QCD phase diagram at $\mu \neq 0$

1. Standard lore:

   One transition, chiral = deconfined, “semicircle”

2. Large number of colors, N_c:

   Two transitions, chiral $\neq$ deconfinement

   “Quarkyonic” matter

   Confined, chirally symmetric baryons: massive, parity doubled.

3. QCD?

   Perhaps: phase intermediate between nuclear matter and “just” quarks

McLerran & RDP, 0706.2191. Hidaka, McLerran, & RDP 0803.0279

The first semicircle

Cabibbo and Parisi ‘75: Exponential (Hagedorn) spectrum limiting temperature,
or transition to new, “unconfined” phase. One transition.

Punchline today: below for chiral transition, deconfinement splits off at finite μ.

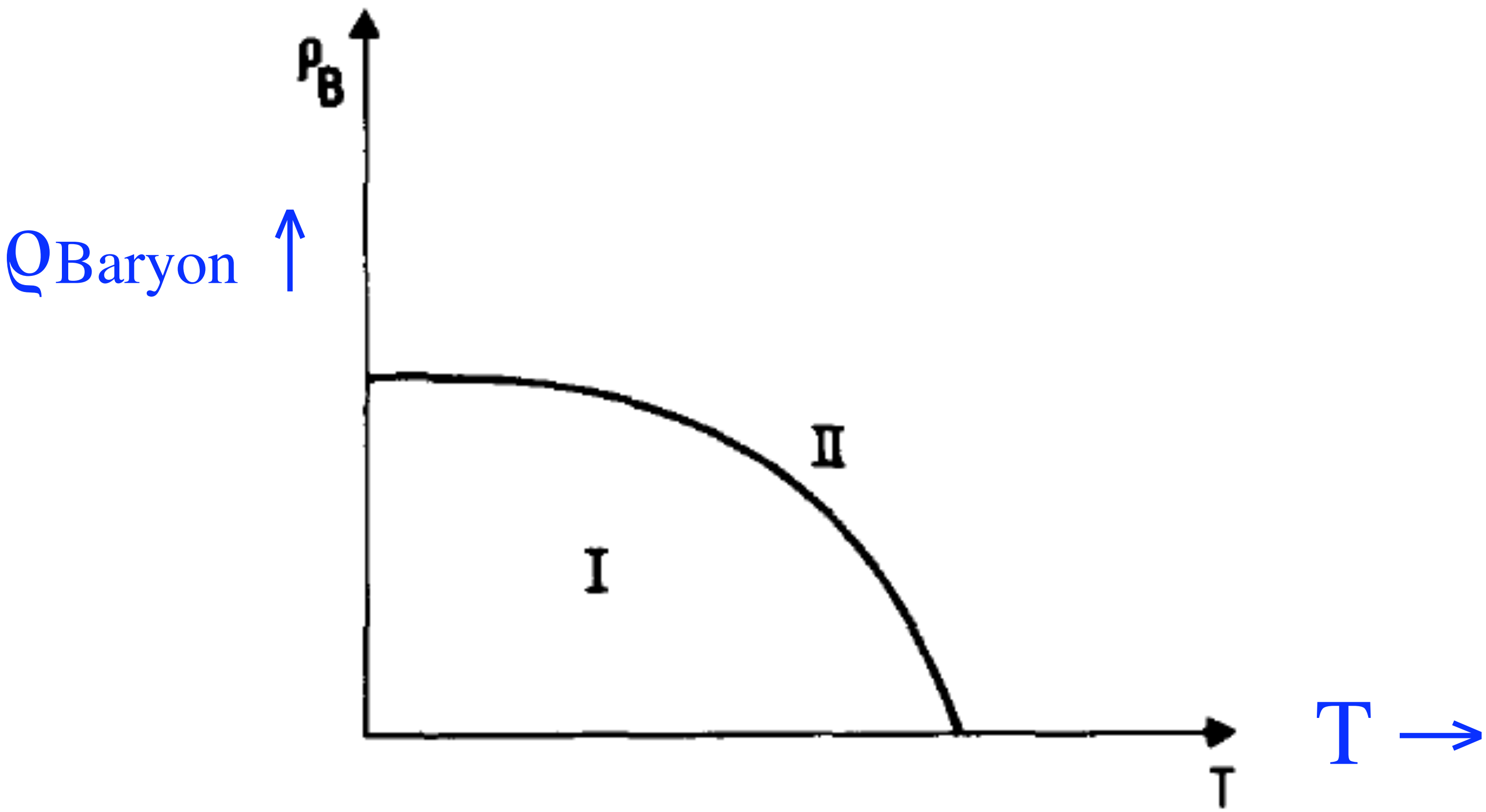


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

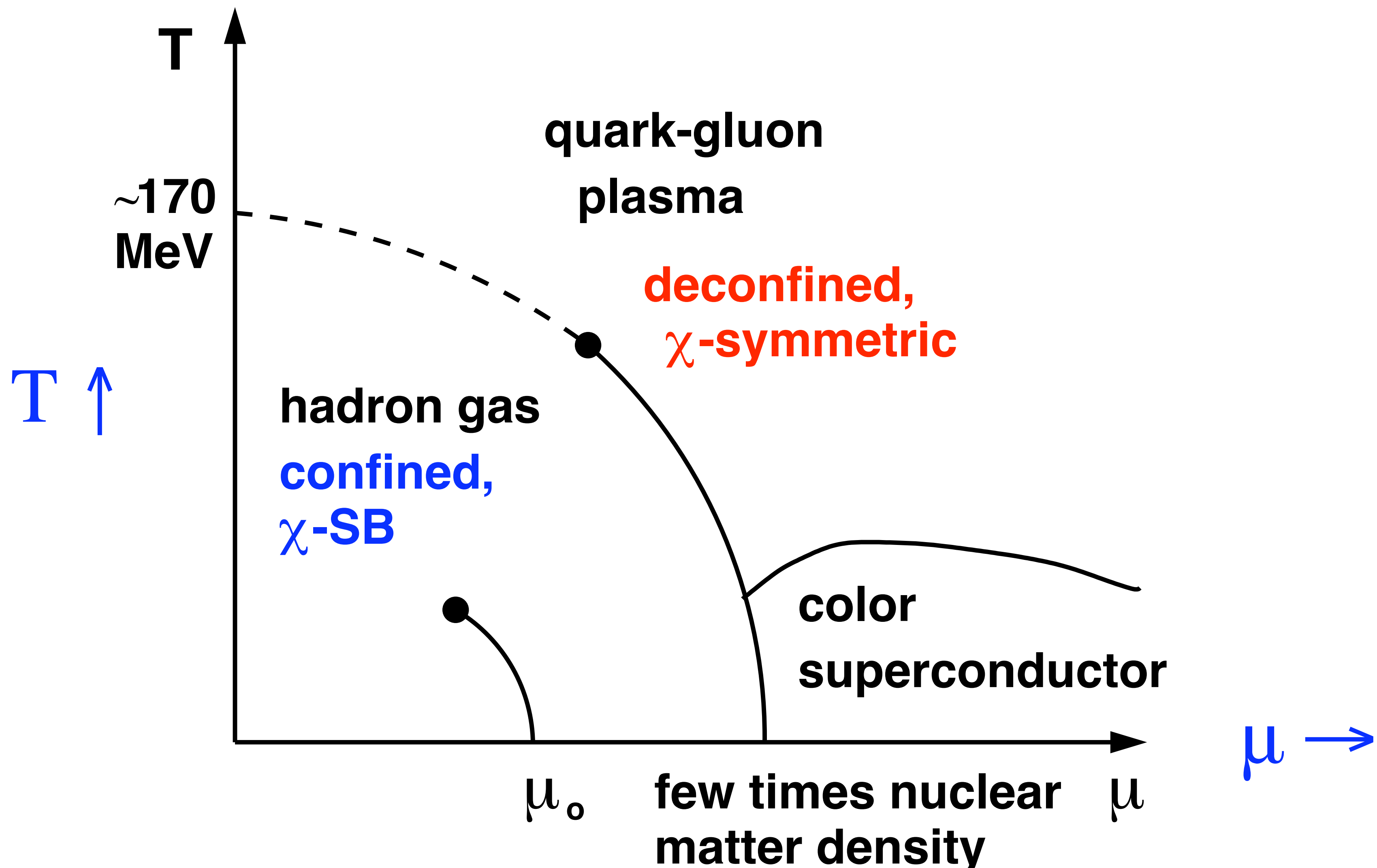
Phase diagram, ~ '06

Lattice, $T \neq 0$, $\mu = 0$: two possible transitions; one crossover, same T.

Karsch hep-lat/0601013. Remains crossover for $\mu \neq 0$?

Stephanov, Rajagopal, & Shuryak hep-ph/9806219, /9903292, /0010100

Critical end point where crossover turns into first order transition

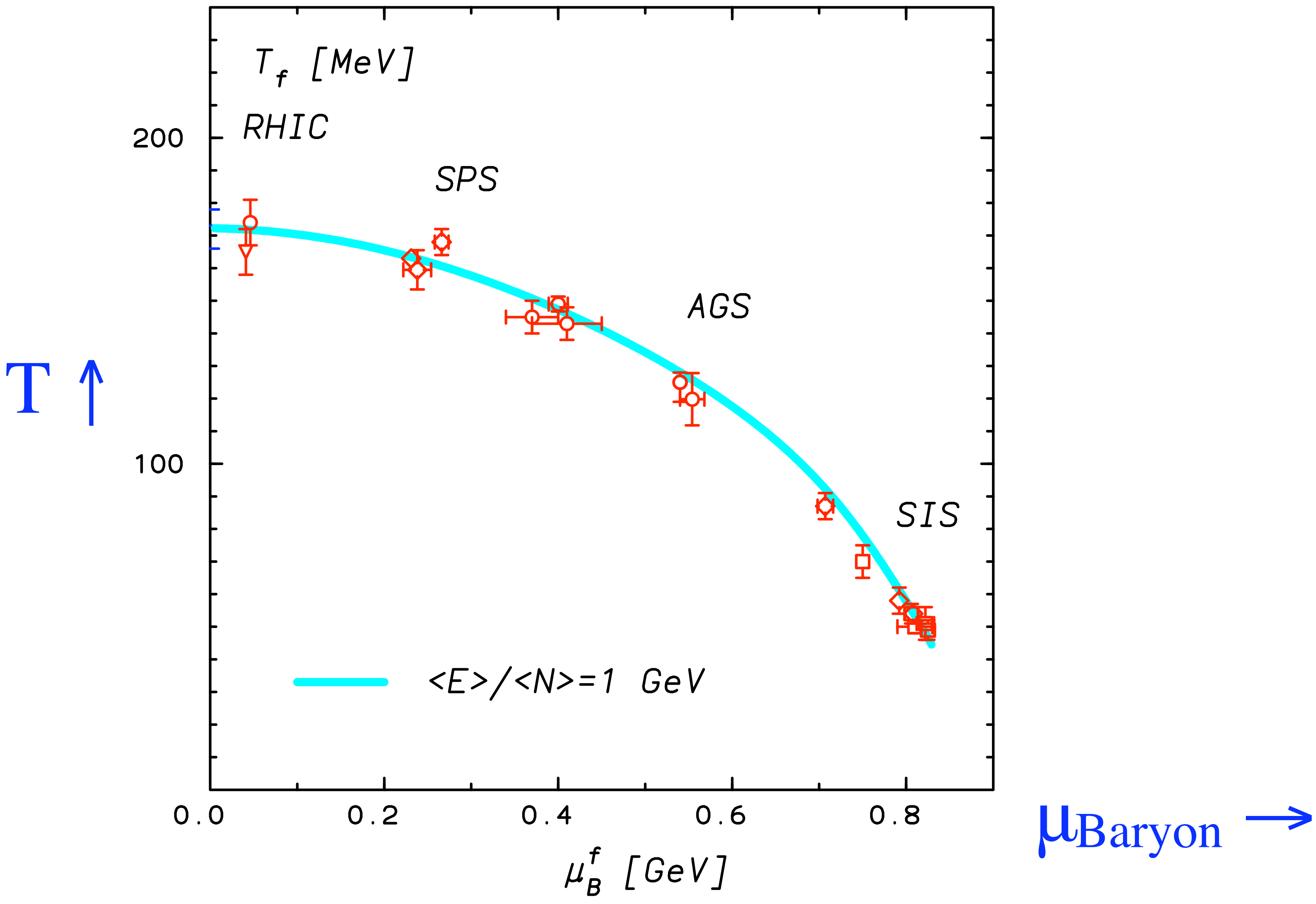


Experiment: freezeout line

Cleymans & Redlich nucl-th/9906065...Kraus, Cleymans, Oeschler, Redlich 0808.0611

Line for chemical equilibriation at freezeout ~ semicircle.

N.B.: for T = 0, goes down to ~ nucleon mass.

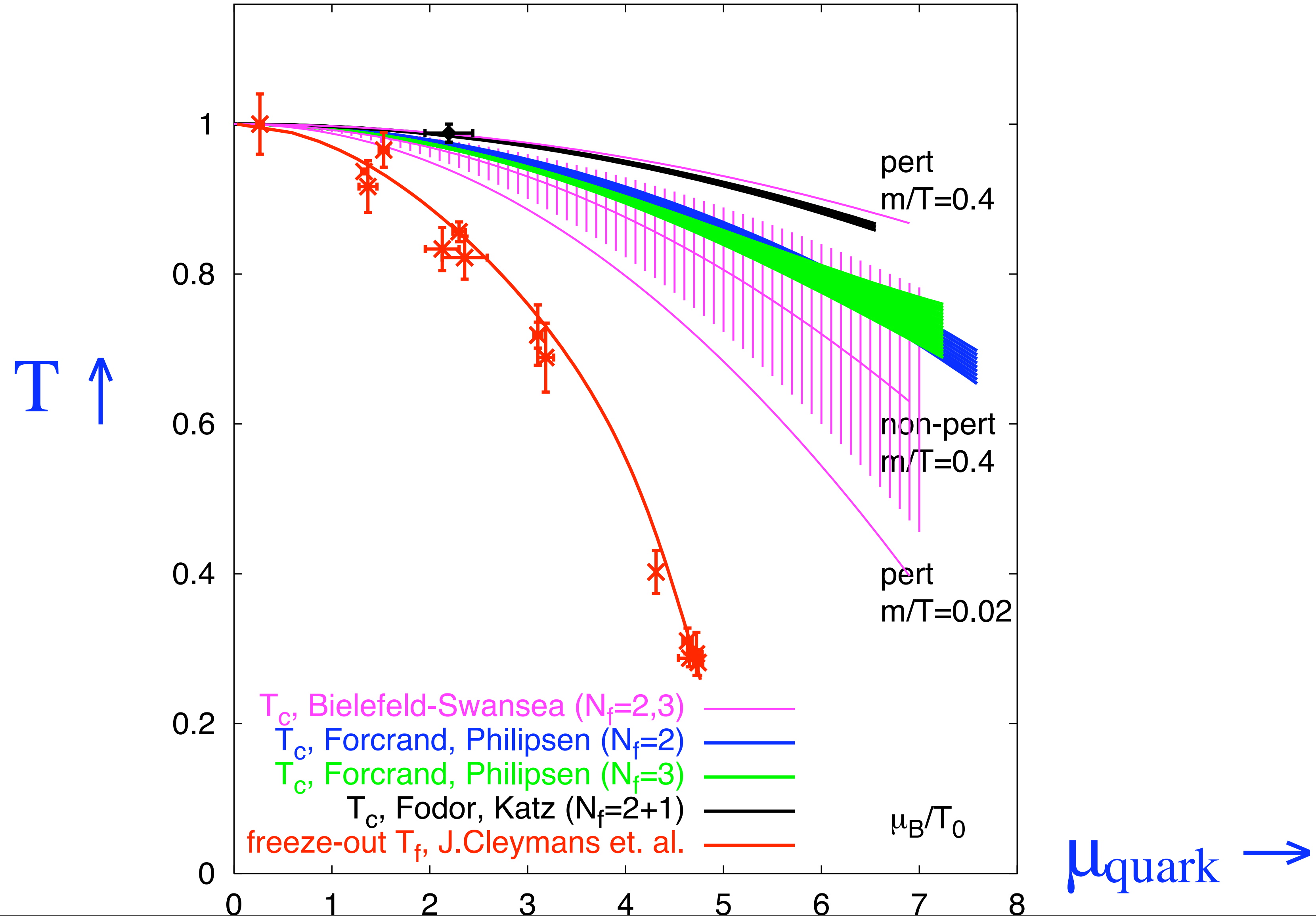


Experiment vs. Lattice

Lattice "transition" appears *above* freezeout line?

Fodor, Katz, & Schmidt hep-lat/0701022; Schmidt '07

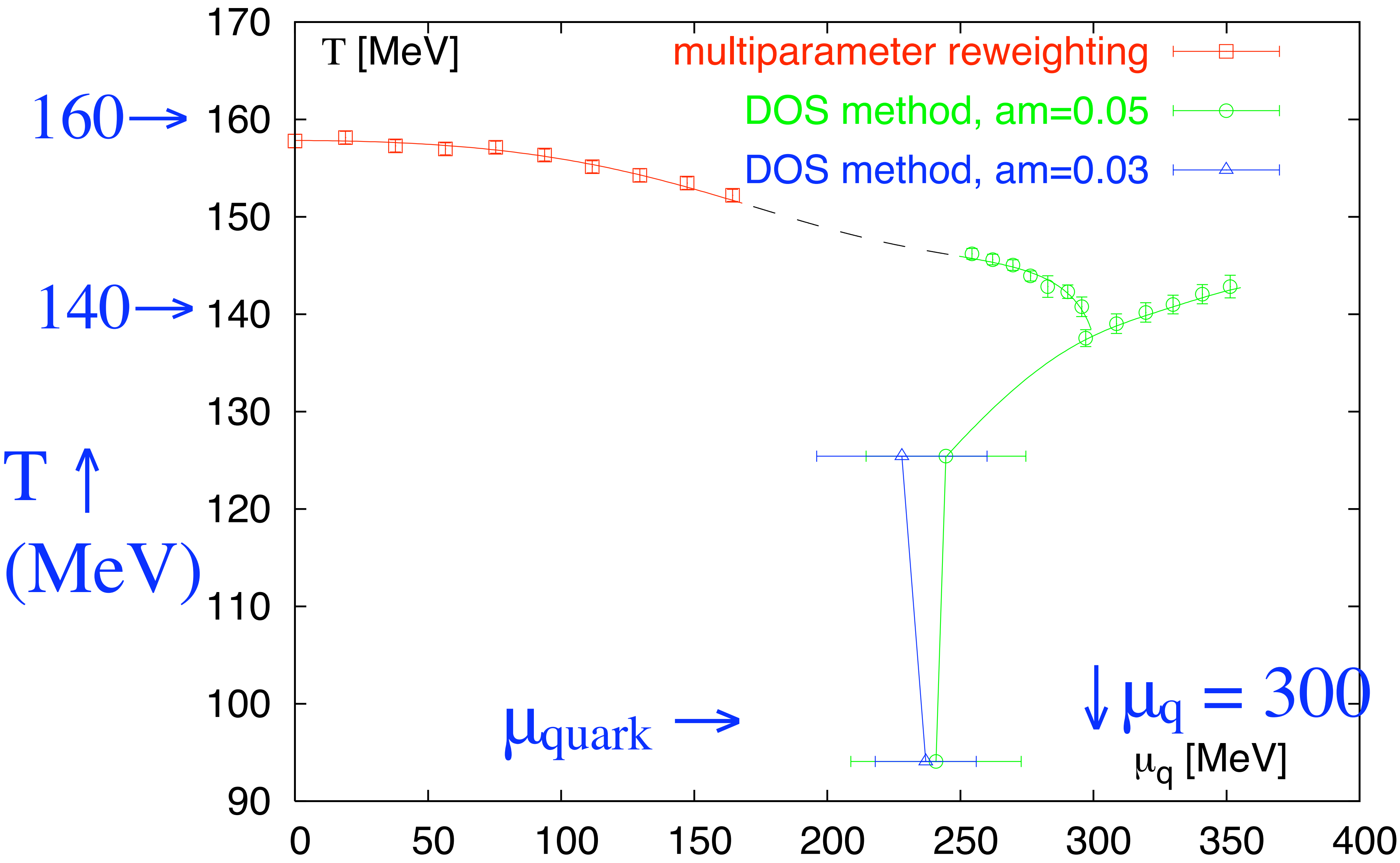
N.B.: *small* change in T_c with μ?



Lattice T_c , vs μ

Rather small change in T_c vs μ? Depends where μ_c is at $T = 0$.

Fodor, Katz, & Schmidt hep-lat/0701022



EoS of nuclear matter

Akmal, Panharipande, & Ravenhall nucl-th/9804027:

Equation of State for nuclear matter, T=0

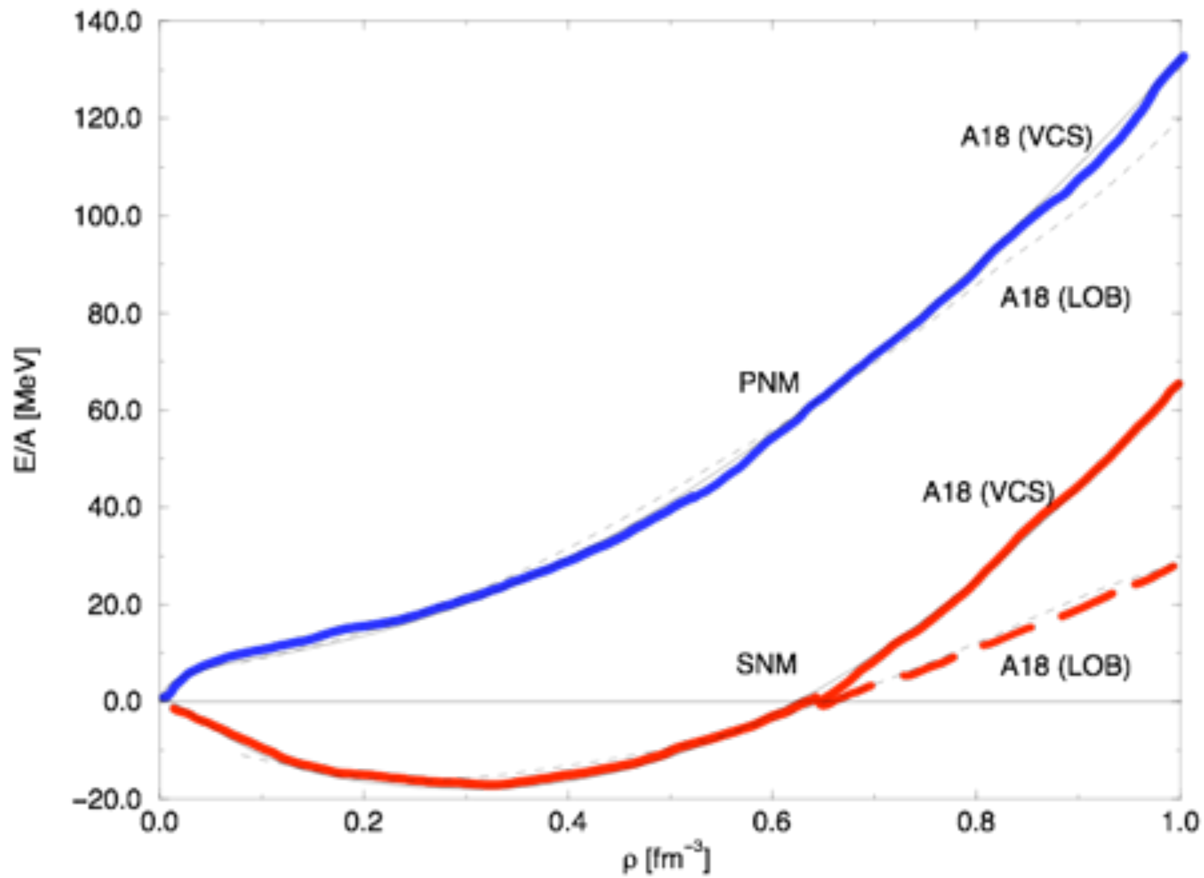
E/A = energy/nucleon. Fits to various nuclear potentials: A18= Argonne 18…

PNM = pure neutron matter. SNM = symmetric nuclear matter (equal #'s n's, p's)

Binding energy of nuclear matter ~ 15 MeV!

Much smaller than any natural hadronic scale: f_π , Λ_{MS}…

E/A ↑



ρ_{Baryon} →

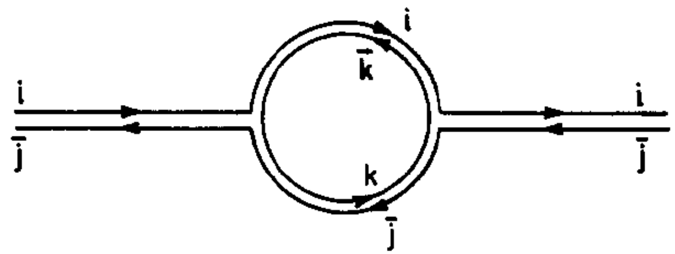
Expansion in large N_c

't Hooft '74: let $N_c \to \infty$, with $\lambda = g^2 N_c$ fixed.

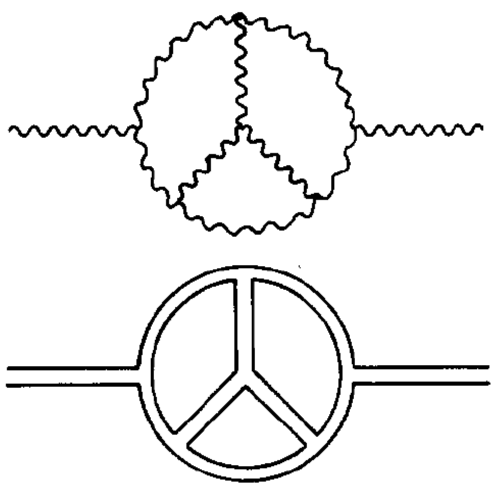
$\sim N_c^2$ gluons in adjoint representation, vs $\sim N_c$ quarks in fundamental rep. $\Rightarrow$

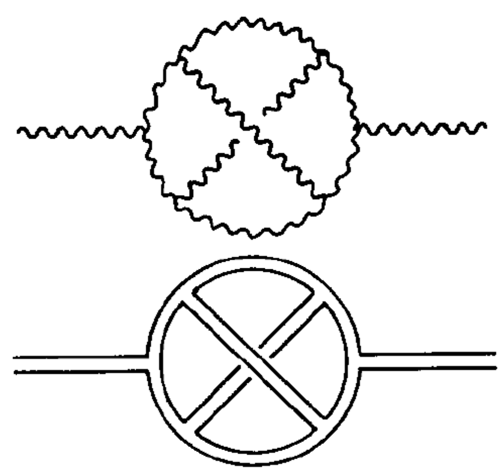
large N_c dominated by *gluons* (iff N_f = # quark flavors *small*)

Double line (birdtrack) notation:



$\sim g^2 N_c = \lambda$



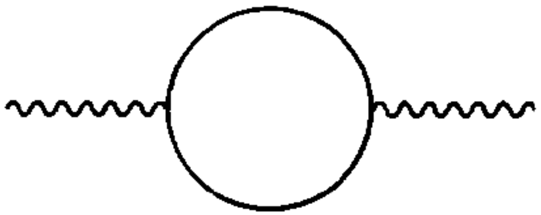


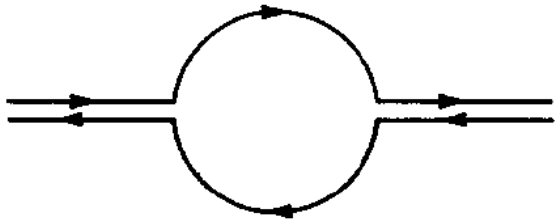
Planar diagram, $\sim \lambda^2$

Non-planar diagram, $\sim \lambda^2 / N_c$

Suppressed by $1/N_c$. Trace terms also $1/N_c$

Quark loops suppressed at large N_c





$$\sim g^2 = \lambda \times \frac{1}{N_c}$$

Quark loops are suppressed at large N_c *if* N_f , # quark flavors, is held fixed

Thus: limit of: large N_c , *small* N_f

Quarks introduced as external sources.

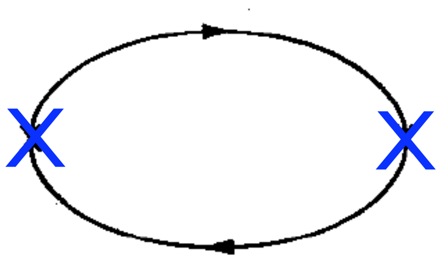
Analogous to “quenched” approximation, expansion about $N_f = 0$.

Veneziano ‘78: take both N_c and N_f large.
Can use baryon number as order parameter: Hidaka, McLerran, & RDP.

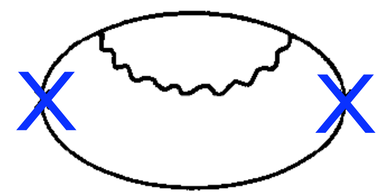
Form factors at large N_c

J ~ (gauge invariant) mesonic current

$$< J(x)J(0) > \sim N_c$$



Infinite # of planar diagrams for $< J\,J >$:



Confinement => sum over mesons, form factors ~ $N_c^{1/2}$

$$< J(x)J(0) > \sim \int d^4p\, e^{ip\cdot x} \sum_n < 0|J|n > \frac{1}{p^2+m_n^2} < n|J|0 >$$

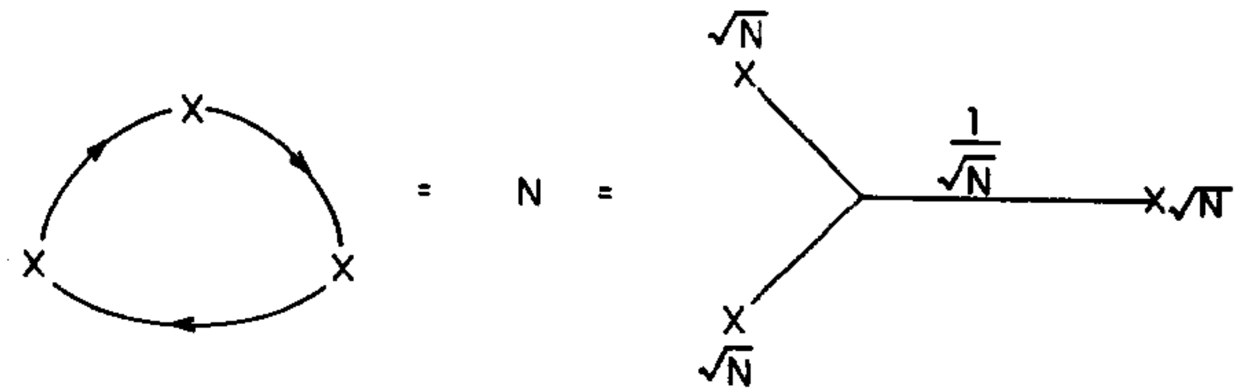
$$< J(x)J(0) > \sim \ N_c \quad \Rightarrow \ < 0|J|n > \sim \sqrt{N_c} \ \text{ if } \ m_n \sim 1$$

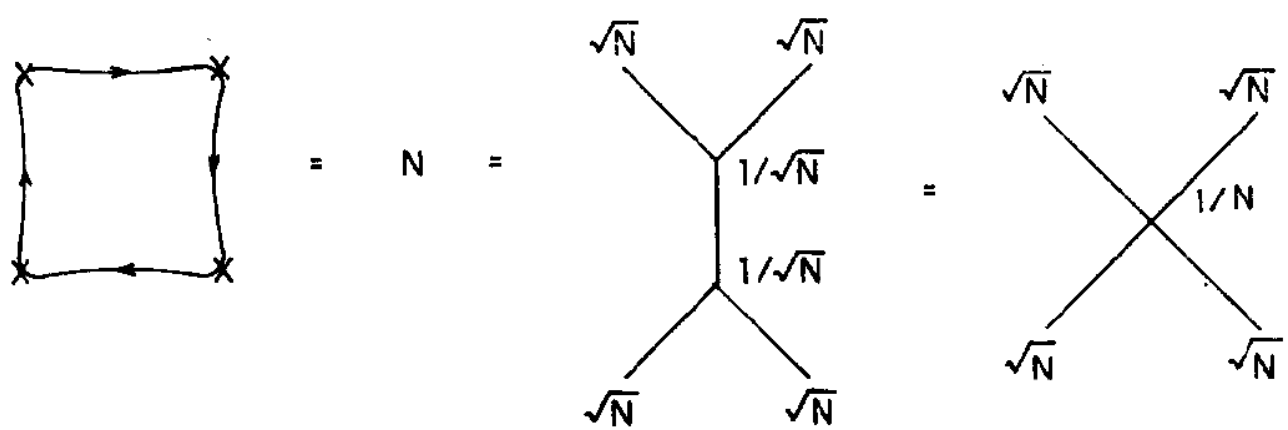
Mesons & glueballs *free* at $N_c = \infty$

With form factors ~ $N_c^{1/2}$, 3-meson couplings ~ $1/N_c^{1/2}$; 4-meson, ~ $1/N_c$
For glueballs, 3-glueball couplings ~ $1/N_c$, 4-glueball ~ $1/N_c^2$

Mesons and glueballs don't interact at $N_c = \infty$.

Large N limit *always* (some) classical mechanics Yaffe '82



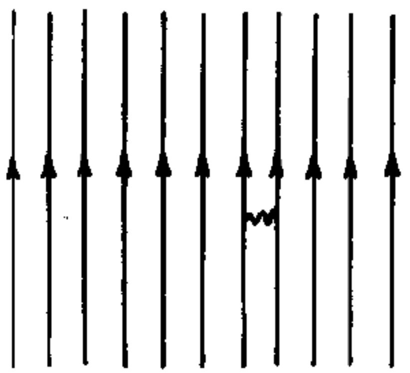


Baryons at large N_c

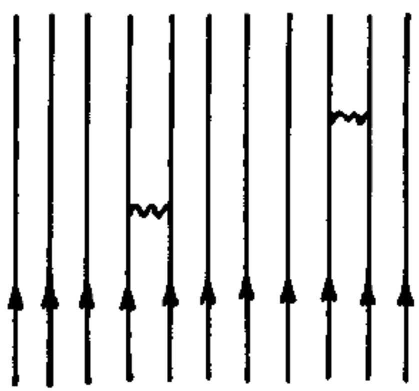
Witten '79: Baryons have N_c quarks, so nucleon mass $M_N \sim N_c \Lambda_{QCD}$.

Baryons like "solitons" of large N_c limit (~ Skyrmion)

Leading correction to baryon mass:



$$g^2 \times N_c \times N_c \sim \lambda N_c$$

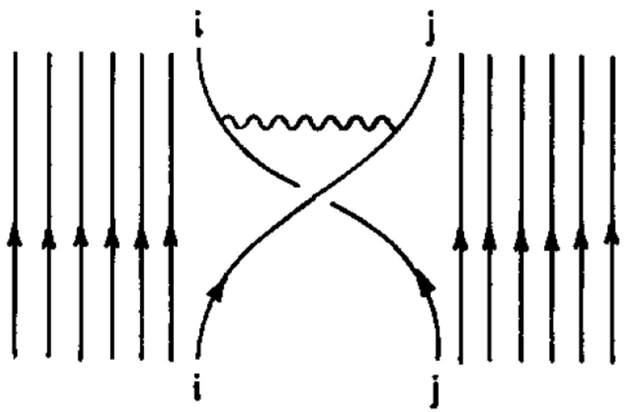


Appears $\sim g^4 N_c^4 \sim \lambda^2 N_c^2$?

No, iteration of average potential, mass still $\sim N_c$.

Baryons are *not* free at $N_c = \infty$

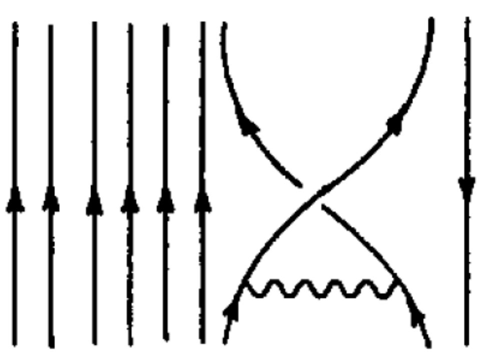
Baryons interact strongly. Two baryon scattering ~ N_c :



$$g^2 \times N_c \times N_c \sim \lambda N_c$$

Scattering of three, four... baryons also ~ N_c

Mesons also interact strongly with baryons, ~ N_c^0 ~ 1



$$g^2 \times N_c \sim \lambda$$

Towards the phase diagram at $N_c = \infty$

As example, consider gluon polarization tensor at zero momentum.
(at leading order, ~ Debye mass^2 , gauge invariant)

$$\Pi^{\mu\mu}(0) = g^2 \left(\left(N_c + \frac{N_f}{2} \right) \frac{T^2}{3} + \frac{N_f \mu^2}{2\pi^2} \right) = \lambda \, \frac{T^2}{3} \ , \ N_c = \infty$$

For $\mu \sim N_c^0 \sim 1$, at $N_c = \infty$ the gluons are blind to quarks.

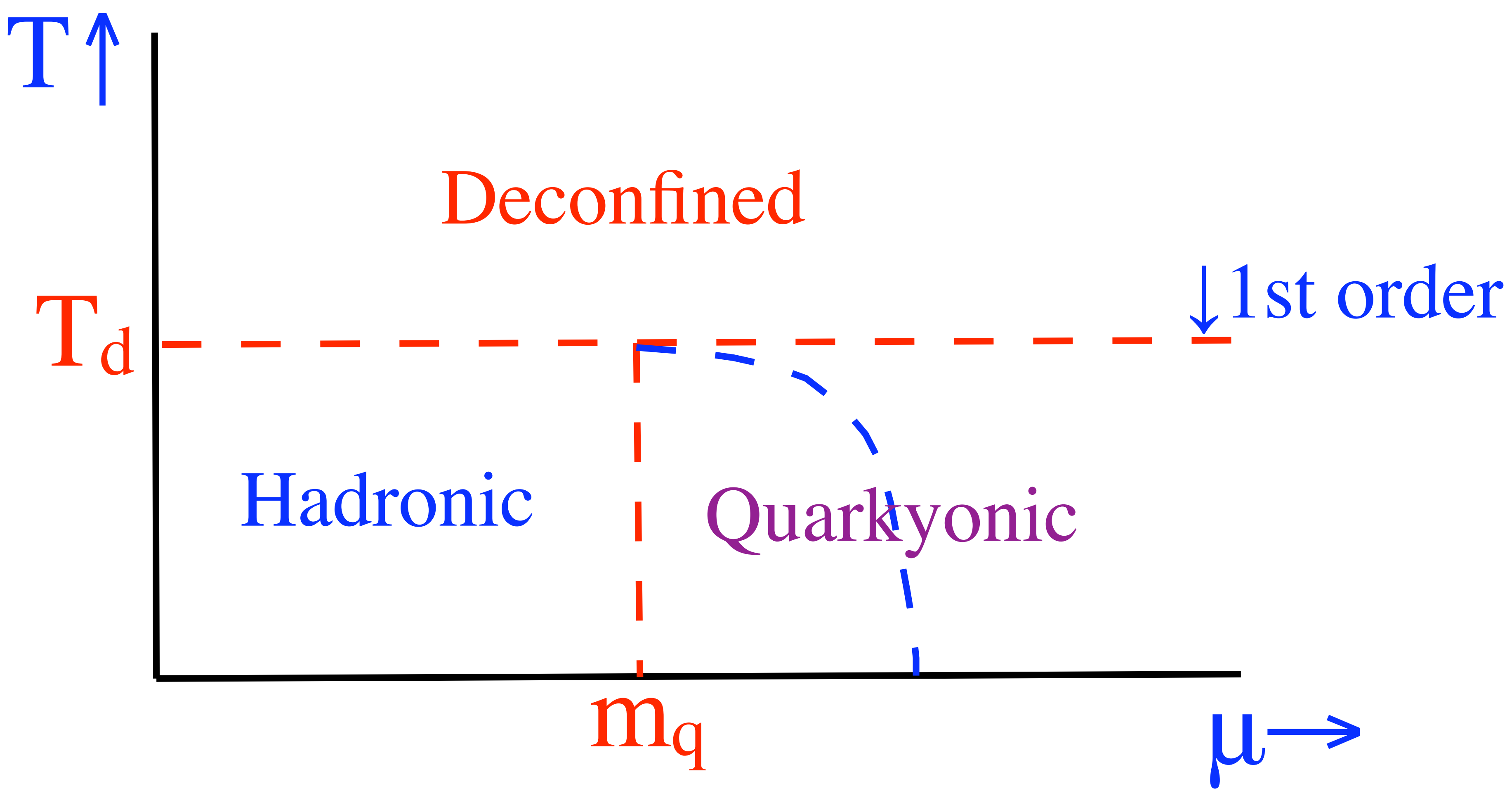
When $\mu \sim 1$, deconfining transition temperature $T_d(\mu) = T_d(0)$

Chemical potential only matters when larger than mass:
$\mu_{Baryon} > M_{Baryon}$. Define $m_{quark} = M_{Baryon}/N_c$; so $\mu > m_{quark}$.

"Box" for $T < T_c$; $\mu < m_{quark}$: confined phase baryon free, since their mass ~ N_c
Thermal excitation ~ $\exp(-m_B/T) \sim \exp(-N_c) = 0$ at large N_c.
So hadronic phase in "box" = mesons & glueballs only, *no* baryons.

Phase diagram at $N_c = \infty$, I

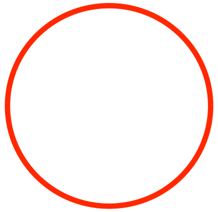
At *least* three phases. At large N_c, can use pressure, P, as order parameter.
Hadronic (confined): $P \sim 1$. Deconfined, $P \sim N_c^2$. Thorn ’81; RDP ’84...
$P \sim N_c$: quarks or baryonic = “quark-yonic”. Chiral symmetry restoration?
L. McLerran & RDP, 0706.2191
N.B.: mass threshold at m_q neglects (possible) nuclear binding, Son.



Nuclear matter at large N_c

$\mu_{Baryon} = \sqrt{k_F^2 + M^2}$, k_F = Fermi momentum of baryons.
Pressure of ideal baryons density times energy of non-relativistic baryons:

$$P_{\text{ideal baryons}} \sim n(k_F)\,\frac{k_F^2}{M} \sim \frac{1}{N_c}\,\frac{k_F^5}{\Lambda_{QCD}}$$

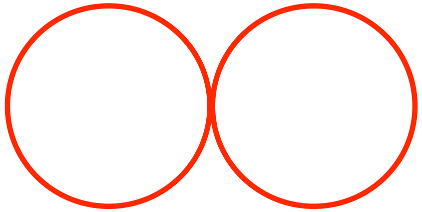


This is small, ~ $1/N_c$. The pressure of the I = J tower of resonances is as small:

$$\delta P_{\text{resonances}} \sim \frac{1}{M}\,\frac{k_F^8}{\Lambda_{QCD}^3} \sim \frac{1}{N_c}\,\frac{k_F^8}{\Lambda_{QCD}^4}$$

Two body interactions are huge, ~ N_c in pressure.

$$\delta P_{\text{two body int.'s}} \sim N_c\,\frac{n(k_F)^2}{\Lambda_{QCD}^2} \sim N_c\,\frac{k_F^6}{\Lambda_{QCD}^2}$$



At large N_c , nuclear matter is dominated by potential, not kinetic terms!
Two body, three body... interactions *all* contribute ~ N_c .

Window of nuclear matter

Balancing $P_{\text{ideal baryons}} \sim P_{\text{two body int.'s}}$, interactions important very quickly,

$$k_F \sim \frac{1}{N_c^2}\,\Lambda_{QCD}$$

For such momenta, only two body interactions contribute.

By the time $k_F \sim 1$, *all* interactions terms contribute $\sim N_c$ to the pressure.

But this is *very* close to the mass threshold,

$$\mu - m_q = \frac{\mu_B - M}{N_c} = \frac{k_F^2}{2MN_c} \sim \frac{1}{N_c^2}\,k_F^2$$

Hence "ordinary" nuclear matter is only in a *very* narrow window.

One quickly goes to a phase with pressure $P \sim N_c$.

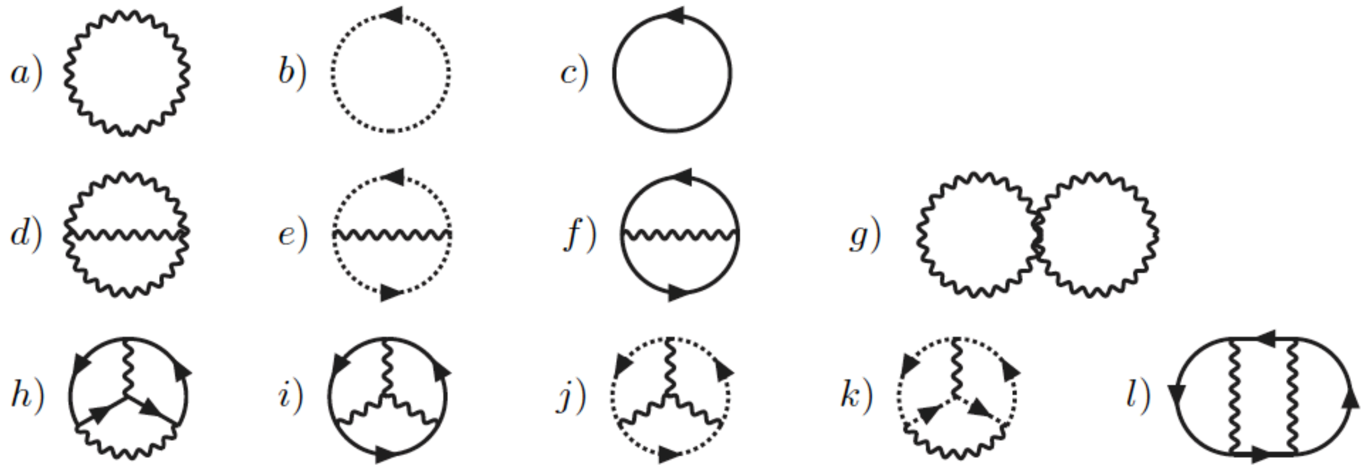
So are they baryons, or quarks?

Perturbative pressure

At high density, $\mu >> \Lambda_{QCD}$, compute $P(\mu)$ in QCD perturbation theory.

To $\sim g^4$, (Freedman & McLerran)[4] '77

Ipp, Kajantie, Rebhan, & Vuorinen, hep-ph/0604060



At $\mu \neq 0$, only diagrams with at least one quark loop contribute. Still...

$$P_{\text{pert.}}(\mu) \sim N_c N_f \ \mu^4 \ F_0(g^2(\mu/\Lambda_{QCD}), N_f)$$

For $\mu >> \Lambda_{QCD}$, but $\mu \sim N_c^0 \sim 1$, calculation reliable.

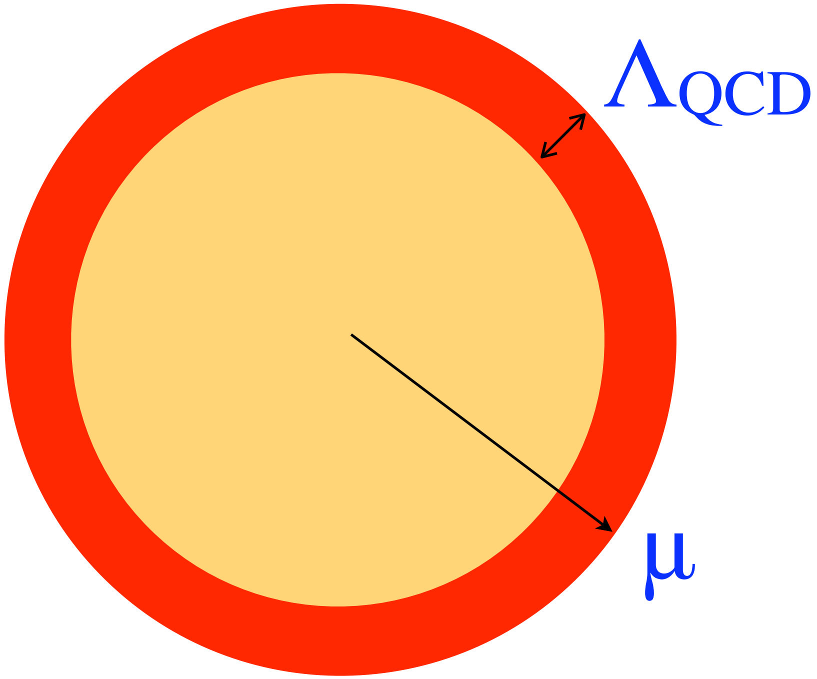
Compute $P(\mu)$ to $\sim g^6$? No "magnetic mass" at $\mu \neq 0$, well defined $\forall\ (g^2)^n$.

"Quarkyonic" phase at large N_c

As gluons blind to quarks at large N_c, for $\mu \sim N_c^0 \sim 1$, *confined* phase for $T < T_d$

This includes $\mu >> \Lambda_{QCD}$! Central puzzle. We suggest:

To the right: Fermi sea =>



Deep in the Fermi sea, $k << \mu$,
looks like quarks.

But: within ~ Λ_{QCD} of the Fermi surface,
confinement => *baryons*

We term combination "*quark-yonic*"

OK for $\mu >> \Lambda_{QCD}$. When $\mu \sim \Lambda_{QCD}$, baryonic "skin" entire Fermi sea.

But what about chiral symmetry breaking?

Skyrmions and $N_c = \infty$ baryons

Witten '83; Adkins, Nappi, Witten '83: Skyrme model for baryons

$$\mathcal{L} = f_\pi^2 \, \mathrm{tr}|V_\mu|^2 + \kappa \, \mathrm{tr}[V_\mu, V_\nu]^2 \; , \; V_\mu = U^\dagger \partial_\mu U \; , \; U = e^{i\pi/f_\pi}$$

Baryon soliton of pion Lagrangian: $f_\pi \sim N_c^{1/2}$, $\kappa \sim N_c$, mass $\sim f_\pi^2 \sim \kappa \sim N_c$.

Above Lagrangian simplest form: surely *infinite* series in V_μ.

Single baryon: at $r = \infty$, $\pi^a = 0$, $U = 1$. At $r = 0$, $\pi^a = \pi \, r^a/r$.
Baryon number topological: Wess & Zumino '71; Witten '83.

Huge degeneracy of baryons: multiplets of isospin and spin, $I = J$: $1/2 \ldots N_c/2$.
Obvious as collective coordinates of soliton, coupling spin & isospin

Dashen & Manohar '93, Dashen, Jenkins, & Manohar '94:
Baryon-meson coupling $\sim N_c^{1/2}$,
Cancellations from extended $SU(2\,N_f)$ symmetry.

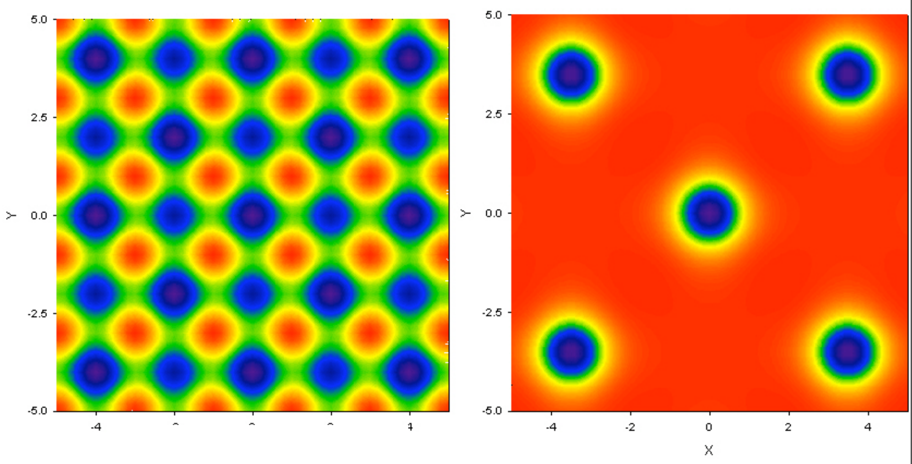
Skyrmion crystals

Skyrmion crystal: soliton periodic in space.

Kutschera, Pethick & Ravenhall (KPR) '84; Klebanov '85 + ...

Lee, Park, Min, Rho & Vento, hep-ph/0302019

Park, Lee, & Vento, 0811.3731:



At low density, chiral symmetry broken by Skyrme crystal, as in vacuum.

Chiral symmetry *restored* at nonzero density: $\langle U \rangle = 0$ in each cell.

Goldhaber & Manton '87: due to "half" Skyrmion symmetry in each cell.

Forkel, Jackson et al, '89: excitations *are* chirally symmetric.

Easiest to understand with "spherical" crystal, KPR '84, Manton '87.

Take same boundary conditions as a single baryon, but for sphere of radius R:

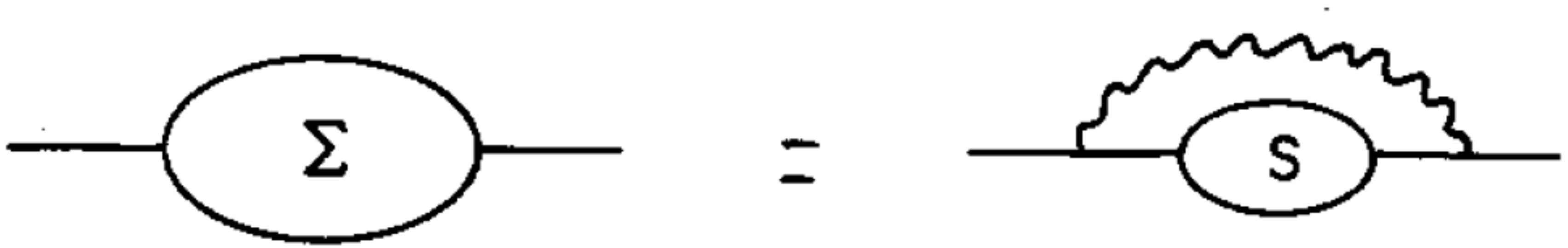
At $r = R$: $\pi^a = 0$. At $r = 0$, $\pi^a = \pi\, r^a/r$. Density one baryon/$(4\pi R^3/3)$.

At high density, term $\sim \kappa$ dominates, so energy density $\sim$ baryon density$^{4/3}$.

Like perturbative QCD! Accident of simplest Skyrme Lagrangian.

Schwinger-Dyson equations at large N_c: 1+1 dim.'s

't Hooft '74: as gluons blind to quarks at large Nc, S-D eqs. simple for quark:
Gluon propagator, and gluon quark anti-quark vertex unchanged.
To leading order in $1/N_c$, only quark propagator changes:



't Hooft '74: in 1+1 dimensions, single gluon exchange generates linear potential,

$$g_{2D}^2 \int dk \, \frac{e^{ikr}}{k^2} \sim g_{2D}^2 \, r$$

In vacuum, Regge trajectories of confined mesons. Baryons?

Solution at $\mu \neq 0$? Should be possible, not yet solved.

M. Thies, hep-th/0601049, C. Boehmer, U. Fritsch, S. Kraus, & M. Thies, 0807.2571
Gross-Neveu model has crystalline structure at $\mu \neq 0$

Quarkyonic matter via Schwinger-Dyson

Glozman & Wagenbrunn 0709.3080, 0805.4799; Glozman 0812.1101:

In 3+1 dimensions, confining gluon propagator, $1/(k^2)^2$ as $k^2 \to 0$:

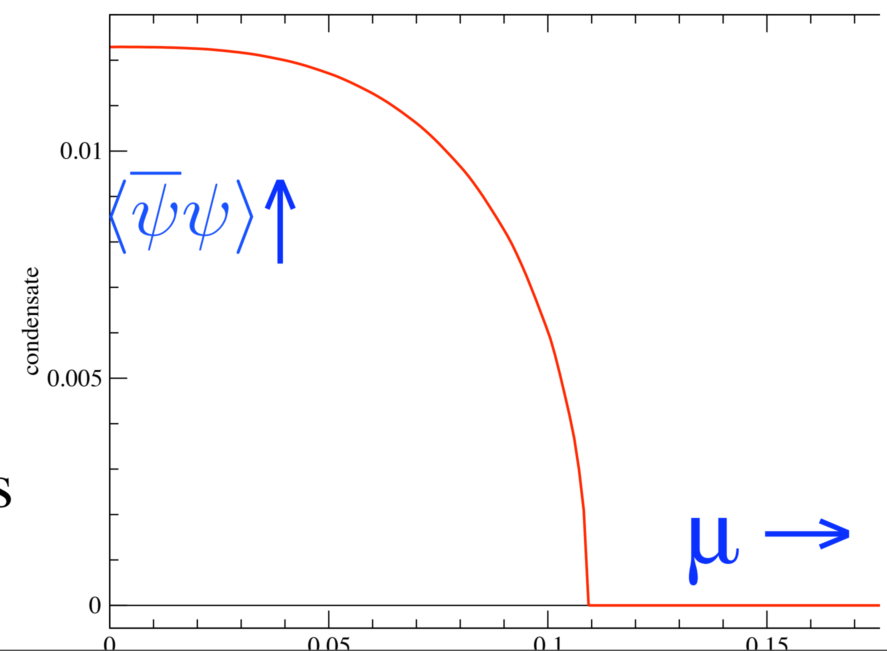
$$g^2 \int d^3k \, \frac{e^{ikr}}{k^2} \left(1 + \frac{\sigma}{k^2} \right) \sim g^2 \, \sigma \, r \ , \ r \to \infty$$

σ = string tension. Very similar to 1+1 dimensions. $\mu = 0$: $\langle \overline{\psi} \psi \rangle = (.23 \sqrt{\sigma})^3$

Take Schwinger-Dyon eq. at large N_c: confinement unchanged by $\mu \neq 0$.
Treat μ by usual cutoff in momentum space: for confining system, same as $\mu \neq 0$?

Chiral symmetry restoration: $\mu_\chi = .11 \sqrt{\sigma}$
Transition *second* order: not evident.
Also: all infrared divergences *cancel*.

No nuclear matter:
restore chiral symmetry before Fermi sea forms



Asymptotically large μ, grows with N_c

For $\mu \sim (N_c)^p$, $p > 0$, gluons feel the effect of quarks. Perturbatively,

$$P_{\text{pert.}}(\mu, T) \sim N_c N_f \, \mu^4 \, F_0 \; , \; N_c N_f \, \mu^2 \, T^2 \, F_1 \; , \; N_c^2 \, T^4 \, F_2 \; .$$

First two terms from quarks & gluons, last only from gluons. Two regimes:

$\mu \sim N_c^{1/4} \Lambda_{QCD}$: $N_c \, \mu^4 \, F_0 \sim N_c^2 \, F_2 \sim N_c^2 >> N_c \, \mu^2 \, F_1 \sim N_c^{3/2}$.

Gluons & quarks contribute equally to pressure; quark cont. T-independent.

$\mu \sim N_c^{1/2} \Lambda_{QCD}$: New regime: $m^2_{Debye} \sim g^2 \mu^2 \sim 1$, so gluons feel quarks.

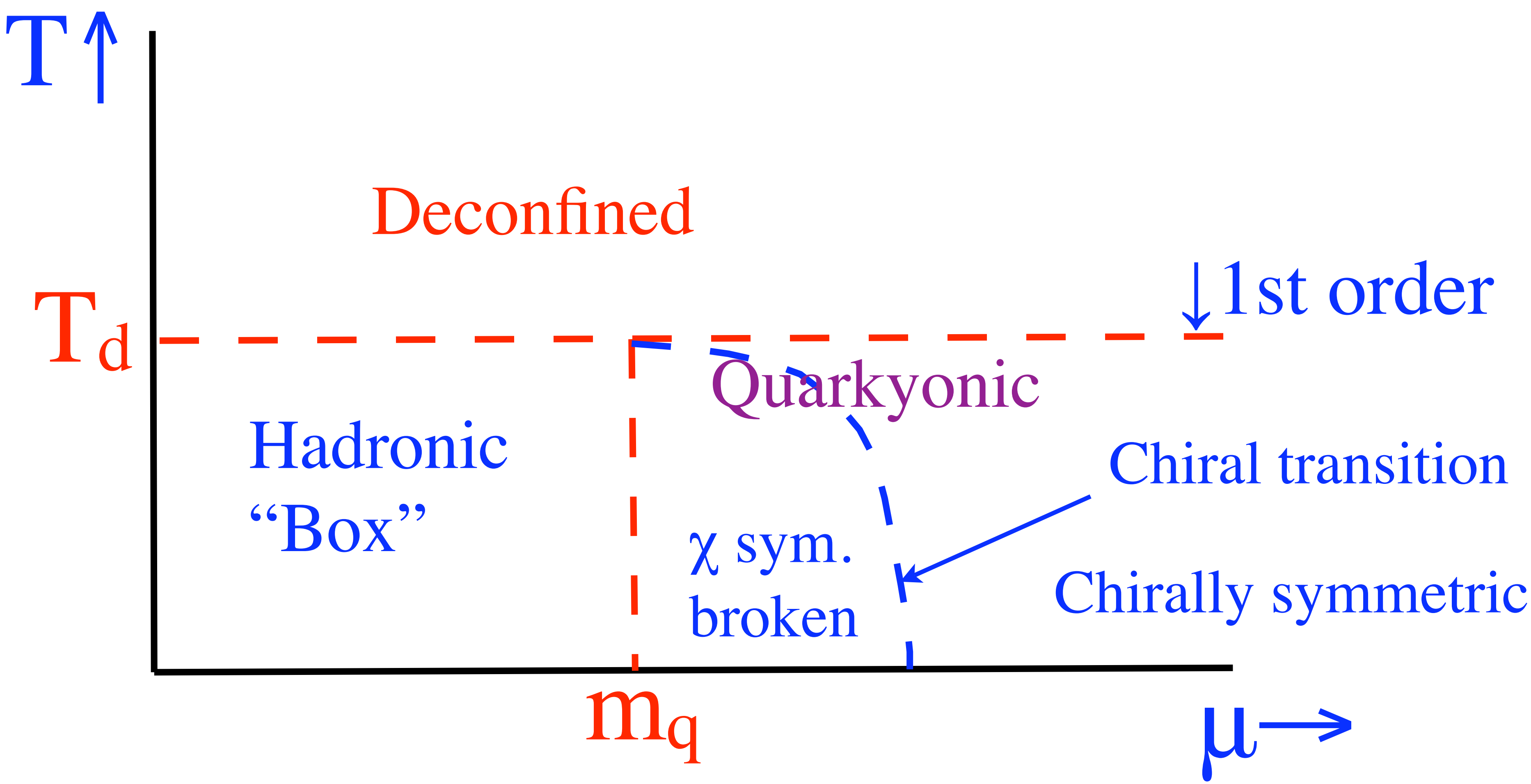
$N_c \, \mu^4 \, F_0 \sim N_c^3 >> N_c \, \mu^2 \, F_1 \; , N_c^2 \, F_2 \sim N_c^2$.

Quarks dominate pressure, T-independent.

Eventually, first order deconfining transition can either:

end in a critical point, or bend over to $T = 0$: ?

Phase diagram at $N_c = \infty$, II



We suggest: quarkyonic phase includes chiral trans. Order by usual arguments.

Mocsy, Sannino & Tuominen hep-th/0308135:

splitting of transitions in effective models

But: quarkyonic phase *confined*. Chirally symmetric baryons?

Chirally symmetric baryons

B. Lee, '72; DeTar & Kunihiro '89; Jido, Oka & Hosaka, hep-ph/0110005; Zschiesche et al nucl-th/0608044. Consider *two* baryon multiplets. One usual nucleon, other parity partner, transforming *opposite* under chiral transformations:

$$\psi_{L,R} \rightarrow U_{L,R}\, \psi_{L,R} \;\; ; \;\; \chi_{L,R} \rightarrow U_{R,L}\, \chi_{L,R}$$

With two multiplets, can form chirally symmetric (parity even) mass term:

$$\psi_L\, \chi_R - \psi_R\, \chi_L + \chi_R\, \psi_L - \chi_L\, \psi_R$$

Also: usual sigma field, $\Phi \rightarrow U_L\, \Phi\, U_R^\dagger$, couplings for linear sigma model:

$$g_1\, \psi_L\, \Phi\, \psi_R + g_2\, \chi_R\, \Phi\, \chi_L$$

Generalized model at $\mu \neq 0$: D. Fernandez-Fraile & RDP '09...

Anomalies?

't Hooft, '80: anomalies rule *out* massive, parity doubled baryons in vacuum:
No massless modes to saturate anomaly condition

Itoyama & Mueller'83; RDP, Trueman & Tytgat hep-ph/9702362:
At $T \neq 0$, $\mu \neq 0$, anomaly constraints *far* less restrictive (many more amplitudes)
E.g.: anomaly unchanged at $T \neq 0$, $\mu \neq 0$, but Sutherland-Veltman theorem *fails*

To do: show parity doubled baryons consistent with anomalies at $\mu \neq 0$.
At $T \neq 0$, $\mu = 0$, no massless modes. Anomalies probably rule out model(s).
But at $\mu \neq 0$, *always* have massless modes near the Fermi surface.
Will constrain mass gaps (superconductivity, superfluidity)

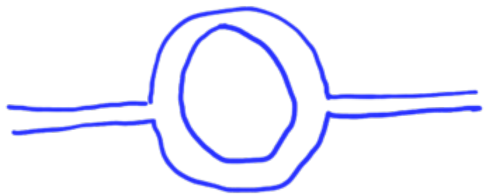
Casher '79: heuristically, confinement => chiral sym. breaking in vacuum
Especially at large N_c, carries over to $T \neq 0$, $\mu = 0$.
Does *not* apply at $\mu \neq 0$: baryons strongly interacting at large N_c.
Banks & Casher '80: chiral sym. breaking from eigenvalue density at origin.
Osborn, Splittorff & Verbaarschot 0807.4584: at $\mu \neq 0$, eigenvalues spread in complex plane; will tend to chiral symmetry restoration at finite μ

Baryons at Large N_f

Veneziano '78: take *both* N_c and N_f large. Mesons M^{ij} : $i,j = 1 \ldots N_f$.

Thus mesons interact weakly, but there are *many* mesons.

Thus in the hadronic phase, mesons interact *strongly*:



$$\Pi \sim N_f \, g_{3\pi}^2 \sim N_f / N_c$$

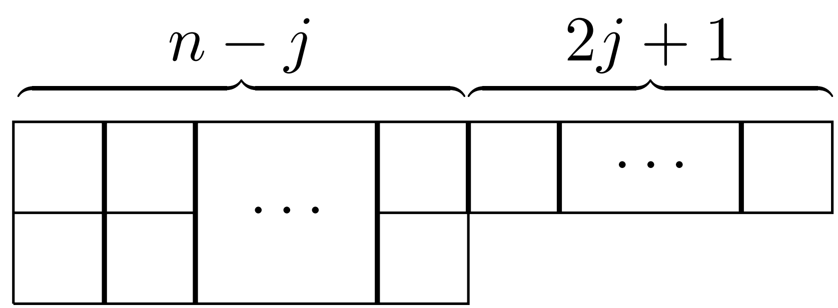
Pressure large in *both* phases:

$\sim N_f^2$ in hadronic phase, $\sim N_c^2$, $N_c \, N_f$ in "deconfined" phase.

Polyakov loop also nonzero in both phases.

Baryons: lowest state with spin j

has Young tableaux ($N_c = 2n + 1$) =>



$$d_j \sim \mathrm{e}^{+N_c \, f(N_f/n)} \; , \; f(x) = (1+x)\log(1+x) - x \log(x)$$

Y. Hidaka, RDP, & L. McLerran, 0803.0279:

degeneracy of baryons increases *exponentially*.

Baryon condensation at large N_f

Use *baryons* as order parameter.

At $T=0$, $\langle B^2 \rangle \neq 0$ when $N_c\, f(N_c/n) = m_B/T$, or

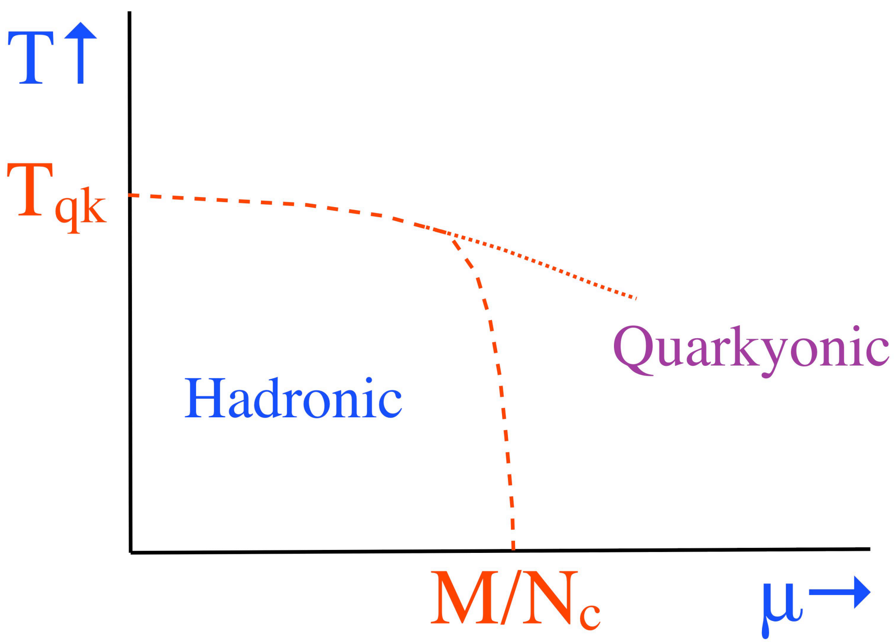
$$T_{qk} = f(N_f/n)\,\frac{m_B}{N_c}$$

At $T \neq 0$, $\langle B \rangle \neq 0$ when $N_c\, f(N_c/n) = (m_B - N_c\, \mu)/T$:

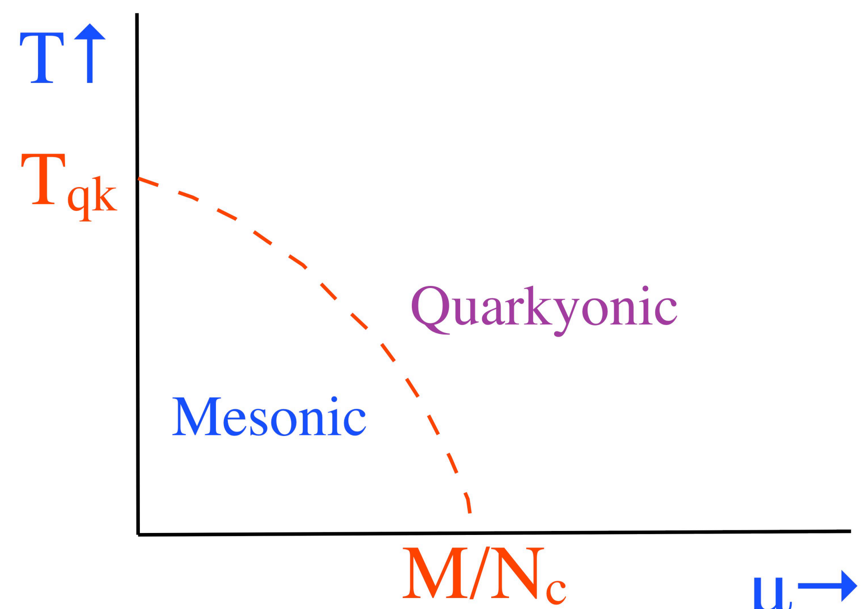
$$T_{qk} = f(N_f/n)\left(\frac{m_B}{N_c} - \mu\right)$$

Argument is heuristic: baryons are strongly interacting. Still, difficult to see how interactions can overwhelm exponentially growing spectrum.

Small N_f

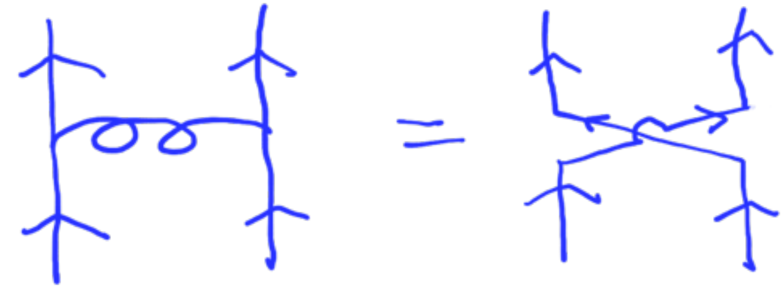


Large N_f



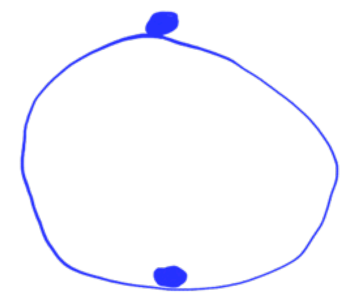
Chiral Density Waves (perturbative)

Excitations near the Fermi surface?



At large N_c, color superconductivity suppressed, ~ $1/N_c$: pairing into two-index state:

Also possible to have "chiral density waves", pairing of quark and anti-quark:
Deryagin, Grigoriev, & Rubakov '92. Shuster & Son, hep-ph/9905448.
Rapp, Shuryak, and Zahed, hep-ph/0008207.



Order parameter $\langle\overline{\psi}(-\vec{p}_f)\,\psi(+\vec{p}_f)\rangle$
Sum over color, so *not* suppressed by $1/N_c$.

Pair quark at + p_f with anti-quark at - p_f : for a *fixed* direction.
Breaks chiral symmetry, with state varying ~ exp(- 2 p_f z).

Wins over superconductivity in low dimensions. Loses in higher.
Shuster & Son '99: in perturbative regime, CDW only wins for $N_c > 1000\, N_f$

Quarkyonic chiral density waves

Consider meson wave function, with kernel:
Confining potential in 3+1 dimensions like
Coulomb potential in 1+1 dim.s:

$$\int dk_0\, dk_z \int d^2k_\perp \, \frac{1}{(k_0^2 + k_z^2 + k_\perp^2)^2} \sim \int dk_0\, dk_z \, \frac{1}{k_0^2 + k_z^2}$$

In 1+1 dim.'s, behavior of massless quarks near Fermi surface maps $\sim \mu = 0$!

Mesons in vacuum naturally map into CDW mesons.

Witten '84: in 1+1 dim.'s, use non-Abelian bosonization for QCD.

a, b= 1...N_c. i,j = 1... N_f.

$$J_+^{ij} = \overline{\psi}^{a,i} \psi^{a,j} \sim g^{-1} \partial_+ g \; ; \; J_+^{ab} = \overline{\psi}^{a,i} \psi^{b,i} \sim h^{-1} \partial_+ h \; .$$

Steinhardt '80. Affleck '86. Frishman & Sonnenschein, hep-th/920717...
Armoni, Frishman, Sonnenschein & Trittman, hep-th/9805155; AFS, hep-th/0011043..
Bringoltz 0901.4035; Galvez, Hietanan, & Narayanan, 0812.3449.

Bosonized quarkyonic matter

After non-Abelian bosonization, action factorizes into sum of g, in $SU(N_f)$, and h, in $SU(N_c)$. Action for g is

$$8\pi \, S_{WZW} = \int d^2 z \, \mathrm{tr} \, B_i^2 + 2/3 \int d^3 y \, \epsilon^{ijk} \, \mathrm{tr} \, B_i B_j B_k \ , \ B_i = g^{-1} \partial_i g \ .$$

Action for h, is a $SU(N_c)$ gauged WZW model. But: g and h *decouple*!
Spectrum of h complicated, involves massive modes, like usual 't Hooft model.

Spectrum of g is that of usual WZW model, with *massless* modes.

Hence in 1+1 dim.'s, CDW are natural, but with *massless* excitations thereof.

In 3+1 dim.'s: have highly anisotropic state, *somen*-state:
Y. Hidaka, T. Kojo, L. McLerran, & RDP '09...

Chiral condensate ~ Λ_{QCD}^2/μ^2. Length of somen-state large, ~ $\exp(N_c)$.
Quantum fluctuations tend to scramble the somen.

Guess for phase diagram in QCD

Pure guesswork: deconfining & chiral transitions split apart at critical end-point?
Line for deconfining transition first order to the right of the critical end-point?
Critical end-point for deconfinement, or continues down to T=0?