HIC for FAIR
Helmholtz International Center

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Helmholtz International Center for FAIR

(HIC for FAIR) 1st Anniversary Colloquium on Thursday, July 2, 2009, 15:00 st

Welcome Address by the Scientific Director **Prof. Dr. Carsten Greiner**
Welcome Address by the Vice-President of the Goethe-Universität **Prof. Dr. Wolf Aßmus**
Welcome Address by the Vice-President of the Helmholtz-Gemeinschaft Deutscher Forschungszentren & Scientific Director of the GSI Helmholtzzentrum für Schwerionenforschung GmbH **Prof. Dr. Horst Stöcker**

Guest speaker: **Prof. Dr. Robert Pisarski** (Brookhaven National Laboratory, Upton, New York/USA)

"Quarkyonic matter, and the triple point in the phase diagram of QCD"

LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

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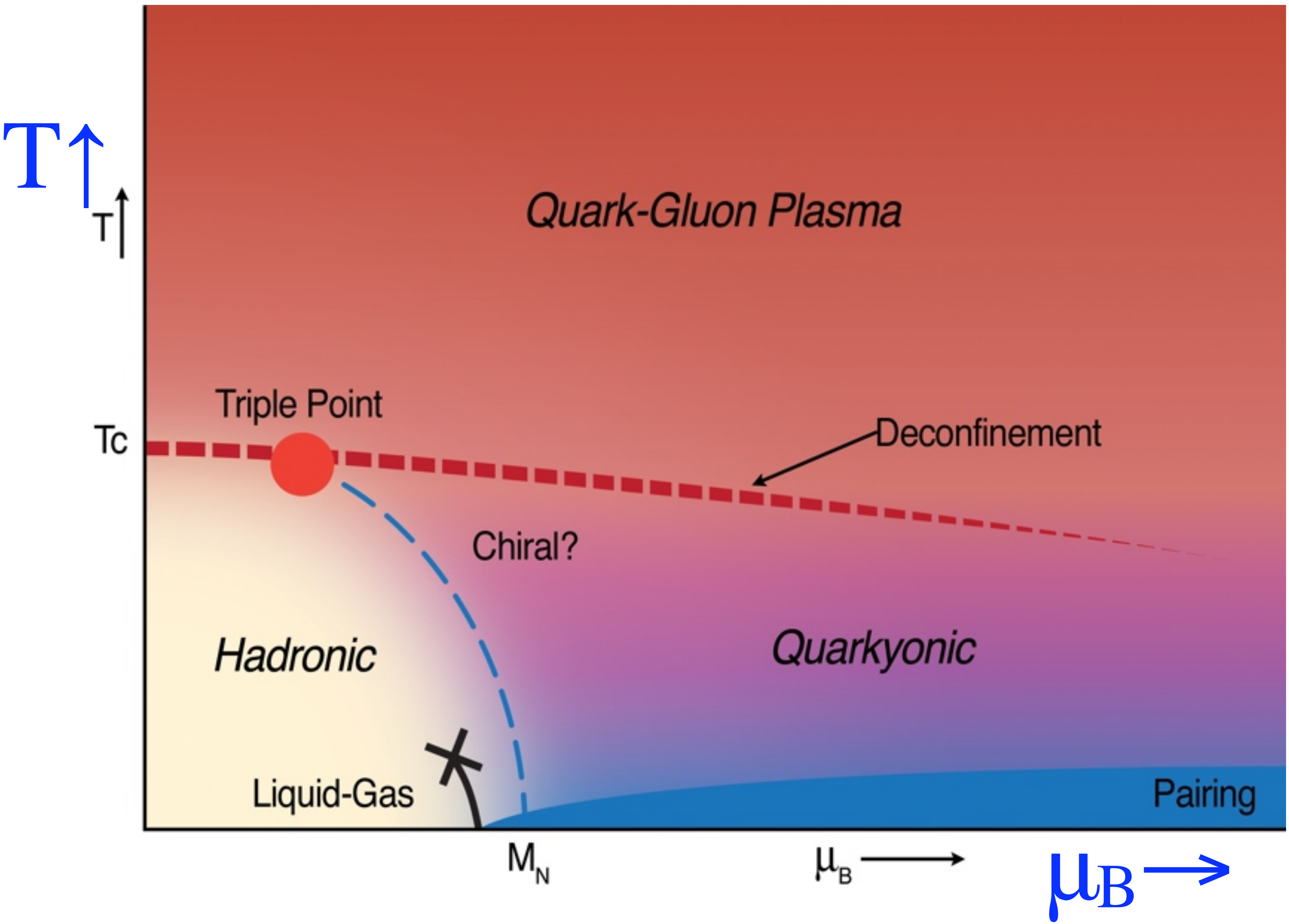
FIAS Frankfurt Institute for Advanced Studies



$T\uparrow$
T
Quark-Gluon Plasma
Triple Point
Tc
Deconfinement
Chiral?
Hadronic
Quarkyonic
Liquid-Gas
Pairing
M_N
μ_B
$\mu_B \longrightarrow$

A triple point in the QCD phase diagram?
From SPS, to RHIC, & (down) to FAIR

1. Large N_c, small N_f:
   Quark-yonic matter - quark Fermi sea *plus* bar-yonic Fermi surface
   Triple point. Deconfining critical end point at *large* $\mu_{qk} \sim N_c^{1/2}$
2. Large N_c, large N_f: baryon density as order parameter
3. New phase diagram for QCD
4. "Purely pionic" effective Lagrangians and nuclear matter:
   The unbearable lightness of being (nuclear matter)?

Gazdzicki's Strange "MatterHorn" ≈ Triple Point?

McLerran & RDP, 0706.2191. Hidaka, McLerran, & RDP 0803.0279
McLerran, Redlich & Sasaki 0812.3585
Hidaka, Kojo, McLerran, & RDP 09......
Blaizot, Nowak, McLerran & RDP 09......

Blashcke, Braun-Munzinger, Cleymans, Fukushima, Oeschler,
RDP, McLerran, Redlich, Sasaki, Stachel (BBMCFOPMRSS) '09....

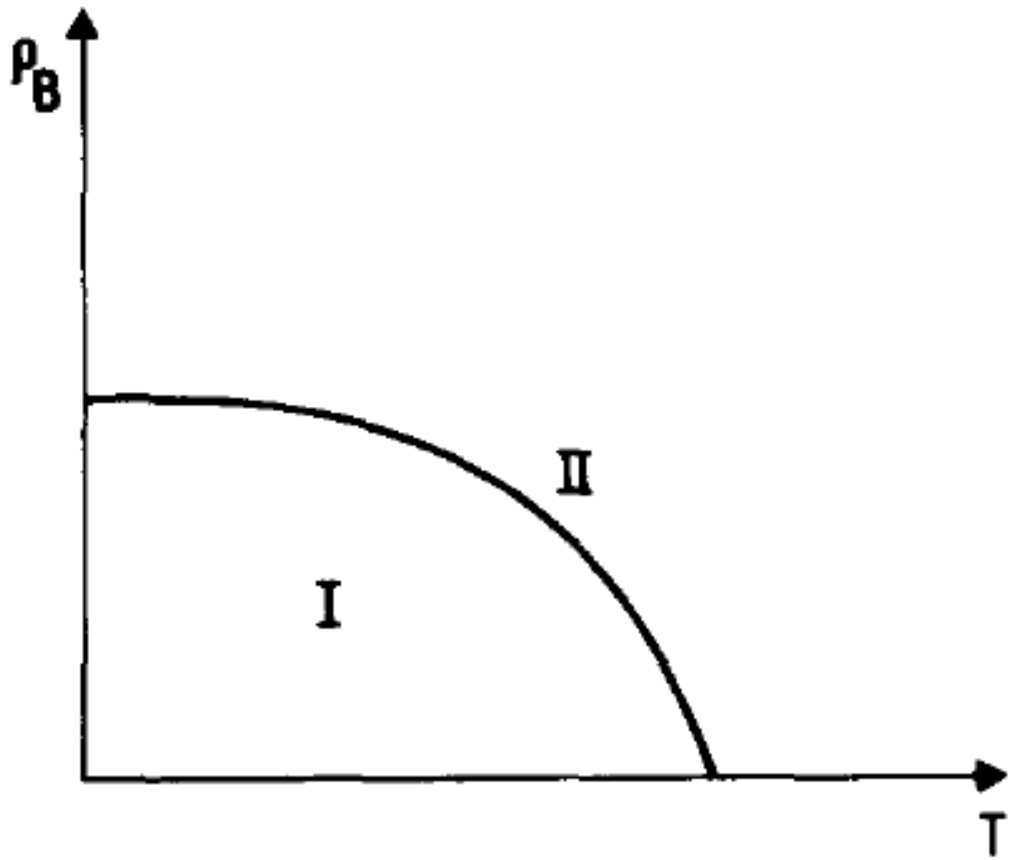
Brief summary of what is to come

The "usual" phase diagram of QCD

Cabibbo and Parisi '75: Hagedorn spectrum not limiting temperature, but transition to "unconfined" phase.

Semi-circle in the μ - T plane (μ = quark chemical potential, T = temperature)

ϱBaryon ↑



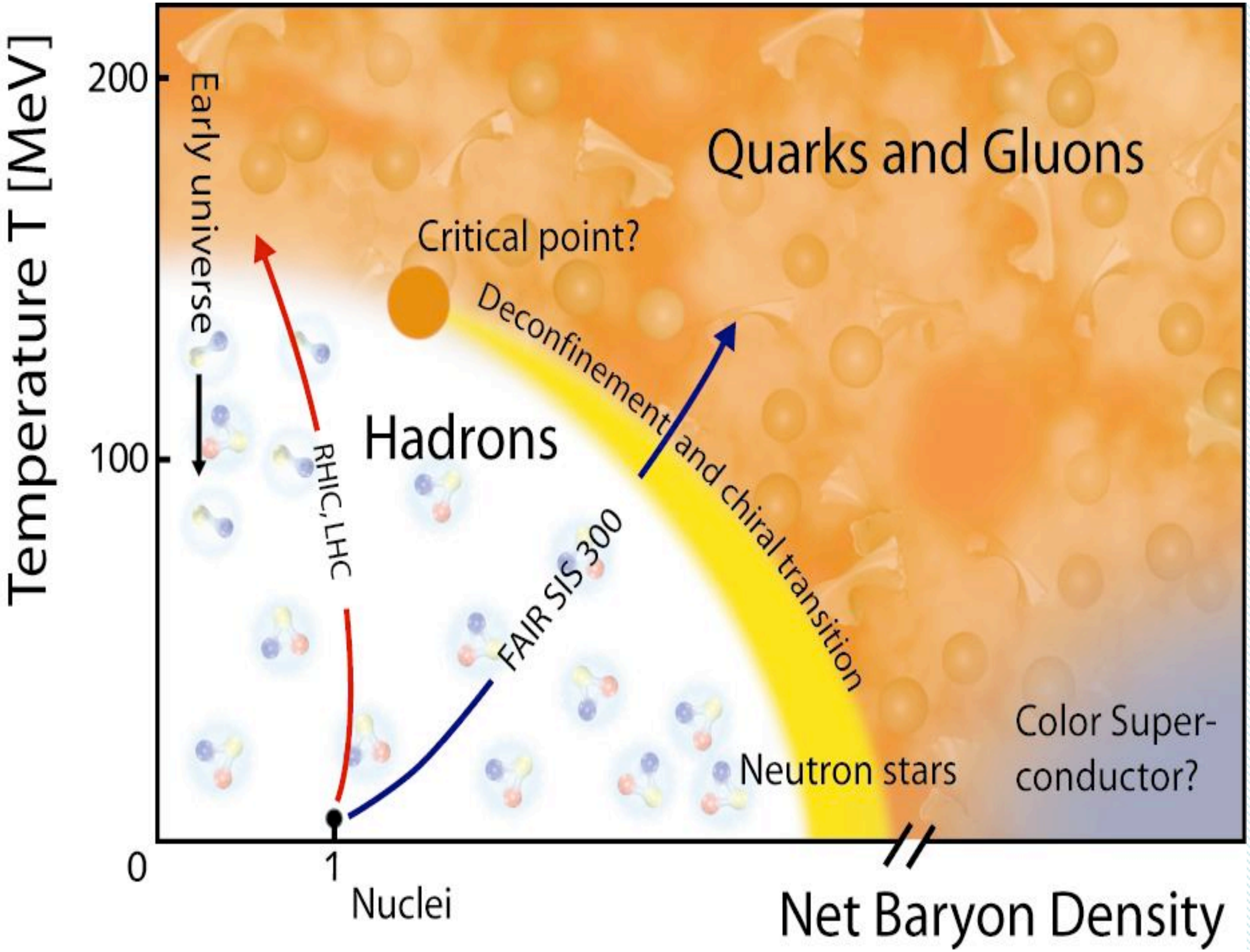
T →

Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

The "usual" phase diagram, updated

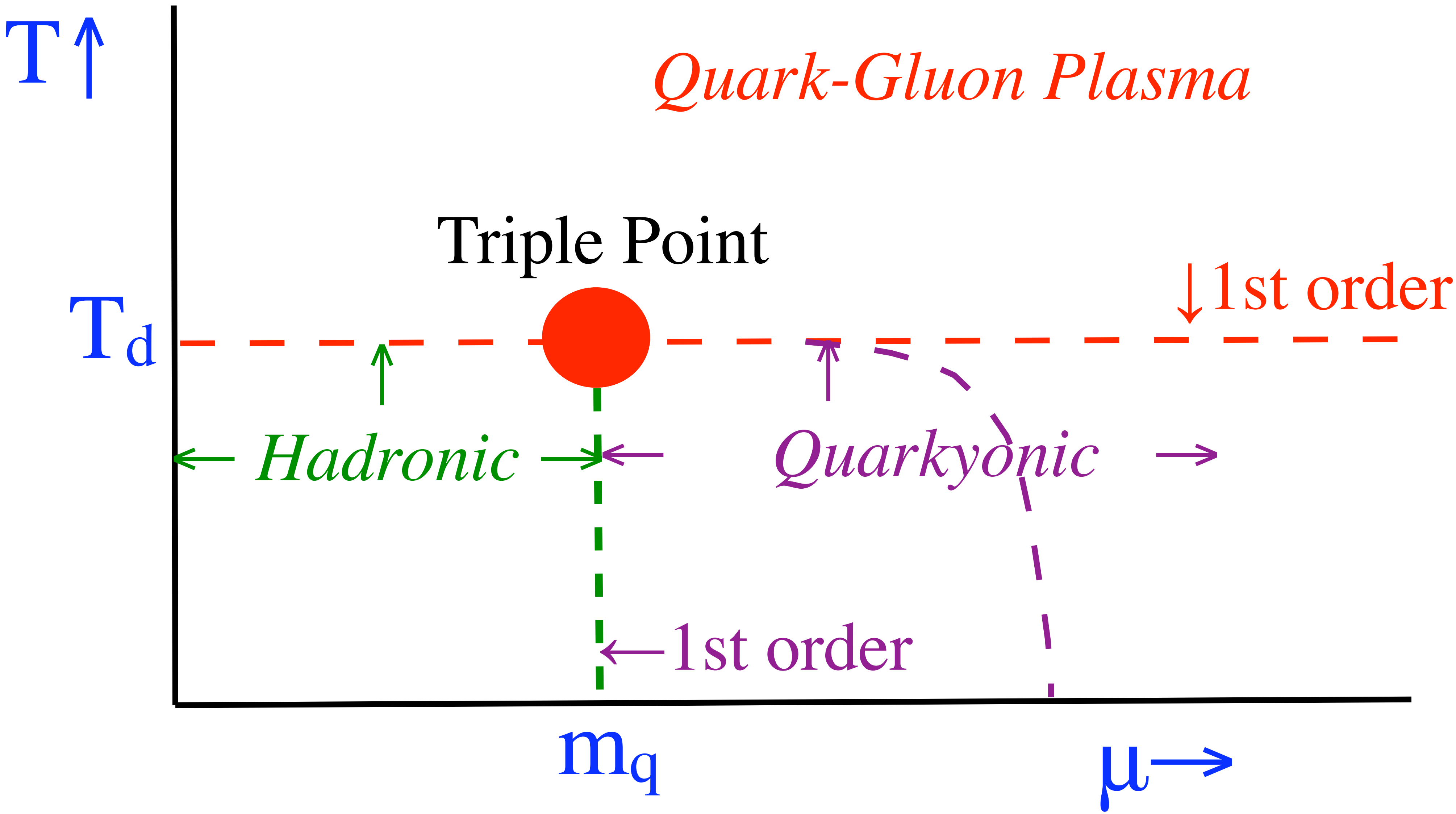
In plane of μ - T plane: *critical end point?* Still semi-circle...

Rajagopal, Shuryak, Stephanov hep-ph/9806219, 9903292



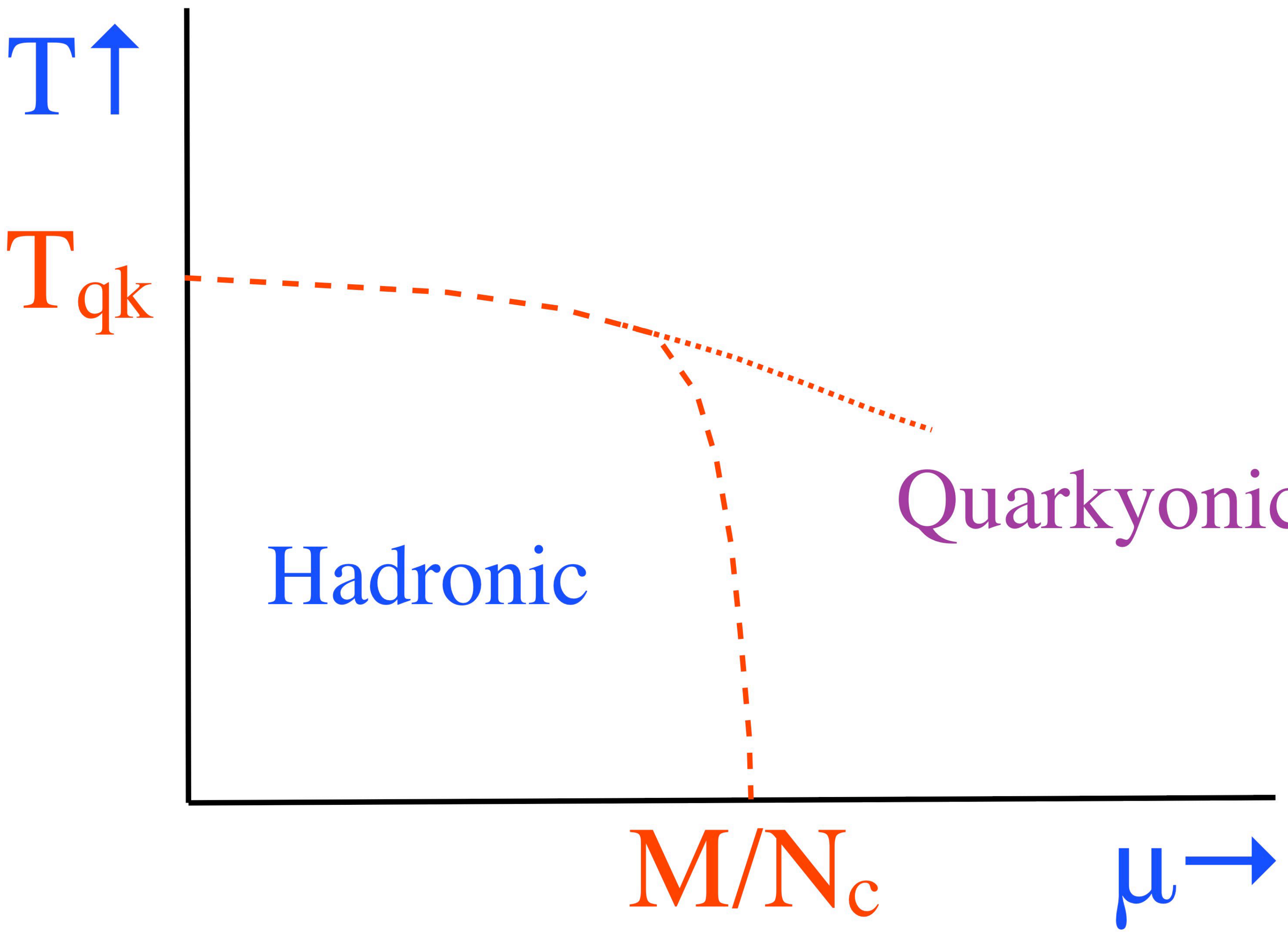
Cartoon Physics:

If you take this... (large N_c, small N_f)



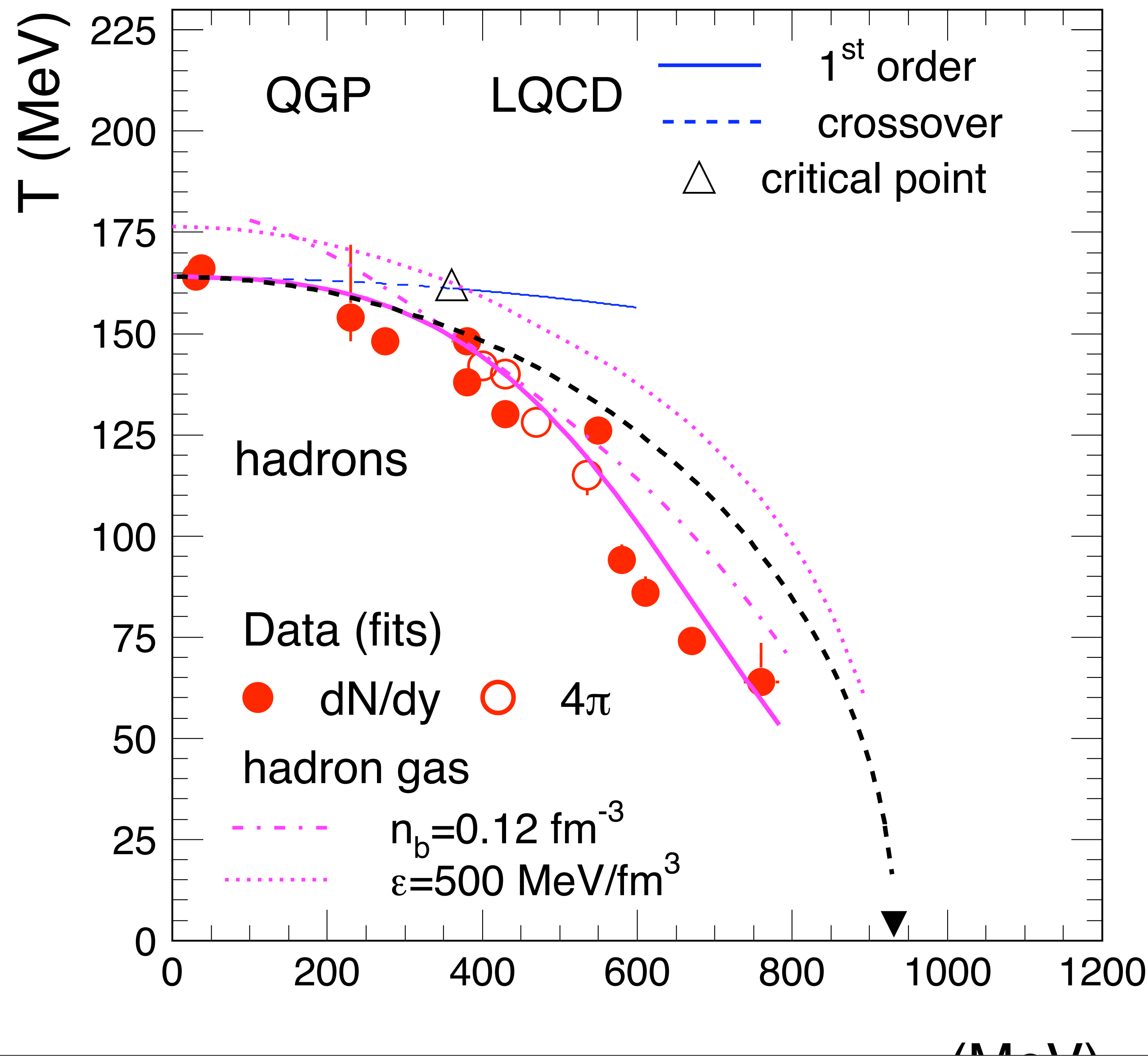
and then this...

(large N_c, large N_f)

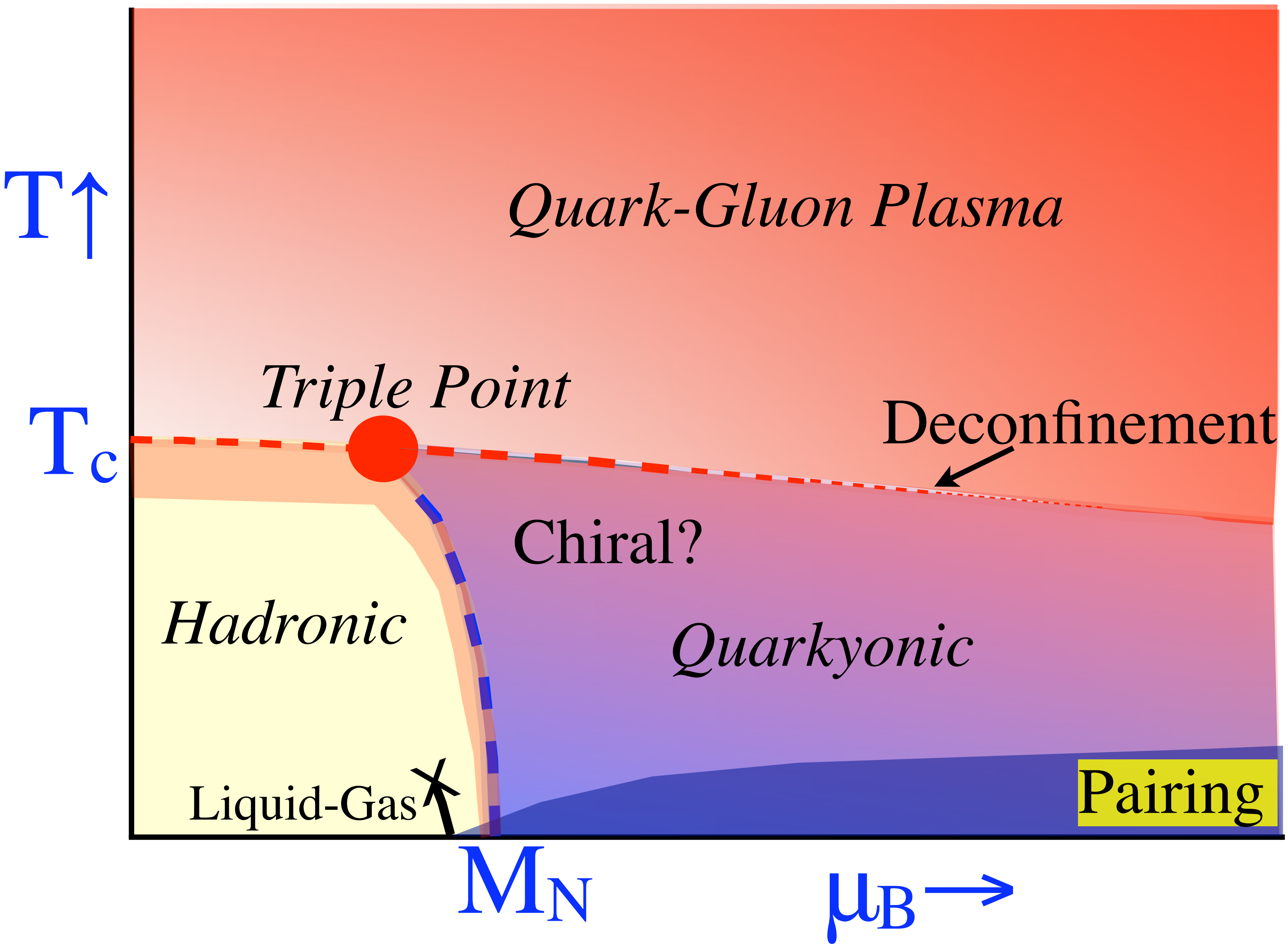


and look at this...

AGS & SPS: “just” baryonic to mesonic freezeout?



You might get...



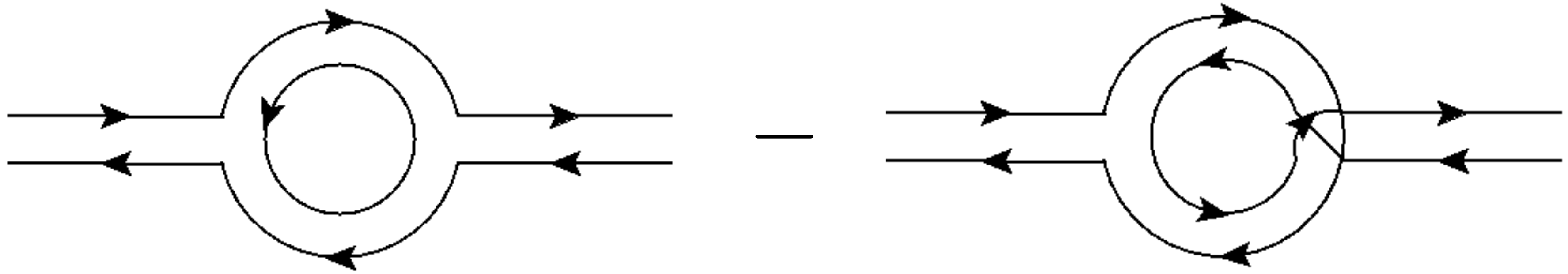
So what really *is* Quarkyonic matter?

QCD at large N_c , small N_f

In $SU(N_c)$, gluons matrices, N_c x N_c, quarks column vectors.
Denote fund. rep. by a line: quarks have one line, gluons have two.

't Hooft '74: let N_c = # colors $\rightarrow \infty$, $\lambda = g^2 N_c$ *fixed*. Keep N_f = # flavors *finite*.

Consider gluon self energy at 1 loop order. For *any* N_c, color structure in all diagrams (3 gluon & 4 gluon vertices) reduces to (Hidaka & RDP 0906.1751)



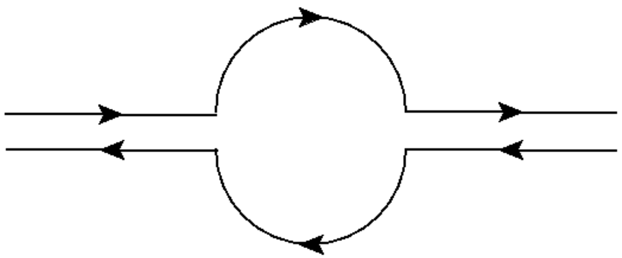
$$\sim g^2 N_c \sim \lambda \qquad\qquad \sim g^2 \sim \frac{\lambda}{N_c}$$

First diagram is "planar". Second, involving trace, is not, is down by $1/N_c$.

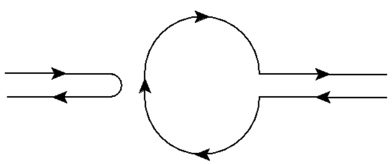
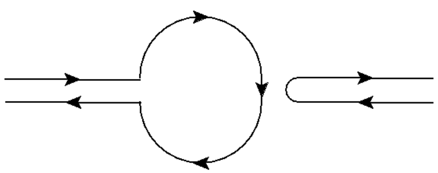
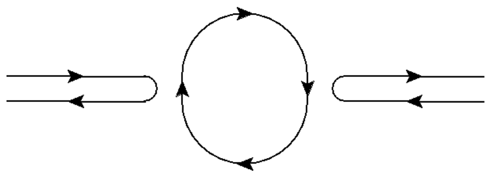
At large N_c and small N_f , planar diagrams dominate.

Large N_c and small N_f: *glue* dominates

Contribution of the quarks to the gluon self energy at 1 loop order, any N_c:



$$\sim g^2 N_f \sim \frac{1}{N_c} N_f \lambda$$

$-\frac{1}{N}$  $-\frac{1}{N}$  $+\frac{1}{N^2}$ 

If $N_f/N_c \to 0$ as $N_c \to \infty$, loops *dominated* by gluons, *blind* to quarks.

Quarks act *something* like external sources, not quite.

N.B.: limit of large N_c, small N_f is *free* of the pathologies of $N_f = 0$ (quenched)

No problems considering nonzero quark density, μ_{qk}:

quarks do *not* affect gluons when $\mu_{qk} \sim 1$!

Phases at large N_c and small N_f

$T = \mu_{qk} = 0$: confinement, only color singlets. Glueballs, meson masses ~ 1.
Baryons *very* heavy, masses ~ N_c, so no virtual baryon anti-baryon pairs.

$T \neq 0$, $\mu_{qk} = 0$:

$T < T_c$: Hadrons. T_c ~ glueball/meson mass ~ 1.
Degeneracy of hadrons ~ N_c^0 ~ 1, so pressure = $p \sim 1$.
$T > T_c$: Quark-Gluon Plasma. Deconfined gluons & quarks.
Degeneracy ~ N_c^2, so $p \sim N_c^2$, dominated by gluons.

$T \neq 0$, $\mu_{qk} \neq 0$: baryons only for $\mu_{qk} > M_N/N_c = m_{qk} \sim N_c^0 \sim 1$.

$T < T_c$, $\mu_{qk} < m_{qk}$: Hadronic "box" in T-μ_{qk} plane: *no* baryons.

$T > T_c$ any μ_{qk} : Quark-Gluon Plasma. Some quarks, so what, $p_{qk} \sim N_c$.

$T < T_c$, $\mu_{qk} > m_{qk}$: *Confined* with Fermi sea of quarks, "*quark-yonic*"
Degeneracy ~ N_c , so $p \sim N_c$. Quarks or (bar)yons?
Dense nuclear matter (*not* dilute)

Lattice: (pure glue) SU(3) close to SU(∞)

Bringoltz & Teper, hep-lat/0506034 & 0508021:

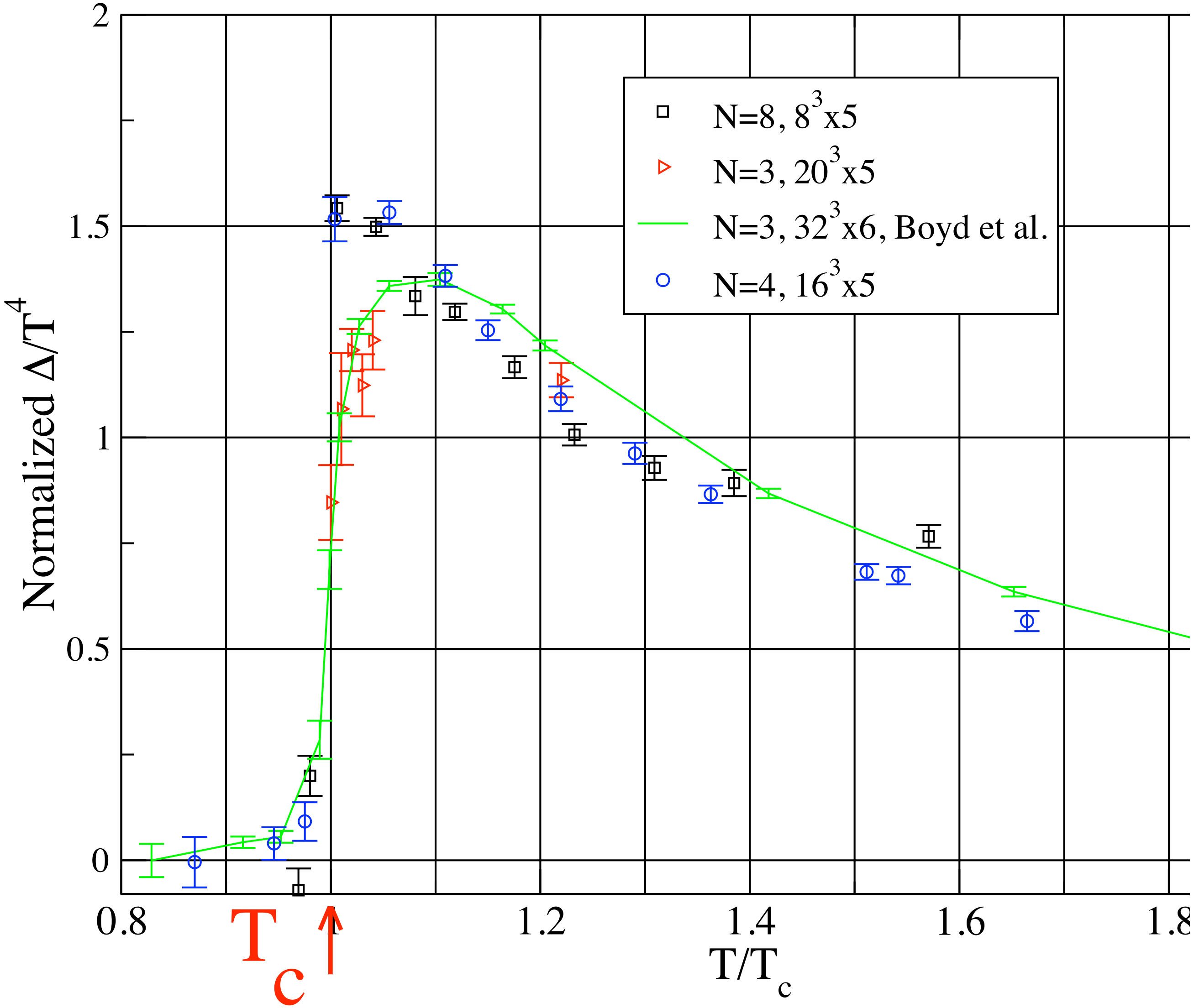
SU(N_c), *no quarks*, N_c = 3, 4, 6, 8, 10, 12.

Deconfining transition first order, latent heat ~ N_c^2.

Hagedorn temperature $T_H \sim 1.116(9)\ T_c$ for $N_c = \infty$

$$\frac{e-3p}{N^2\ T^4} \sim \text{const.}$$

$$\frac{e-3p}{N^2\ T^4} \uparrow$$



$T_c \uparrow$

$T/T_c \rightarrow$

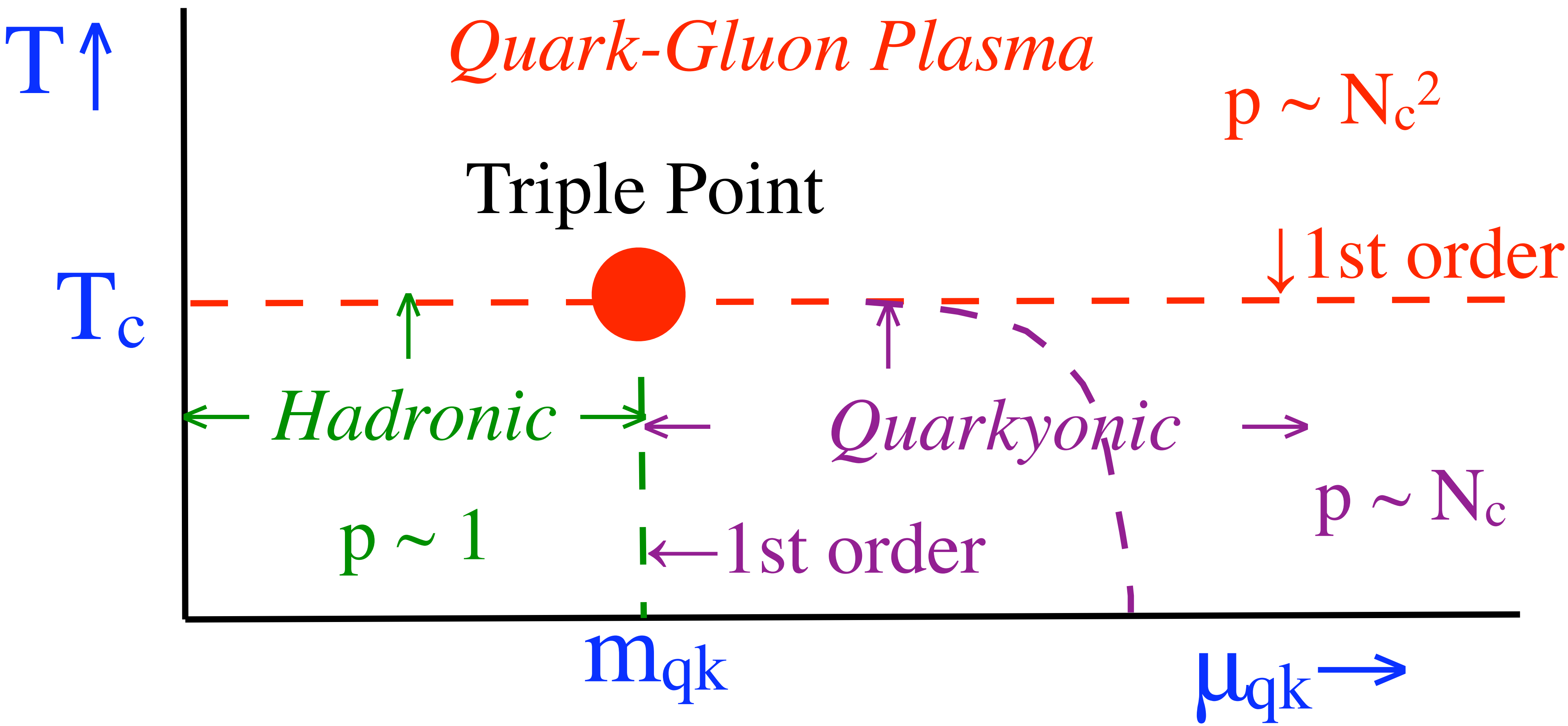
Phase diagram at large N_c and small N_f

Lattice (Teper, 0812.0085): deconfining transition 1st order at $T \neq 0$, $\mu_{qk} = 0$.
must remain so when $\mu_{qk} \neq 0$. *Straight* line in T - μ_{qk} plane.

Hadronic/Quarkyonic transition: energy density jumps by N_c, 1st order?

Chiral transition: in Quarkyonic phase?

True triple point!



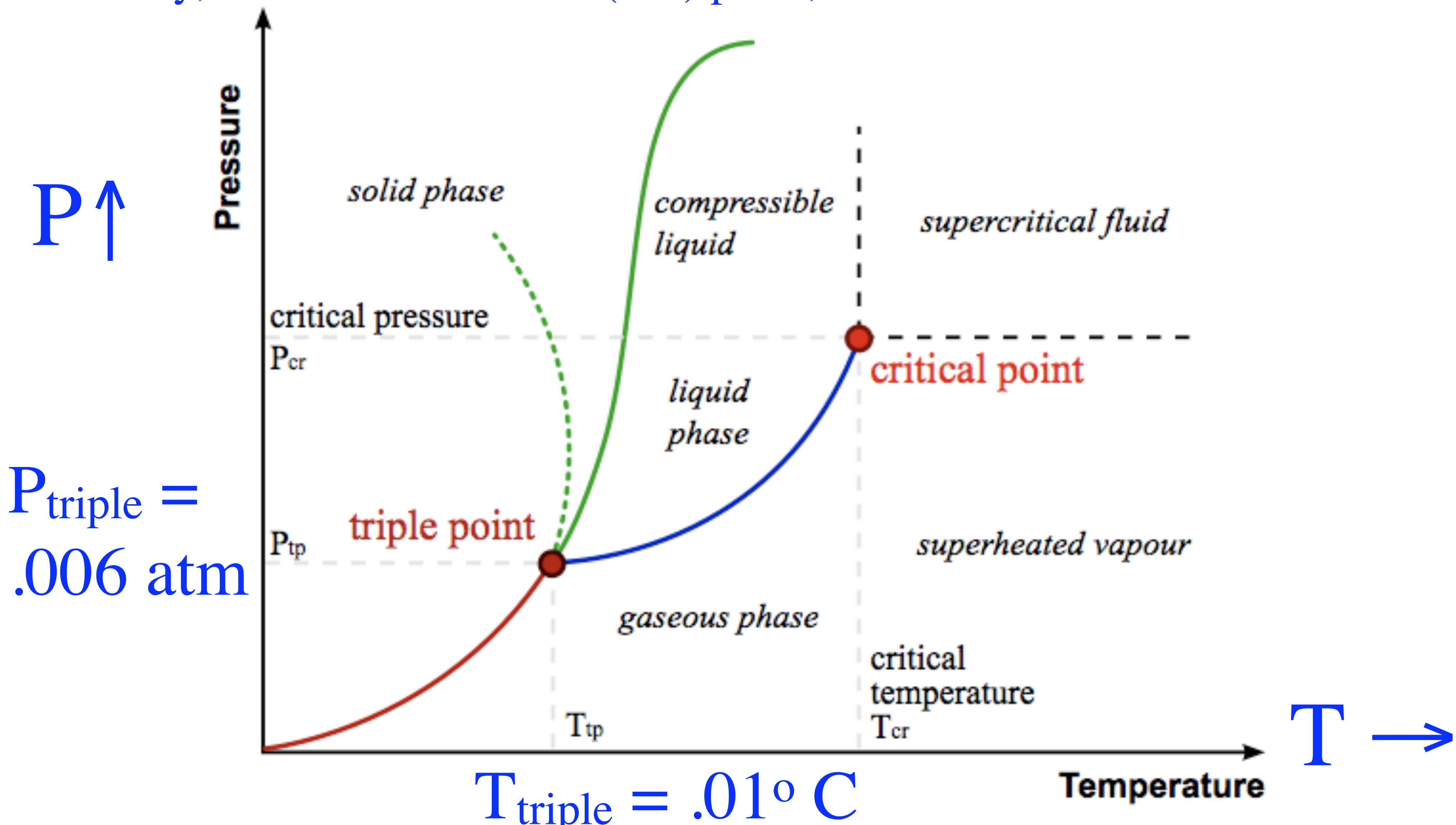
Triple point for water

Triple point where three lines of first order transitions meet.

E.g., for ice/water/steam, in plane of temperature and pressure.

(Generalizes: four lines of first order transitions meeting is a quadruple point.)

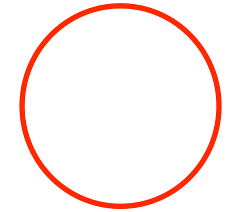
Generically, *distinct* from critical (end) point, where one first order line ends.



Nuclear matter at large N_c, no miracles

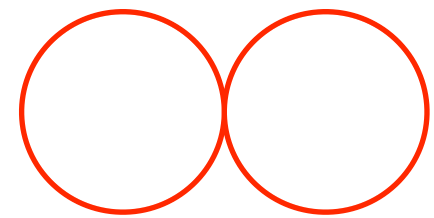
$\mu_{Baryon} = \sqrt{k_F^2 + M^2}$, k_F = baryon Fermi mom. Ideal pressure small, ~ $1/N_c$:

$$P_{\text{ideal baryons}} \sim n(k_F)\,\frac{k_F^2}{M} \sim \frac{1}{N_c}\,\frac{k_F^5}{\Lambda_{QCD}}$$



Usual large N counting: two body int.'s *big*, contribute ~ N_c to pressure:

$$\delta P_{\text{two body int.'s}} \sim N_c\,\frac{n(k_F)^2}{\Lambda_{QCD}^2} \sim N_c\,\frac{k_F^6}{\Lambda_{QCD}^2}$$



At large N_c , (dilute) nuclear matter dominated by potential terms: crystal

Two body, three body... interactions *all* contribute ~ N_c .

Pressure ideal ~ two body interactions for small momenta, $k_F \sim \dfrac{1}{N_c^2}\,\Lambda_{QCD}$

$$\mu - m_q = \frac{\mu_B - M}{N_c} = \frac{k_F^2}{2MN_c} \sim \frac{1}{N_c^2}\,k_F^2$$

Hence dilute nuclear matter only in a *very* narrow window.

Quarkyonic phase at large N_c, large μ?

Let $\mu >> \Lambda_{QCD}$ but $\sim N_c^0$. Coupling runs with μ, so pressure $\sim N_c$ is close to perturbative! How can the pressure be (nearly) perturbative in a confined theory?

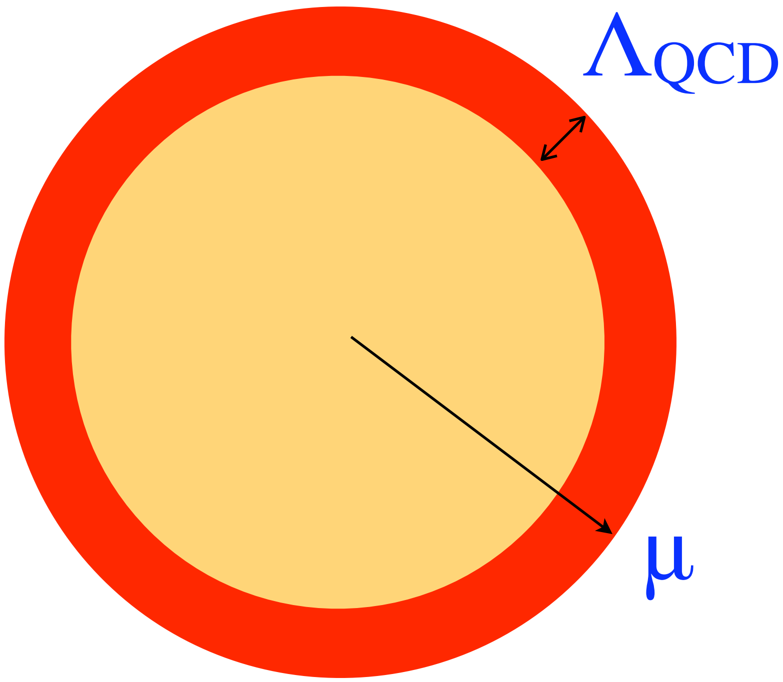
Pressure: dominated by quarks far from Fermi surf.: perturbative,

$p_{qk} \sim N_c \, \mu^4 \, (1 + g^2(\mu) + g^4(\mu) \log(\mu) + \ldots.)$

Within Λ_{QCD} of Fermi surface: *confined* states.

$p_{qk} \sim N_c \, \mu^4 \, (\Lambda_{QCD}/\mu)^2$, *non*-perturbative.

Within skin, only confined states contribute.



Fermi sea of quarks + Fermi surface of bar-yons

= "quark-yonic". *N=3?*

Pressure dominated by quarks.

But transport properties *dominated* by confined states near Fermi surface!

For QCD: what is (cold) nuclear matter like at high density?

Just a quark NJL model?

Deconfining critical end point at (large) $\mu_{qk} \sim N_c^{1/2}$

Semi-QGP theory of deconfinement: Hidaka & RDP 0803.0453

$$A_0 = \frac{T}{g}\, Q$$

For large μ: compute one loop determinant in background field.
Korthals-Altes, Sinkovics, & RDP hep-ph/9904305

$$S_{qk} = \mathrm{tr}\,(\mu + i\,T\,Q)^4\;,\; T^2\;\mathrm{tr}\,(\mu + i\,T\,Q)^2\;,\; N_c^2\,T^4\,V(Q)$$

RDP '09: for large μ, expand:

$$S^{qk}_{\mu\sim\sqrt{N_c},T\sim 1} \sim N_c\,\mu^4 - 6\,\mu^2\,T^2\,tr\,Q^2 + \ldots \sim N_c^3\;,\; N_c^2\;(tr\,Q^2/N_c)$$

Consider $\mu \sim N_c^{1/2}$, $T \sim 1$: gluons *do* feel quarks.

Term $\mu^4 \sim N_c^3$ dominates, but *independent* of Q *and* temperature.

Term $\mu^2 \sim N_c^2$ Q-dependent. Breaks $Z(N_c)$ symmetry, so washes out 1st order deconfining transition: Deconfining Critical End Point (CEP)

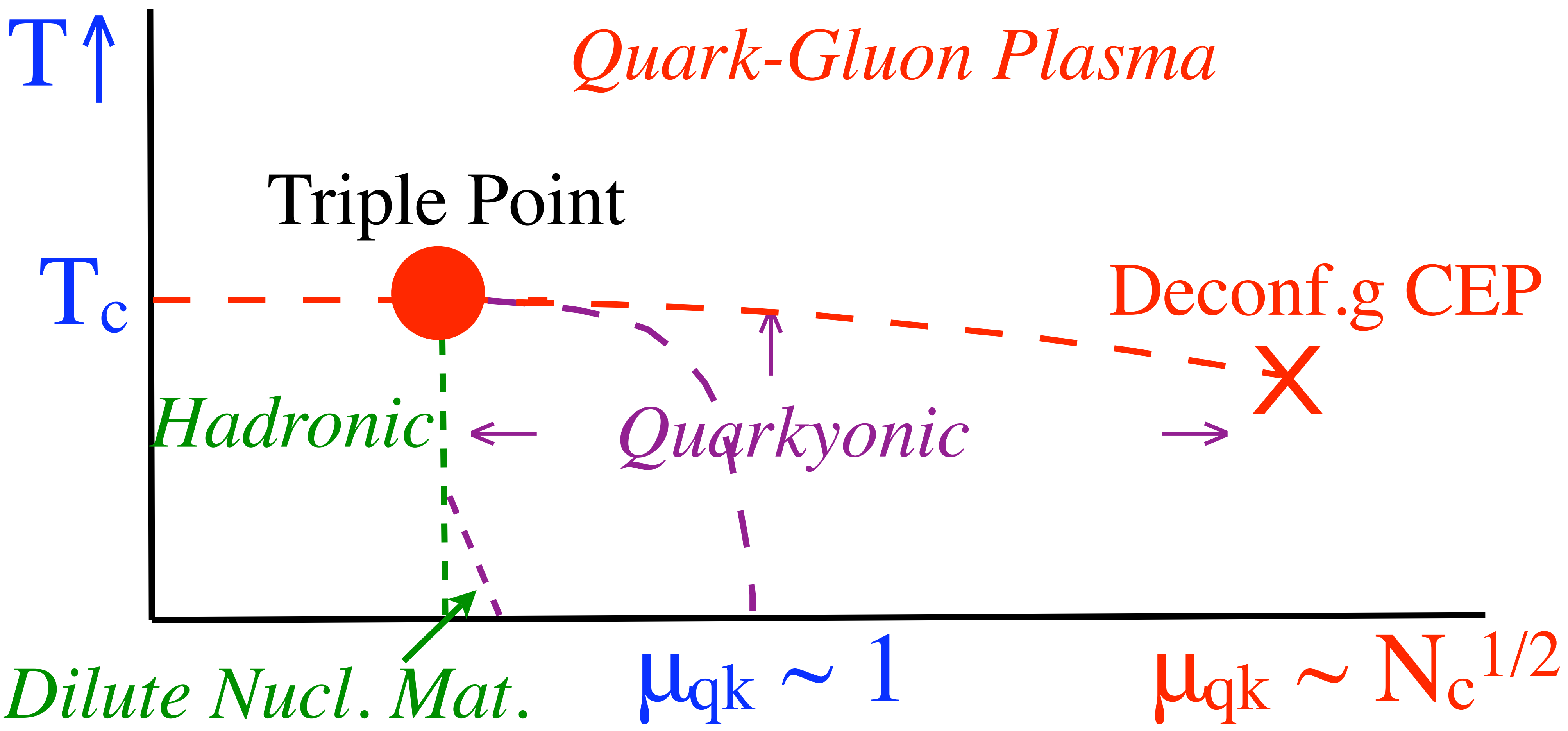
Phase diagram at large N_c and small N_f, II

About deconfining Critical End Point (CEP), smooth transition between deconfined and quarkyonic phases.

Since gluons are sensitive to quarks for such large μ, expect curvature in line.

Triple point still well defined, as coincidence of three 1st order lines.

Chiral transition?



Deconfining Critical End Point on a $N_c = \infty$ femto-sphere

Sundborg, hep-th/9908001; Aharony, Marsano, Minwalla, Papadodimas, & Van Raamsdonk, hep-th/0310285 & 0502149; Schnitzer, hep-th/0402219; Dumitru, Lenaghan, & RDP, hep-ph/0410294.

Consider (pure gauge) SU(N) on a *very* small sphere: radius R, with $g^2(R) << 1$.

(Sphere because constant modes simple, spherically symmetric)

At $N = \infty$, can have a phase transition even in a *finite* volume.

At $g^2 = 0$: *precisely* defined Hagedorn temperature, T_H . Density of states:

$$\rho(E \to \infty) \sim \mathrm{e}^{E/T_H} \;, \; T_H = \frac{1}{\log(2+\sqrt{3})}\,\frac{1}{R}$$

$g^2 = 0$: 1st order deconfining transition at $T_d = T_H$. $T_d < T_H$ to $\sim g^4$.

At deconfining transition, tr L = exp(i Q)/N (only) becomes massless.

Add quarks. Since $\mu >> 1/R$, can use pressure in infinite volume.

Previous term dominates, acts like background field.

Washes out 1st order deconfining transition for any value of background field.

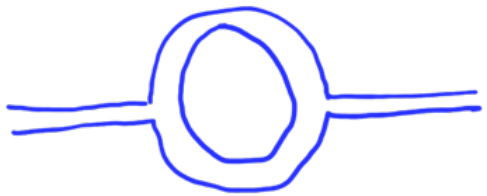
$N = \infty$ on a small sphere singular, deconf.'g CEP exists at any finite N.

Baryons at large N_c and large N_f

Veneziano '78: take *both* N_c and N_f large. Mesons M^{ij} : i,j = 1…N_f .

Thus mesons interact weakly, but there are *many* mesons.

Thus in the hadronic phase, mesons interact *strongly*:



$$\Pi \sim N_f \, g_{3\pi}^2 \sim N_f / N_c$$

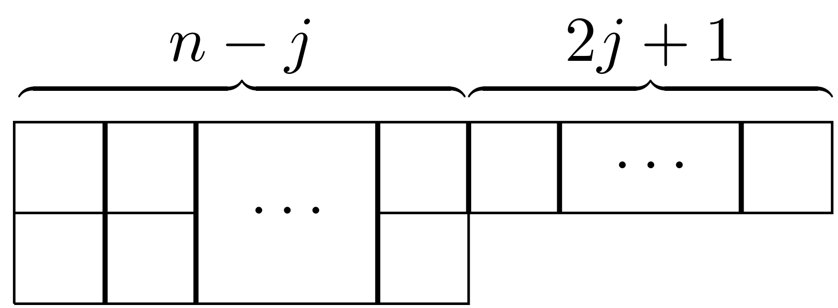
Pressure large in *both* phases:

~ N_f^2 in hadronic phase, ~N_c^2, N_c N_f in "deconfined" phase.

Polyakov loop also nonzero in both phases.

Baryons: lowest state with spin j

has Young tableaux (N_c = 2n + 1) =>



$$d_j = \frac{(2j+2)\,(N_f+n+j)!\,(N_f+n-j-2)!}{(N_f-1)!\,(N_f-2)!\,(n+j+2)!\,(n-j)!}$$

Baryons at Large N_f: order parameters

Y. Hidaka, L. McLerran & RDP, 0803.0279: Use Sterling's formula,

$$d_j \sim \mathrm{e}^{+N_c\, f(N_f/n)} \,, \;\; f(x) = (1+x)\log(1+x) - x\log(x)$$

Degeneracy of baryons increases *exponentially*.

Argument is heuristic: baryons are strongly interacting.
Still, difficult to see how interactions can overwhelm exponentially growing spectrum, even for the lowest state.

Use *baryons* as order parameter. At T=0, fluctuations in baryon number,

$\langle B^2 \rangle \neq 0$ when $N_c\, f(N_c/n) = m_B/T$, or

$$T_{qk} = f(N_f/n)\, \frac{m_B}{N_c}$$

At $\mu \neq 0$, baryon number itself:

$\langle B \rangle \neq 0$ when $N_c\, f(N_c/n) = (m_B - Nc\, \mu)/T$:

$$T_{qk} = f(N_f/n)\left(\frac{m_B}{N_c} - \mu\right)$$

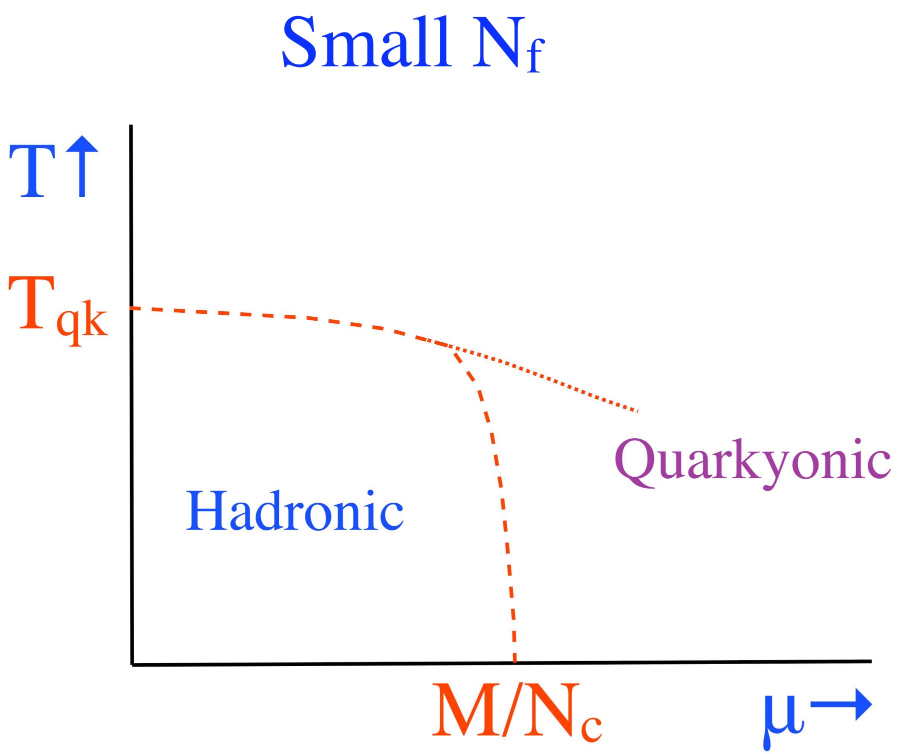
Possible phase diagrams at large N_f

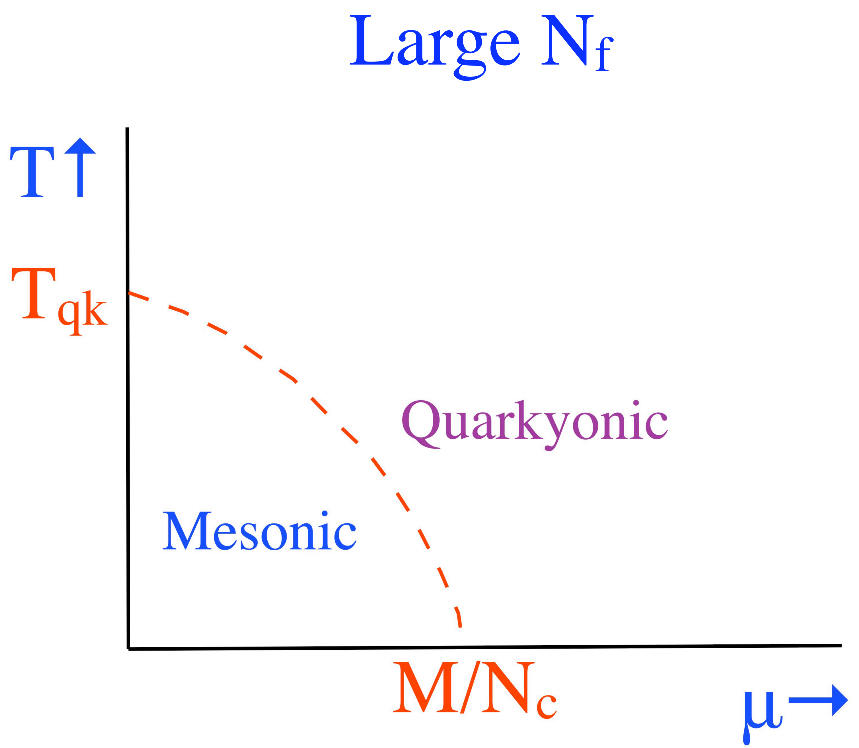
The "rectangle" for small N_f becomes smoothed.

Eventually, maybe the quarkyonic line merges with that for baryon condensation.

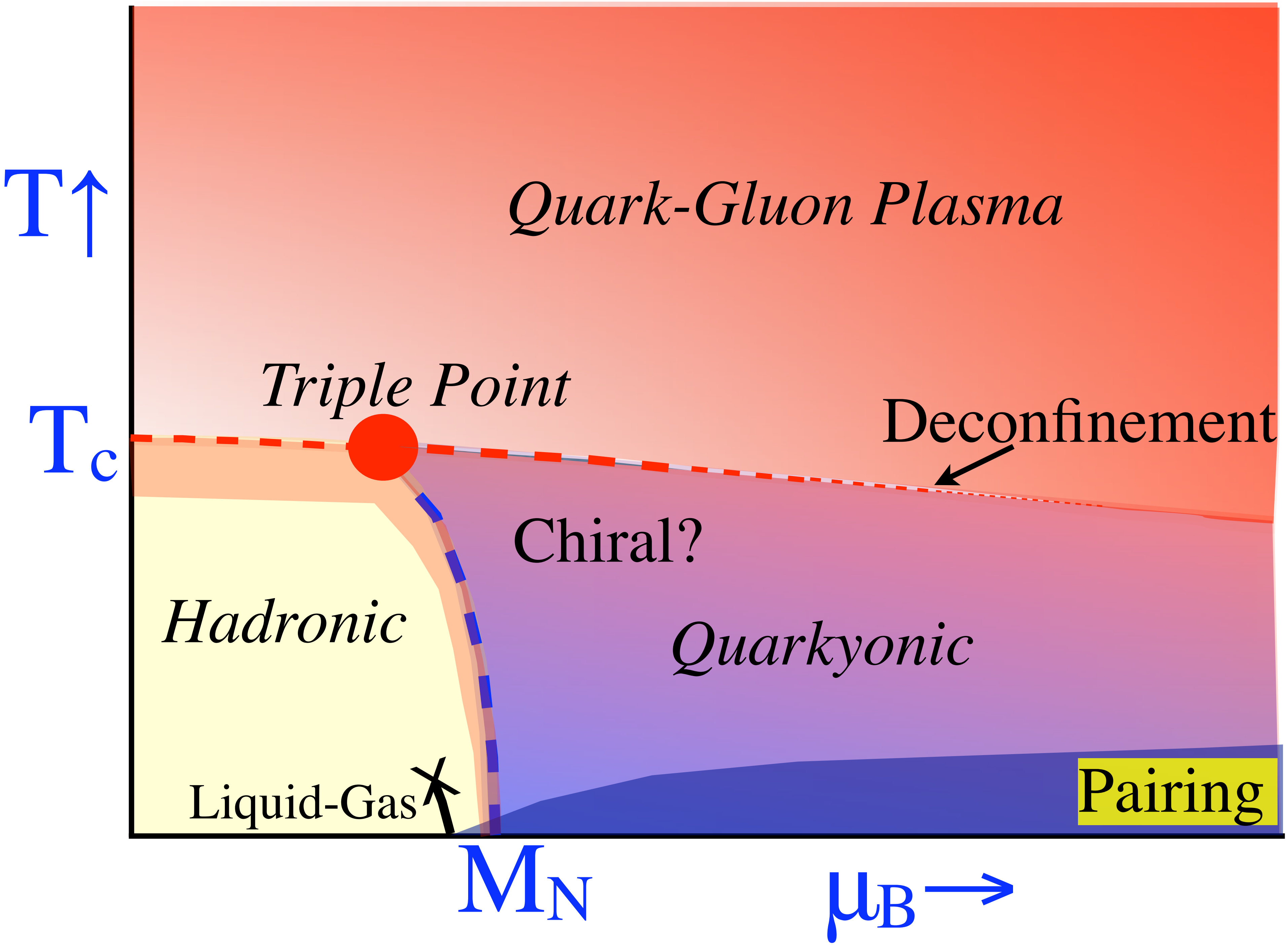
All *VERY* qualitative. Clearly many possible phase diagrams!

With SUSY: condensation of Higgs fields as well.





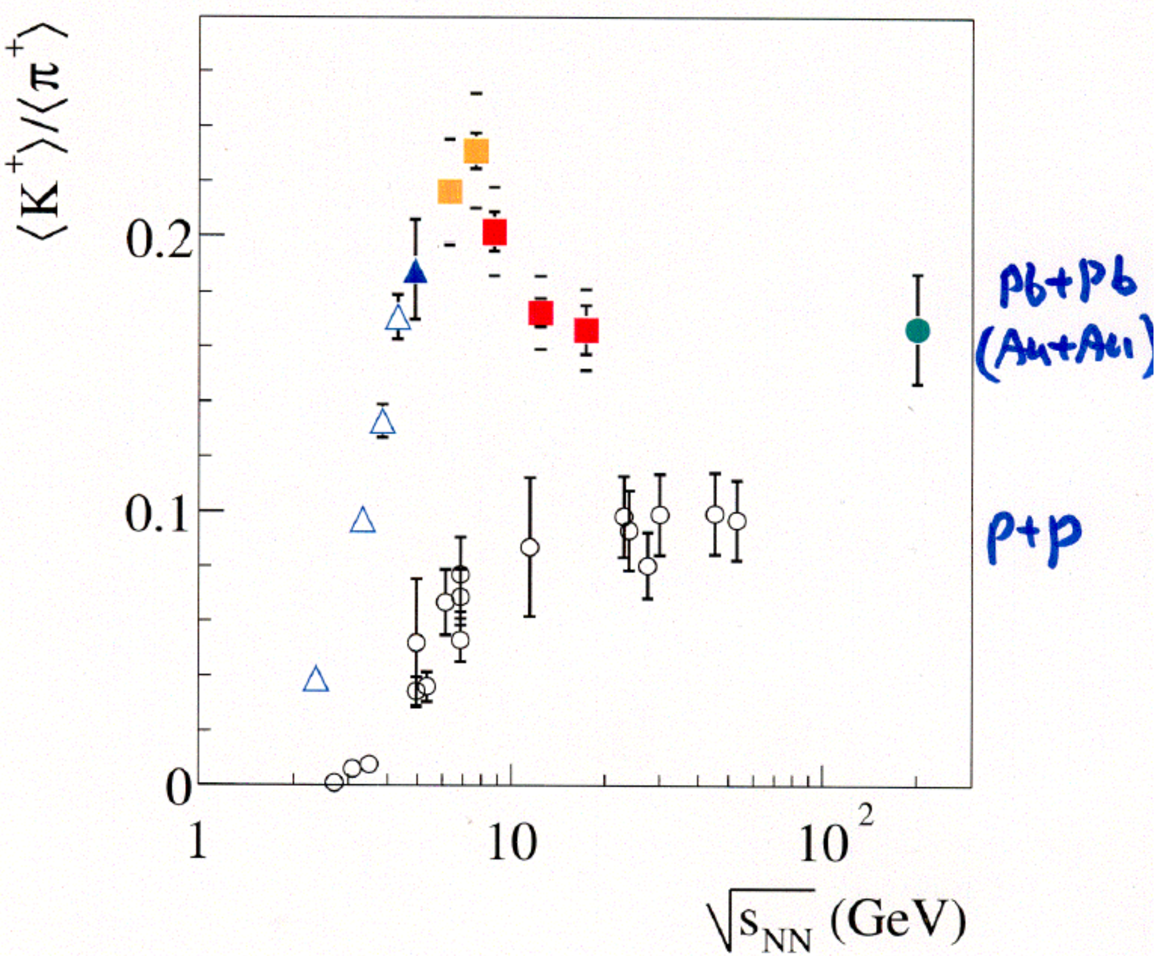
Possible phase diagram for QCD

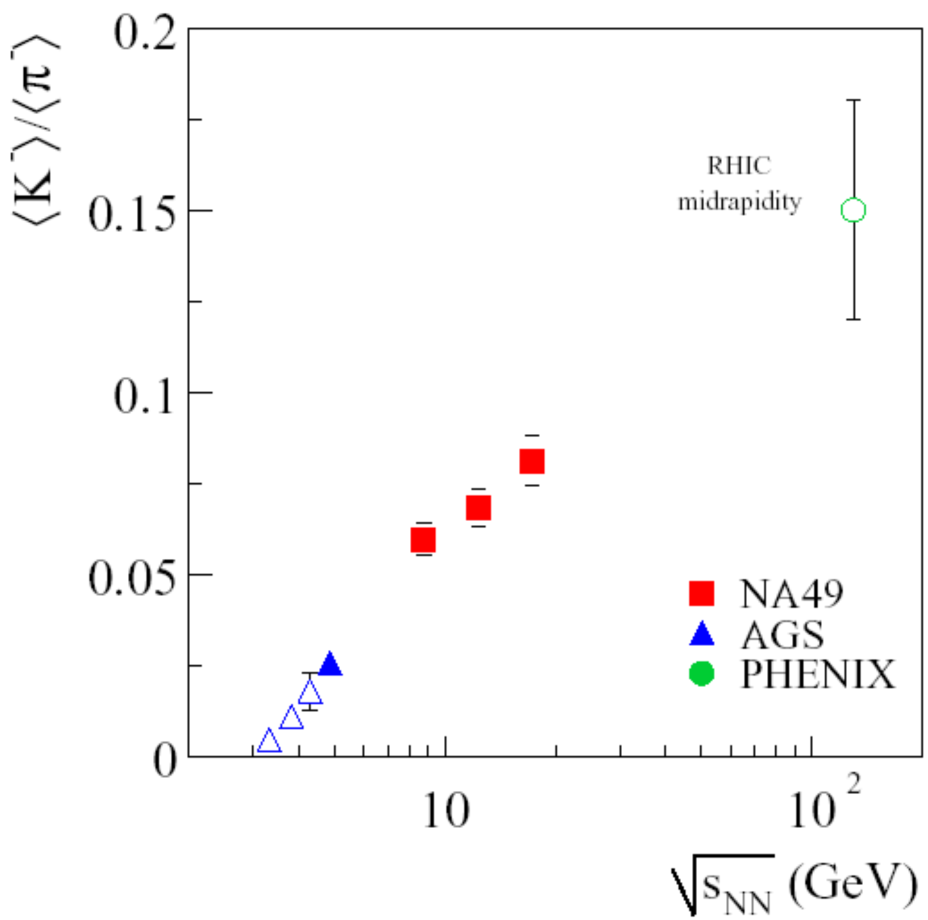


So what does this have to do with experiment?

RDP, Review of Quark Matter 2004:

Is this related to the *narrow* peak in K+/pi+ @ SPS? The "MatterHorn" of NA49 (Gazdzicki)



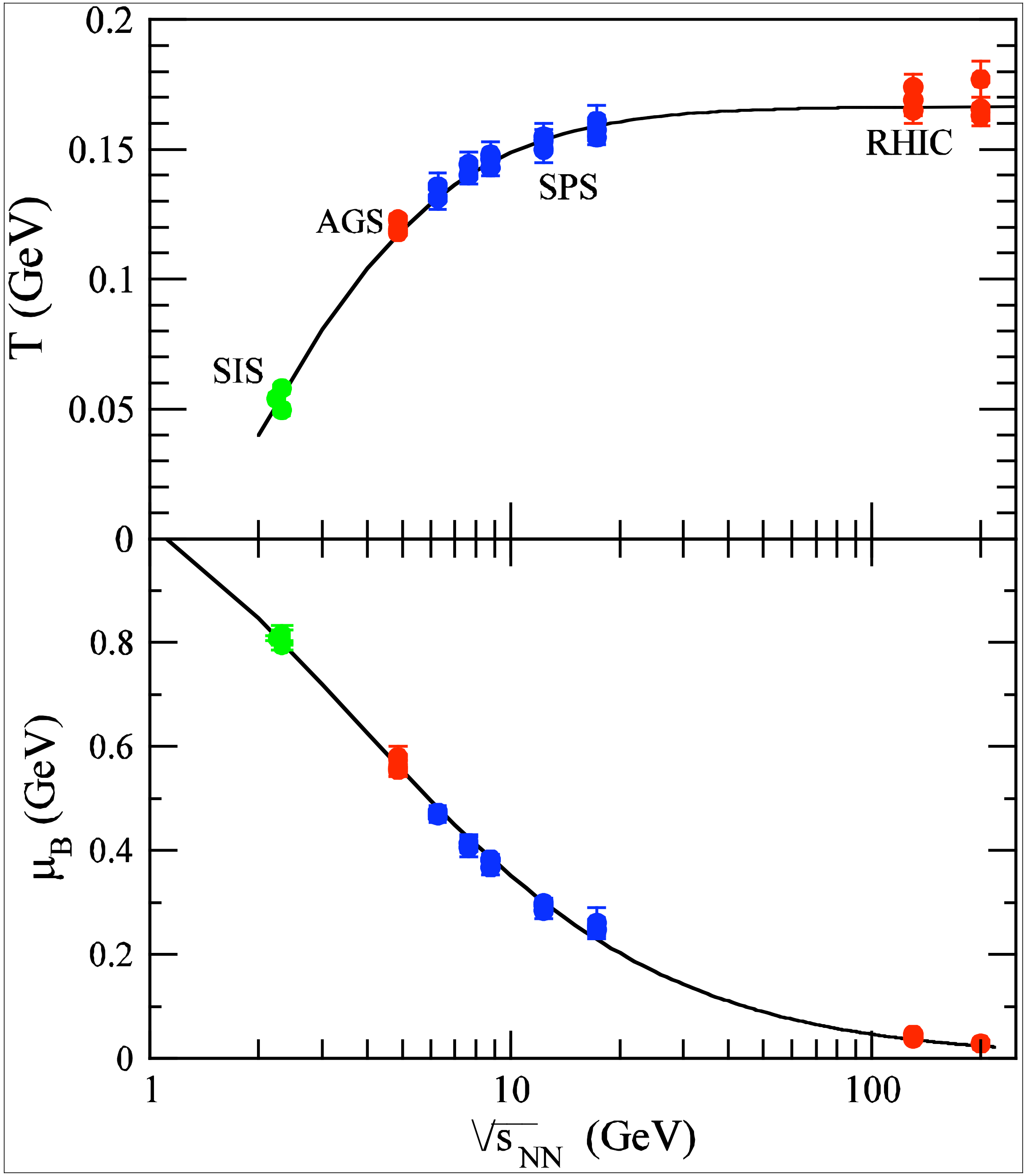


Peak not confirmed by other groups, not seen in other ratios...

Smooth evolution in T, μ_{Baryon} with $\sqrt{s_{NN}}$

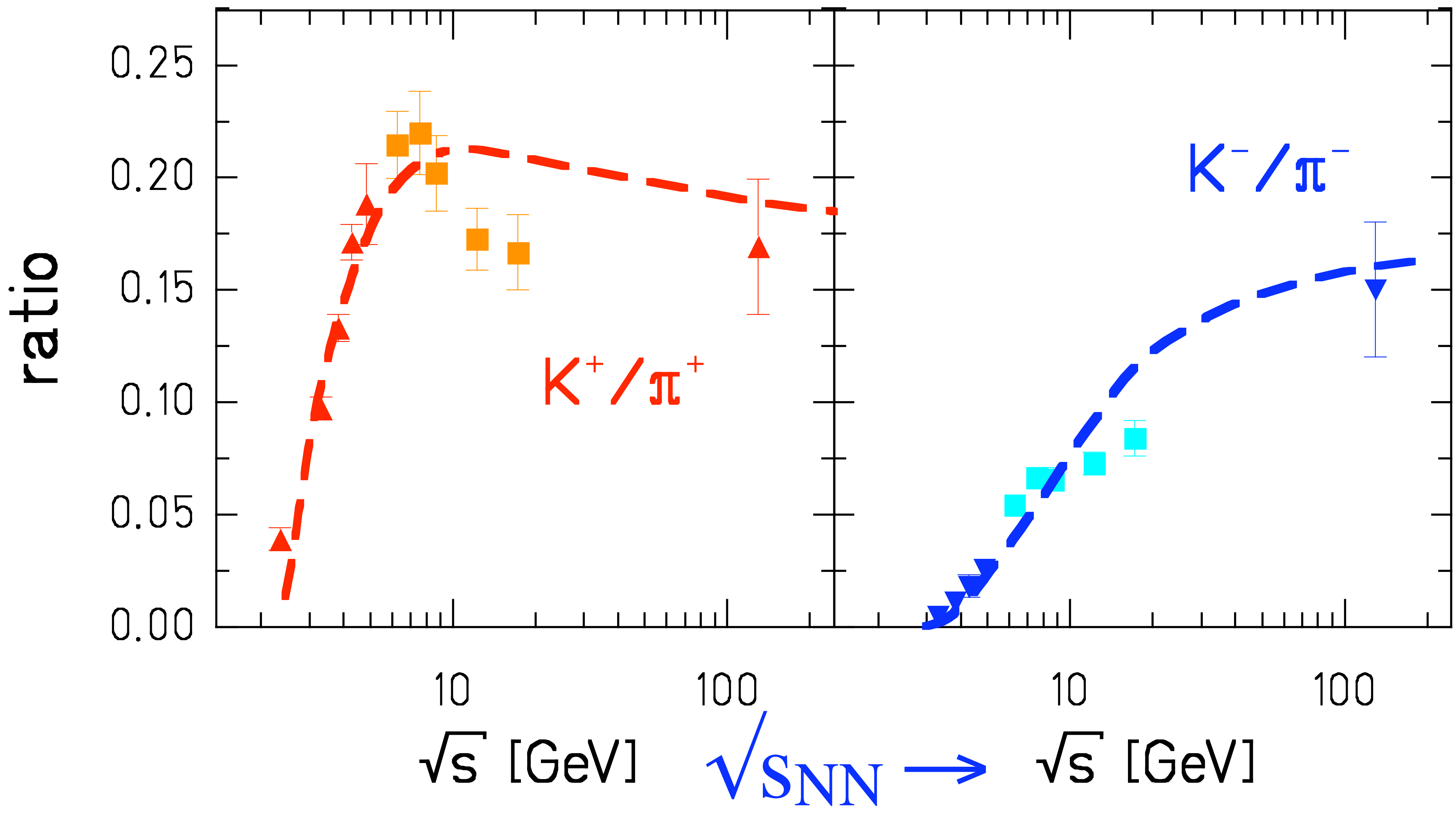
$T \uparrow$

$\mu_B \uparrow$



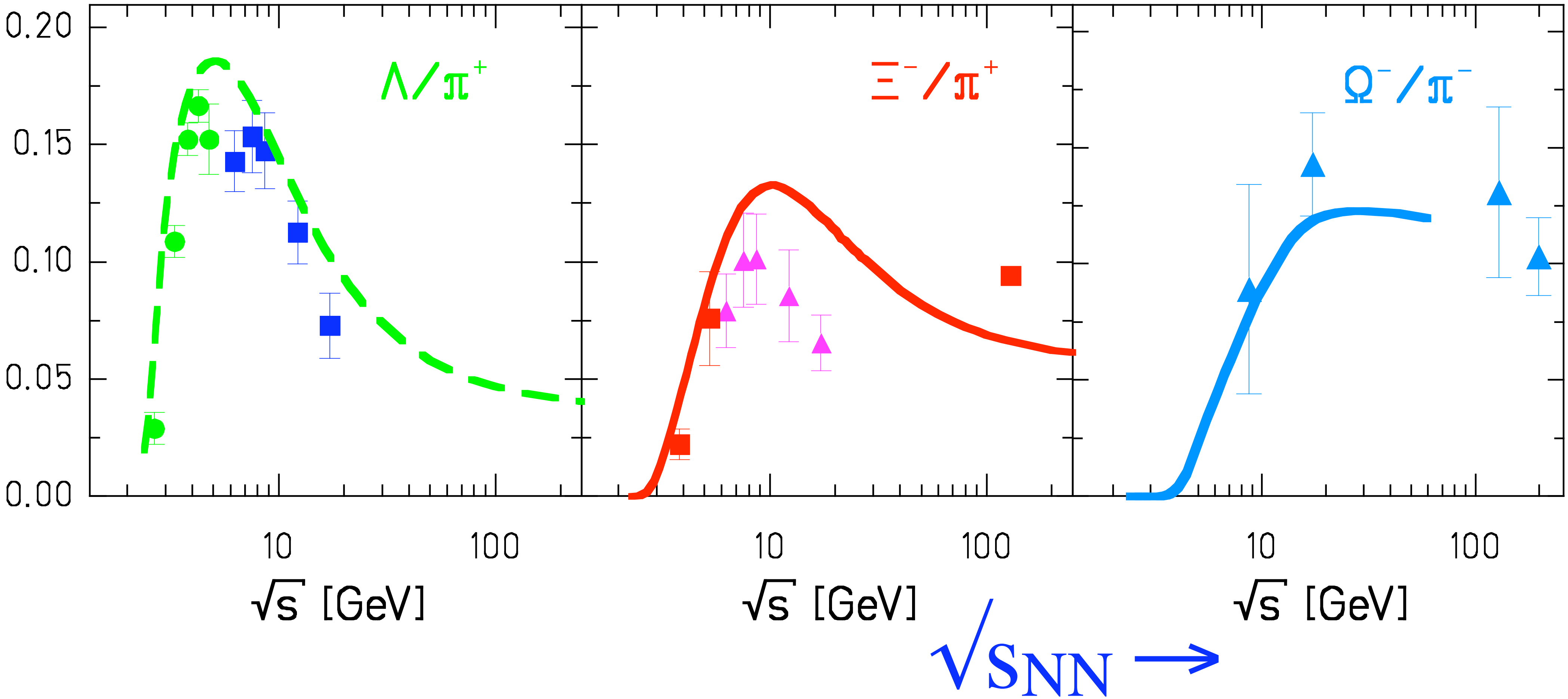
$\sqrt{s_{NN}} \rightarrow$

But strange MatterHorn: peak in K^+/π^+, *not* K^-/π^-

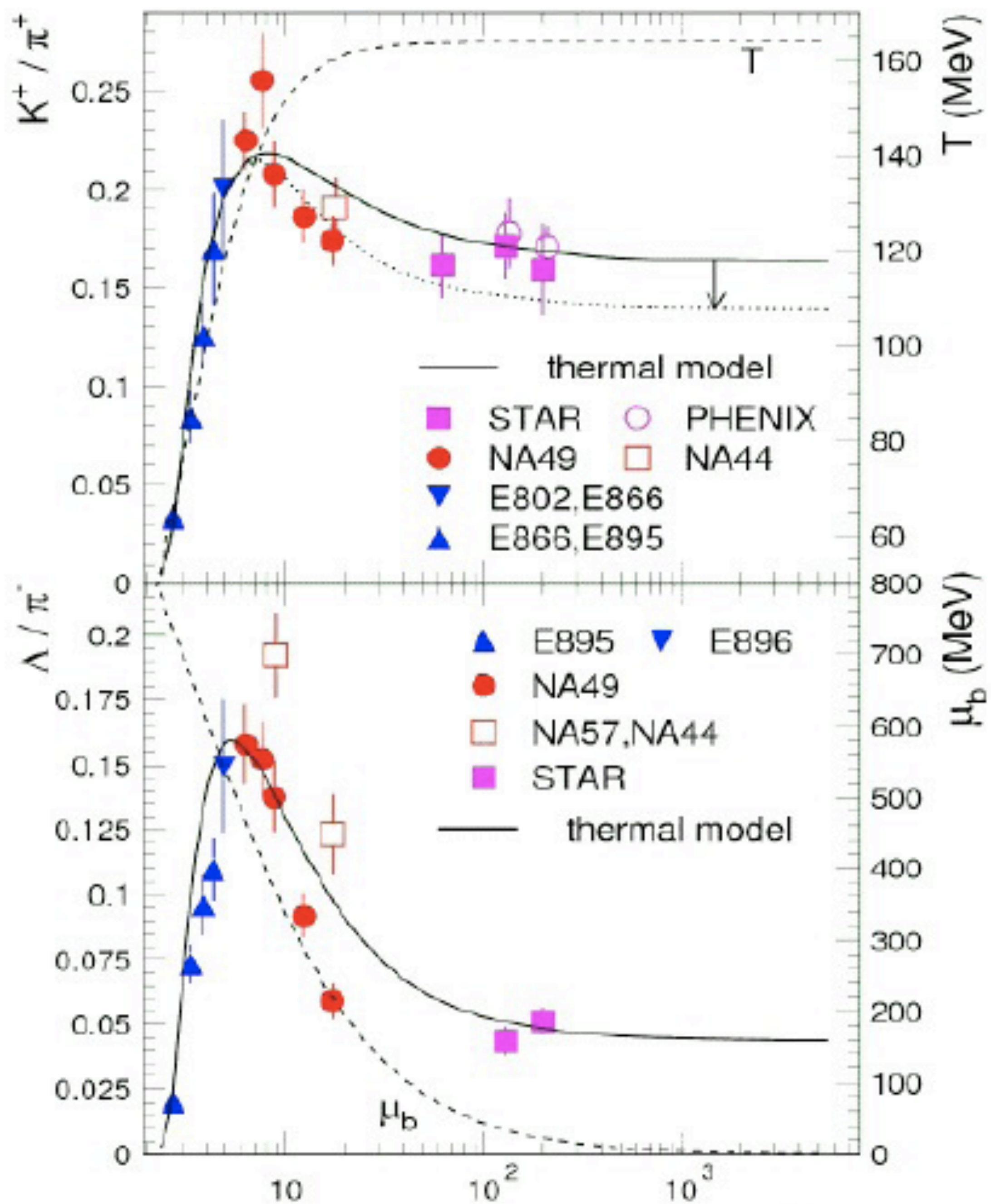


Strange MatterHorn: also in baryons

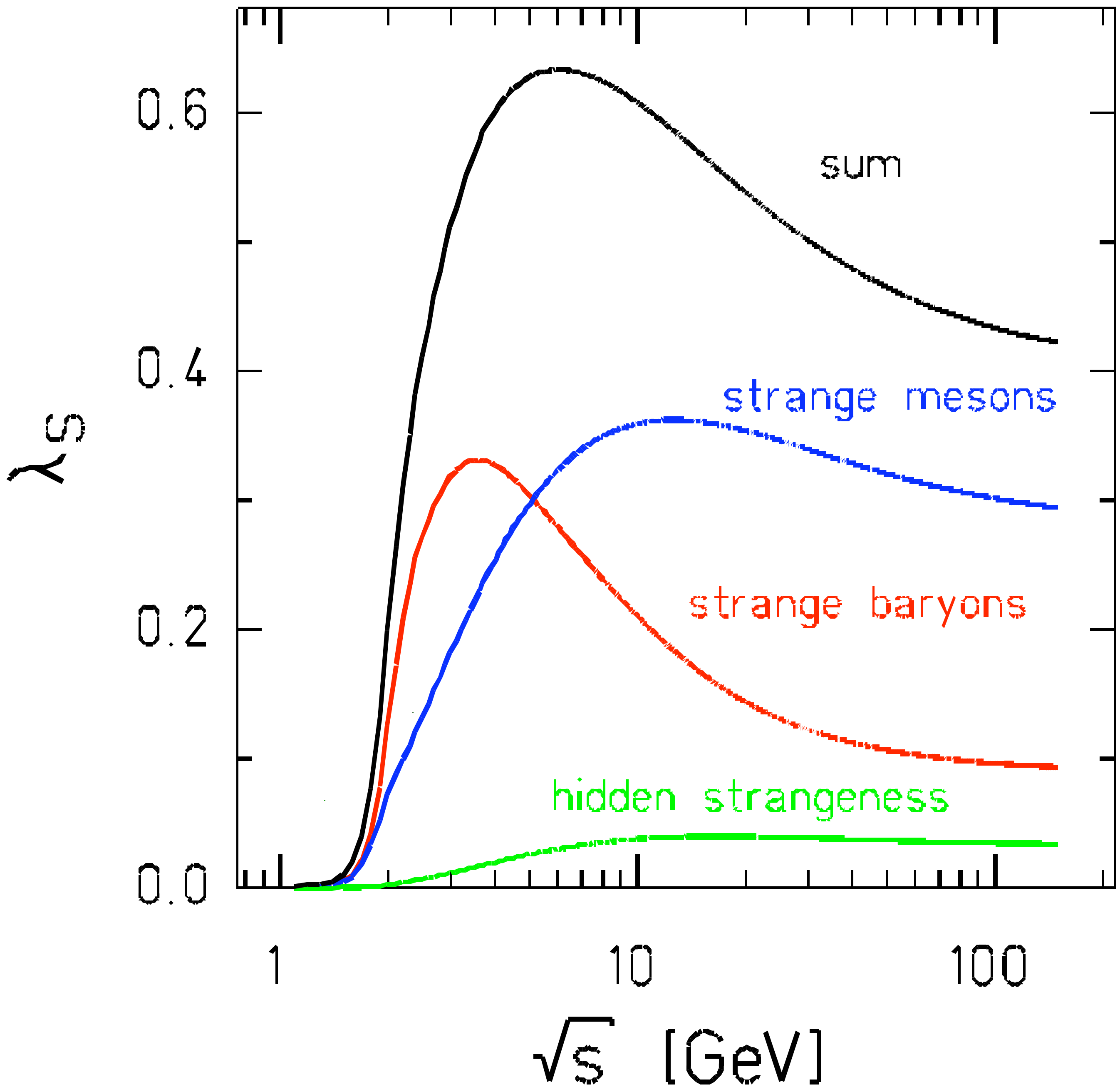
Natural to have peaks in K^+/π^+, strange baryons: start with (s s-bar) pairs.
At $\mu \neq 0$, strange quarks combine into baryons, anti-strange into pions.
For different baryons, peaks do not occur at same energy, but nearby, so not true phase transition, but approximate.



Strange MatterHorn: confirmed up to RHIC



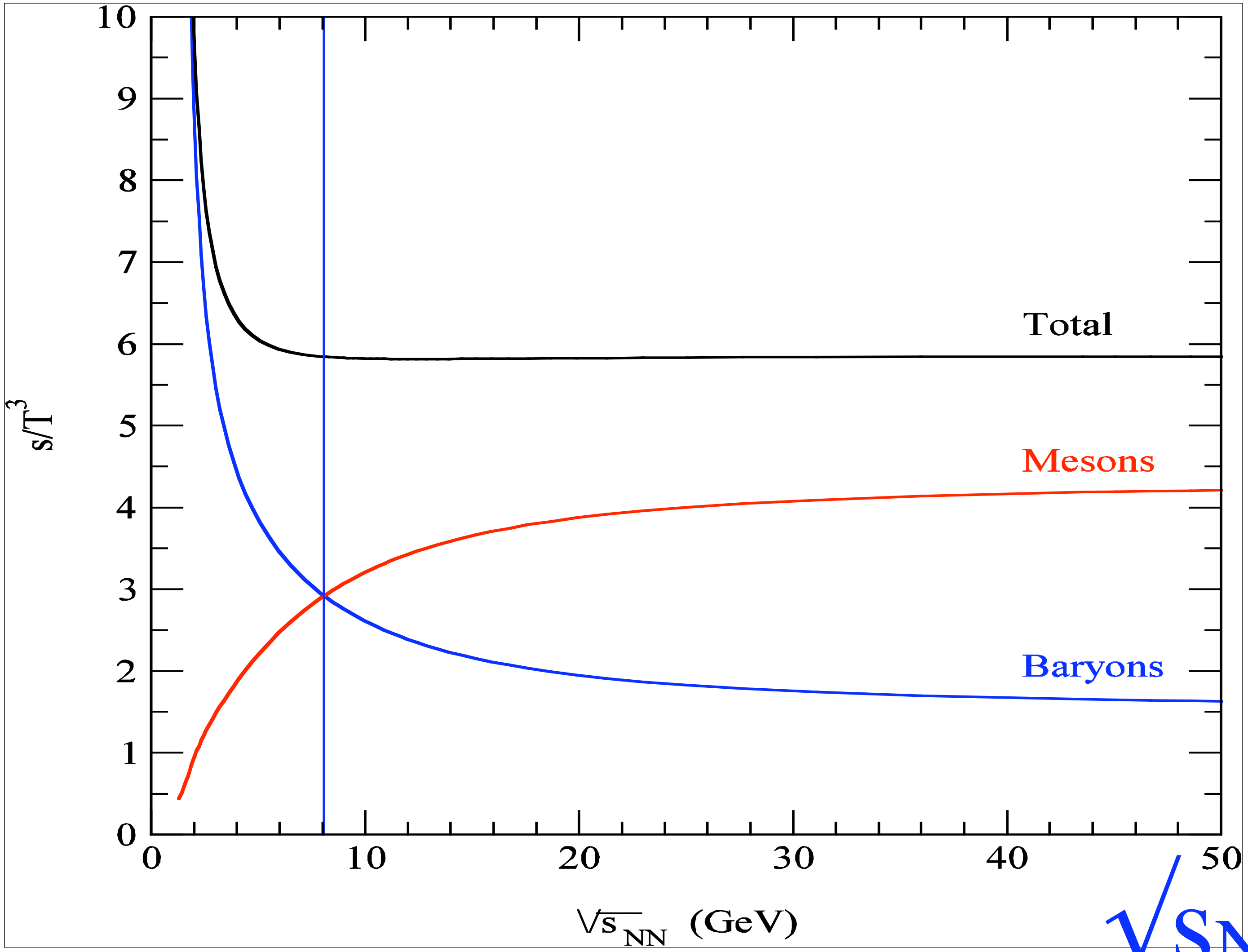
Strange MatterHorn: different for mesons and baryons



Strange MatterHorn and the triple point?

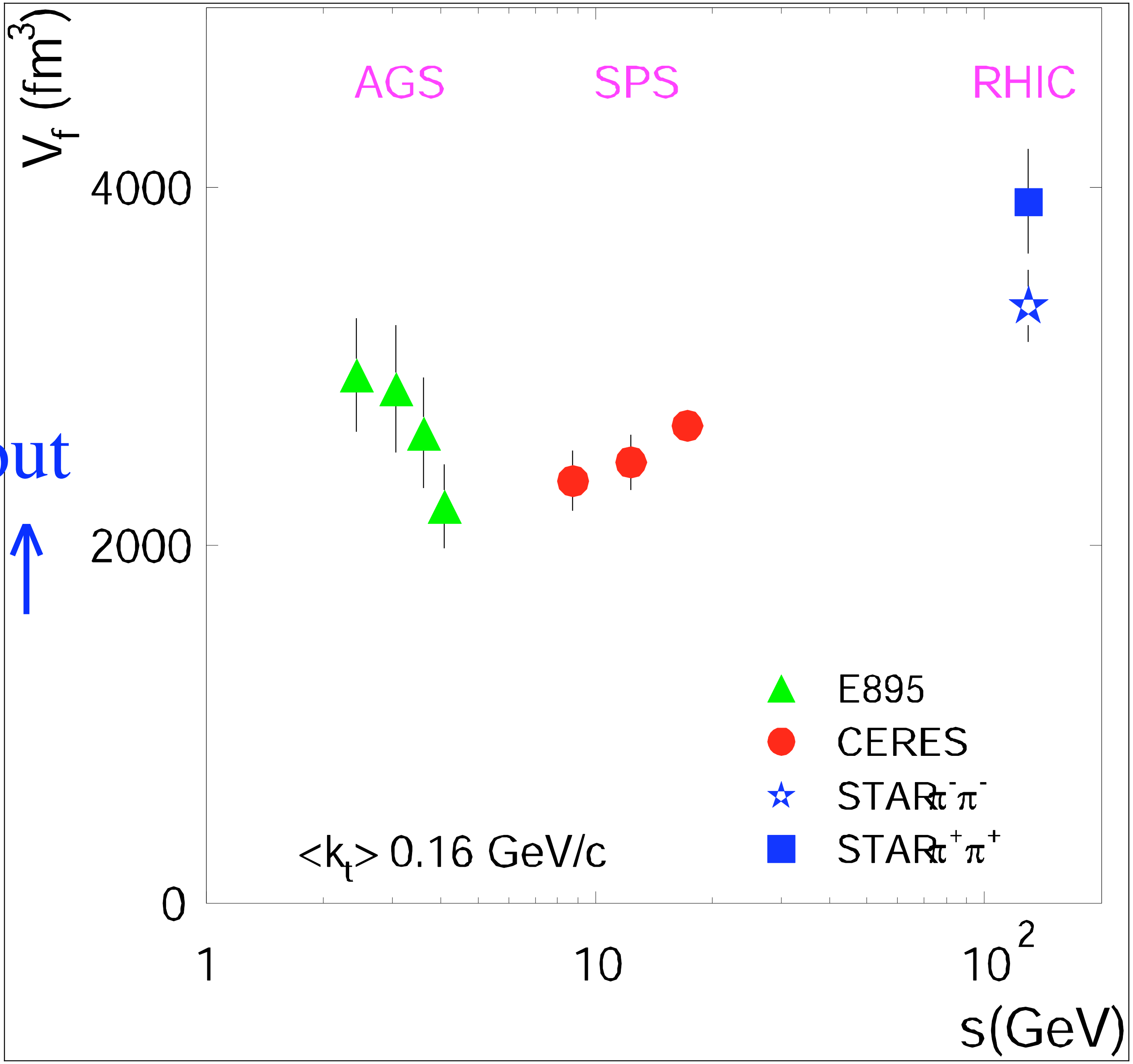
Usual explanation of MatterHorn: transition from baryons to mesons at freezeout
Yes. But also natural if one goes from Quarkyonic line to Quark-Gluon Plasma line.

entropy density/T^3 ↑



$\sqrt{s_{NN}}$ →

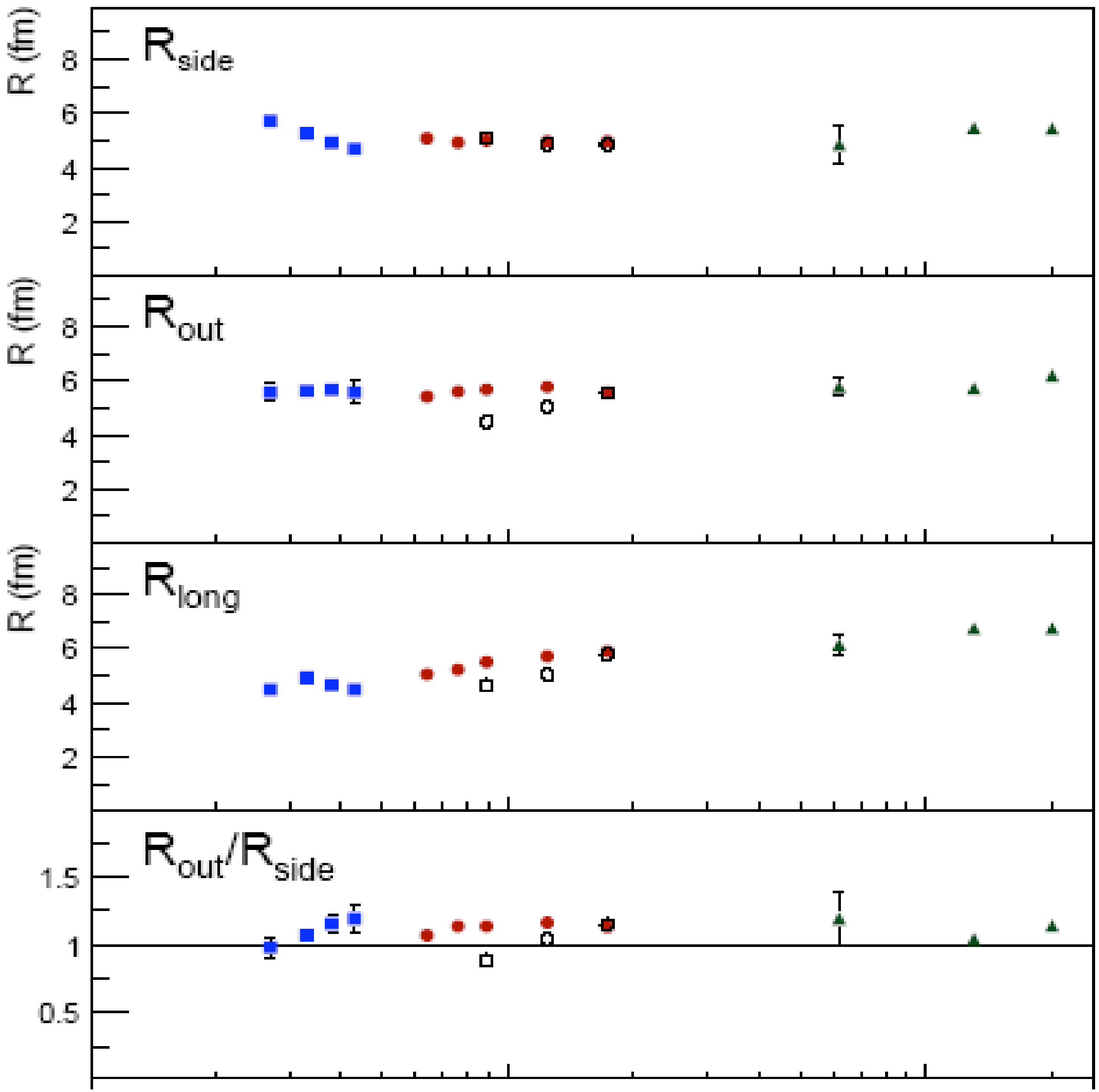
HBT radii: minimum near strange MatterHorn?



freeze out volume ↑

$\sqrt{s_{NN}}$ →

HBT radii: flat from NA49.



$\sqrt{s_{NN}} \rightarrow$

Triple point versus Critical End Point

Critical endpoint: correlation lengths diverge.

Hence: HBT radii should *increase*.

Effects should be greatest on the lightest particles, *not* the heaviest:

K^+/π^+ should *de*crease, not *in*crease. *Neither* seen in the data.

Assume that at triple point, chiral transition splits from deconfining.

Leading operator which couples the two transitions is

Mocsy, Sannino, & Tuominen, hep-ph/0301229, 0306069, 0308135, 0403160:

$$c_1 \, \ell \; \mathrm{tr} \, \Phi^\dagger \Phi \sim c_1 \, \ell \, (\pi^2 + K^2 \ldots)$$

If this coupling c_1 flips sign, transitions diverge. Hence $c_1 = 0$ at triple point?

If so, leading coupling then becomes

$$c_2 \, \ell \; \mathrm{tr} M \, \Phi \sim c_2 \, \ell \, (m_\pi^2 \pi^2 + m_K^2 K^2 + \ldots)$$

This coupling is proportional to mass squared: *bigger* for kaons than pions!

Enchancement of K^+/π^+, strange baryons due to dense environment.

What about chiral symmetry?

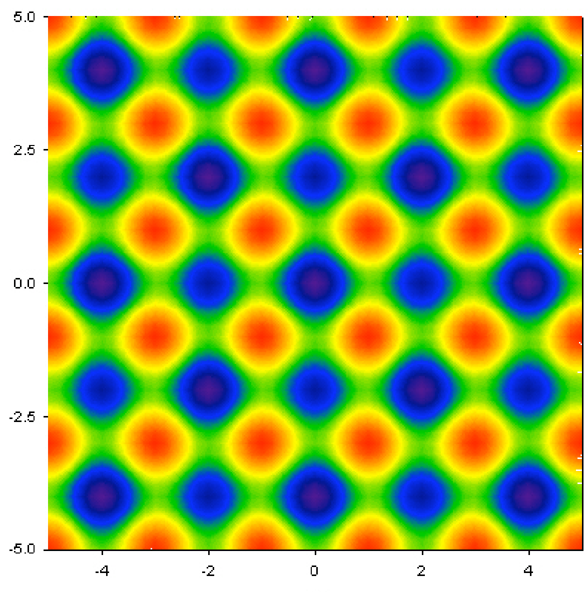
How can you *restore* chiral symmetry in a *confined* phase?

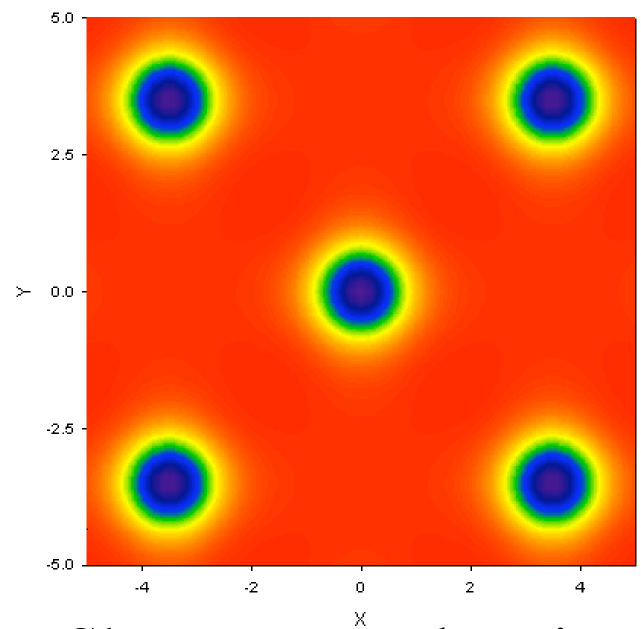
Best example? Skyrmions!

Skyrmion crystals

Kutschera, Pethick & Ravenhall (KPR) '84; Klebanov '85 + ...
Lee, Park, Min, Rho & Vento, hep-ph/0302019; Park, Lee, & Vento, 0811.3731:

At large N_c, baryons are heavy, so form a crystal.
Form Skyrmion crystal by taking periodic boundary conditions in a box.
LPMRV '03 : box of size L, in units of $1/(\sqrt{\kappa}\, f_\pi)$, plot baryon number density:

L=2.0→




←L=3.5

At low density, chiral symmetry broken by Skyrme crystal, as in vacuum.

But chiral symmetry *restored* at nonzero L (density): $< U > = 0$ in *each* cell.

Skyrmion crystals as Quarkyonic matter

Why chiral symmetry restoration in a Skyrmion crystal?
Goldhaber & Manton '87: due to "half" Skyrmion symmetry in each cell.

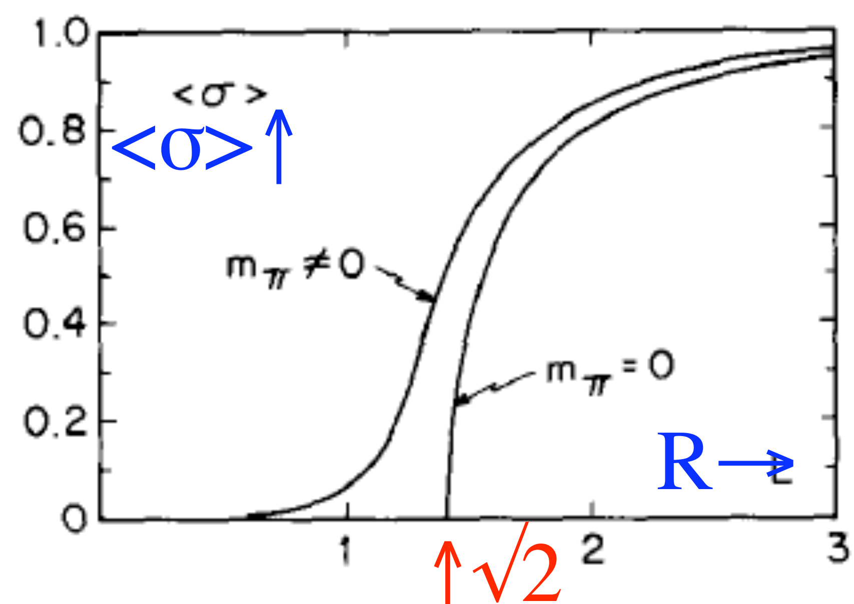
Easiest to understand with "spherical" crystal: sphere instead of cube...
KPR '84, Ruback & Manton '86, Manton '87. Consider the "trivial" map:

$$U(r) = \exp(i\, f(r)\, \hat{r}\cdot\tau)\ ;\ \ f(r) = \pi\left(1 - \frac{r}{R}\right)$$

Solution has unit baryon number per unit sphere, and so is a crystal.
Solution is minimal when $R < \sqrt{2}$ $(*\ 1/(\sqrt{\kappa}\, f_\pi)$.

Forkel, Jackson, Rho, & Weiss '89 =>
looks like standard chiral transition!
Excitations *are* chirally symmetric.

But Skyrmions are *not* deconfined.
Example of Quarkyonic matter,
chirally broken and chirally symmetric.



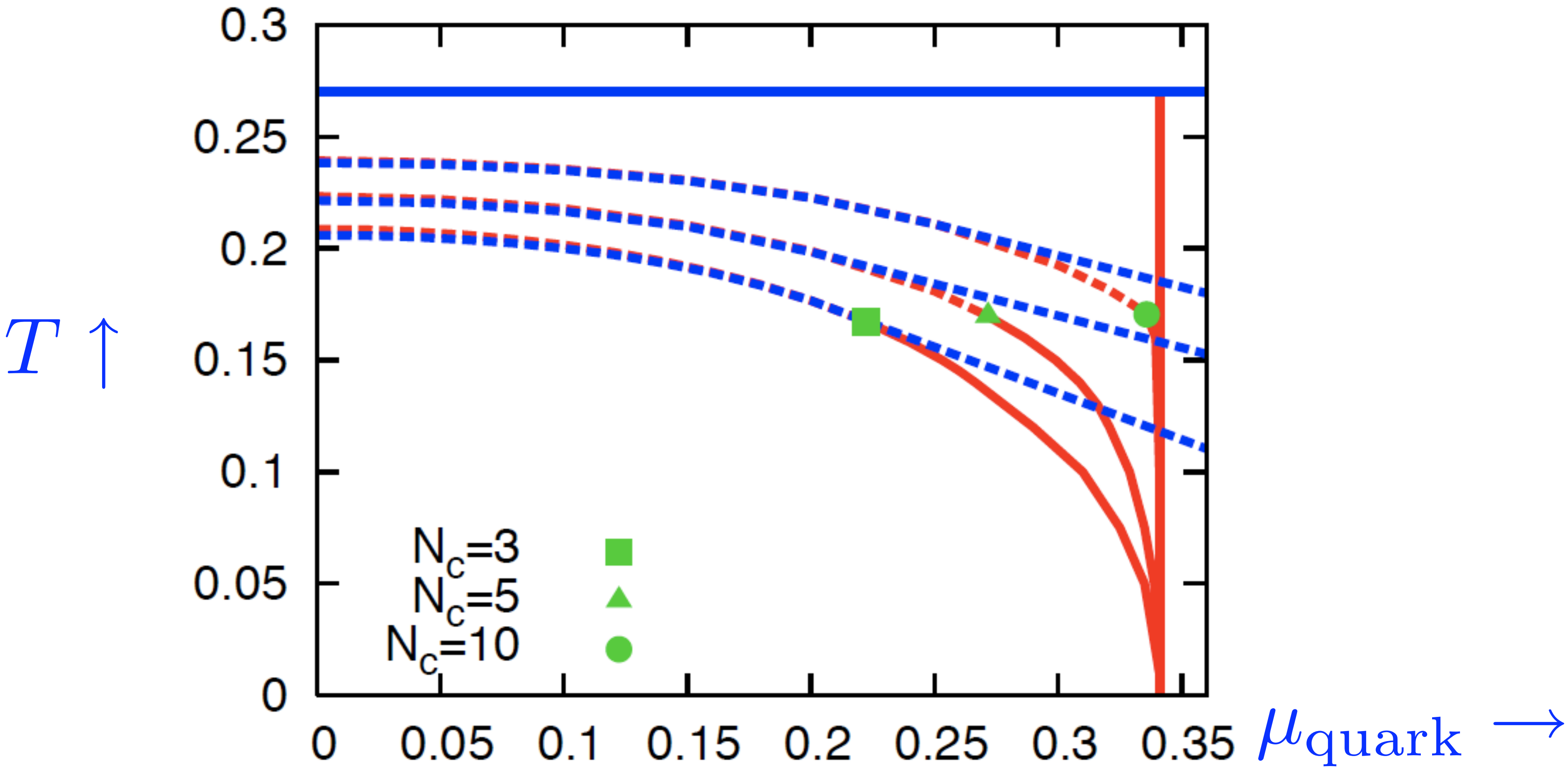
Examples of *quadruple* point in P-NJL models:

deconfining and chiral transitions split

Quarkyonic matter in the Polyakov NJL model

McLerran, Redlich, & Sasaki 0812.3585 Use Polyakov NJL model:
K. Fukushima hep-ph/0303225, hep-ph/0310121, 0803.3318, 0809.3080, 0901.4821
Ratti, Thaler, & Weise ph/0506234, nucl-th/0604025, ph/0609281, 0712.3152, 0810.1099
Sasaki, Friman, & Redlich, hep-ph/0611143, hep-ph/0611147, 0806.4745, 0811.4708

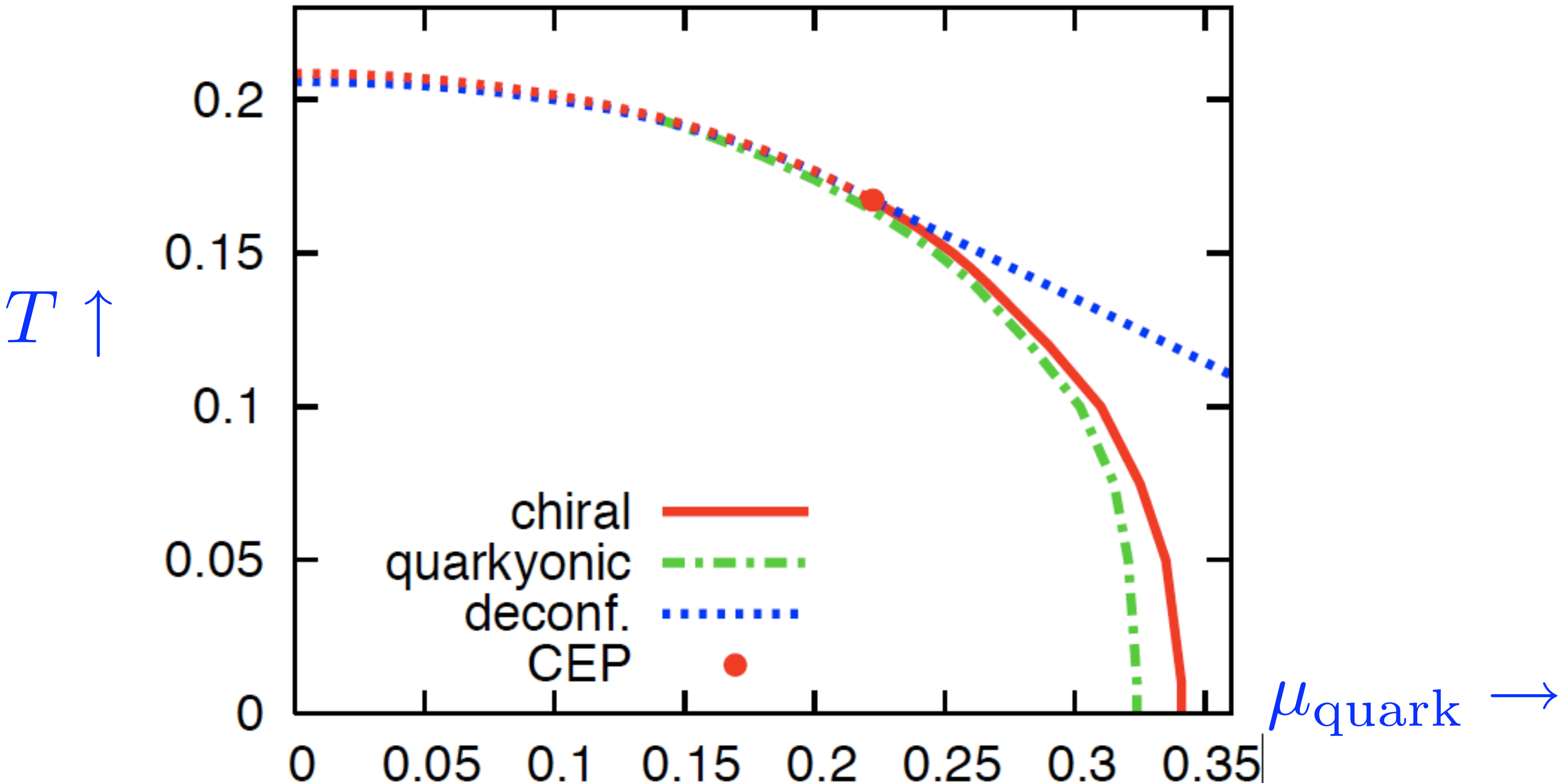
"Straightening" of the line for deconfinement as N_c increases:



Chiral transition and Quarkyonic matter in the P-NJL model

QCD: chiral Critical End Point (CEP), Shuryak, Stephanov, & Rajagopal '99 '00.

Chiral, quarkyonic, & deconfining transitions split at chiral CEP.
Deconfining CEP at larger densities?



(Lunatic) ideas about nuclear matter:

"The unbearable lightness of being (nuclear matter)"

Nucleon-nucleon potentials from the lattice

Ishii, Aoki & Hatsuda, PACS-CS, 0903.5497

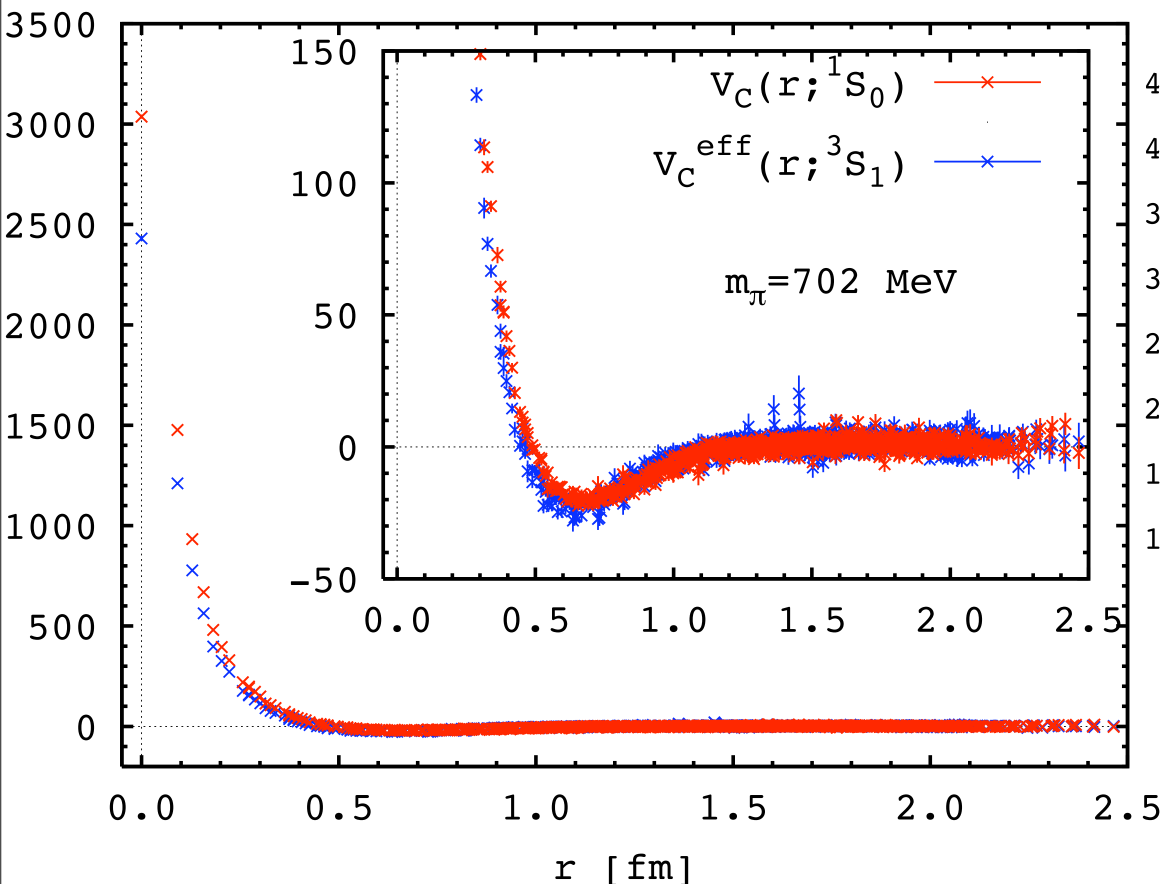
Nucleon-nucleon potentials from quenched and 2+1 flavors.

Pions heavy: 700 MeV (left) and 300 MeV (right)

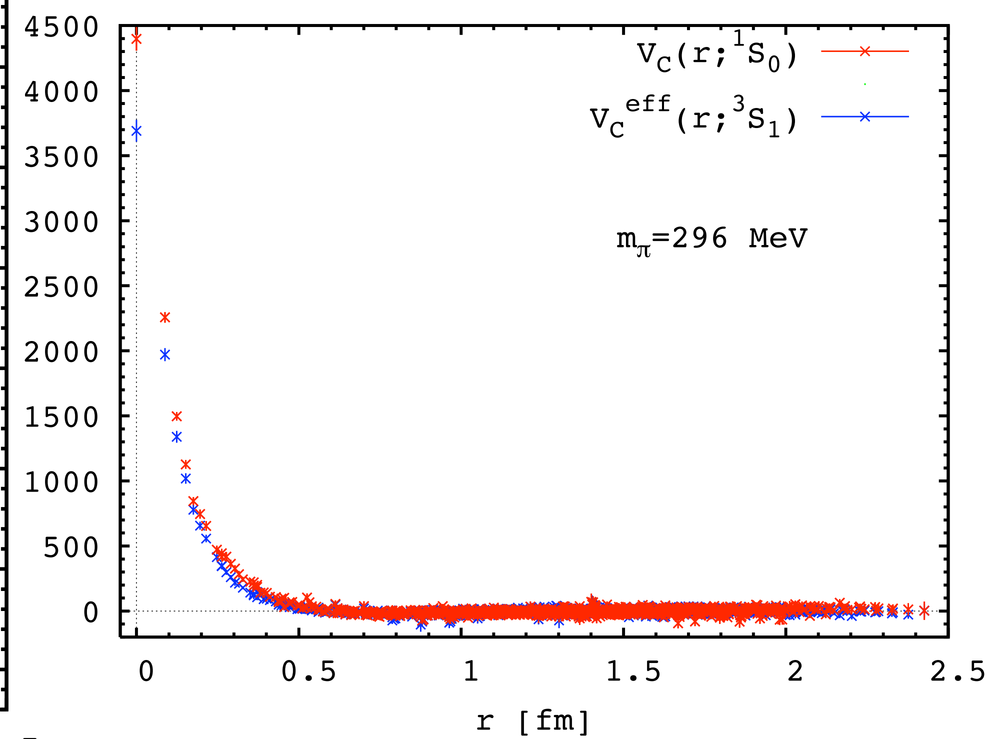
Standard lore: delicate cancellation. *So why independent of pion mass?*

Essentially *zero* potential plus strong hard core repulsion

$m_\pi = 702$ MeV



$m_\pi = 296$ MeV



Purely pionic nuclear matter

J.-P. Blaizot, L. McLerran, M. Nowak, & RDP '09....

At infinite N_c, integrate out *all* degrees of freedom *except* pions:

Lagrangian power series in $U = \mathrm{e}^{i\pi/f_\pi}\ ,\ V_\mu = U^\dagger \partial_\mu U$

Infinite # couplings: Skyrme *plus* complete Gasser-Leutwyler expansion,

$$\mathcal{L}_\pi = f_\pi^2\, V_\mu^2 + \kappa\, [V_\mu, V_\nu]^2 + c_1\, (V_\mu^2)^2 + c_2\, (V_\mu^2)^3 + \ldots$$

All couplings ~ N_c , every mass scale ~ typical hadronic.

Need *infinite* series, but nothing (special) depends upon exact values

Valid for momenta < f_π, masses of sigma, omega, rho...

Useful in (entire?) phase with chiral symmetry breaking?

Higher time derivatives, but no acausality at low momenta.

Purely pions give free baryons

From purely pionic Lagrangian, take baryon as stationary point.

Find baryon mass ~ N_c, some function of couplings.

Couplings of baryon dictated by chiral symmetry:

$$\overline{\psi}\left(i\partial\!\!\!/ + M_B\, \mathrm{e}^{i\tau\cdot\pi\gamma_5/f_\pi}\right)\psi$$

By chiral rotation, $W = \exp(-i\pi\gamma_5/2f_\pi)$

$$\mathcal{L}_B = \overline{\psi}\left(iW^\dagger\partial\!\!\!/ W + M_B\right)\psi \sim \frac{1}{f_\pi}\overline{\psi}\gamma_5\partial\!\!\!/\pi\psi + \ldots$$

At large ~ N_c , $f_\pi \sim N_c^{1/2}$ is *big*. Thus for momenta k < hadronic, interactions are *small*, ~ $1/f_\pi^2 \sim 1/N_c$.

Thus: baryons from chiral Lag. free at large N_c , down to distances $1/f_\pi$.

Manifestly special to chiral baryons. True for u, d, s, but *not* charm?

The Unbearable Lightness of Being (Nuclear Matter)

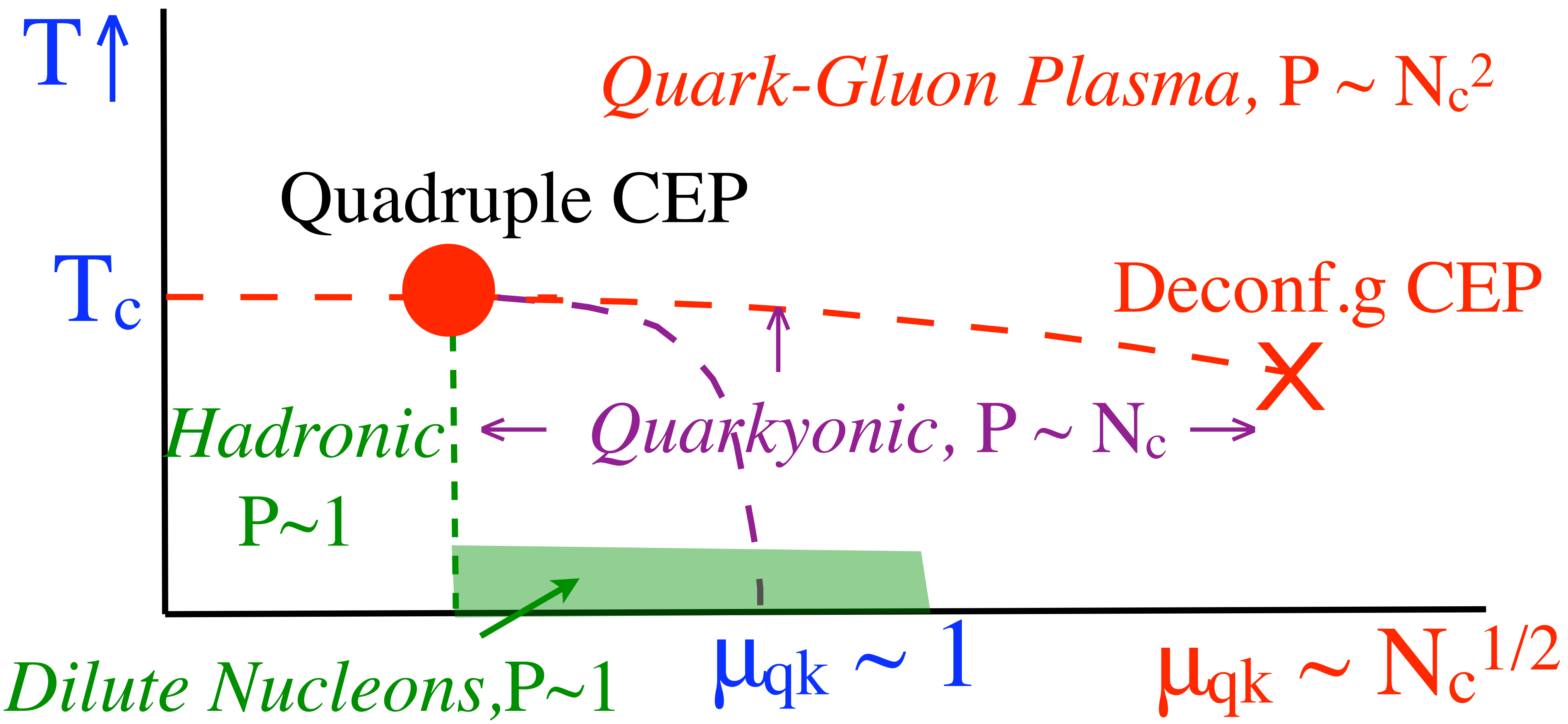
Nuclear matter: crystal of baryons.

Use purely pionic Lagrangian for all of nuclear matter?

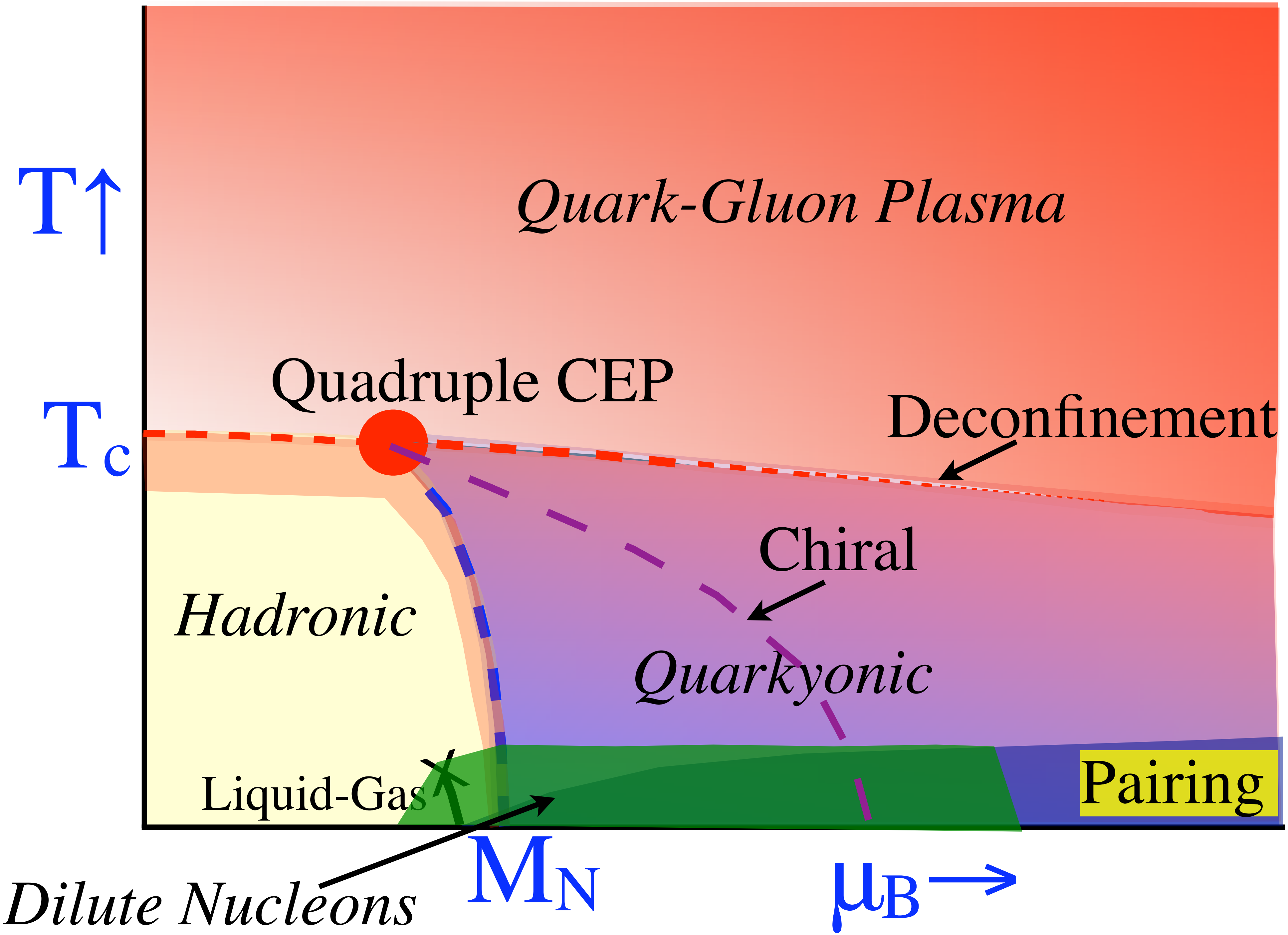
Then pressure ~ 1, and *not* N_c. Like hadronic phase, *not* quarkyonic.

*Un*like standard lore, where pressure(nucl mat) grows quickly, ~ N_c

Red line: 1st order. Green line: Baryons condense. Purple: chiral trans.



An unbearably light phase diagram for QCD

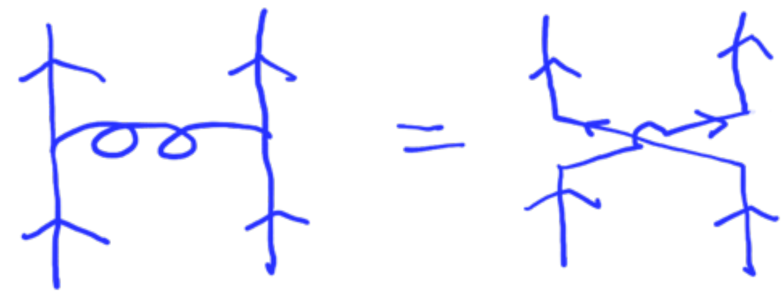


Large N_c: not color superconductivity,

but chiral density waves (pion condensation)

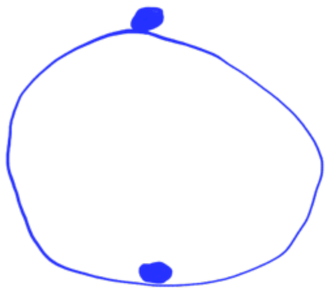
Chiral Density Waves (perturbative)

Excitations near the Fermi surface?



At large N_c, color superconductivity suppressed,
$\sim 1/N_c$: pairing into two-index state:

Also possible to have "chiral density waves", pairing of quark and anti-quark:
Deryagin, Grigoriev, & Rubakov '92. Shuster & Son, hep-ph/9905448.
Rapp, Shuryak, and Zahed, hep-ph/0008207.



Order parameter $\langle\overline{\psi}(-\vec{p}_f)\,\psi(+\vec{p}_f)\rangle$
Sum over color, so *not* suppressed by $1/N_c$.

Pair quark at $+\,p_f$ with anti-quark at $-\,p_f$: for a *fixed* direction.
Breaks chiral symmetry, with state varying $\sim \exp(-\,2\,p_f\,z)$.

Wins over superconductivity in low dimensions. Loses in higher.
Shuster & Son '99: in perturbative regime, CDW only wins for $N_c > 1000\,N_f$

Quarkyonic chiral density waves

Consider meson wave function, with kernel:
Confining potential in 3+1 dimensions like
Coulomb potential in 1+1 dim.s:

$$\int dk_0\, dk_z \int d^2k_\perp \; \frac{1}{(k_0^2+k_z^2+k_\perp^2)^2} \sim \int dk_0\, dk_z \; \frac{1}{k_0^2+k_z^2}$$

In 1+1 dim.'s, behavior of massless quarks near Fermi surface maps ~ $\mu = 0$!

Mesons in vacuum naturally map into CDW mesons.

Witten '84: in 1+1 dim.'s, use non-Abelian bosonization for QCD.

a, b= 1...N_c. i,j = 1... N_f.

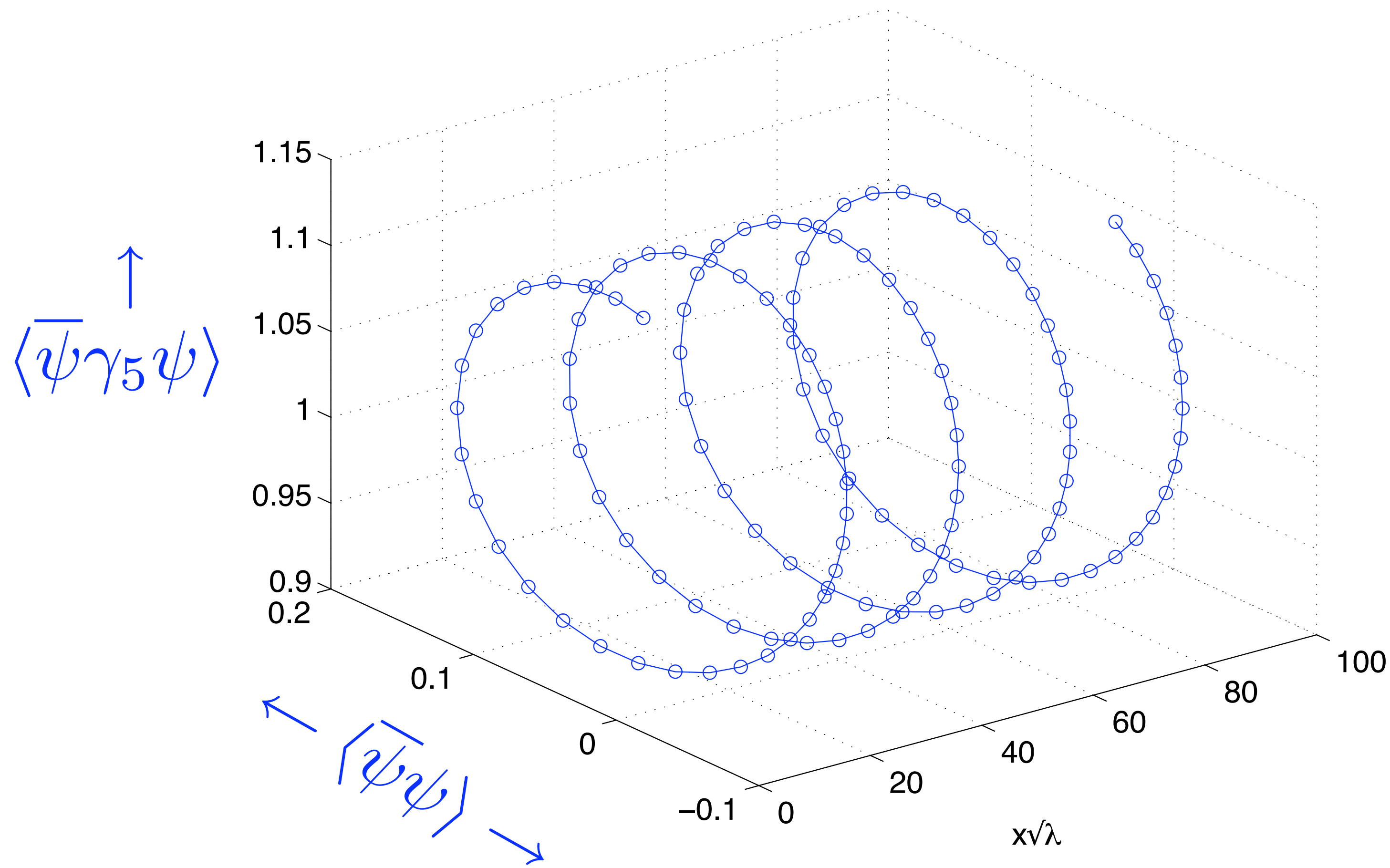
$$J_+^{ij} = \overline{\psi}^{a,i}\psi^{a,j} \sim g^{-1}\partial_+ g \; ; \; J_+^{ab} = \overline{\psi}^{a,i}\psi^{b,i} \sim h^{-1}\partial_+ h \; .$$

Steinhardt '80. Affleck '86. Frishman & Sonnenschein, hep-th/920717...
Armoni, Frishman, Sonnenschein & Trittman, hep-th/9805155; AFS, hep-th/0011043..
Bringoltz 0901.4035; Galvez, Hietanan, & Narayanan, 0812.3449.

Solution to dense QCD in 1+1 dimensions

Bringoltz, 0901.4035: ‘t Hooft model, with massive quarks.
Works in Coulomb gauge, in *canonical* ensemble: fixed baryon number.
Solves numerically equations of motion under constaint of nonzero baryon #
Finds chiral density wave.

N.B.: for massive quarks, should have massless excitations, but with energy $\sim 1/N_c$.



Quarkyonic matter for *two* colors?

i.e., are large N_c arguments even ok for $N_c = 2$?

Quarkyonic matter for two colors?

Hands, Ilgenfritz, Kenny, Kim, Mueller-Preussker, Schubert, Sitch, & Skullerud,
HIKKMPSSS 09....

$N_c = N_f = 2$, Wilson fermions. *No* sign problem, measure real.

Mesons and baryons are both bosons. Baryon μ_{qk} like isospin μ_{qk} for 3 colors.

μ_{qk} only matters when $> \mu_0 = m_\pi/2$ (not $> m_{Baryon}$).

Expect Bose-Einstein condensate for $\mu_{qk} > \mu_0$, compare to Chiral Pert. Theory.

Find: $\mu_0\, a = 0.2$: BEC turns on, good agreement with CPT only *very* near μ_0 .

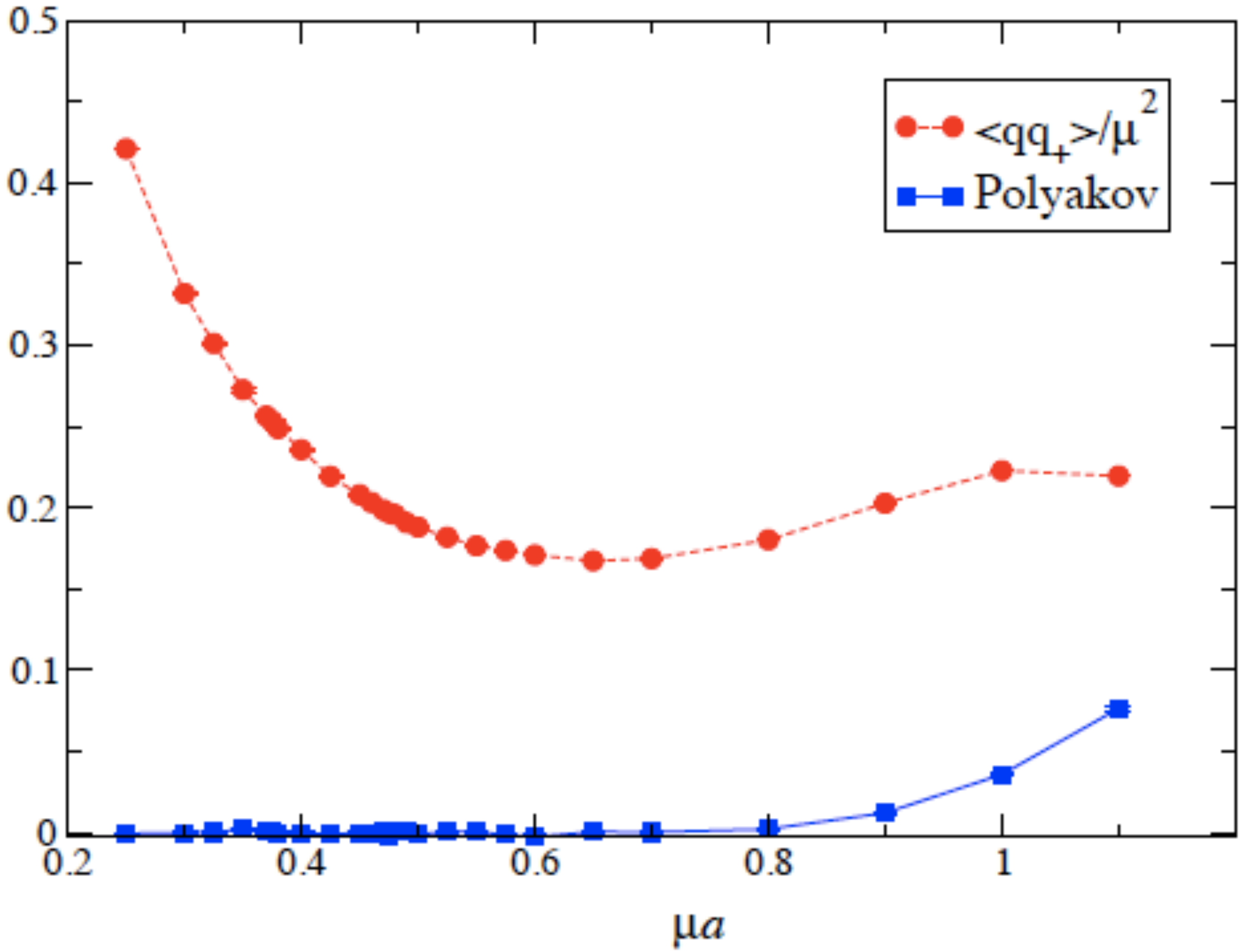
$\mu_t\, a = 0.4$: *big* jump in energy density - ?

$\mu_d\, a = 0.65$: Polyakov loop nonzero, deconfined quarks,
only at high density

Suggests: Quarkyonic matter for a large range, between μ_0 and μ_d , for $N_c = 2$!

Order Parameters

HIKKMPSSS 09...



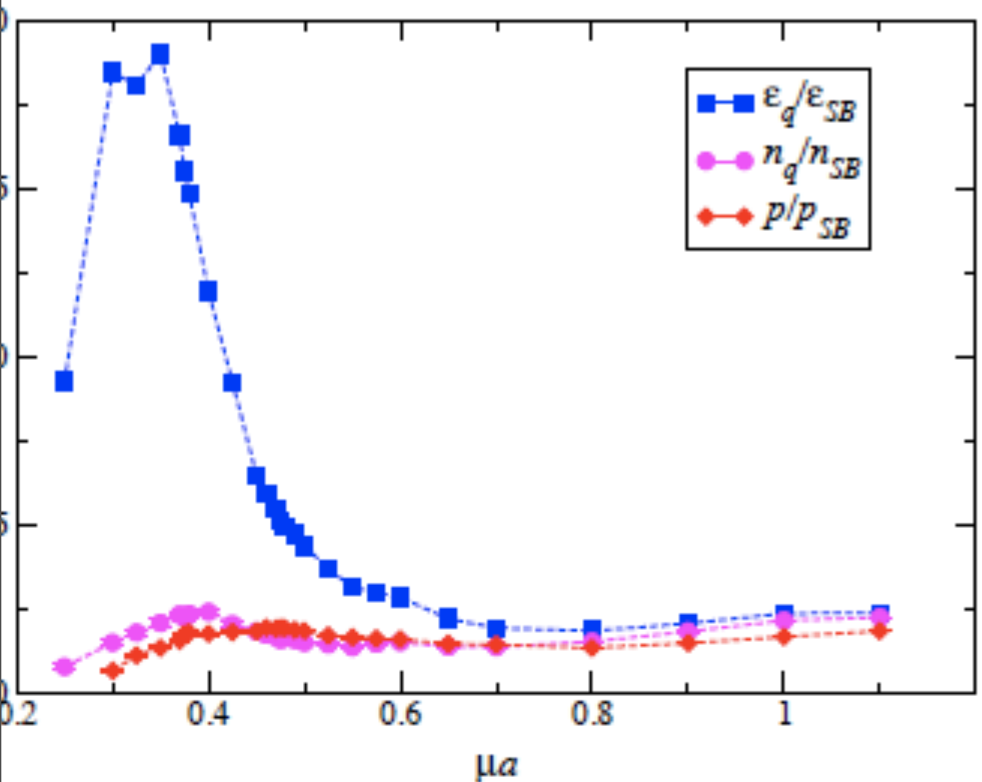
Superfluid condensate scaling ≈ BCS for $\mu_Q \lesssim \mu \lesssim \mu_d$

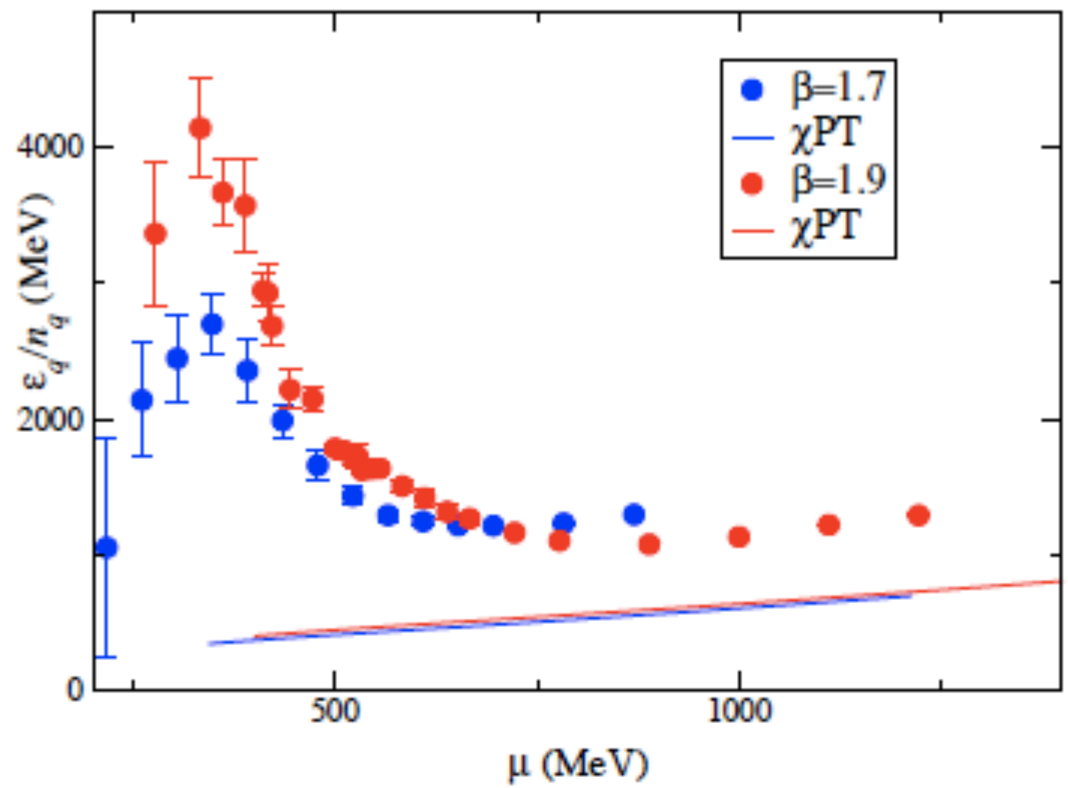
Polyakov loop ≈ 0 for $\mu < \mu_d$, but then rises from zero

⇒ Deconfinement at $\mu \approx 900$MeV, $n_q \approx 35\,\text{fm}^{-3}$

Quark Energy Density

HIKKMPSSS 09....





In contrast to χPT prediction, (unrenormalised) quark energy density ε_q greatly exceeds SB value as $\mu \searrow \mu_{o+}$

$\Rightarrow$

Energy per quark ε_q/n_q has shallow minimum for $\mu > \mu_Q$

Or maybe we’re just full of it...

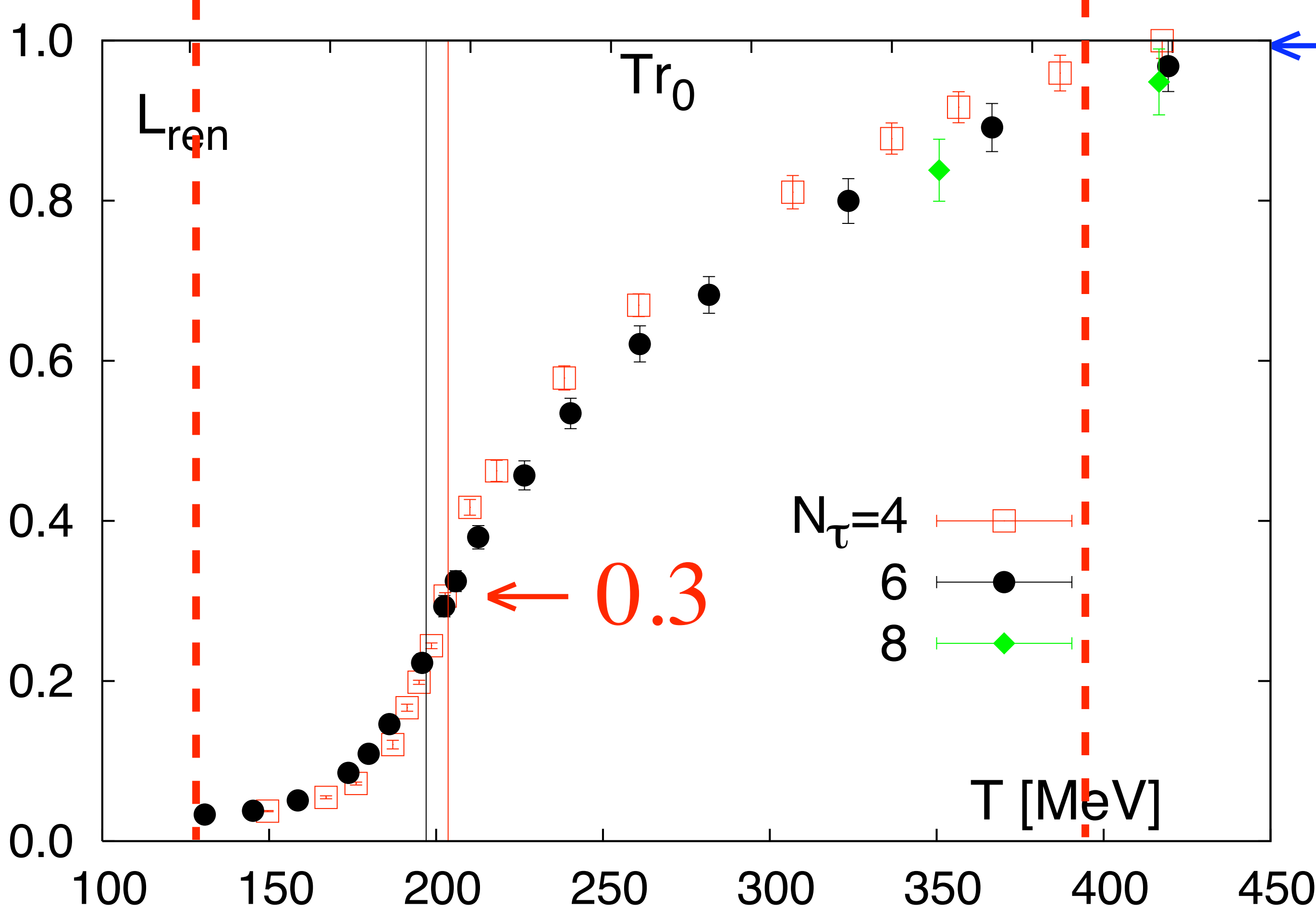
Lattice: renormalized loop, with quarks

Cheng et al, 0710.0354: ~ QCD, 2+1 flavors. T_c ~ 190 MeV, crossover.

<*Polyakov loop*>: nonzero from ~ 0.8 T_c; ~ 0.3 at T_c; ~ 1.0 at 2 T_c.

Deconfinement from ~ 0.8 T_c , *below T_c* (Semi-QGP starts from 0.8 T_c)

← Confined →← Semi-QGP →←Complete QGP



No quark Fermi-surface for even N_c?

Langfeld, Wellegehausen & Wipf, 0906.5554:

Baryons are bosons for even N_c, fermions for odd N_c. LWW say at nonzero T:

"…for even N_c, and in the confinement phase, the quark determinant is independent of the boundary conditions, periodic or anti-periodic ones"

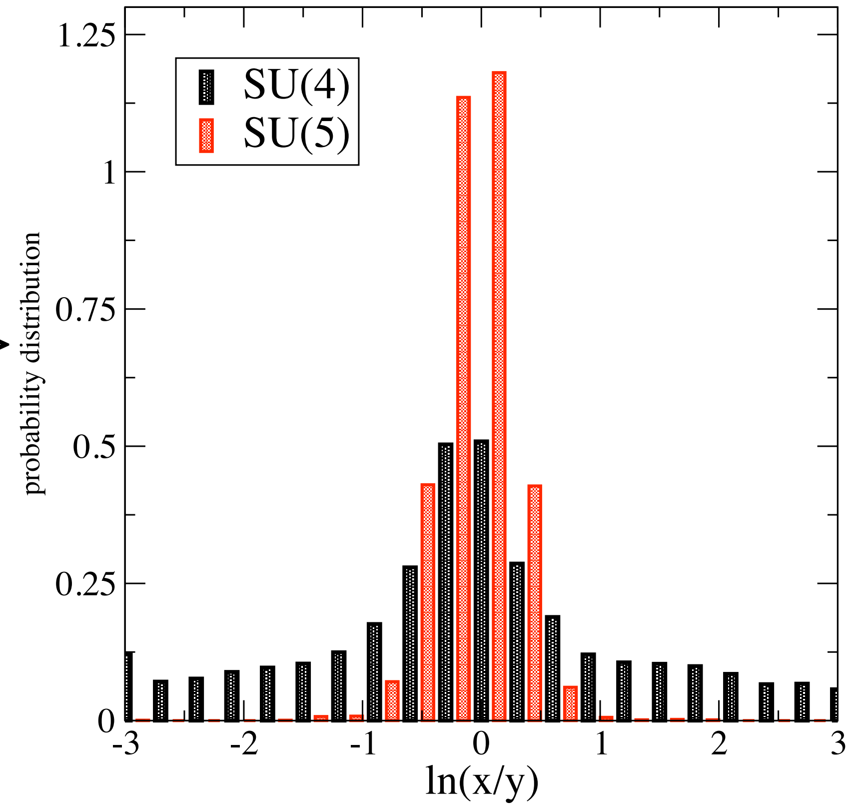
But - there is *no* "confined" phase with *dynamical* quarks.

For any number of flavors, the Polyakov loop is *always* nonzero, even *below* T_c.

With dynamical quarks, the "semi"-QGP extends *below* T_c.

Above claim by LWW fails by - ? - .95 T_c.

Quasiparticles are different, because baryons differ. Seen by LWW in eigenvalues =>



So? For large μ_{qk}, baryons differ, still quarkyonic phase. *But down to $N_c = 2$?*