Matrix models in QCD

Chiral matrix model: marrying
   a linear sigma model, for the chiral transition
   plus a “matrix model”, to characterize deconfinement
RDP & VV Skokov, 1604.00002

Finite size effects for baryon # cumulants:
   G Almasi, VV Skokov, & RDP, 1612.04416
Tetraquarks in QCD: two chiral order parameters, two chiral transitions?
   RDP & VV Skokov 1606.04111
Solution for SU($\infty$): RDP & VV Skokov; 1205.0545
   S Lin, RDP & VV Skokov, 1301.7432;
   H Nishimura, RDP & VV Skokov, 1712.04465
Matrix model for deconfinement

Polyakov Loop: \[ \ell = \frac{1}{3} \text{tr} \mathcal{P} \exp \left( ig \int_0^{1/T} A_0 \, d\tau \right) \]

*Simplest* approximation to give a non-trivial loop: constant, diagonal \( A_0 \):

\[ A_0^{cl} = \frac{2\pi T}{3g} \lambda_3 \, q(T) \]

\[ \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

Depends upon single function, \( q(T) \), fixed from pressure(T).

Only need two parameters to fit pressure, then compute
Matrix model for pure glue

To one loop order, Stefan-Boltzmann + potential for q

$$\mathcal{V}_{pert}(q) = \frac{2\pi^2}{3} T^4 \left( -\frac{4}{15} + \sum_{a,b} q_{ab}^2 (1 - q_{ab})^2 \right), \quad q_{ab} = |q_a - q_b|_{mod\,1}$$

From lattice data for pure glue, assume non-pert. potential $\sim T^2$:

$$\mathcal{V}_{non}(q) = \frac{2\pi^2}{3} T^2 T_d^2 \sum_{a,b} \left( -c_1 q_{ab}(1 - q_{ab}) - c_2 q_{ab}^2(1 - q_{ab})^2 + c_3 \right)$$

From lattice for pure glue: $T_d = 270$ MeV.

Constant term $\sim c_3$ most important for $T > 1.2 T_d$.

q’s only matter for $T < 1.2 T_d$ : narrow transition region

Dumitru, Guo, Hidaka, Korthals-Altes & RDP, 1011.3820 & 1205.0137 + ….
Chiral symmetry

For 3 flavors of massless quarks,

\[ \mathcal{L}^{q_k} = \bar{q} \not{D} q = \bar{q}_L \not{D} q_L + \bar{q}_L \not{D} q_L , \quad q_{L,R} = \frac{1 \pm \gamma_5}{2} q \]

Classically, global flavor symmetry of SU(3)_L x SU(3)_R x U(1)_A,

\[ q_L \rightarrow e^{-i\alpha/2} U_L q_L , \quad q_R \rightarrow e^{+i\alpha/2} U_R q_R \]

Simplest order parameter for \( \chi \) sym. breaking: \( a,b\ldots = \) flavor. \( A,B\ldots = \) color

\[ \Phi^{ab} = \bar{q}_L^b A q_R^a A \quad \Phi \rightarrow e^{+i\alpha} U_R \Phi U_L^\dagger \]

Quantum mechanically, axial U(1)_A is broken by instantons +…. to Z(3)_A at T=0

't Hooft instanton vertex is invariant under Z(3)_A:

\[ \det \Phi \rightarrow e^{3i\alpha} \det \Phi \]

As \( T \rightarrow \infty \), U(1)_A approximately restored as \( 1/T^{7-9} \).
Usual linear sigma model

Linear sigma model for $\Phi$:

$$\mathcal{V}_\Phi = m^2 \text{tr} (\Phi^\dagger \Phi) - c_A (\text{det} \Phi + c.c.) + \lambda \text{tr} (\Phi^\dagger \Phi)^2$$

Drop ($\text{tr} \Phi \Phi^\dagger$): fits show coefficient is *really* small

Mass, quartic terms $U(1)_A$ invariant; $\text{det} \Phi$ *only* under $Z(3)_A$.

For light but massive quarks, need to add

$$\mathcal{V}_H^0 = - \text{tr} (H (\Phi^\dagger + \Phi))$$

So $m_\pi^2 \sim H$, etc. Standard linear sigma model.
Quarks generate potential in “q”, so must couple $\Phi$ to quarks: $P_{L,R} = (1 \pm \gamma_5)/2$

$$L_{\Phi}^{q_k} = \bar{q} \left( \mathcal{D} + \mu \gamma^0 + y \left( \Phi \mathcal{P}_L + \Phi^\dagger \mathcal{P}_R \right) \right) q$$

Use matrix model from pure glue, with same $T_d = 270$ MeV.

With quarks, $T_d$ is just a parameter in a potential, not deconfining $T_c$.

From quark loop, need logarithmic term in $\Phi$:

$$\mathcal{V}_{\Phi}^{log} = \kappa \text{tr} \left( (\Phi^\dagger \Phi)^2 \log \left( \frac{M^2}{\Phi^\dagger \Phi} \right) \right)$$

To 1 loop order, $\kappa = 3y^4/(16 \pi^2)$; $y$ is a free parameter, fit to $T_\chi$.

Log term complicates things, results similar to that for $\kappa = 0$. 

**Chiral matrix model**
New symmetry breaking term

With usual symmetry breaking, at high $T$,

$$V^{\text{eff}} \approx -h \phi + \frac{y^2 T^2}{12} \phi^2 + \ldots, \quad T \to \infty$$

1\textsuperscript{st} term SB’g, 2\textsuperscript{nd} quark fluctuations. But then at high $T$, no symmetry breaking!

$$\phi \sim \frac{12h}{y^2 T^2}, \quad m_{qk} \sim y \phi \sim \frac{1}{T^2}$$

Solve by adding a new term \textit{by hand}

$$V^{\text{eff}} \sim h \phi - \frac{y}{6} m_0 T^2 \phi + \frac{y^2 T^2}{12} \phi^2 + \ldots$$

So $\phi \sim m_0/y$ at high $T$, $m_{qk} \sim m_0$. In QCD,

$$V^T_h = -\frac{m_{qk}}{V} \left( \text{tr} \left( \frac{1}{D + \mu \gamma^0 + y \Phi_{ii}} \right)_{T \neq 0} - (T = 0) \right)$$
Solution at $T = 0$

Consider first the SU(3) symmetric case, $h_u = h_d = h_s$.

Spectrum. 0−: singlet $\eta'$ & octet $\pi$. 0+: singlet $\sigma$ and octet $a_0$.

Satisfy a ’t Hooft relation:

$$m_{\eta'}^2 - m_{\pi}^2 = m_{a_0}^2 - m_{\sigma}^2$$

The anomaly moves $\eta'$ up from the $\pi$, but also moves $\sigma$ down from the $a_0$.

QCD: $\langle \Phi \rangle = (\Sigma_u, \Sigma_u, \Sigma_s)$. From:

$$f_\pi = 93, \ m_\pi = 140, \ m_K = 495, \ m_\eta = 540, \ m_{\eta'} = 960$$

Determine:

$$\Sigma_u = 46, \ \Sigma_s = 76, \ h_u = (97)^3, \ h_s = (305)^3, \ c_A = 4560$$

$$m^2 = (538)^2 - 121y^4, \ \lambda = 18 + 0.04y^4$$

One free parameter, Yukawa coupling “y”, fix from $T_\chi$. 
Varying the Yukawa coupling

$T_\chi$ defined from maximum in light quark suscep., $d\Sigma_u/dT$

$\leq$ Grey band: vary $T_d$ from 260 → 280

$\leq$ Yellow band = $y$: 4.5 → 5.5

Grey band: experimental uncertainty in the mass of the $a_0 \Rightarrow$

$y=5 \uparrow$
To eliminate u.v. divergences, lattice uses substracted condensates

\[
\Delta^\text{lattice}_{u,s}(T) = \frac{\langle \bar{q}q \rangle_{u,T} - (m_u/m_s)\langle \bar{q}q \rangle_{s,T}}{\langle \bar{q}q \rangle_{u,0} - (m_u/m_s)\langle \bar{q}q \rangle_{s,0}}
\]

In the chiral-matrix (\(\chi\)-M) model use this to fix \(y = 5\).

\[
\Delta^\chi-M_{u,s}(T) = \frac{\Sigma_u(T) - (h_u/h_s)\Sigma_s(T)}{\Sigma_u(0) - (h_u/h_s)\Sigma_s(0)}
\]

Bazavov et al, 1407.6387 1701.03548
Usual pattern for $m_u = m_d \neq m_s$. $y = 5$.

$U(1)_A$ breaking persists to high $T$, unphysical.
Pressure, interaction measure vs T

Pressure and interaction measure, \((e-3p)/T^4\)
\(\chi\)-M model,
Lattice, Bazavov et al, 1407.6387
Hard Thermal Loop (HTL)
(blue region = change ren. scale)
Andersen et al, 1511.04660
Order parameters, chiral and deconfining

Chiral matrix model:

Chiral and deconfining order parameters are strongly correlated

But Polyakov loop from lattice Petreczky & Schadler, 1509.07874 is much smaller than in model.

Persistent discrepancy, also in pure gauge. What’s up with lattice loop?
Susceptibilities, chiral and deconfining

Largest peak for up-up; strange-strange small.
In QCD, notable peaks for loop-up & loop-loop, strongly correlated with up-up

In chiral limit: loop-up susceptibility diverges. Sasaki, Friman, Redlich ph/0611147
loop-loop and loop-antiloop finite
In $\chi$-M model, $\chi_6$ shows non-monotonic behavior near $T_\chi$.

In HTL, $\chi_6$ is very small (because $m=0$).

$\sigma$ model: including change in $\Sigma_u$, but not in loop. Change in $\chi_6$ much smaller.
Baryon susceptibilities: 2nd & 4th

As evaluated at $\mu = 0$, lattice ok.
Baryon $\mu_B = 3 \mu_q$.

\[
\chi_n^B(T) = T^{n-4} \left. \frac{\partial^n}{\partial \mu_B^n} p(T, \mu_B) \right|_{\mu_B=0}
\]

Lattice: Bazavov et al, 1701.04325
Ratios of moments, vs Columbia lattice

Left: ratio of $\chi_4/\chi_2$ and $\chi_6/\chi_2$ in $\chi$-M model

Bazavov et al, 1701.04325

$\chi_6/\chi_2$ ↑
Lattice moments, Frankfurt

Vovchenko, Steinheimer, Philipsen, Stoecker 1711.0126:

\[ \frac{\chi_4}{\chi_2} \]

\[ \frac{\chi_6}{\chi_2} \]
What’s up with the lattice loop?

Looked at wide variety of variations on $\chi$-M models.

Below: $\chi_2$ from chiral matrix model, lattice, and fitting the loop to the lattice value, then computing $\chi_2$.  

*If the lattice loop is right, then $\chi_2$ is too small.*