

Light tetraquarks and the chiral phase transition

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XYZ, P_c states: *strong* evidence for tetraquark & pentaquark states,
composed of both light and *heavy* quarks

Why do we need heavy quarks to see tetraquark states?

Jaffe '79...Schechter...Close...Tornqvist...Maiani...Giacosa...Peleaz '15:
“the” σ meson is “a” tetraquark (diquark anti-diquark). But situation murky....

Punchline: tetraquarks *must* be included to understand the phase diagram of QCD
(versus quark mass, plane of temperature T and baryon chemical potential μ)

Tetraquarks: for three (*not* two) flavors of *very* light quarks,
tetraquarks *may* generate a second chiral phase transition

For QCD, in the plane of T & μ plane:

direct connection between tetraquarks and color superconductivity
in effective modes!, *need* tetraquarks to determine the Critical EndPoint

Tetraquarks for two light flavors:
meh, fuh gedda 'boud it

Chiral symmetry for two flavors

Classically, chiral symmetry $G_{cl} = SU(2)_L \times SU(2)_R \times U(1)_A = O(4) \times O(2)$

Use ϕ , complex 4-component vector
Linear σ model for exact χ symmetry:

$$\phi = (\sigma + i\eta, \vec{a}_0 + i\vec{\pi})$$

$$\mathcal{L}^{cl} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m^2 \phi^* \cdot \phi + \lambda (\phi^* \cdot \phi)^2 + \dots$$

Quantum mechanically, axial anomaly reduces $U(1)_A \rightarrow Z(2)_A : \phi \rightarrow (-) \phi$
Simplest term which is only $Z(2)_A$ invariant:

$$\mathcal{L}^A = +m_A^2 (\text{Im } \phi)^2 + \dots$$

With G_{cl} , η degenerate with π . With axial anomaly, η splits from massless π
Directly induced by instantons + ...

Diquarks and tetraquarks for two flavors

Jaffe '79: most attractive channel for quark-quark scattering is antisym. in *both* flavor and color.

Color: $3 \times 3 = \underline{3}$ (antisym) + 6 (sym)

Two flavors: $2 \times 2 = 1$ (antisym) + 3 (sym)

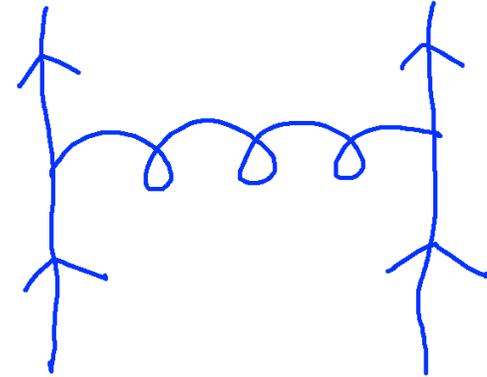
For two flavors diquark is a color triplet, flavor *singlet*,

$$\chi_L^A = \epsilon^{ABC} \epsilon^{ab} (q_L^{aB})^T C^{-1} q_L^{bC}$$

(A, B, C = color; a, b = flavor) Also χ_R . χ_L and χ_R *singlets* under $Z(2)_A$.

One complex valued tetraquark field:

$$\zeta = (\chi_R^A)^* \chi_L^A$$



Sigma models and tetraquarks for two flavors

The tetraquark field ζ is a *singlet* under flavor and $Z(2)_A$.

Split complex ζ into its real and imaginary parts, ζ_r and ζ_i .

QCD is even under parity, so only even powers of ζ_i can appear, forget ζ_i .

But *any* powers of ζ_r can!

$$\mathcal{V}_{\zeta_r}^A = h_r \zeta_r + m_r^2 \zeta_r^2 + \kappa_r \zeta_r^3 + \lambda_r \zeta_r^4$$

Hence $\langle \zeta_r \rangle$ is *always nonzero*!

Couplings to ϕ start with $U(1)_A$ inv.:

$$\mathcal{V}_{\zeta\phi} = \kappa \zeta \phi^* \cdot \phi + \dots$$

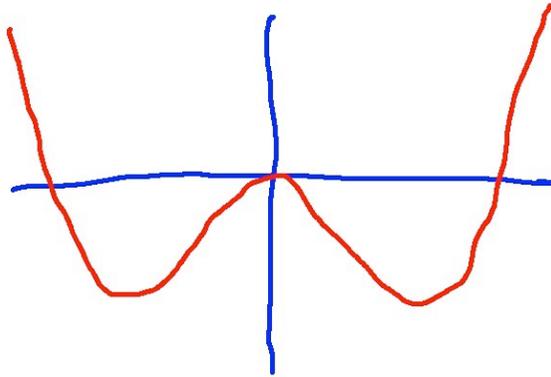
The tetraquark ζ_r is just a massive field with a v.e.v. : *boring*!

Chiral transition for two flavors

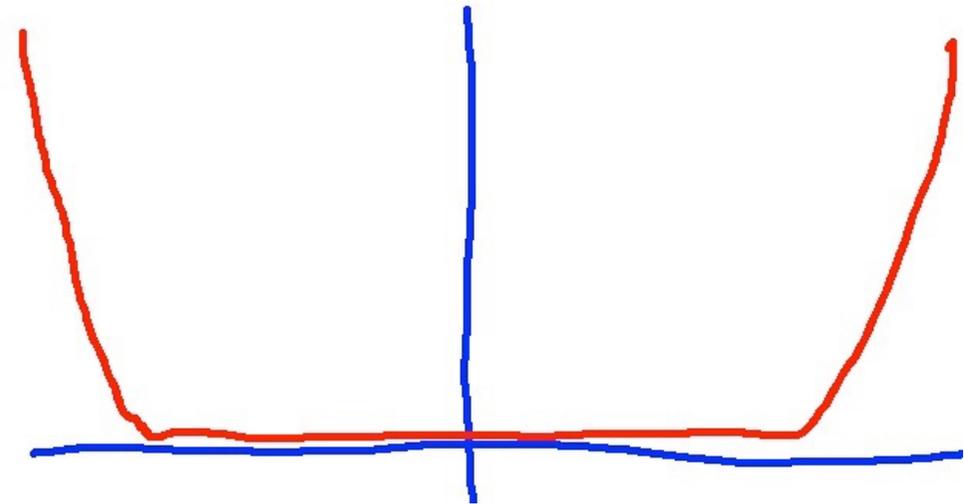
Assume anomaly is large up to T_χ .

$$\mathcal{V}_\phi = \frac{m^2}{2} \phi^2 + \lambda \phi^4$$

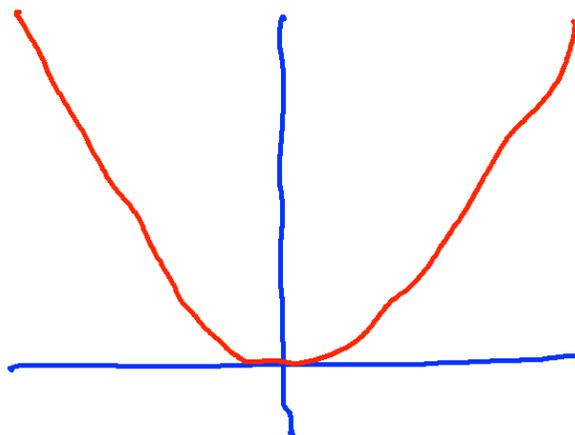
$T = 0: m^2 < 0,$
 $\langle \phi \rangle \neq 0 \Rightarrow$



$T = T_\chi: m^2 = 0, \langle \phi \rangle = 0 \Downarrow$



$T \gg f_\pi: m^2 > 0,$
 $\langle \phi \rangle = 0 \Rightarrow$



Chiral transition second order,
 $m^2 = 0 \Rightarrow$ infinite correlation length
 $O(4)$ universality class.

Tetraquark field just doesn't matter:
massive field with cubic couplings

Tetraquarks for three light flavors:
must be included

Chiral symmetry, three flavors

Quark Lagrangian

$$\mathcal{L}^{qk} = \bar{q} \not{D} q = \bar{q}_L \not{D} q_L + \bar{q}_R \not{D} q_R \quad , \quad q_{L,R} = \frac{1 \pm \gamma_5}{2} q$$

Classical $G_{cl} = SU(3)_L \times SU(3)_R \times U(1)_A$:

$$q_L \rightarrow e^{-i\alpha/2} U_L q_L \quad , \quad q_R \rightarrow e^{+i\alpha/2} U_R q_R$$

Order parameter for χ symmetry breaking (a, b... = flavor; A,B... = color)

$$\Phi^{ab} = \bar{q}_L^{bA} q_R^{aA} \quad \Phi \rightarrow e^{+i\alpha} U_R \Phi U_L^\dagger$$

Axial anomaly reduces $U(1)_A$ to $Z(3)_A$.

σ models for χ symmetry

$U(1)_A$ invariant terms:

$$\mathcal{V}_\Phi = m^2 \text{tr} (\Phi^\dagger \Phi) + \lambda \text{tr} (\Phi^\dagger \Phi)^2 + \lambda_{O(18)} (\text{tr} (\Phi^\dagger \Phi))^2$$

Drop last term, $\lambda_{O(18)} \sim \lambda/50$.

χ sym. broken by background field:

$$\mathcal{V}_H^0 = - \text{tr} (H (\Phi^\dagger + \Phi))$$

To split the η' from the π , K , & η ,
add $Z(3)_A$ invariant term

$$\mathcal{V}^A = \kappa (\det \Phi + \text{c.c.})$$

Example: $SU(3)$ symmetric case, $H = h \mathbf{1}$.

$J^P = 0^-$: octet $\pi = K = \eta$ & singlet η' . $J^P = 0^+$: octet $a_0 = \kappa = \sigma_8$ & singlet σ
 $h \neq 0$ so π 's massive . Satisfy (like 't Hooft '86):

$$m_{\eta'}^2 - m_\pi^2 = m_{a_0}^2 - m_\sigma^2$$

The anomaly moves η' *up* from the π , but also moves singlet σ *down* from the a_0 !
Light σ is a problem for σ models

Tetraquarks for three flavors

Three flavors: $3 \times 3 = \underline{3} + 6$.

Diquark field flavor *anti-triplet*, $\underline{3}$

$$\chi_L^{aA} = \epsilon^{abc} \epsilon^{ABC} (q_L^{bB})^T C^{-1} q_L^{cC}$$

LR tetraquark field ζ transforms *identically* to Φ under $G_\chi = \text{SU}(3)_L \times \text{SU}(3)_R$

$$\zeta^{ab} = (\chi_R^{aA})^* \chi_L^{bA}$$

Under $U(1)_A$, Φ has charge +1, ζ charge -2.

Since ζ & Φ in *same* representation of G_χ , *direct* mixing term. $Z(3)_A$ invariant:

$$\mathcal{V}_{\zeta\Phi,2}^A = \tilde{m}^2 \text{tr} (\zeta^\dagger \Phi + \Phi^\dagger \zeta)$$

Black, Fariborz, Schechter ph/9808415 +;

't Hooft, Isidori, Maiani, Polosa 0801.2288 +

An extra *dozen* couplings.

e.g., $U(1)_A$ inv. cubic term:

$$\mathcal{V}_{\zeta\Phi,3}^\infty = \kappa_\infty \epsilon^{abc} \epsilon^{a'b'c'} (\zeta^{aa'} \Phi^{bb'} \Phi^{cc'} + \text{c.c.})$$

“Mirror” model, $T = 0$

Spectrum : $\Phi = \pi, K, \eta, \eta'; a_0, \kappa, \sigma_8, \sigma_0$; $\zeta = \tilde{\pi}, \tilde{K}, \tilde{\eta}, \tilde{\eta}'; \tilde{a}_0, \tilde{\kappa}, \tilde{\sigma}_8, \tilde{\sigma}_0$.

General model has 20 couplings:

Fariborz, Jora, & Schechter: [ph/0506170](#); [0707.0843](#); [0801.2552](#). Pelaez, [1510.00653](#)

Instead study “mirror” model, where Φ and ζ start with *identical* couplings

$$\mathcal{V}_\Phi = m^2 \text{tr} (\Phi^\dagger \Phi) - \kappa (\det \Phi + \text{c.c.}) + \lambda \text{tr} (\Phi^\dagger \Phi)^2$$

$$\mathcal{V}_\zeta = m^2 \text{tr} (\zeta^\dagger \zeta) - \kappa (\det \zeta + \text{c.c.}) + \lambda \text{tr} (\zeta^\dagger \zeta)^2$$

Assume only $\zeta\Phi$ coupling is mass term: $\mathcal{V}_{\zeta\Phi,2}^A = \tilde{m}^2 \text{tr} (\zeta^\dagger \Phi + \Phi^\dagger \zeta)$

Simple, because mass *only* mixes: $\pi \leftrightarrow \tilde{\pi}, K \leftrightarrow \tilde{K} \dots$

Spectrum of the mirror model

In the chiral limit, the mass eigenstates: (need to assume $\tilde{m}^2 < 0$)

$$\pi, \tilde{\pi} = 0, -2\tilde{m}^2 ; \quad \eta', \tilde{\eta}' = 3\kappa\phi, 3\kappa\phi - 2\tilde{m}^2$$

$$a_0, \tilde{a}_0 = m^2 + \kappa\phi + 6\lambda\phi^2 \pm \tilde{m}^2 ; \quad \sigma_0, \tilde{\sigma} = m^2 - 2\kappa\phi + 6\lambda\phi^2 \pm \tilde{m}^2$$

All states are mixtures of Φ and ζ . Of course 8 Goldstone bosons.

Satisfy *generalized* 't Hooft relation

$$m_{\eta'}^2 + m_{\tilde{\eta}'}^2 - m_{\pi}^2 - m_{\tilde{\pi}}^2 = m_{a_0}^2 + m_{\tilde{a}_0}^2 - m_{\sigma}^2 - m_{\tilde{\sigma}}^2$$

Since *every* multiplet is doubled, this can easily be satisfied (unlike if just one).

Even with same couplings, all masses are *split* by the mixing term.

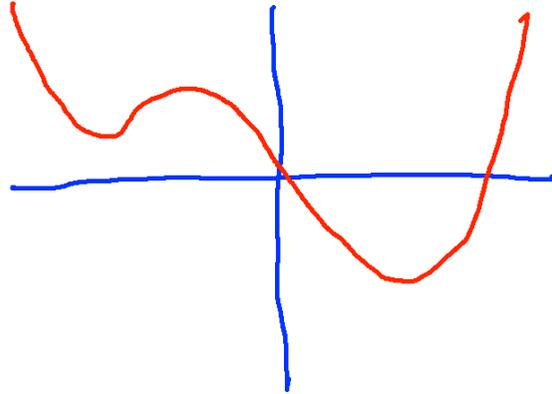
At nonzero T, the thermal masses of the Φ and ζ *cannot* be equal!

Chiral transition for three flavors, no tetraquark

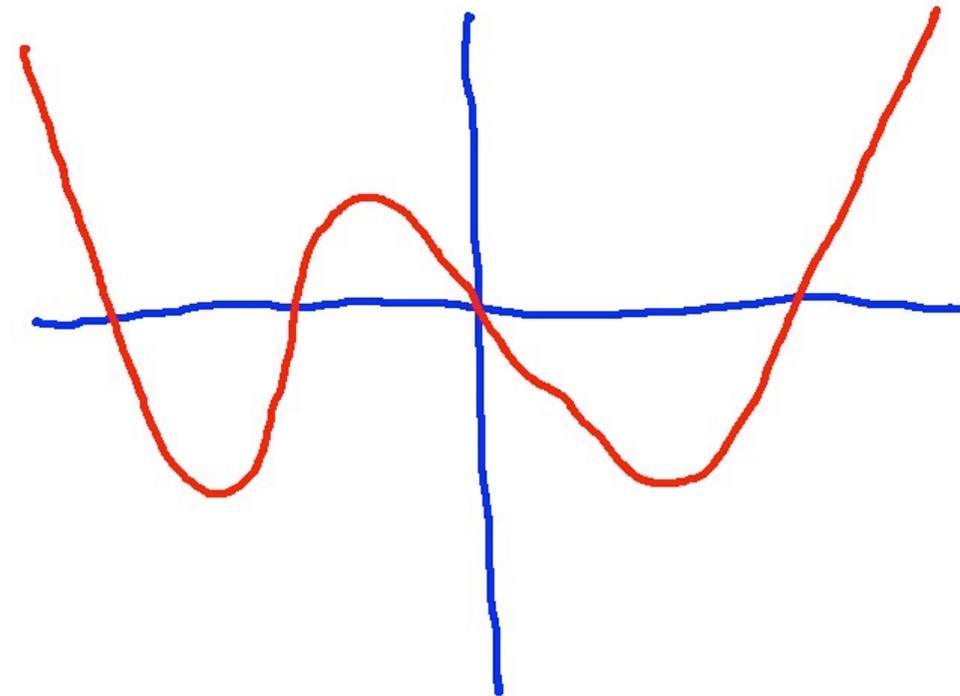
Cubic terms *always* generate *first* order transitions.

$$\mathcal{V}_\phi = \frac{m^2}{2} \phi^2 - \kappa \phi^3 + \lambda \phi^4$$

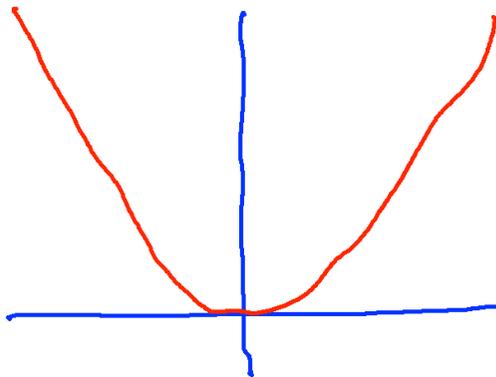
$T = 0: m^2 < 0,$
 $\langle \phi \rangle \neq 0 \Rightarrow$



$T = T_\chi: \text{cannot flatten the potential} \Rightarrow$



$T \gg f_\pi: m^2 > 0,$
 $\langle \phi \rangle = 0 \Rightarrow$



At T_χ , two degenerate minima,
with barrier between them.
Transition is first order.

With tetraquarks, *maybe* two chiral transitions

In chiral limit, *may* have have *two* chiral phase transitions. =>

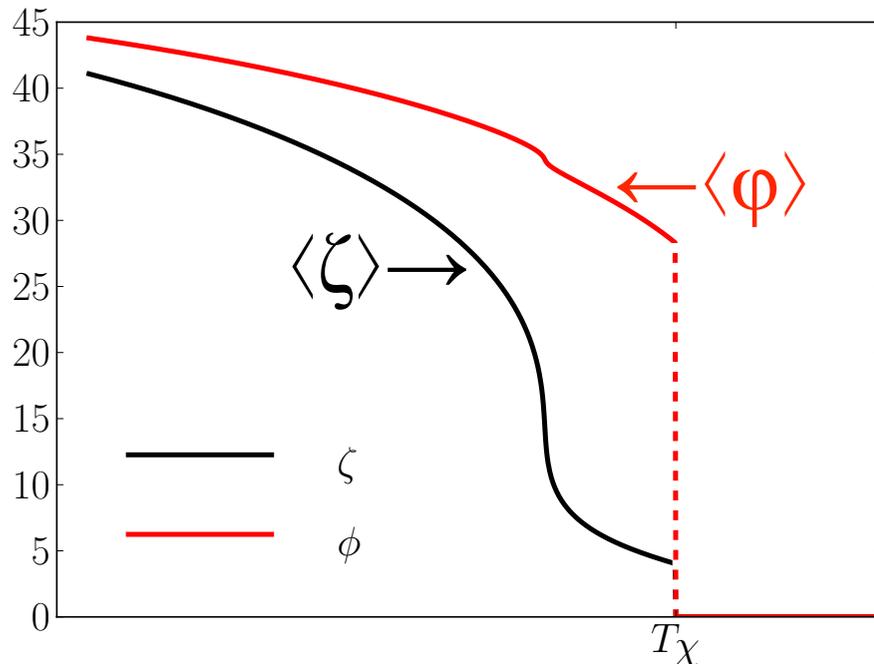
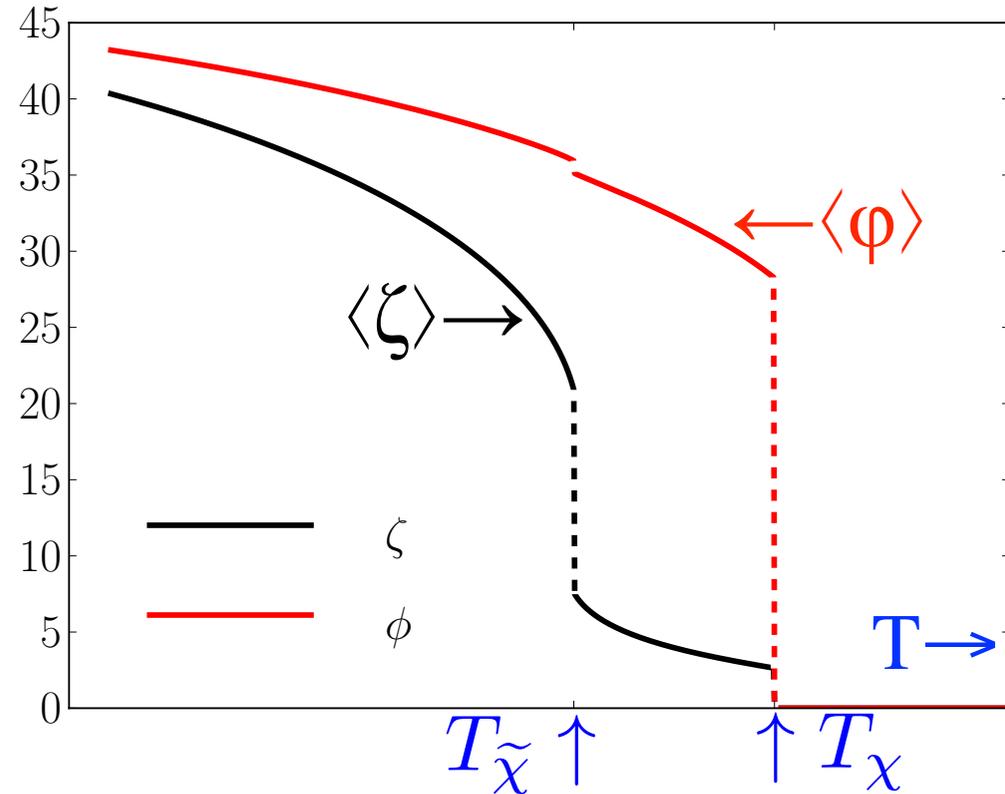
At first, both jump, remain nonzero.

At second, both jump to zero.

$$m_\phi^2(T) = 3T^2 + m^2$$

$$m_\zeta^2(T) = 5T^2 + m^2$$

$$\tilde{m}^2 = -(100)^2$$



<= Also possible to have single chiral phase transition, tetraquark crossover

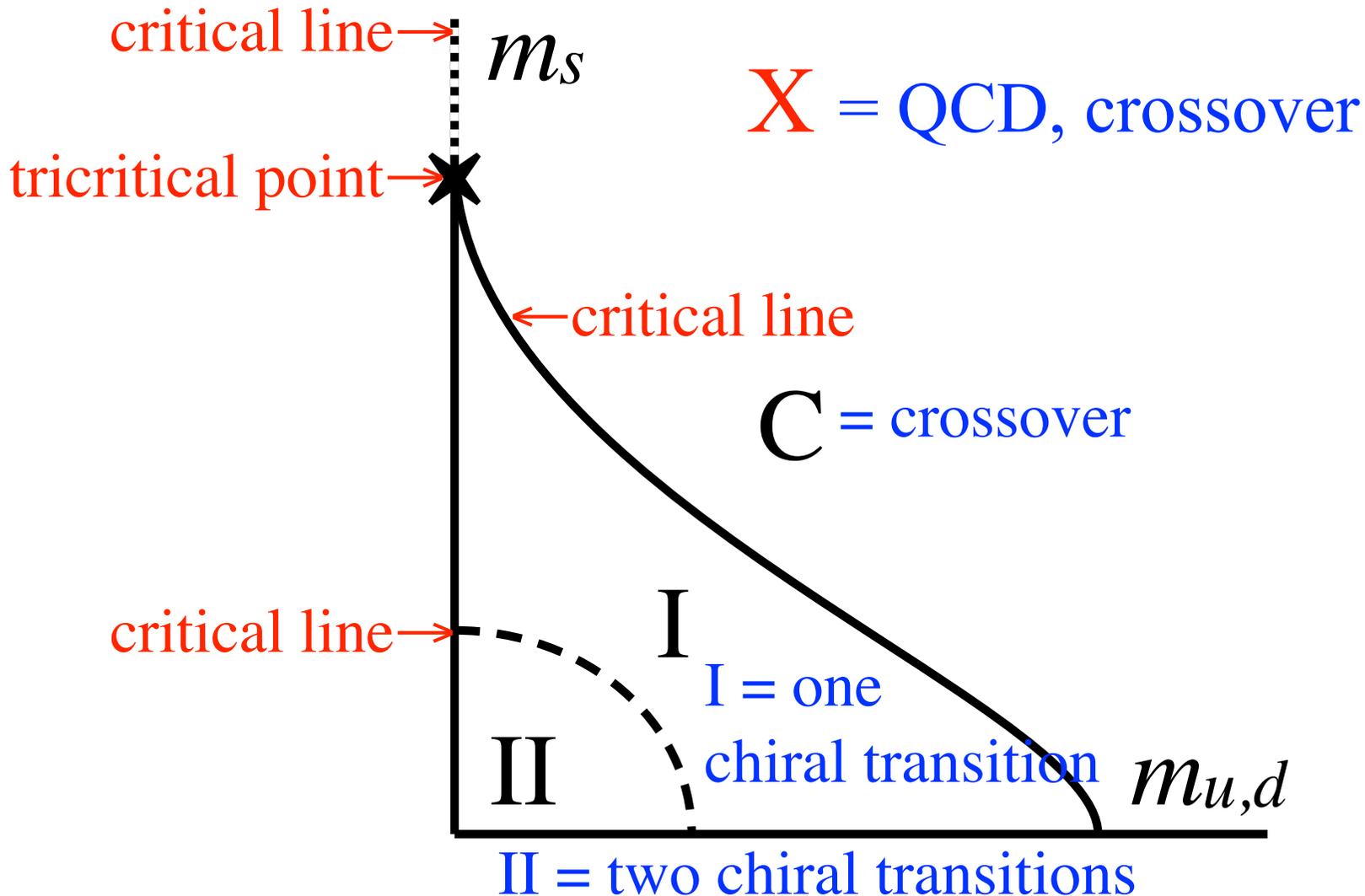
$$\tilde{m}^2 = -(120)^2$$

"Columbia" phase diagram for light quarks

Lattice: chiral transition crossover in QCD

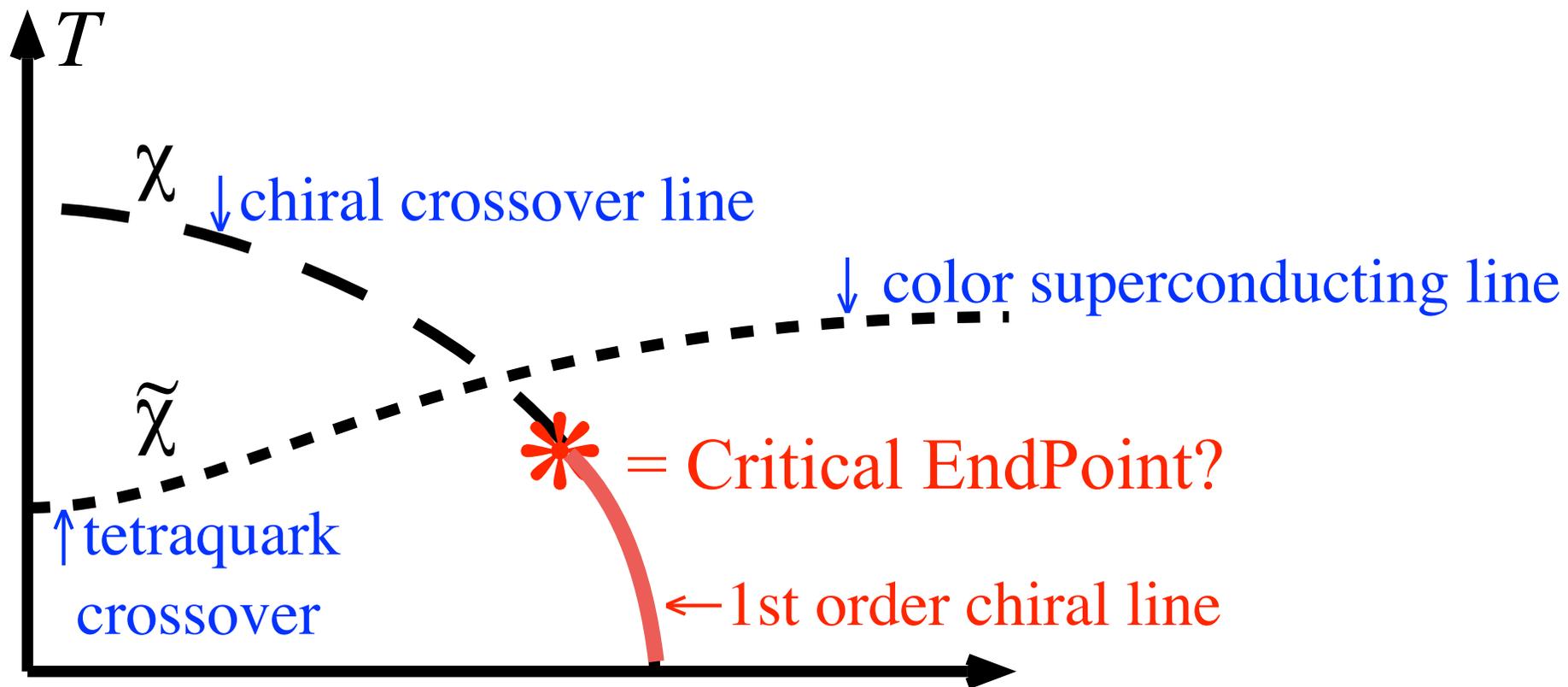
If two chiral phase transitions for three massless flavors, persists for nonzero mass

Implies new phase diagram in the plane of $m_u = m_d$ versus m_s :



Tetraquarks in the plane of T and μ

Diquark fields are *identical* to the order parameters for color superconductivity. Tetraquark condensate = gauge invariant square of CS condensate. Suggests:



Line for chiral crossover *might* end, meet line for *first* order chiral transition at *Critical EndPoint* (CEP). Massless σ at CEP. Rajagopal, Shuryak & Stephanov, '99

In *effective* models, to find the CEP, *must* include tetraquarks: need the *right* σ !

Using the lattice to fix the σ model?

Briceno, Dudek, Edwards, & Wilson (Hadron Spectrum Coll.), 1607.05900

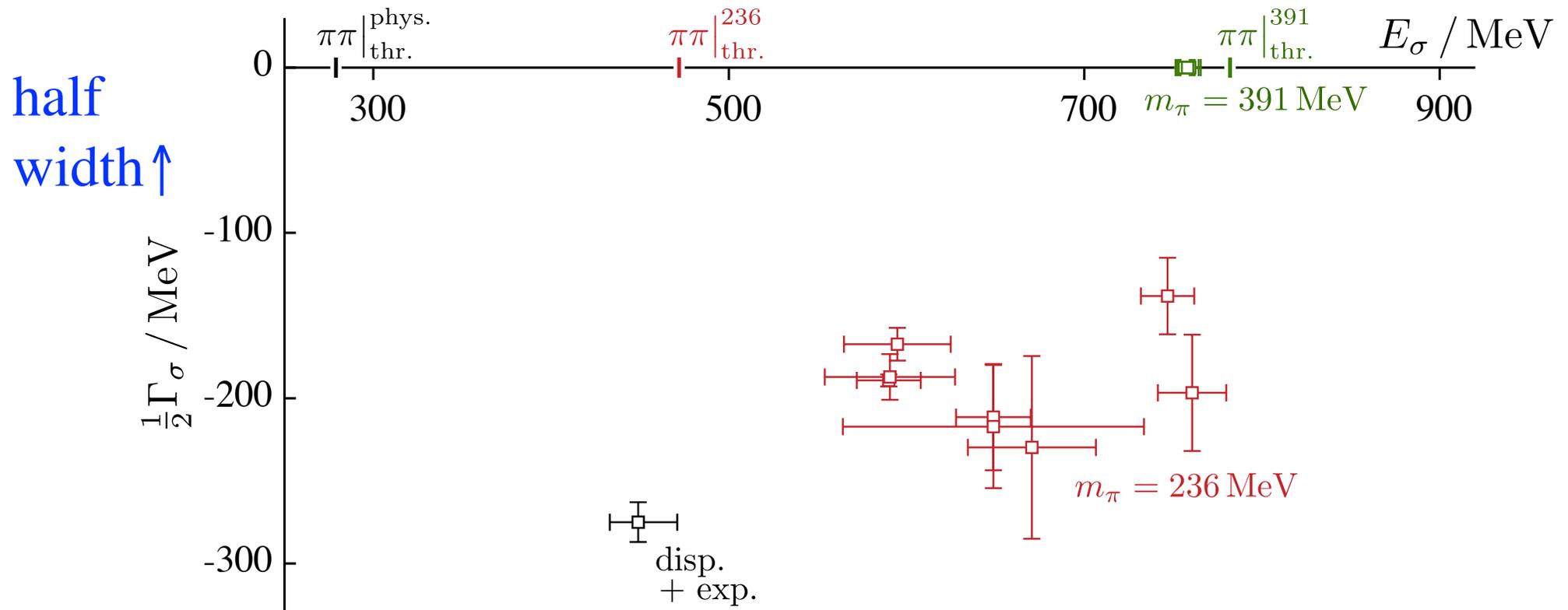
First study of the σ meson on the lattice with light, dynamical quarks

$m_\pi = 391$ MeV: σ bound state resonance just below $\pi\pi$ threshold

$m_\pi = 236$ MeV: σ broad resonance, well above $\pi\pi$ threshold

Perhaps: fix the parameters of σ model plus tetraquarks from the lattice

Very difficult, but of direct relevance for the CEP.



Four flavors, three colors: *hexaquarks*

Diquark 2-index antisymmetric tensor:

$$\chi_L^{(ab)A} = \epsilon^{abcd} \epsilon^{ABC} (q_L^{cB})^T C^{-1} q_L^{dC}$$

So LR tetraquark is same:

$$\zeta^{(ab),(cd)} = \left(\chi_R^{(ab)A} \right)^\dagger \chi_L^{(cd)A}$$

Tetraquark couples to usual Φ through cubic, quadratic terms, *so what*.

Instead, consider *triquark* field:

$$\chi_L^a = \epsilon^{abcd} \epsilon^{ABC} q_L^{bA} (q_L^{cB})^T C^{-1} q_L^{dC}$$

Triquark is a color singlet, fundamental rep. in flavor.

Hence a LR *hexaquark* field is just like the usual Φ , and mixes *directly* with it.

$$\xi^{ab} = (\chi_R^a)^\dagger \chi_L^b$$

Analysis for general numbers of flavors and colors is *not* trivial.

Like color superconductivity.