for (small) Wilson loops; "QQbar" ren.

DLHOP: work at different N_t, same T: "direct" ren.

GHK: for pure gauge SU(3), two methods agree to numerical accuracy:

$Z(g^2)$ ↑
triplet loop

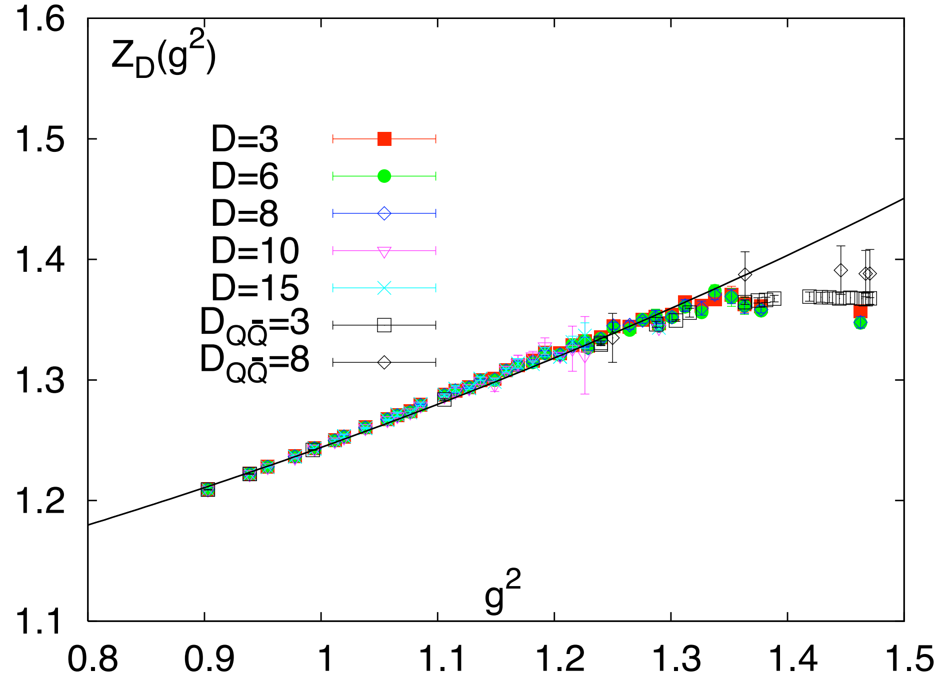


g^2 ⟶

Casimir scaling: bare *and* ren.'d loops

GHK: SU(3), pure gauge. In principle, each representation has different Z_R
Numerically find *very* useful relation,
Casimir scaling of ren. constants:
all Z's follow from fundamental rep.

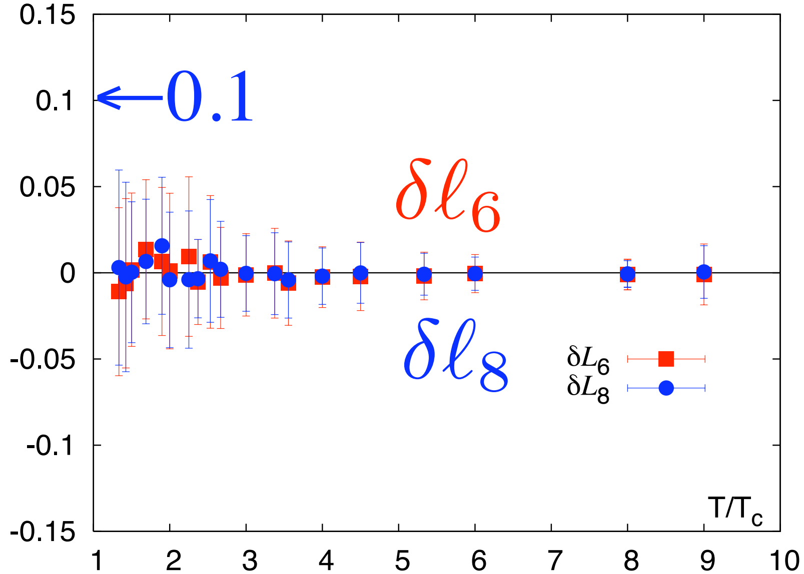
$$\mathcal{Z}_R(g^2) = \left(\mathcal{Z}(g^2)\right)^{C_R}$$



GHK: Renormalized loops satisfy *approximate* Casimir scaling.

$$< \ell_R^{\rm ren} > \approx < \ell_3^{\rm ren} >^{C_R/C_3}$$

$$\delta\ell_R \approx < \ell_3^{\rm ren} > - < \ell_R^{\rm ren} >^{C_3/C_R}$$

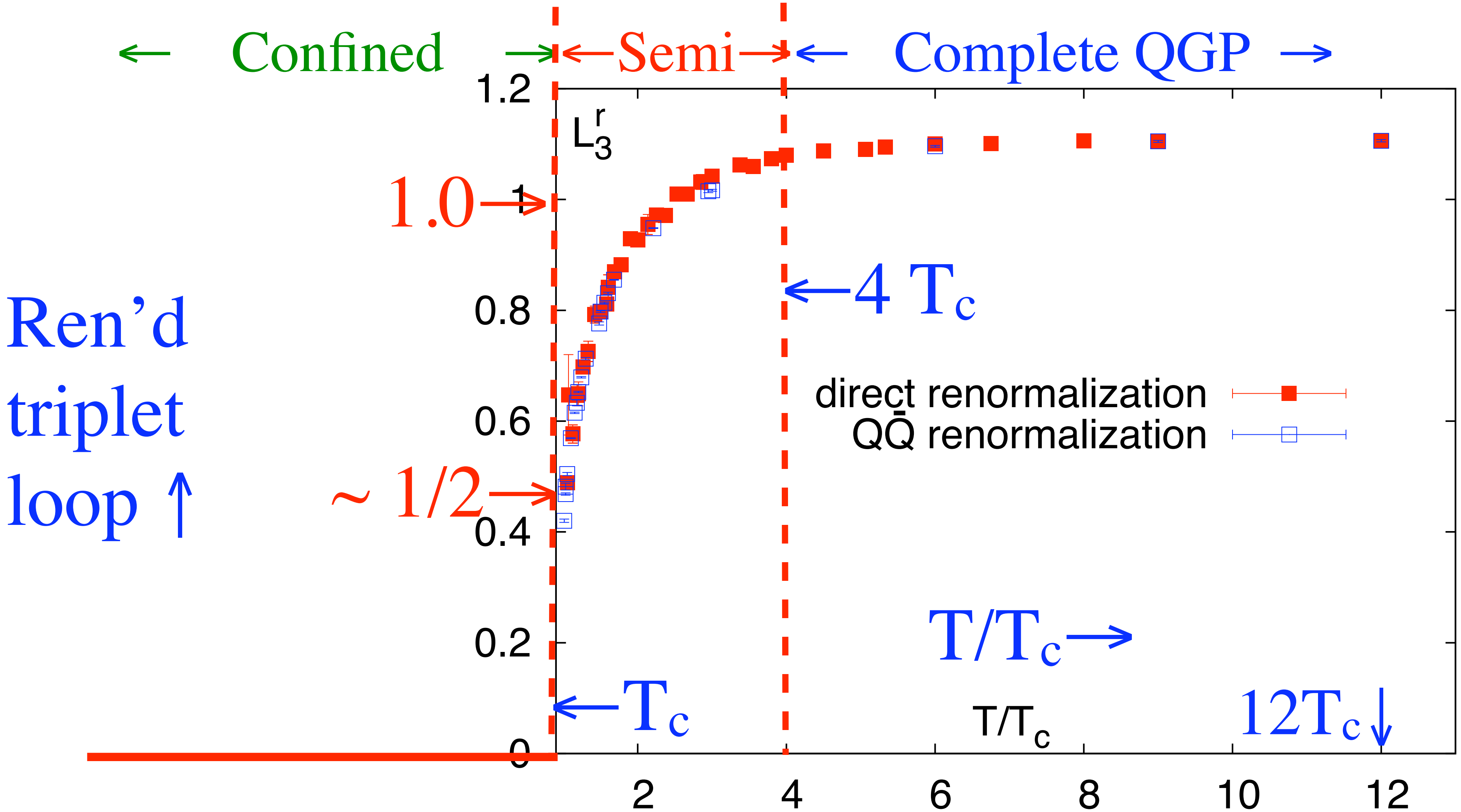


Not exact, breaks down near T_c for SU(2) (2nd order) Huebner & Pica 0809.3933

Lattice: renormalized loop

GHK: Lattice SU(3), *no* quarks, triplet loop
<*loop*> ~ 1/2 at T_c^+. N=3 close to Gross-Witten point?

semi-QGP: from (*exactly*) T_c^+ to 2 - 4 T_c (?). <*loop*> ~ constant above 4 T_c.

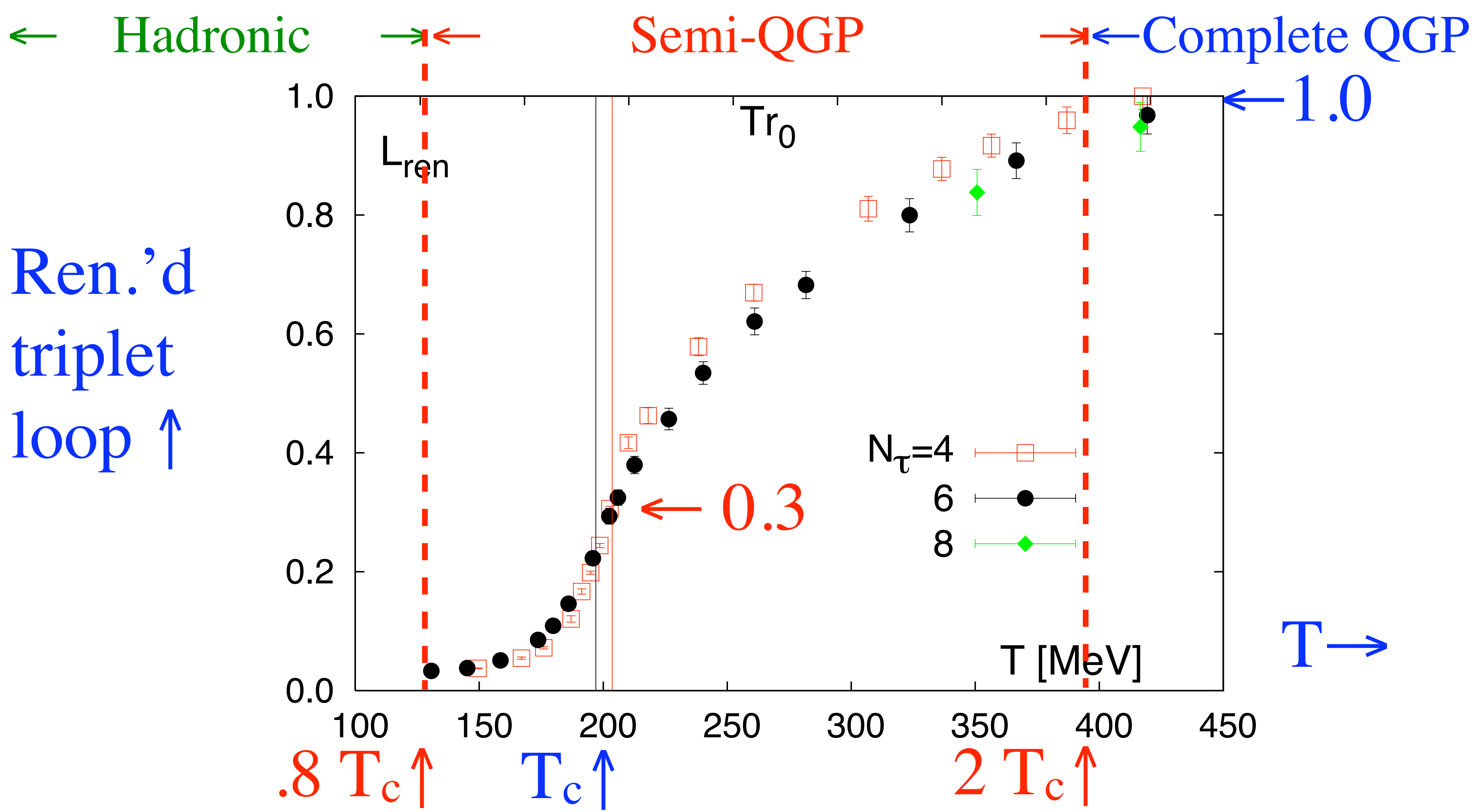


Lattice: renormalized loop, with quarks

Cheng et al, 0710.0354: ~ QCD, 2+1 flavors. T_c ~ 190 MeV, crossover.

<*loop*>: nonzero from ~ 0.8 T_c; ~ 0.3 at T_c; ~ 1.0 at 2 T_c.

Semi-QGP from ~ 0.8 T_c (*below* T_c) to ~ 2-3 T_c (?). <*loop*> *small* at T_c .



Lattice: ren.'d adjoint loop *small* below T_c

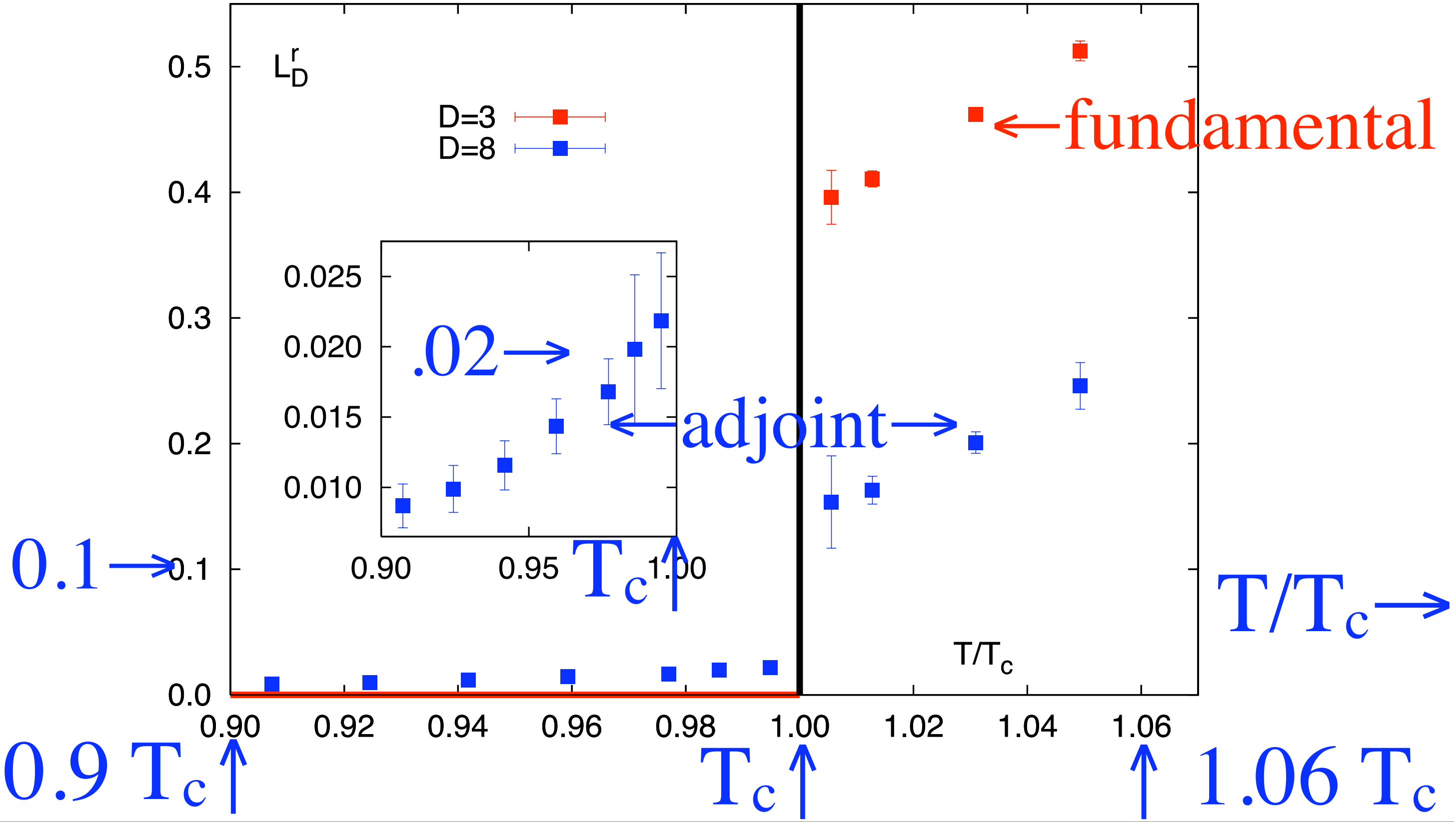
GHK: SU(3), pure gauge. Adjoint loop:
$$\ell_{adj.} = \frac{1}{N^2-1}\left(|\mathrm{tr}\,\mathbf{L}|^2 - 1\right)$$

Below T_c: Z(N) charged loops vanish. <*loop*$_{\text{adjoint}}$> Z(N) neutral, can be *non*-zero

Large N factorization:
$$\langle \ell_{adj} \rangle \sim |\langle \ell_N \rangle|^2 + 1/N^2$$

So <*loop*$_{\text{adjoint}}$> ~ 10% at T_c^-? No, ~ 2% !

Only true in matrix model, where *all* loops vanish in a confined phase.



5. Goal: (complete) effective theory near T_c

Complete effective theory for semi-QGP

Lattice shows <*loop*> $\neq 1$ near T_c => large $A_0 \sim T/g$. *Effective theory for large A_0?*

Euclidean theory: determine loop potential from numerical simulations?

Wozar, Kaestner, Wellegehausen, Wipf, & Heinzl hep-lat/0605012;0711.0868; 0808.4046

Dumitru & Smith 0711.0868; Velytsky, 0805.4450.

$$\mathcal{V} = m^2 \, \ell_{adj} + \sum_j \kappa_j \, \ell_j$$

Simple guess: temperature dependent Vandermonde potential, does not work.

Also corrections to kinetic terms, etc.

Hope: gluon potential, where quarks can be incorporated perturbatively.

Alternately: Polyakov NJL model. Fit potential to lattice data. Most useful.

Fukushima: hep-ph/0303225; 0803.3318; 0809.3080

Sasaki, Friman, & Redlich hep-ph/0611147

Ratti, Thaler & Weise hep-ph/0505256; Roessner, Hell, Thaler, & Weise 0712.3152

Hell, Roessner, Cristoforetti, & Weise, 0810.1099

Lattice: pressure & "flavor independence"

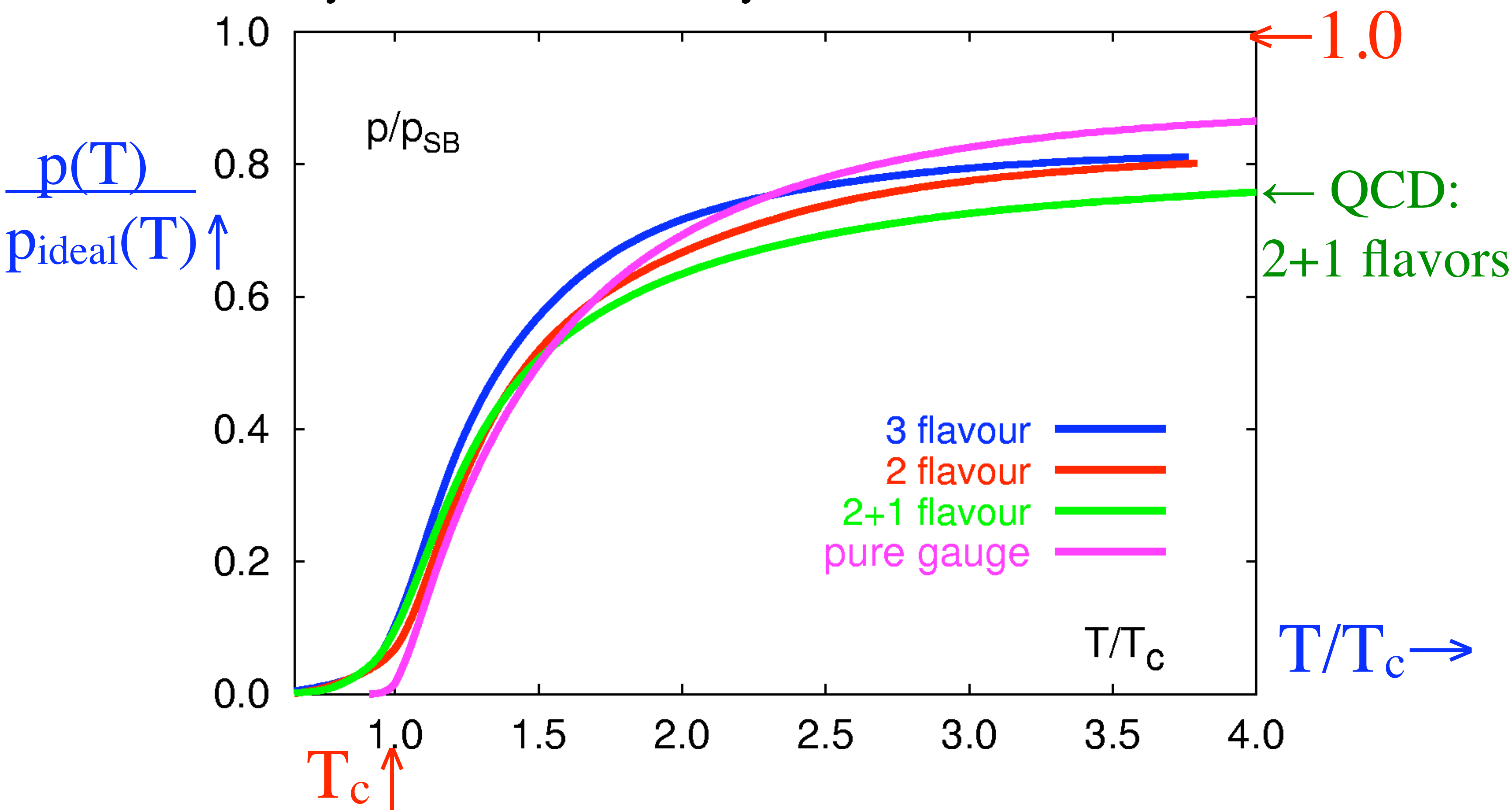
Pure SU(3): *weakly* 1st order QCD: crossover

Bielefeld: properly scaled, $\approx$ *universal* pressure

$$\frac{p}{p_{ideal}}\left(\frac{T}{T_c}\right) \approx \text{const.}$$

Not exact, but *severe* constraint on any effective theory

Ideal: increases by ~ 3. T_c: decreases by ~ 1/3.



6. Effective theory, near T_c, in Euclidean spacetime

Effective theory for large A_0

Lattice shows <*loop*> $\neq 1$ near T_c => large $A_0 \sim T/g$. *Effective theory for large A_0?*

For small A_0, effective theory is just QCD_3 + massive (adjoint) scalar, A_0:

$$\mathcal{L}^{eff} = \frac{1}{2}\,\mathrm{tr}\,G_{ij}^2 + \mathrm{tr}\,|D_i A_0|^2 + m_D^2\,\mathrm{tr}\,A_0^2 + \kappa\,\mathrm{tr}\,A_0^4$$

Symmetries for large A_0? Certainly, invariance under static gauge transf.'s.

Plus: "large" gauge transformations - spatially constant, time *de*pendent. For SU(N):

$$\Omega(\tau) = \mathrm{e}^{2\pi i \tau T\, t^{NN}} \quad , \quad t^{NN} = \frac{1}{N}\begin{pmatrix} \mathbf{1}_{N-1} & 0 \\ 0 & -(N-1) \end{pmatrix}$$

This $\Omega(\tau)$ is *only* valid c/o quarks: $\Omega(1/T) = \mathrm{e}^{2\pi i/N}\,\Omega(0)$: center symmetry

With quarks, consider *strictly* periodic transf: $\Omega_p(\tau) = \Omega^N(\tau)$.

All theories must respect invariance under such *strictly* periodic gauge transf.'s.

For any gauge group, with any matter fields; even for QED.

Effective electric field?

Want 3D effective thy. for large $A_0 \sim T/g$.

Valid for $r > 1/T$, so A_0 varies slowly in space, momenta $p < T$.

Original electric field $E_i^{4D} = D_i A_0 - \partial_0 A_i$. So: $E_i^{3D} = D_i A_0$?

For large gauge transf. $\Omega_p(\tau)$:

$$A_0^{diag} \to A_0^{diag} + \frac{2\pi T}{g} N \, t^{NN} \; , \; A_i \to \frac{1}{-ig} \, \Omega_p^\dagger(\tau) A_i \, \Omega_p(\tau)$$

Constant shift in A_0 , time *de*pendent rotation of A_i .

$D_i A_0 = (\partial_i - i g [A_i ,) A_0$ *not* invariant for A_i^{aN} as $[t^{aN}, t^{NN}] \neq 0$.

Of course, E_i^{4D} *invariant* under $\Omega_p(\tau)$.

$E_i^{3D} = D_i A_0$ at small A_0, but *not* at large A_0!

Periodicity in $q \to q+1$ violated: true only for kinetic, *not* potential terms.

Diakonov & Oswald hep-ph/0303129; hep-ph/0312126; hep-ph/0403108

Megias, Arriola, & Salcedo, hep-ph/0312133

Form E_i^{3D} from Wilson lines?

Electric field of Wilson lines

Wilson line SU(N) matrix, so diagonalize: $$\mathbf{L}(x) = \Omega(x)^\dagger e^{\,i\lambda(x)}\,\Omega(x)$$

Static gauge transf.'s: diagonal matrix λ invariant, Ω changes.

Strictly periodic $\Omega_p(\tau)$: $\lambda_a \to \lambda_a + 2\pi \times$ integer: λ_a periodic.

Use just eigenvalues, $E_i^{3D} \sim \partial_i \lambda$? No, $E_i^{3D} \neq D_i A_0$ at small A_0

E_i^{3D} hermitean, so: $$E_i^{3D}(x) = \frac{T}{ig}\mathbf{L}^\dagger(x) D_i \mathbf{L}(x)(1 + c_1|\mathrm{tr}\mathbf{L}|^2 + \ldots)$$

Small A_0 OK, but does *not* fix c_1, c_2...

Large but *abelian* A_0, $A_i = 0$: if $E_i^{3D} = \partial_i A_0$, *must* have c_1=c_2=...=0.

Necessary for interfaces to match at *leading* order. Beyond: c_1, c_2 ... $\sim g^2$.

In general, *infinite* number of terms enter.

Calculable perturbatively, match through interfaces, Z(N) or U(1).

$\mathcal{L}_{\mathrm{eff}}$ of Wilson lines at 0th order

To leading order, $$E_i^{3D} = \frac{T}{ig}\, \mathbf{L}^\dagger\, D_i\, \mathbf{L}$$

Gauge covariant "average" in time: $\mathbf{L}(\tau) = e^{ig\int_0^\tau A_o(\tau')d\tau'}\ ;\ \mathbf{L} = \mathbf{L}(1/T)$

$$E_i^{3D}/T = \int_0^{1/T} d\tau\, \mathbf{L}(\tau)^\dagger\, \partial_i A_0(\tau)\, \mathbf{L}(\tau) - \mathbf{L}^\dagger[A_i, \mathbf{L}]$$

Lagrangian continuum form of Banks and Ukawa '83, on lattice:

$$\mathcal{L}_{cl}^{eff} = \frac{1}{2}\,\mathrm{tr}\, G_{ij}^2 + \frac{T^2}{g^2}\,\mathrm{tr}\,|\mathbf{L}^\dagger D_i \mathbf{L}|^2$$

To 0th order, Lagrangian for SU(N) principal chiral field.
Non-renormalizable in 3D, but only effective theory for r > 1/T.
Instanton number in 4D = winding number of **L** in 3D

Linear model: *many* more terms. Interfaces match *approximately*, not exactly.

Vuorinen & Yaffe hep-ph/0604100. Kurkela, 0704.1416.

de Forcrand, Kurkela, & Vuorinen, 0801.1566. Korthals-Altes 0810.3325

Confinement & adjoint Higgs phase?

Diagonalize $L = \Omega^\dagger \, e^{i\lambda} \, \Omega$

Static gauge transf.'s U: $e^{i\lambda}$ invariant, Ω not: $\Omega \to \Omega \, \mathcal{U} \; , \; D_i \to \mathcal{U}^\dagger \, D_i \, \mathcal{U}$

Electric field term:

$$\mathrm{tr} \, |\mathbf{L}^\dagger D_i \mathbf{L}|^2 \; = \; \mathrm{tr} \, (\partial_i \lambda)^2 \; + \; \mathrm{tr} \, |[\Omega \, D_i \, \Omega^\dagger , \mathrm{e}^{i\lambda}]|^2$$

1st term same as abelian

2nd term gauge *in*variant coupling of electric & magnetic sectors

$\langle e^{i\lambda} \rangle = 1$: no Higgs phase. True in perturbation theory, order by order in g^2

If $\langle e^{i\lambda} \rangle \neq 1$, Higgs phase,

In weak coupling, diagonal gluons massless,
off diagonal massive (a,b = 1...N)

$$m^2_{ab} = g^2 |\mathrm{e}^{i\lambda_a} - \mathrm{e}^{i\lambda_b}|^2$$

But for 3D theory, gluons couple *strongly*. Effects of Higgs phase?

N.B.: above 't Hooft's abelian projection for Wilson line.

Loop potential, perturbative & not.

U(N): constant **L**, 1 loop order:

$$\mathcal{L}^{eff}_{1\ loop} = -\,\frac{2\,T^4}{\pi^2}\,\sum_{m=1}^{\infty}\,\frac{1}{m^4}\,\left|\mathrm{tr}\,\mathbf{L}^m\right|^2\ .$$

Perturbative vacuum $<e^{i\lambda}> = 1$,
stable to leading order, to *any* finite order in g^2 .

Can compute corrections to effective Lagrangian at next to leading order, NLO.
At NNLO, $\sim g^3$, need to resum m_{Debye} . Eventually, $m_{magnetic}$

SU(3) lattice: near T_c , pressure(T) $\sim T^4$ *and* $\sim T^2$.

To represent: add, *by hand:*

$$\mathcal{L}^{eff}_{non-pert.}(\mathbf{L}) = +\,B_f\,T^2\ \left|\mathrm{tr}\,\mathbf{L}\right|^2$$

$B_f \sim \#\, T_c^2$ “fuzzy” bag const. Non-pert., infinity of possible terms.

$B_f \neq 0 \Rightarrow <e^{i\lambda}> \neq 1 \Rightarrow$ Higgs phase near T_c

Confinement in $L_{\rm eff}$

SU(N), no quarks: in confined state, all Z(N) charged loops vanish:

$$\langle {\rm tr}\, \mathbf{L}^j_{\rm conf} \rangle = 0 \;, \;\; j = 1 \ldots (N-1)$$

Satisfied by "center symmetric" vacuum:

$$\mathbf{L}_{\rm conf} = {\rm diag}(1, z, z^2 \ldots z^{N-1}) \;, \;\; z = {\rm e}^{2\pi i/N} .$$

At finite N, perturbative pressure($\mathbf{L}_{\rm conf}$) *negative*. Not so good.

Large N: pressure($\mathbf{L}_{\rm conf}$) ~ 1, vs. ~ N^2 in deconfined phase.

At N=∞, center sym. state *can* represent confined vacuum.

$\mathbf{L}_{\rm conf}$ familiar from random matrix models:
completely *flat* eigenvalue distribution, from eigenvalue repulsion.

Where does eigenvalue repulsion arise *dynamically*?

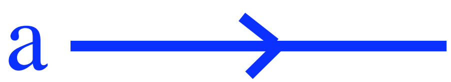
7. Effective theory, near T_c, in Minkowski spacetime

Semi-QGP in weak coupling

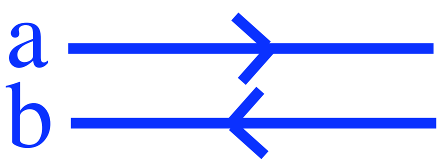
Y. Hidaka & RDP 0803.0453. Semi-classical expansion of the semi-QGP:

$$A_\mu = A_\mu^{\rm cl} + B_\mu \;\; , \;\; A_0^{\rm cl} = Q/g \;\; .$$

$Q \neq 0$: just like semi-classical calc. of 't Hooft loop. $Q = Q^a$, *diagonal* matrix.
Work at large N, large N_f, use double line notation. (Finite N ok, messy.)



$$i D_0^{\rm cl} = p_0 + Q^a = p_0^a$$



$$i D_0^{\rm cl} = p_0 + Q^a - Q^b = p_0^{ab}$$

Perturbation theory in B_μ's same as $Q = 0$, but with "shifted" p_0's.
Amplitudes in real time: $p_0^a \rightarrow i\,\omega$, etc. Furuuchi, hep-th/0510056

Q (imaginary) chemical potential
for (diagonal) color charge.
e.g., for quarks:

$$\widetilde{n}(E - iQ^a) = \frac{1}{{\rm e}^{(E - iQ^a)/T} + 1}$$

How color evaporates in the semi-QGP

AMMPV: simple trick.

$$\mathrm{tr}\, \frac{1}{\mathrm{e}^{(E-iQ^a)/T}-1} = \mathrm{tr} \sum_{j=1}^{\infty} \mathrm{e}^{-j(E-iQ^a)/T} = \sum_{j=1}^{\infty} \mathrm{e}^{-jE/T}\, \mathrm{tr}\, \mathbf{L}^j$$

$\mathrm{L} = \mathrm{e}^{i\,Q/T}$ = Wilson line. Obtain expressions in terms of moments of L, L^j.

We *don't* know (yet) effective theory for Q's. *So we guess.*

Take first moment, l = <*loop*> = < tr L>/N, from lattice for N = 3.
For higher moments, given l, assume either: 1. Gross-Witten, or 2. step function.

L ~ propagator of *infinitely* heavy (test) quark.
In *this* semi-cl. expansion, for colored fields of *any* momentum and mass,
As $l \rightarrow 0$, *all* quarks suppressed ~ l ; *all* gluons, ~ l^2 : *universal* color evaporation

Smells right: *all* colored fields *should* evaporate as <*loop*> → 0.

Shear viscosity in the semi-QGP

Shear viscosity, η, in the complete QGP:

Arnold, Moore & Yaffe, hep-ph/0010177 & 0302165 = AMY.

Generalize to Q ≠ 0: Boltzmann equation in background field.

$$\eta = \frac{\mathcal{S}^2}{\mathcal{C}}$$

S = source, C = collision term. *Two* ways of getting small η:

"Strong" QGP, *large* coupling $S \sim 1$, $C \sim (\text{coupling})^2 >> 1$.

$\mathcal{N} = 4$ SU(N), $g^2 N = N = \infty$: $\eta/s = 1/4\pi$. Kovtun, Son & Starinets hep-th/0405231

"Semi" QGP: small *loop* at *moderate* coupling:

Pure glue: $S \sim \langle loop \rangle^2$, $C \sim g^4 \langle loop \rangle^2$

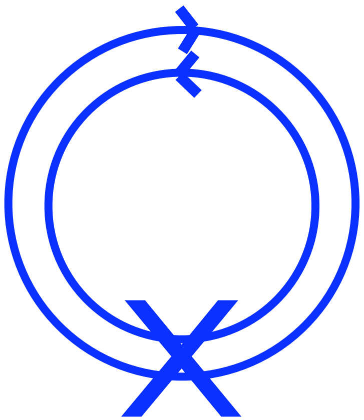
With quarks: $S \sim \langle loop \rangle$, $C \sim g^4$

Both: $\eta \sim \langle loop \rangle^2$

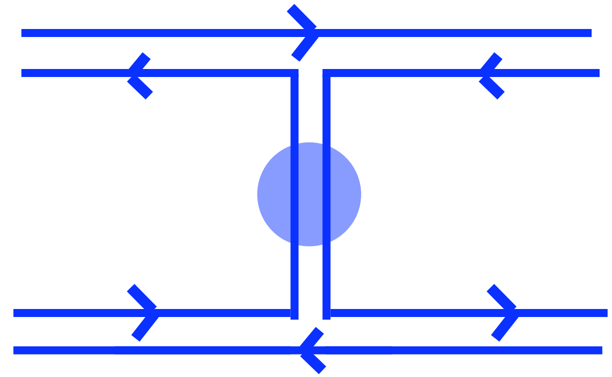
To leading log order: # from AMY, constant "c" beyond leading log

$$\frac{\eta}{T^3} = \frac{\#}{g^4 \log(c/g)} \mathcal{R}(\ell) \quad ; \quad \mathcal{R}(\ell \to 0) \sim \ell^2$$

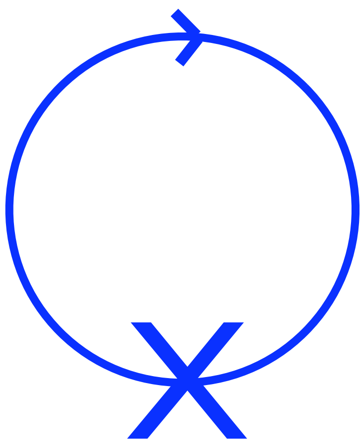
Counting powers of <*loop*> = $l \to 0$



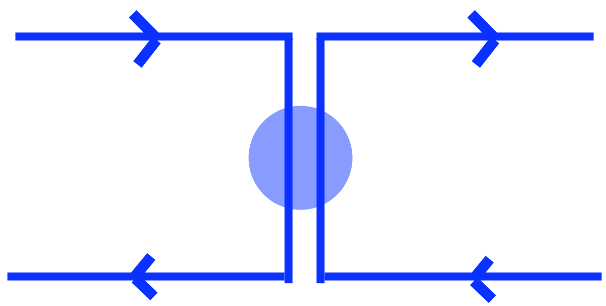
$\mathcal{S} \sim \ell^2$



$\mathcal{C} \sim \ell^2$

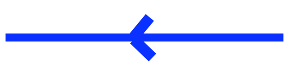


$\mathcal{S} \sim \ell$



$\mathcal{C} \sim 1$

$\sim e^{+iQ^a/T}$



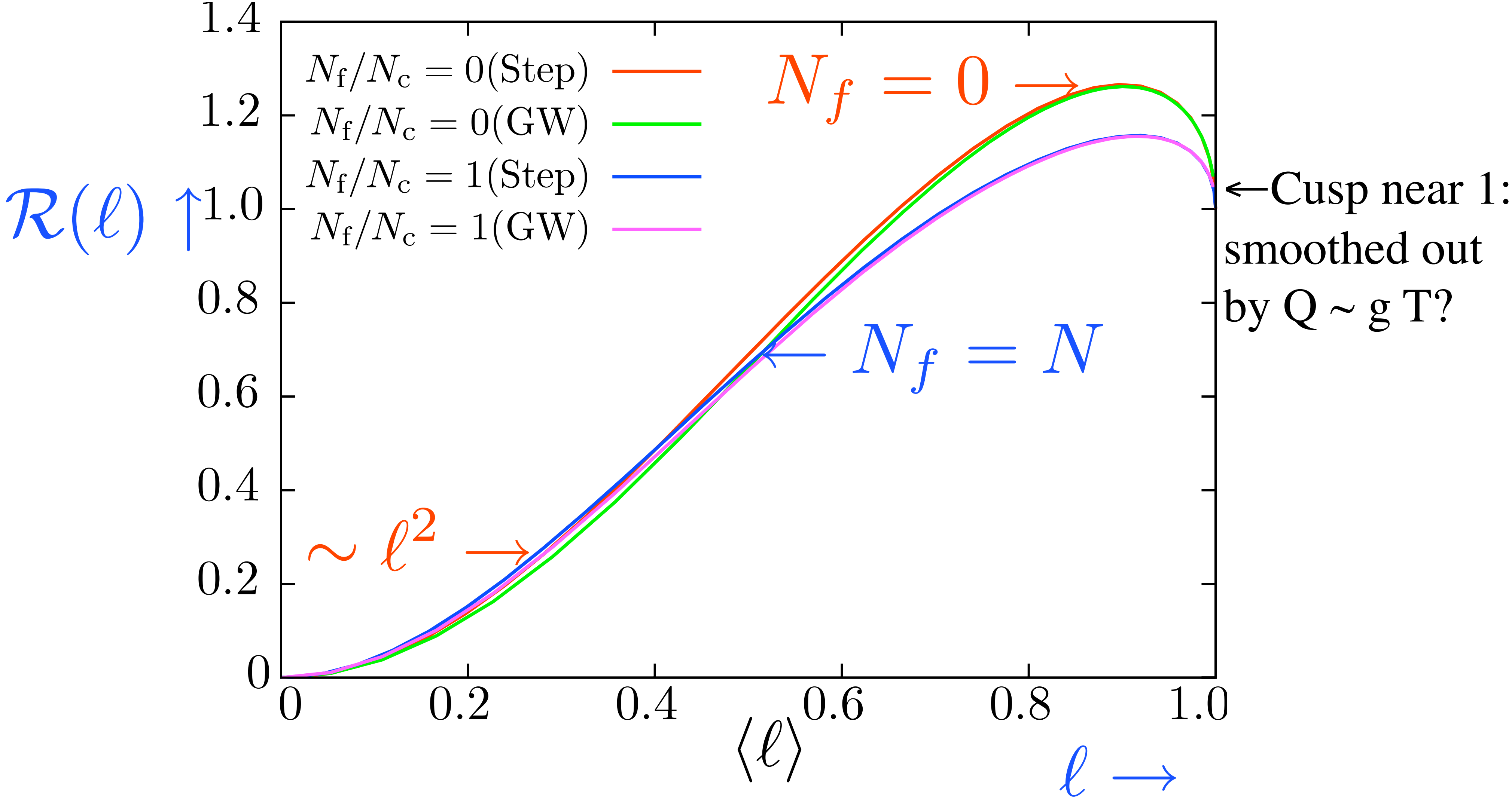
$\sim e^{-iQ^a/T}$

Small shear viscosity from color evaporation

R = ratio of shear viscosity in semi-QGP/complete-QGP at *same* g, T.

Two different eigenvalue distributions give *very* similar results!

When <*loop*> ~ 0.3, *R* ~ 0.3.

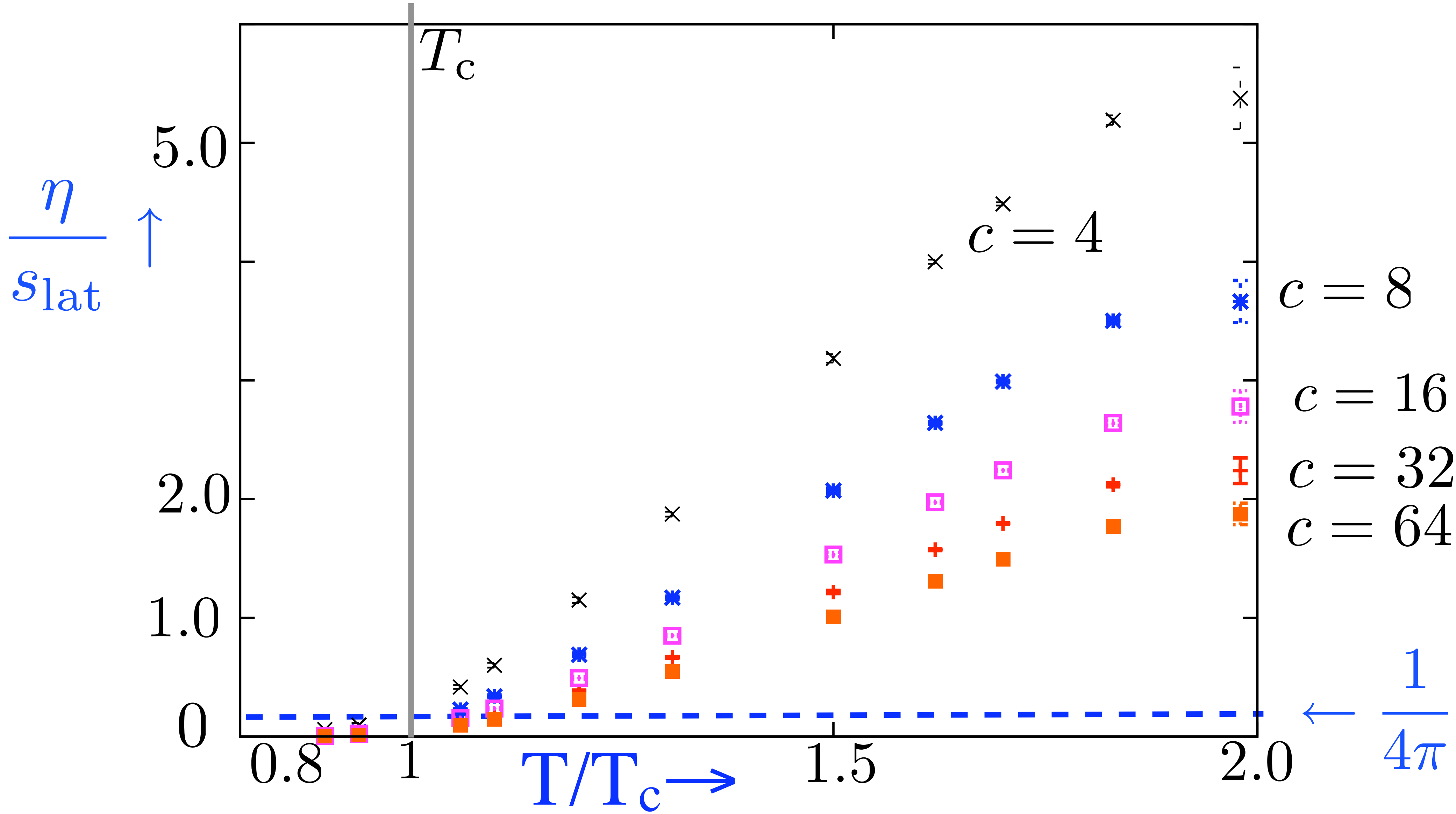


Shear viscosity/entropy

Leading log shear viscosity/lattice entropy. $\alpha_s(T_c) \sim 0.3$.

Large increase from T_c to 2 T_c. *Clearly* need results beyond leading log.

Also need to include: quarks and gluons *below* T_c, hadrons *above* T_c. *Not* easy.



Strong- vs. Semi-QGP at the LHC

At RHIC, $\eta/s \sim 0.1 \pm 0.1$
Luzum & Romatschke, 0804.4015

Close to $\mathcal{N} = 4$ SU(∞), $\eta/s = 1/(4\pi)$.

Strong-QGP: in $\mathcal{N} = 4$ SU(∞),
add scalar potential to fit lattice pressure
But η/s *remains* $= 1/4\pi$!
Evans & Threlfall, 0805.0956
Gubser & Nellore, 0804.0434
Gursoy, Kiritsis, Mazzanti & Nitti 0804.0899
So LHC nearly ideal, like RHIC.

Semi-QGP, *and* non-relativistic systems →
Large change in η/s from T_c to $2\,T_c$.
At *early* times, LHC *viscous*,
*un*like RHIC

Lacey, Ajitnand, Alexander, Chung, Holzman, Issah, Taranenko, Danielewicz & Stocker, nucl-ex/0609025 ↓