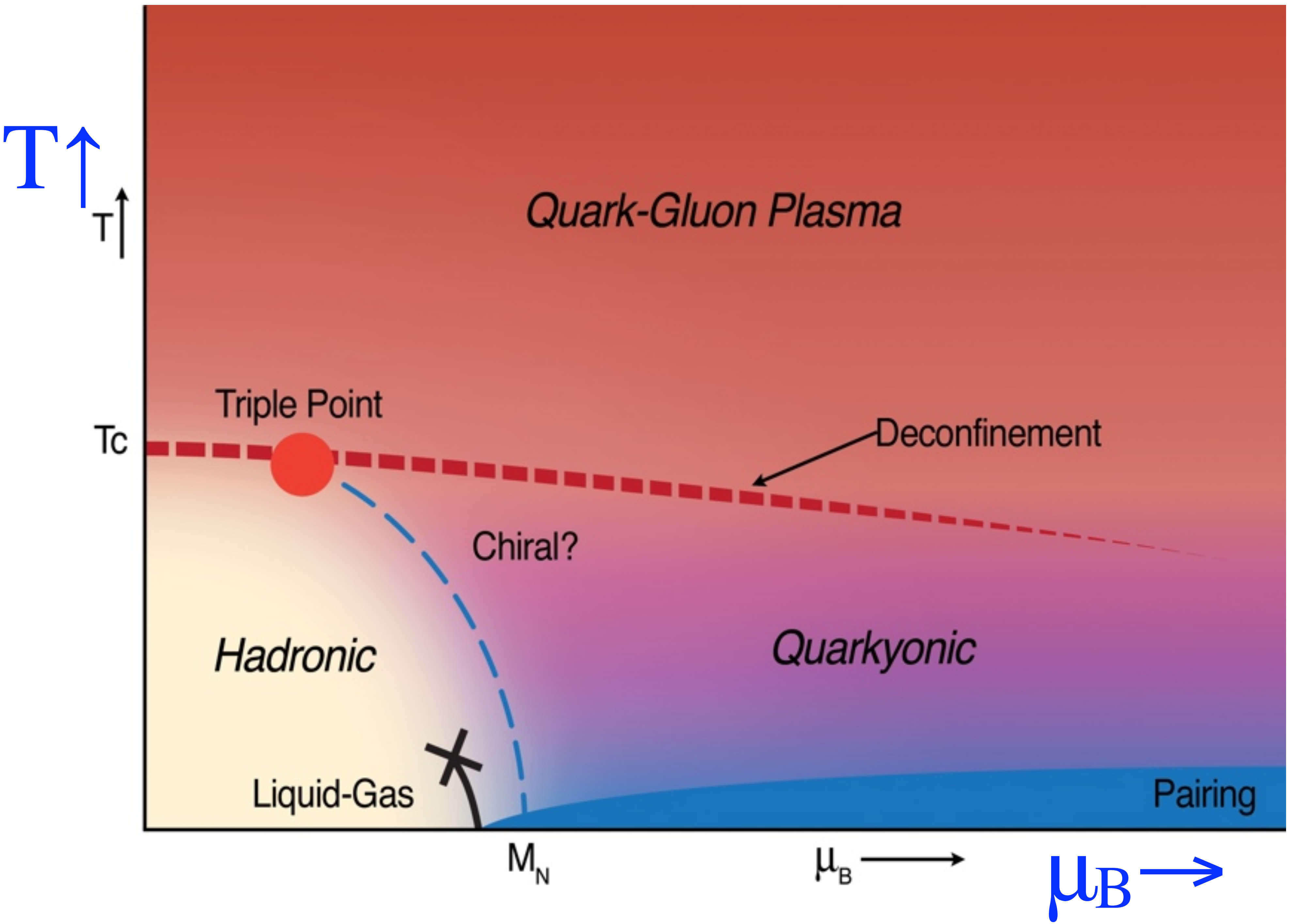

T↑
T
Quark-Gluon Plasma
Triple Point
Tc
Deconfinement
Chiral?
Hadronic
Quarkyonic
Liquid-Gas
Pairing
M_N
μ_B
μ_B →

A triple point in the QCD phase diagram?
From SPS, to RHIC, & (down) to FAIR

1. Large N_c, small N_f:
   Quark-yonic matter - quark- Fermi sea *plus* bar-yonic Fermi surface
   Triple point. Deconfining critical end point at *large* $\mu_{qk} \sim N_c^{1/2}$
2. New phase diagram for QCD
3. "Purely pionic" effective Lagrangians and nuclear matter:
   The unbearable lightness of being (nuclear matter)?

Strange "MatterHorn" ≈ Triple Point?

McLerran & RDP, 0706.2191. Hidaka, McLerran, & RDP 0803.0279
McLerran, Redlich & Sasaki 0812.3585
Hidaka, Kojo, McLerran, & RDP 09......
Blaizot, Nowak, McLerran & RDP 09......

Blaschke, Braun-Munzinger, Cleymans, Fukushima, Oeschler,
RDP, McLerran, Redlich, Sasaki, & Stachel (BBMCFOPMRSS) '09....

So what *is* Quarkyonic matter?

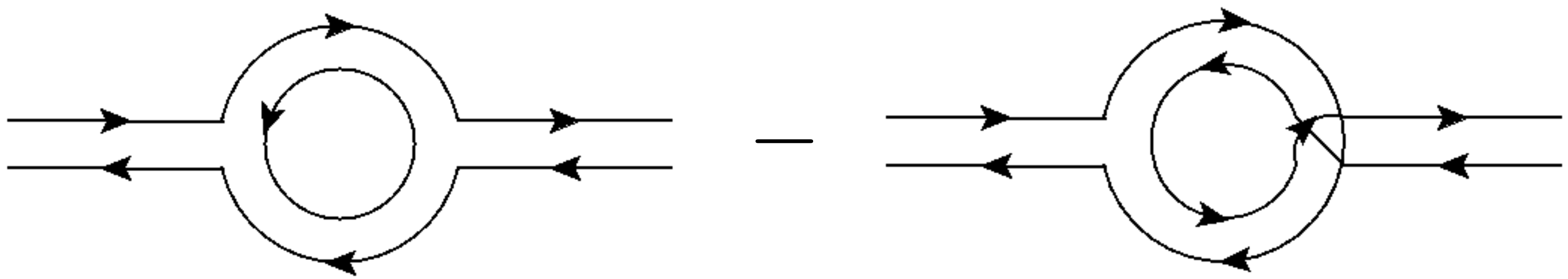
Dense nuclear matter

QCD at large N_c (small N_f)

In $SU(N_c)$, gluons matrices, N_c x N_c, quarks column vectors.
Denote fund. rep. by a line: quarks have one line, gluons have two.

't Hooft '74: let N_c = # colors $\rightarrow \infty$, $\lambda = g^2 N_c$ *fixed*. Keep N_f = # flavors *finite*.

Consider gluon self energy at 1 loop order. For *any* N_c, color structure in all diagrams (3 gluon & 4 gluon vertices) reduces to (Hidaka & RDP 0906.1751)



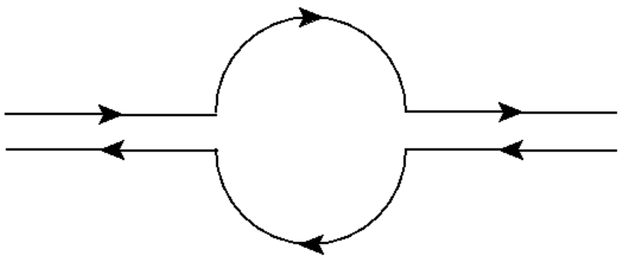
$$\sim g^2 N_c \sim \lambda \qquad\qquad \sim g^2 \sim \frac{\lambda}{N_c}$$

First diagram is "planar". Second, involving trace, is not, is down by $1/N_c$.

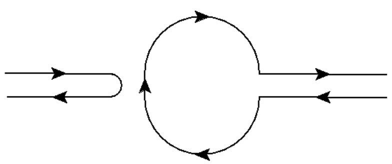
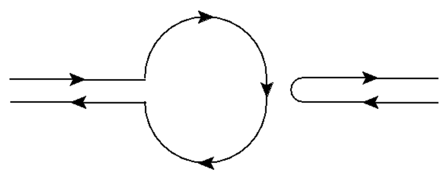
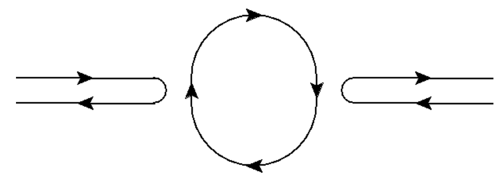
At large N_c and small N_f, planar diagrams dominate.

Large N_c and small N_f: *glue* dominates

Contribution of the quarks to the gluon self energy at 1 loop order, any N_c:



$$\sim g^2 N_f \sim \frac{1}{N_c} N_f \, \lambda$$

$-\frac{1}{N}$  $-\frac{1}{N}$  $+\frac{1}{N^2}$ 

If $N_f/N_c \to 0$ as $N_c \to \infty$, loops *dominated* by gluons, *blind* to quarks.

Quarks act *something* like external sources, not quite.

N.B.: limit of large N_c, small N_f is *free* of the pathologies of $N_f = 0$ (quenched)

No problems considering nonzero quark density, μ_{qk}:

quarks do *not* affect gluons when $\mu_{qk} \sim 1$!

Phases at large N_c : *pressure* as an order parameter

$T = \mu_{qk} = 0$: confined, only color singlets. Glueballs, meson masses ~ 1.
Baryons *very* heavy, masses ~ N_c, so no virtual baryon anti-baryon pairs.

$T \neq 0$, $\mu_{qk} = 0$:

$T < T_c$: Hadrons. T_c ~ mass ~ 1. # hadrons ~ 1, so pressure = $p \sim 1$: *small*.

$T > T_c$: Quark-Gluon Plasma. Deconfined gluons & quarks.
gluons ~ N_c^2, so $p \sim N_c^2$: *big*. Dominated by gluons.

$T \neq 0$, $\mu_{qk} \neq 0$: usual mass threshold, baryons only when $\mu_{qk} > M_N/N_c = m_{qk} \sim 1$.

$T < T_c$, $\mu_{qk} < m_{qk}$: Hadronic "box" in T-μ_{qk} plane: *no* baryons.

$T > T_c$ any μ_{qk} : Quark-Gluon Plasma. Some quarks, so what, $p_{qk} \sim N_c$.

$T < T_c$, $\mu_{qk} > m_{qk}$: # quarks ~ N_c , so $p \sim N_c$: *dense* nuclear matter (*not* dilute)
Confined phase! But Fermi sea of *quarks*? *"Quark-yonic"*

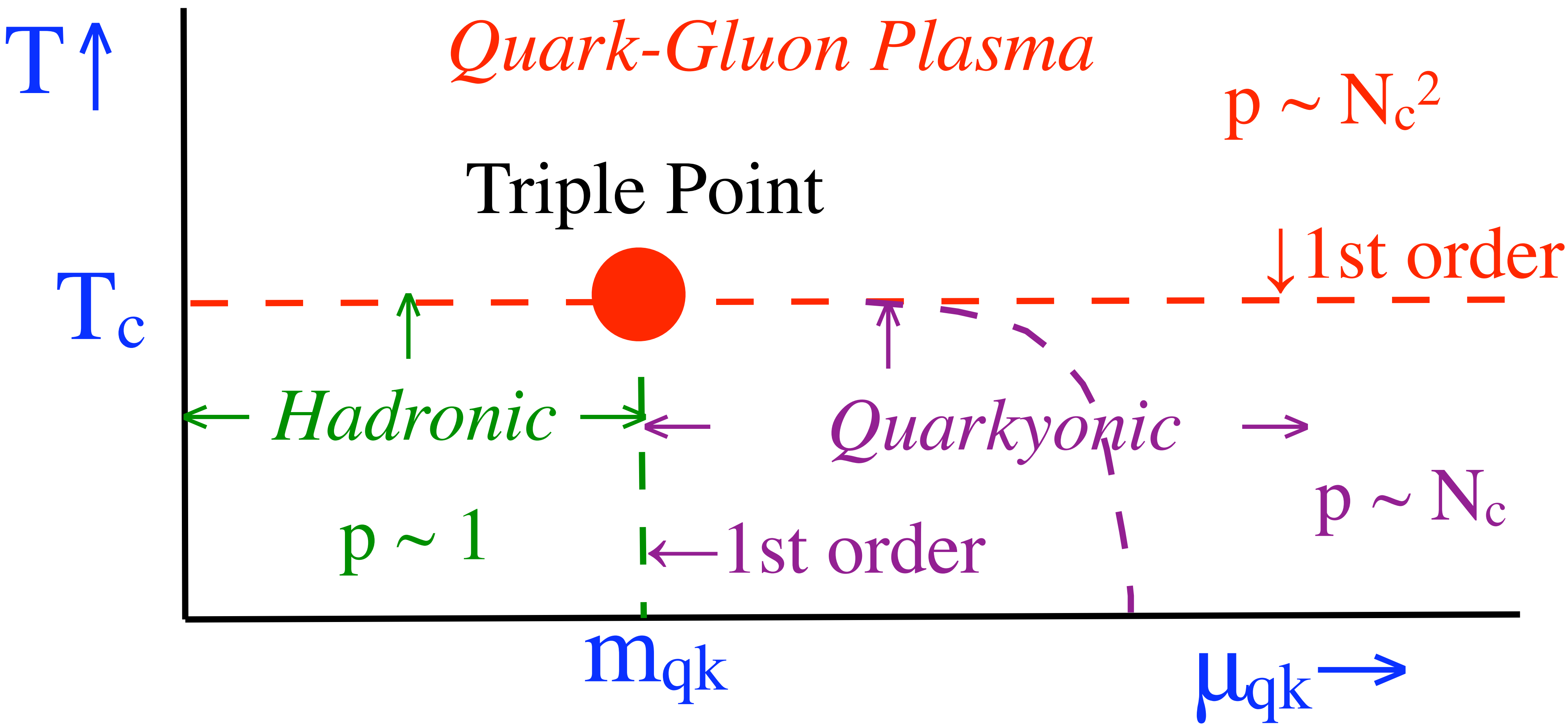
Phase diagram at large N_c and small N_f

Lattice (Teper, 0812.0085): deconfining transition 1st order at $T \neq 0$, $\mu_{qk} = 0$.
must remain so when $\mu_{qk} \neq 0$. *Straight* line in T - μ_{qk} plane.

Hadronic/Quarkyonic transition: energy density jumps by N_c, 1st order?

Chiral transition: in Quarkyonic phase?

True triple point!



Lattice: (pure glue) SU(3) close to SU(∞)

Bringoltz & Teper, hep-lat/0506034 & 0508021:

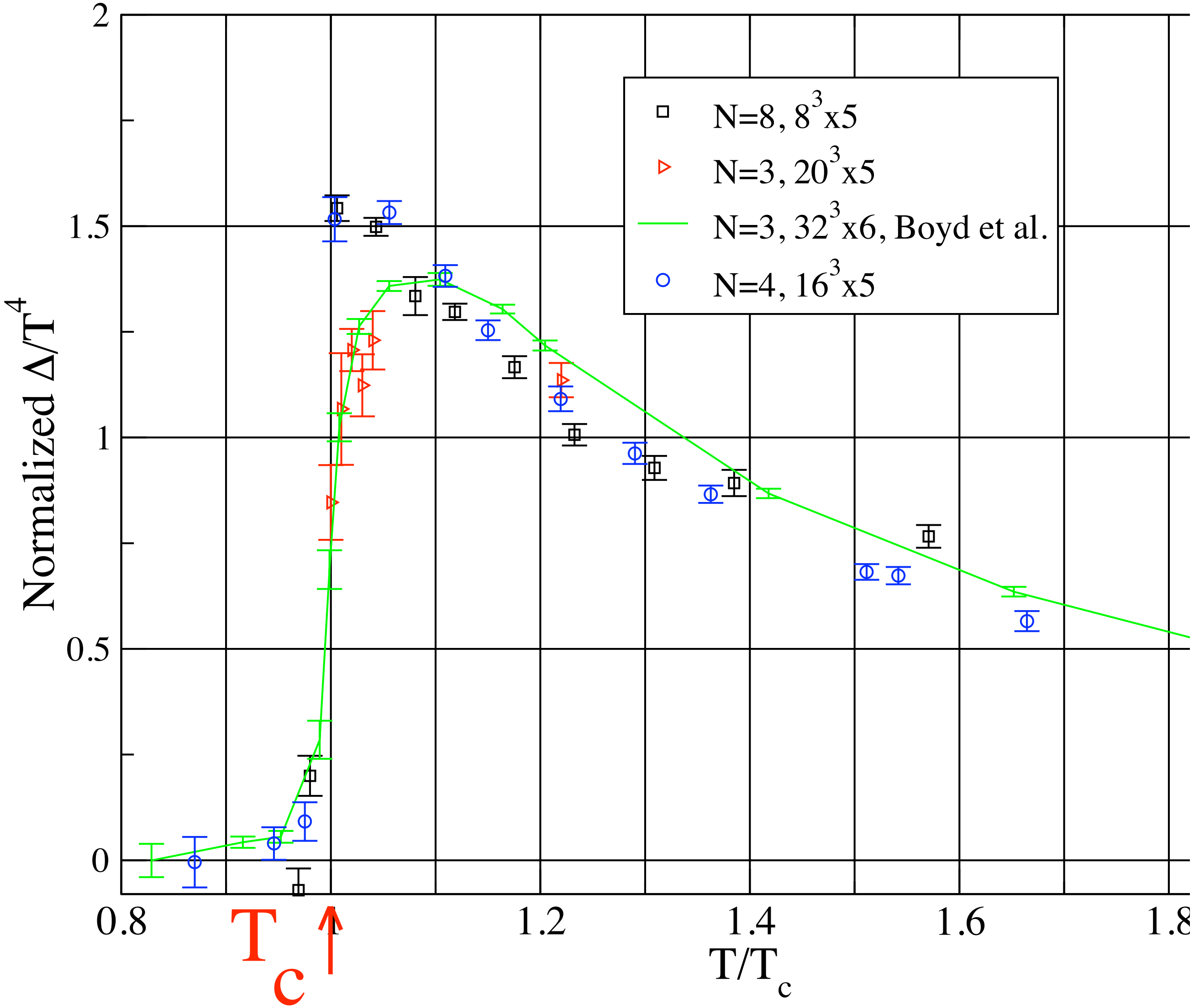
SU(N_c), *no quarks*, $N_c = 3, 4, 6, 8, 10, 12$.

Deconfining transition first order, latent heat ~ N_c^2.

Hagedorn temperature $T_H \sim 1.116(9)\ T_c$ for $N_c = \infty$

$$\frac{e - 3p}{N^2\ T^4} \sim \text{const.}$$

$$\frac{e - 3p}{N^2\ T^4} \uparrow$$



$T/T_c \rightarrow$

Triple point for water

Triple point where three lines of first order transitions meet.

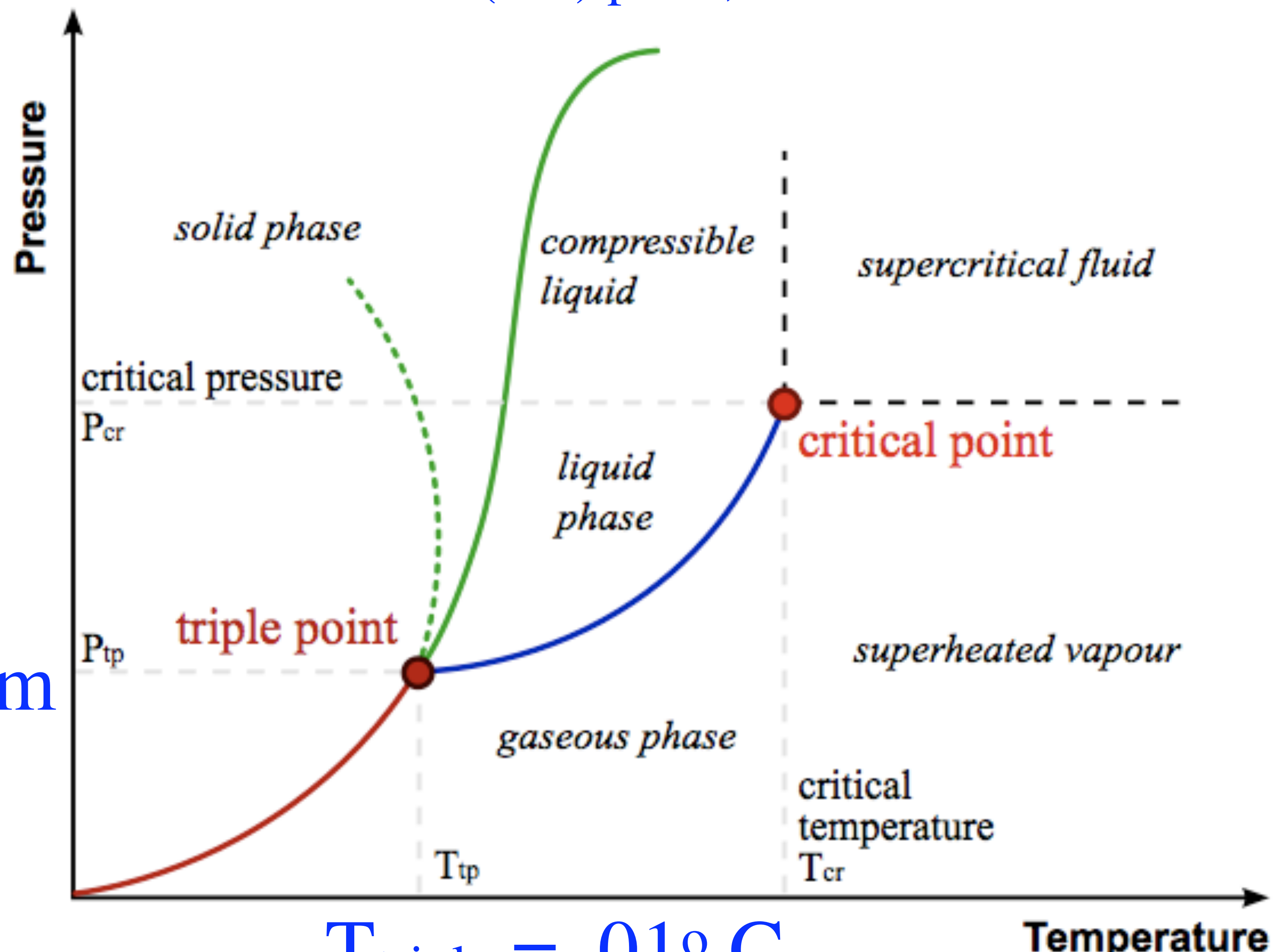
E.g., for ice/water/steam, in plane of temperature and pressure.

(Generalizes: four lines of first order transitions meeting is a quadruple point.)

Generically, *distinct* from critical (end) point, where one first order line ends.

$P\uparrow$

$P_{\text{triple}} =$ $.006$ atm



$T \rightarrow$

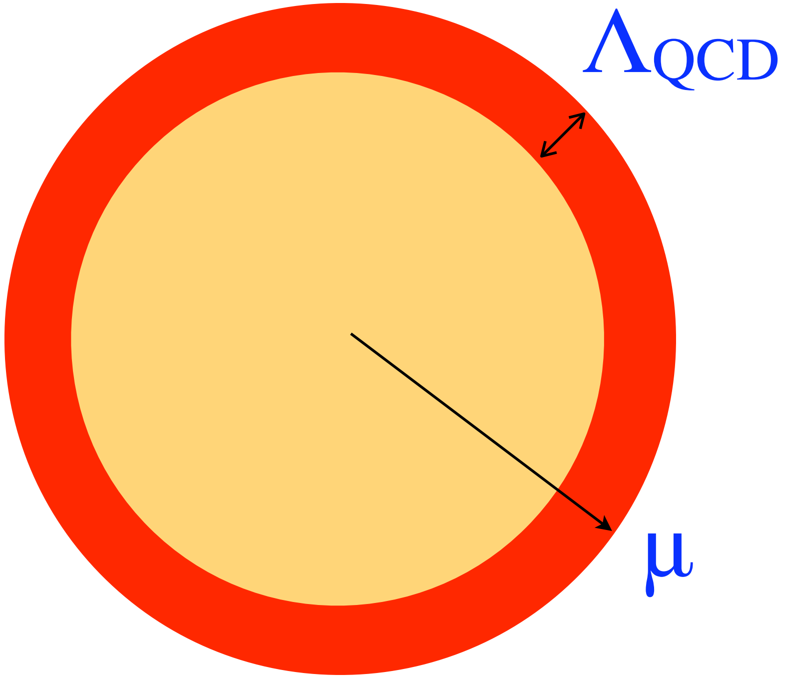
$T_{\text{triple}} = .01^{\circ}$ C

Quarkyonic phase at large N_c, large μ?

Let $\mu >> \Lambda_{QCD}$ but $\sim N_c^0$. Coupling runs with μ, so pressure $\sim N_c$ is close to perturbative! How can the pressure be (nearly) perturbative in a confined theory?

Pressure: dominated by quarks far from Fermi surf.: perturbative,

$p_{qk} \sim N_c\, \mu^4\, (1 + g^2(\mu) + g^4(\mu) \log(\mu) + ...)$



Within Λ_{QCD} of Fermi surface: *confined* states.

$p_{qk} \sim N_c\, \mu^4\, (\Lambda_{QCD}/\mu)^2$, *non*-perturbative.

Within skin, only confined states contribute.

Fermi sea of quarks + Fermi surface of bar-yons

= "quark-yonic". *N=3?*

Pressure dominated by quarks.

But transport properties *dominated* by confined states near Fermi surface!

For QCD: what is (cold) nuclear matter like at high density?

Just a quark NJL model?

Deconfining critical end point at (large) $\mu_{qk} \sim N_c^{1/2}$

Semi-QGP theory of deconfinement: Hidaka & RDP 0803.0453

$$A_0 = \frac{T}{g}\, Q$$

For large μ: compute one loop determinant in background field.
Korthals-Altes, Sinkovics, & RDP hep-ph/9904305

$$S_{qk} = \mathrm{tr}\,(\mu + i\,T\,Q)^4 \;,\; T^2\;\mathrm{tr}\,(\mu + i\,T\,Q)^2 \;,\; N_c^2\,T^4\,V(Q)$$

RDP ‘09: for large μ, expand:

$$S^{qk}_{\mu\sim\sqrt{N_c},T\sim 1} \sim N_c\,\mu^4 - 6\,\mu^2\,T^2\,tr\,Q^2 + \ldots \sim N_c^3 \;,\; N_c^2\;(tr\,Q^2/N_c)$$

Consider $\mu \sim N_c^{1/2}$, $T \sim 1$: gluons *do* feel quarks.

Term $\mu^4 \sim N_c^3$ dominates, but *independent* of Q *and* temperature.

Term $\mu^2 \sim N_c^2$ Q-dependent. Breaks $Z(N_c)$ symmetry, so washes out 1st order deconfining transition: Deconfining Critical End Point (CEP)

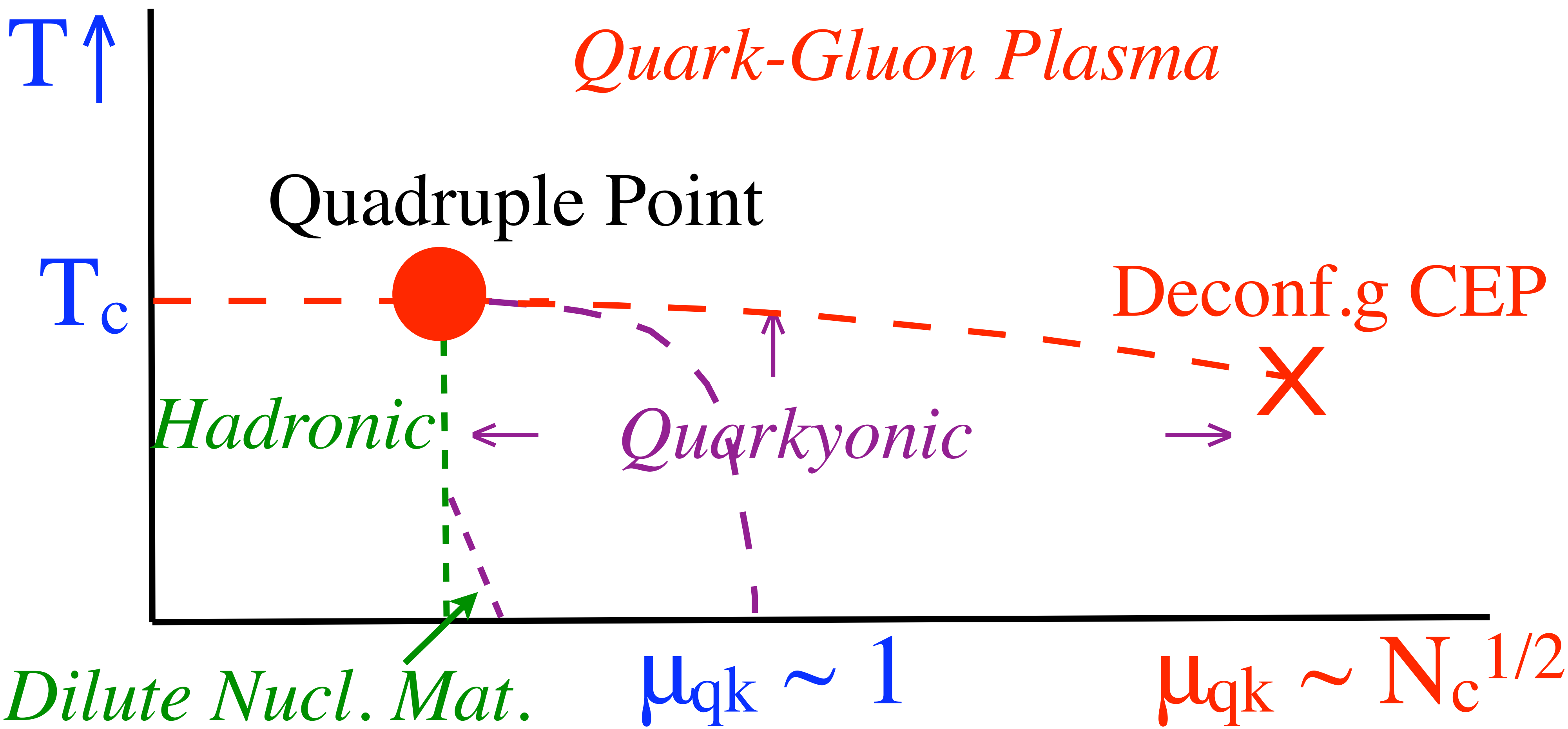
Phase diagram at large N_c and small N_f, II

About deconfining Critical End Point (CEP), smooth transition between deconfined and quarkyonic phases.

Since gluons are sensitive to quarks for such large μ, expect curvature in line.

Triple point still well defined, as coincidence of three 1st order lines.

Chiral transition?



So what does this have to do with experiment?

Strange “MatterHorn” $\approx$ Triple Point?

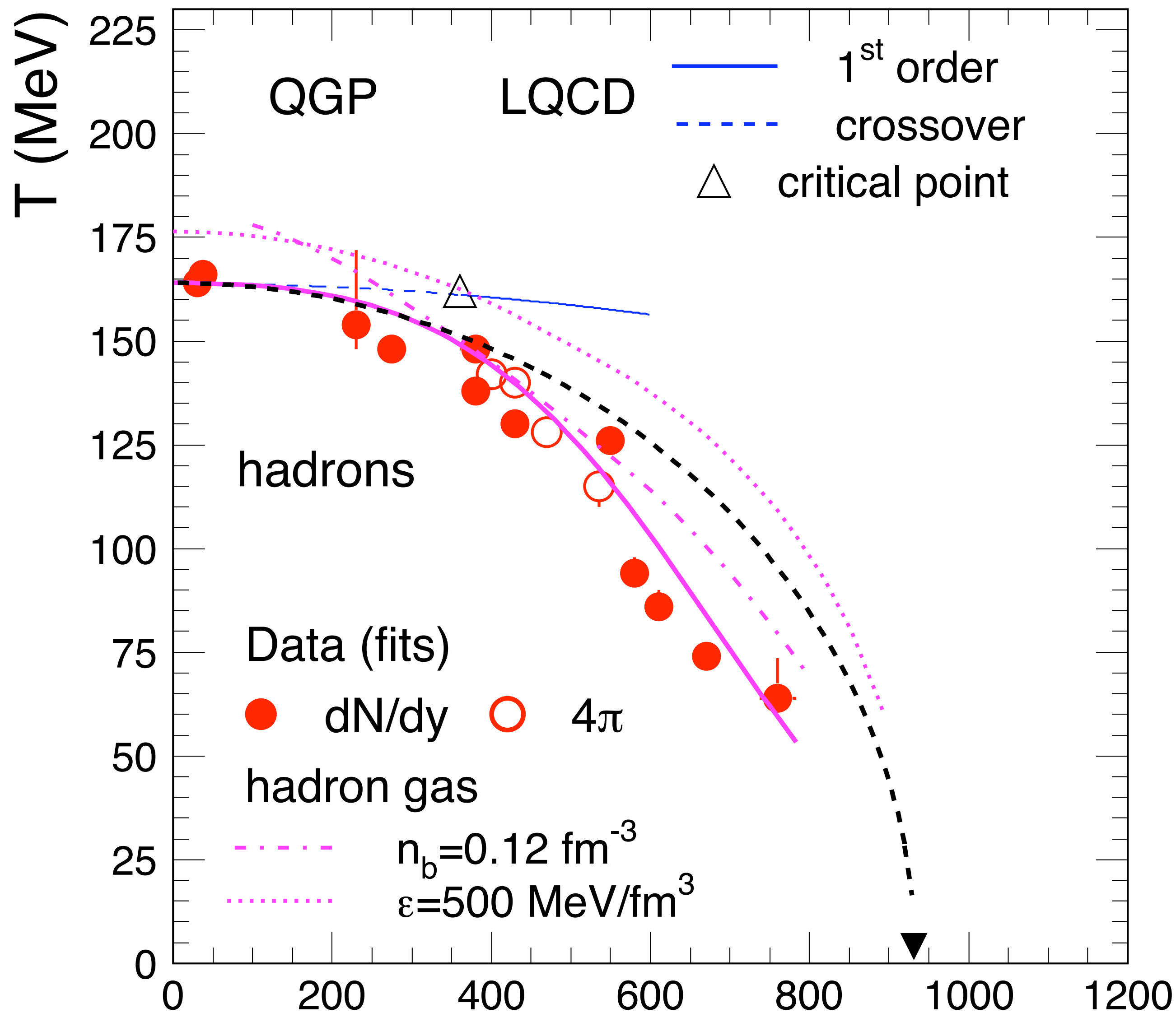
Wonderous utility of statistical/hadron resonance gas models

Chemical equilibribriation at SIS, AGS, SPS, RHIC, and onto NICA and FAIR:

Braun-Munzinger, Cleymans, Oeschler, Redlich, Stachel

plus: Bialas, Biro, Broniowski, Florkowski, Levai, Ko, Satz + ...

T↑

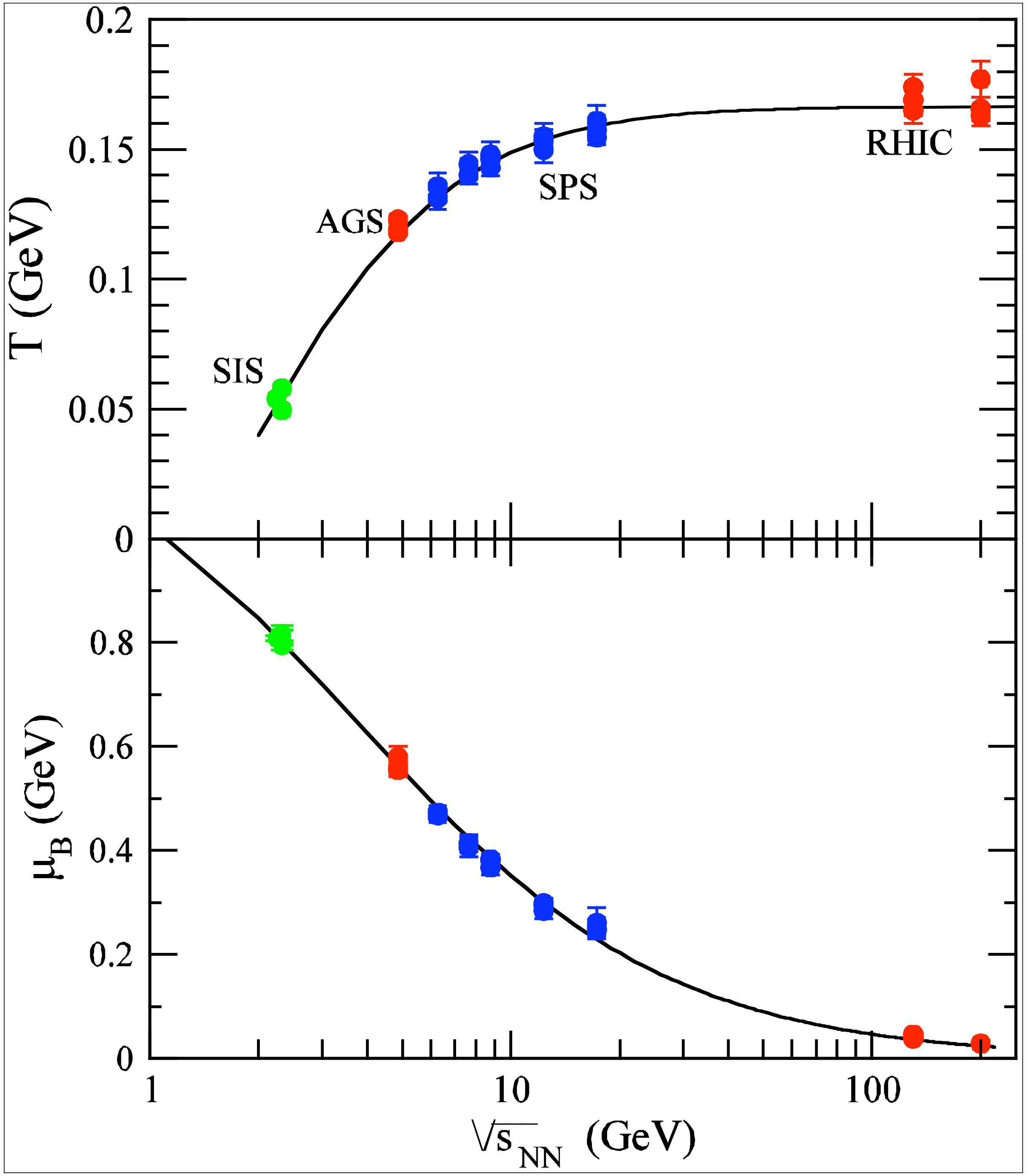


$\mu_B \rightarrow$

Smooth evolution in T, μ_{Baryon} with $\sqrt{s_{NN}}$

$T \uparrow$

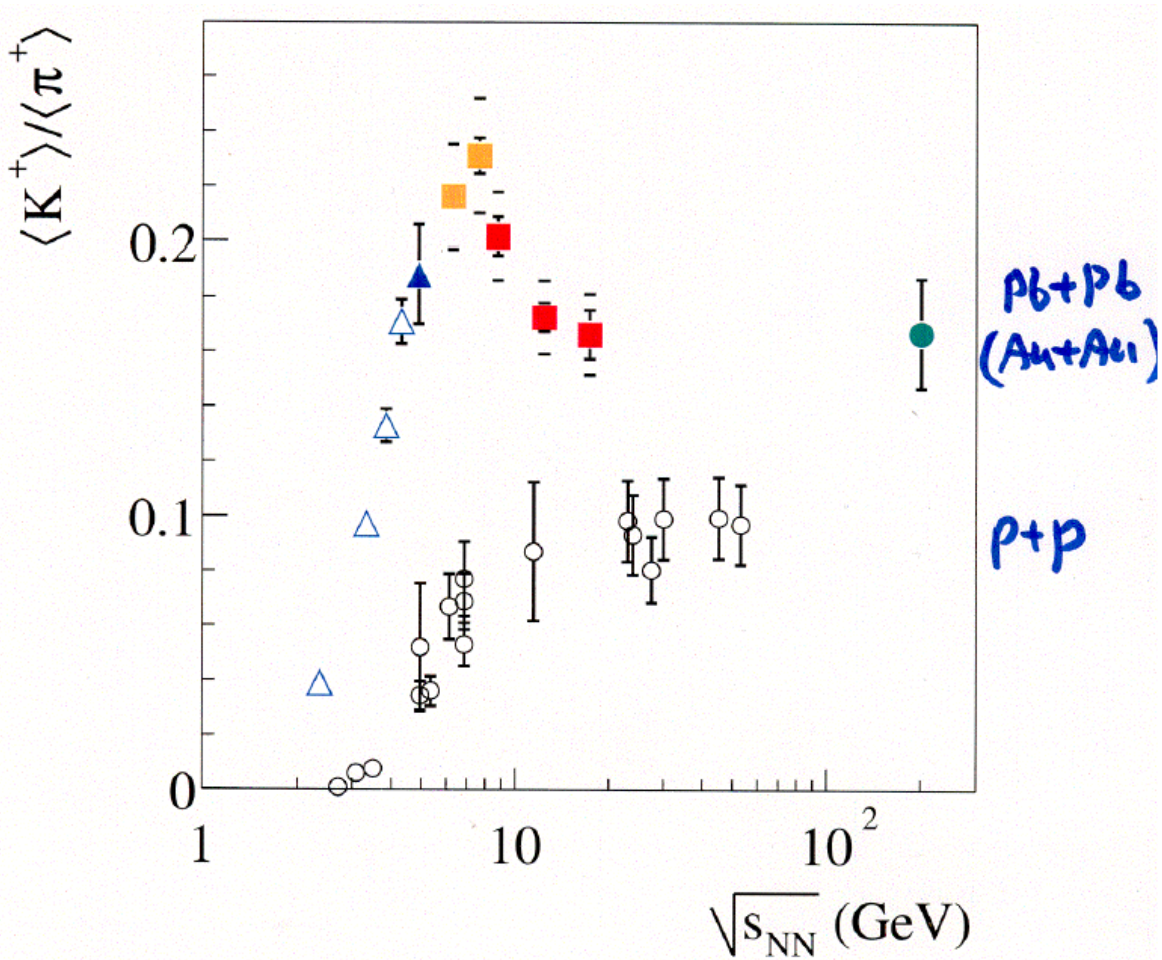
$\mu_B \uparrow$

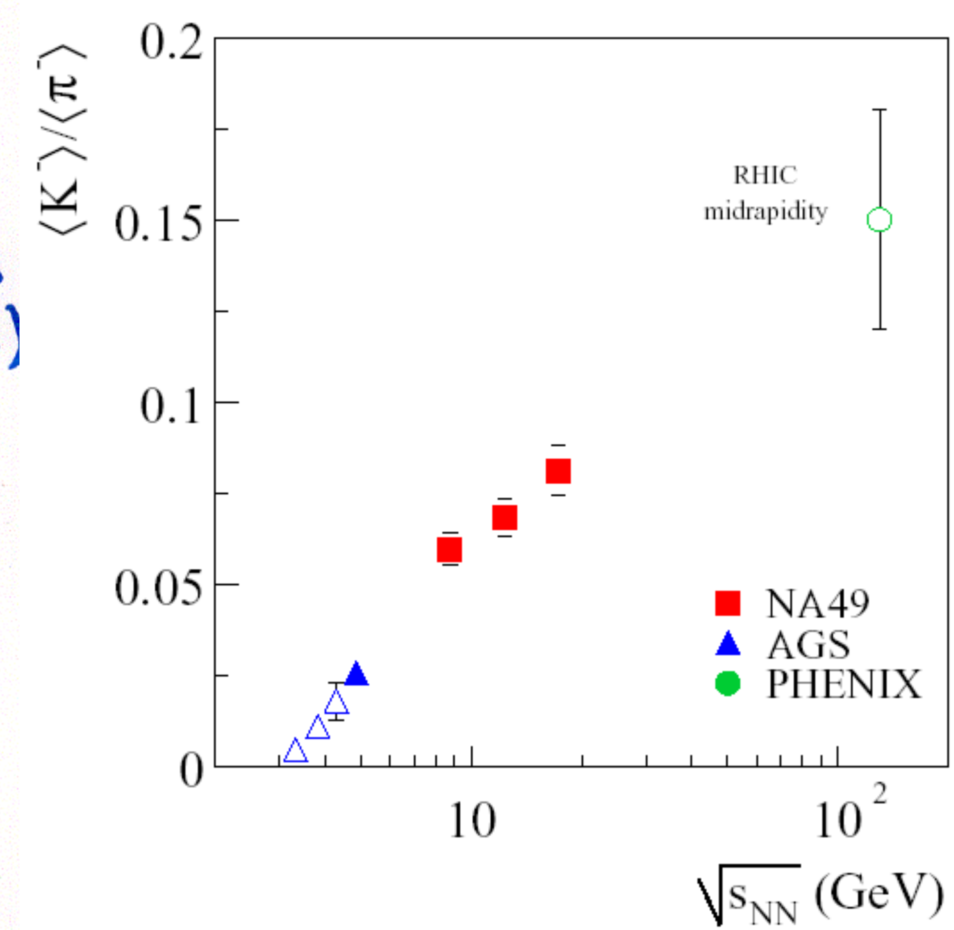


$\sqrt{s_{NN}} \rightarrow$

Devil is in the details: RDP, Review of Quark Matter 2004:

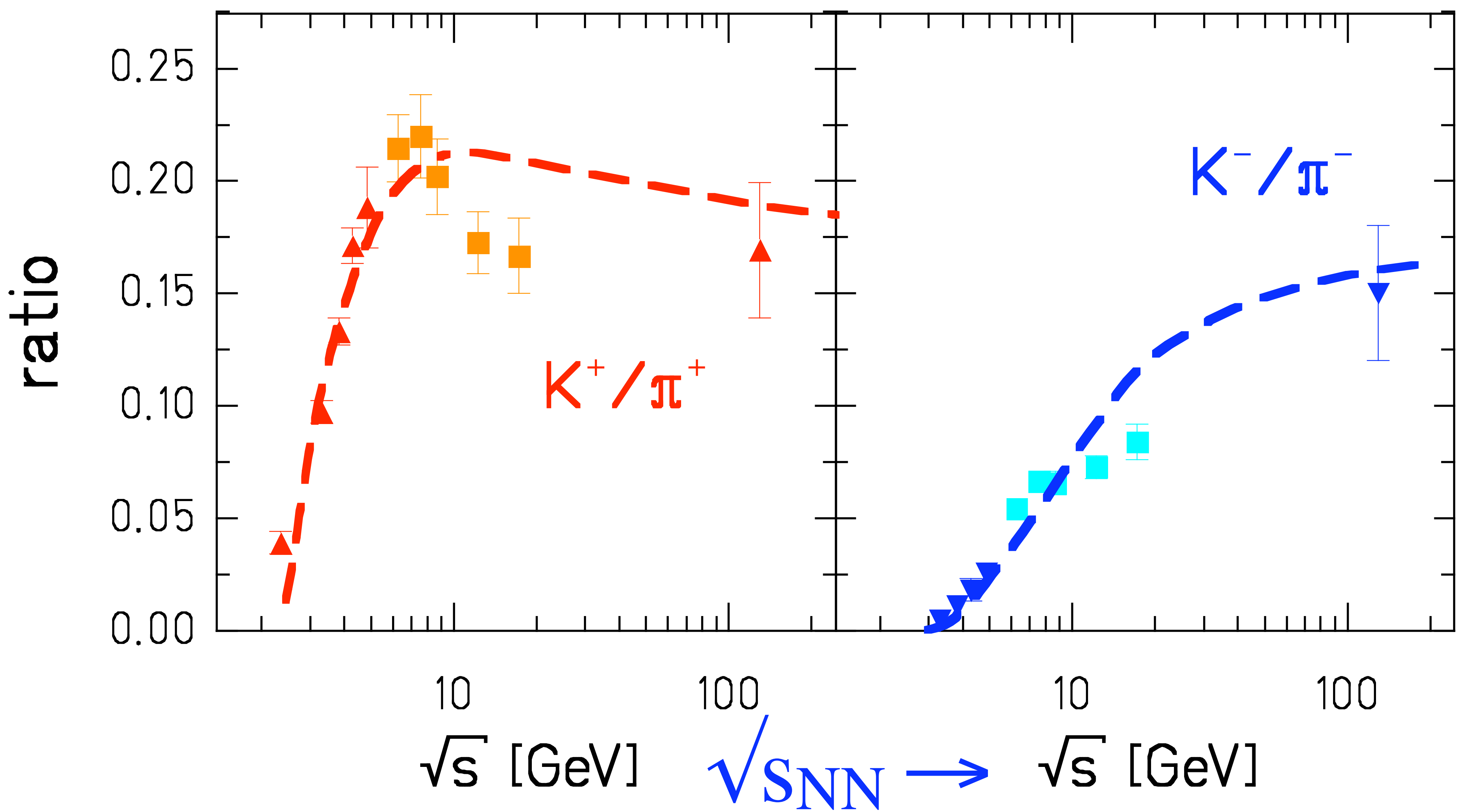
Is this related to the *narrow* peak in K+/pi+ @ SPS? The "MatterHorn" of NA49 ?





Peak not confirmed by other groups, not seen in other ratios...

Strange MatterHorn: peak in K^+/π^+, *not* K^-/π^-

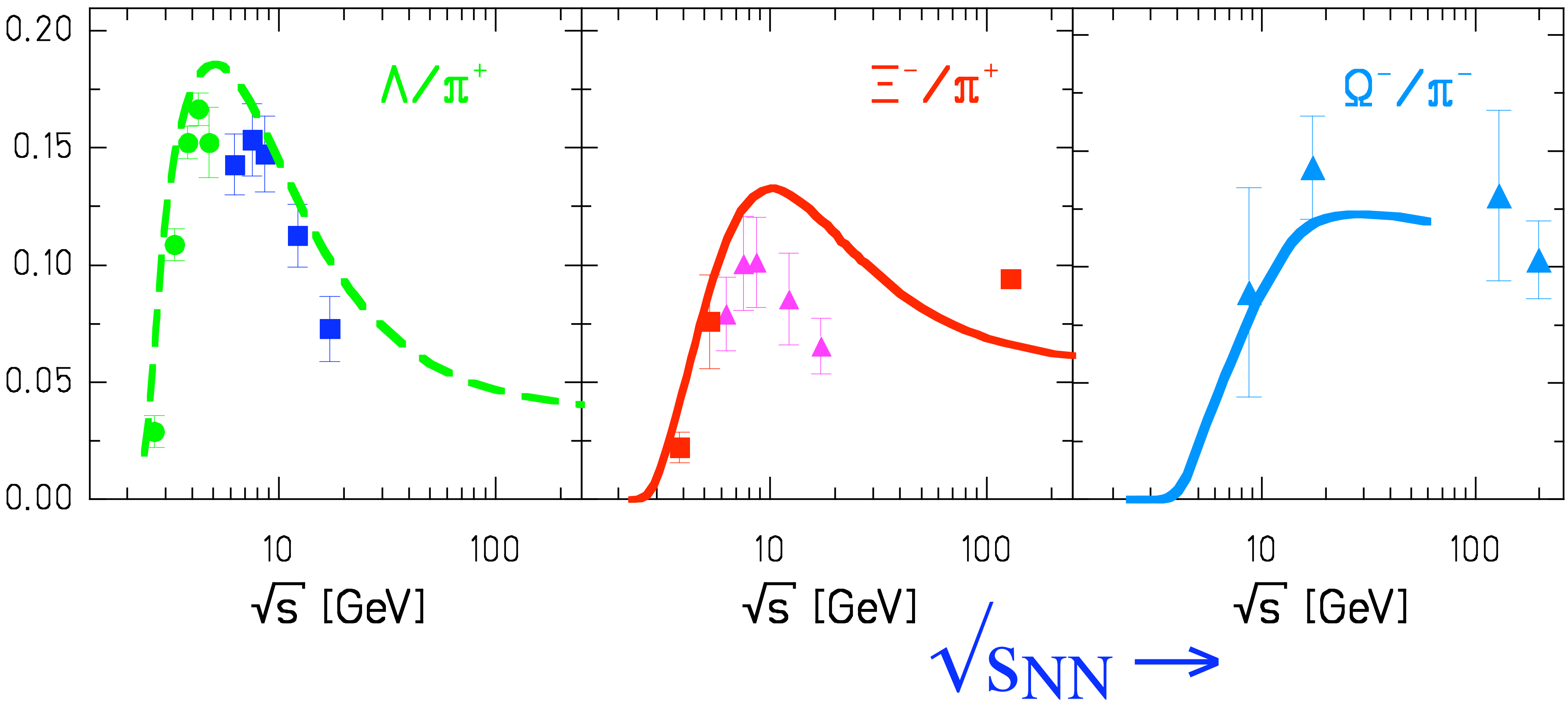


Strange MatterHorn: also in baryons

Natural to have peaks in K^+/π^+, strange baryons: start with (s s-bar) pairs.

At $\mu \neq 0$, strange quarks combine into baryons, anti-strange into pions.

For different baryons, peaks do not occur at same energy, but nearby, so not true phase transition, but approximate.

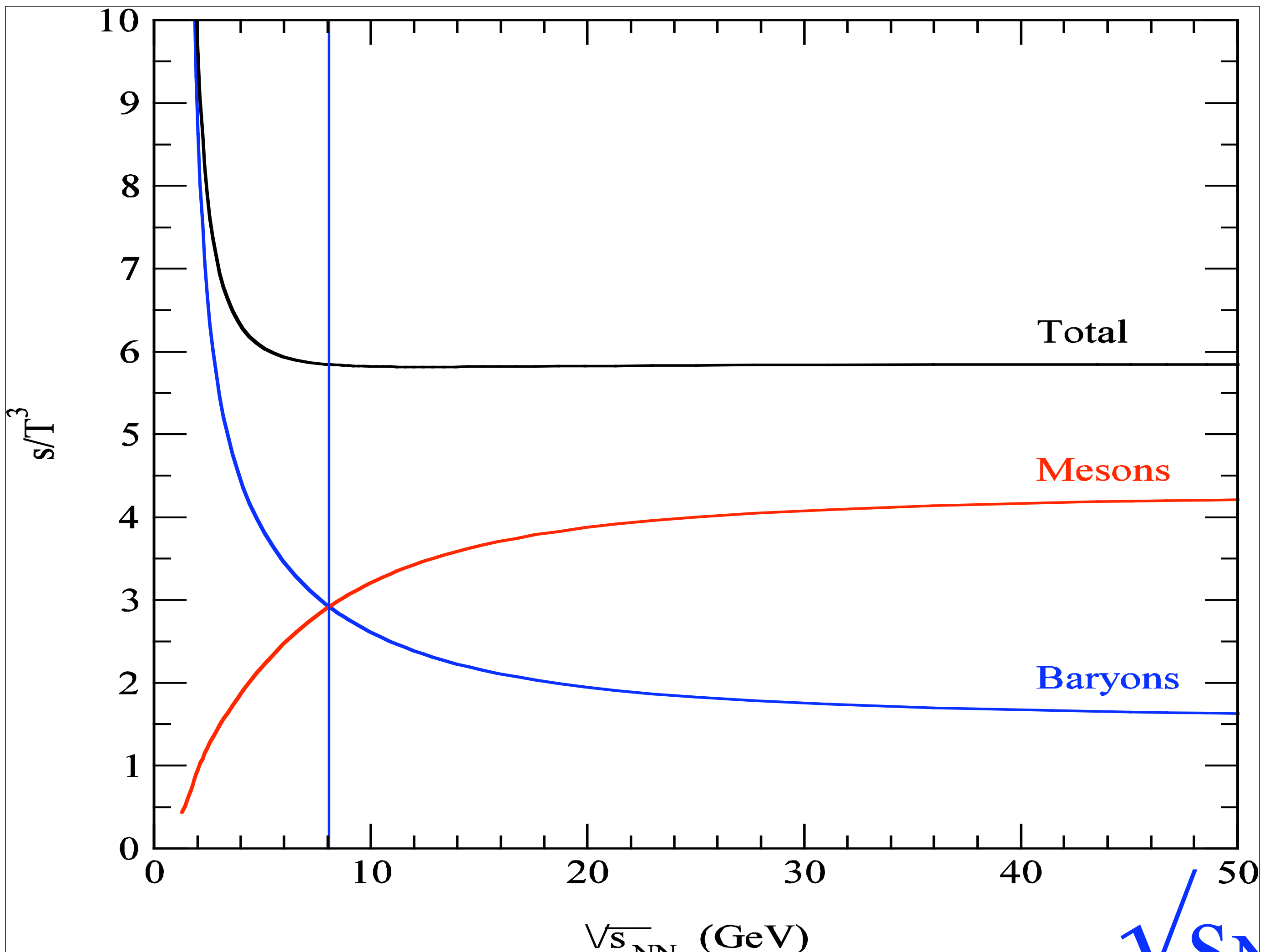


Strange MatterHorn and the triple point?

Usual explanation of MatterHorn: transition from baryons to mesons at freezeout.

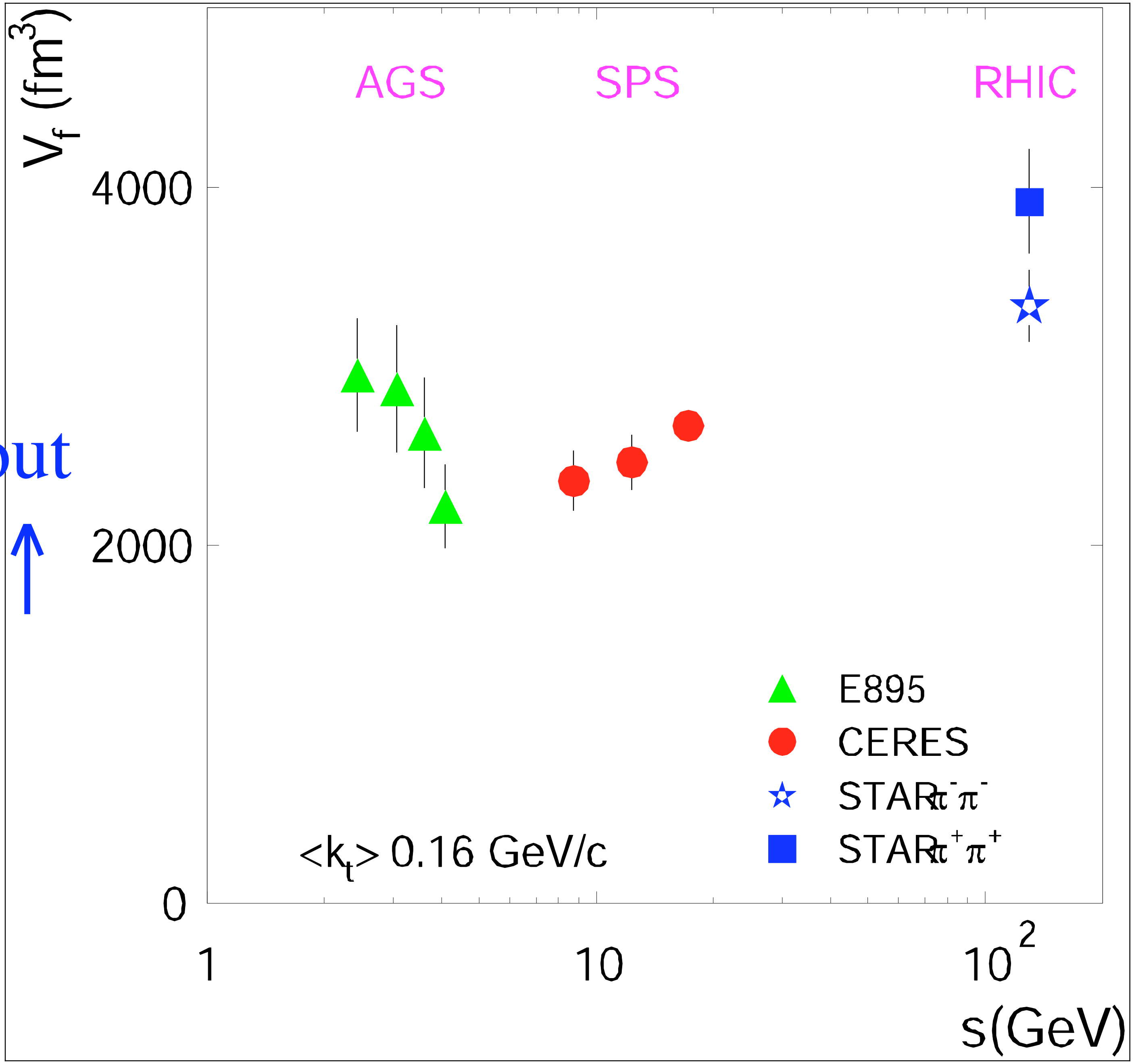
Or: changing from Hadronic/Quarkyonic boundary to Hadronic/QGP boundary: i.e., (approximate) triple point.

entropy density/T^3 ↑



$\sqrt{s_{NN}}$ →

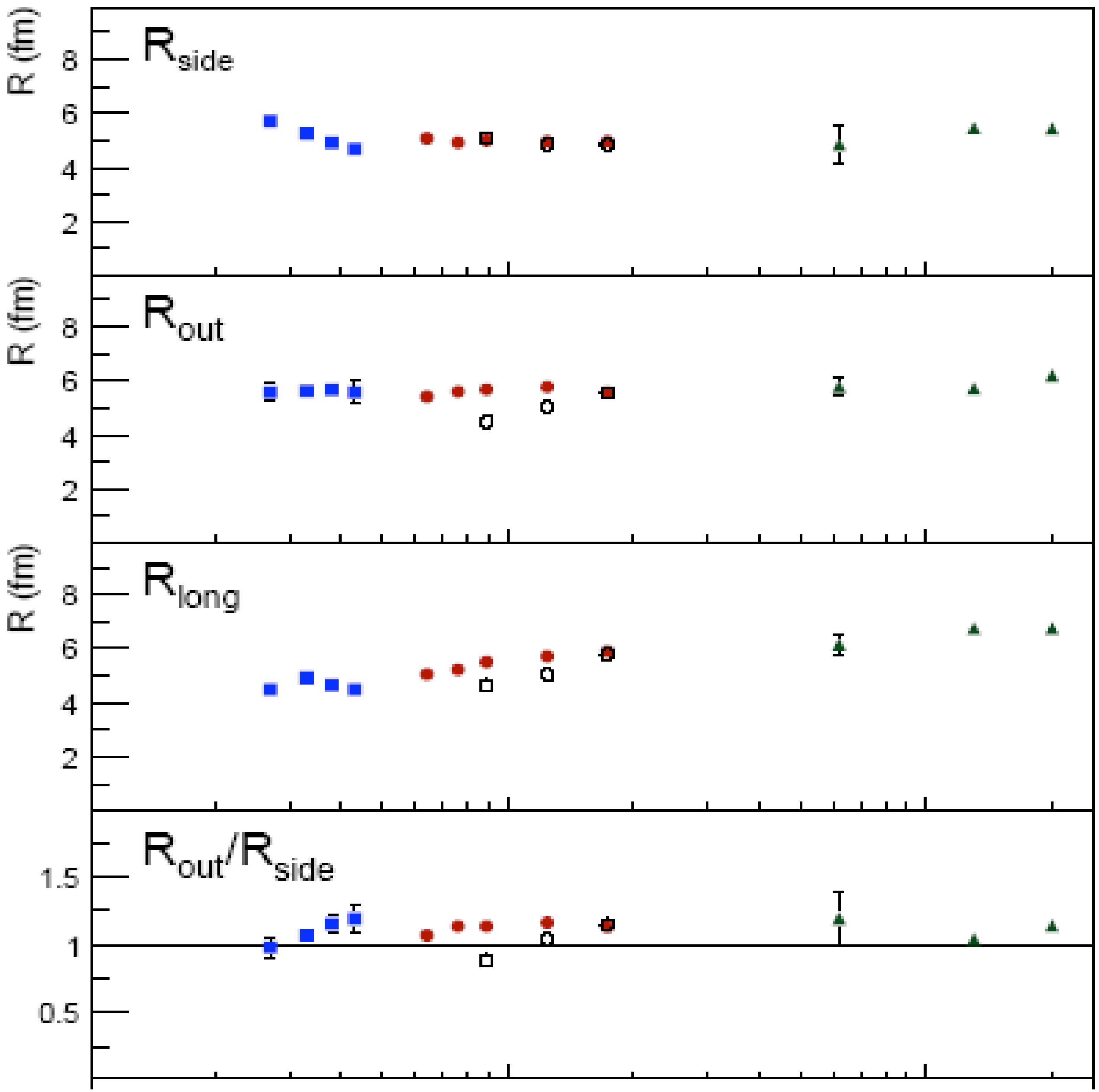
HBT radii: minimum near strange MatterHorn?



freeze out volume ↑

$\sqrt{s_{NN}}$ →

HBT radii: flat from NA49.



$\sqrt{s_{NN}} \rightarrow$

Triple point versus critical end point

Critical endpoint: correlation lengths *diverge*.

Hence: HBT radii should *increase*.

Effects should be greatest on the lightest particles, *not* the heaviest:

K^+/π^+ should *de*crease, not *in*crease. *Neither* is seen in the data.

Assume that at triple point, chiral transition splits from deconfining.
Leading operator which couples the two transitions is
Mocsy, Sannino, & Tuominen, hep-ph/0301229, 0306069, 0308135, 0403160:

$$c_1\,\ell\;\mathrm{tr}\,\Phi^\dagger\Phi \sim c_1\,\ell\,(\pi^2 + K^2 \ldots)$$

If this coupling c_1 flips sign, transitions diverge. Hence $c_1 = 0$ at triple point?
If so, leading coupling then becomes

$$c_2\,\ell\;\mathrm{tr} M\,\Phi \sim c_2\,\ell\,(m_\pi^2\pi^2 + m_K^2 K^2 + \ldots)$$

This coupling is proportional to mass squared: *bigger* for kaons than pions!
Enchancement of K^+/π^+, strange baryons due to dense environment.
Implicitly: line for chiral transition crossover, not 1st order.

(Lunatic) ideas about nuclear matter:

"The unbearable lightness of being (nuclear matter)"

Nucleon-nucleon potentials from the lattice

Ishii, Aoki & Hatsuda, PACS-CS, 0903.5497

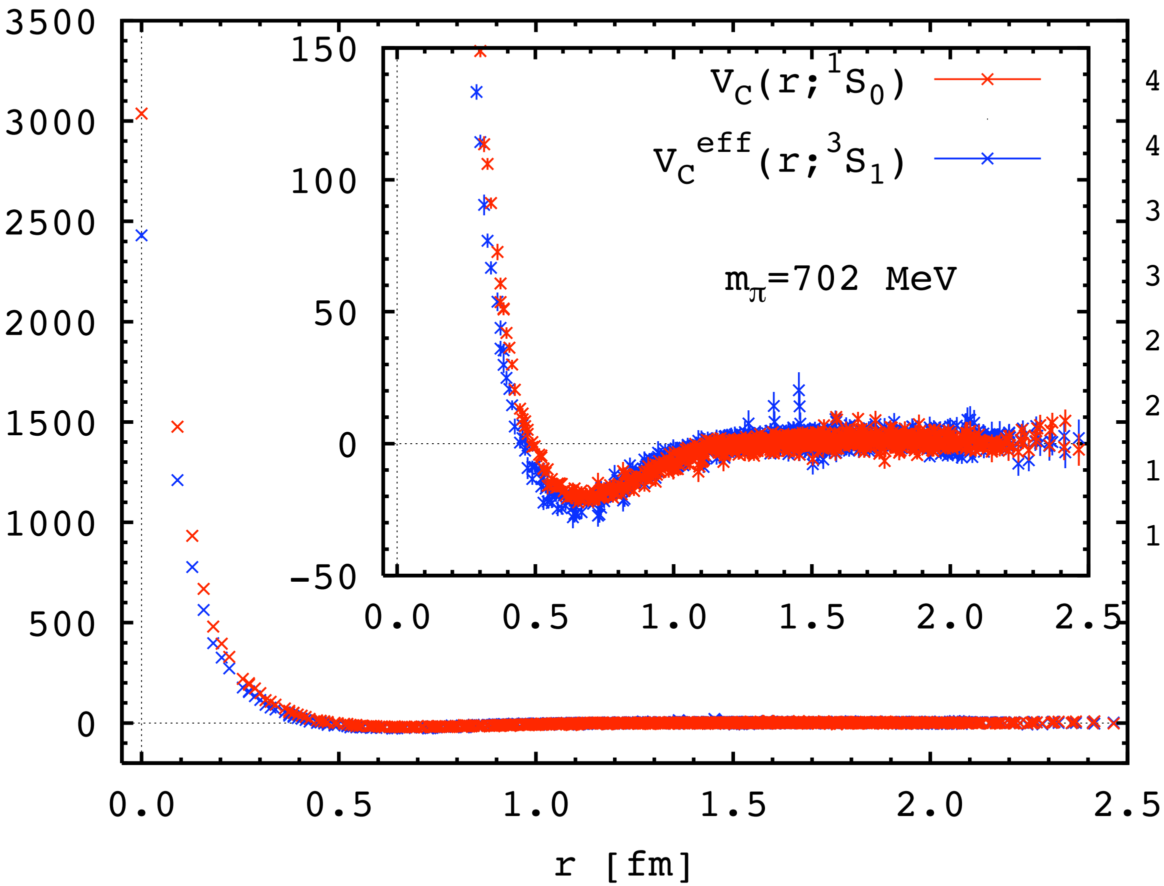
Nucleon-nucleon potentials from quenched and 2+1 flavors.

Pions heavy: 700 MeV (left) and 300 MeV (right)

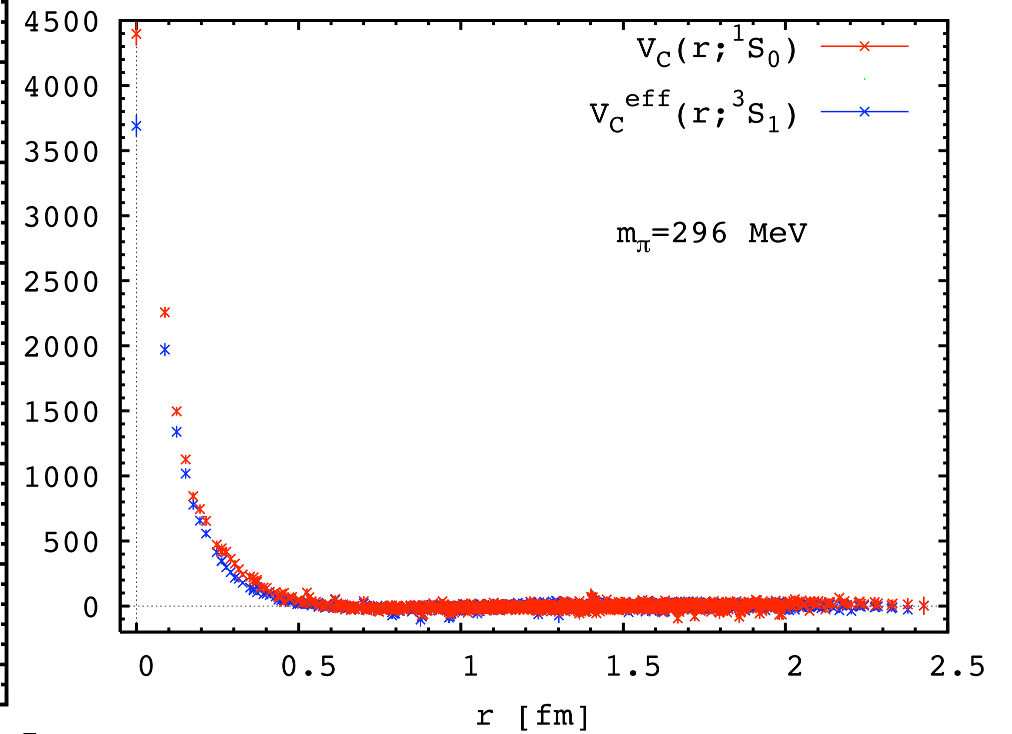
Standard lore: delicate cancellation. *So why independent of pion mass?*

Essentially *zero* potential plus strong hard core repulsion

$m_\pi = 702$ MeV



$m_\pi = 296$ MeV



Purely pionic nuclear matter

J.-P. Blaizot, L. McLerran, M. Nowak, & RDP '09....

At infinite N_c, integrate out *all* degrees of freedom *except* pions:

Lagrangian power series in $U = \mathrm{e}^{i\pi/f_\pi}$, $V_\mu = U^\dagger \partial_\mu U$

Infinite # couplings: Skyrme *plus* complete Gasser-Leutwyler expansion,

$$\mathcal{L}_\pi = f_\pi^2 \, V_\mu^2 + \kappa \, [V_\mu, V_\nu]^2 + c_1 \, (V_\mu^2)^2 + c_2 \, (V_\mu^2)^3 + \ldots$$

All couplings ~ N_c , every mass scale ~ typical hadronic.

Need *infinite* series, but nothing (special) depends upon exact values

Valid for momenta < f_π, masses of sigma, omega, rho...

Useful in (entire?) phase with chiral symmetry breaking?

Higher time derivatives, but no acausality at low momenta.

Purely pions give free baryons

From purely pionic Lagrangian, take baryon as stationary point.

Find baryon mass ~ N_c, some function of couplings.

Couplings of baryon dictated by chiral symmetry:

$$\overline{\psi}\left(i\partial\!\!\!/ + M_B\, \mathrm{e}^{i\tau\cdot\pi\gamma_5/f_\pi}\right)\psi$$

By chiral rotation, $W = \exp(-i\pi\gamma_5/2f_\pi)$

$$\mathcal{L}_B = \overline{\psi}\left(iW^\dagger \partial\!\!\!/ W + M_B\right)\psi \sim \frac{1}{f_\pi}\overline{\psi}\gamma_5\partial\!\!\!/\pi\psi + \ldots$$

At large ~ N_c , $f_\pi \sim N_c^{1/2}$ is *big*. Thus for momenta k < hadronic, interactions are *small*, ~ $1/f_\pi^2 \sim 1/N_c$.

Thus: baryons from chiral Lag. free at large N_c , down to distances $1/f_\pi$.

Manifestly special to chiral baryons. True for u, d, s, but *not* charm?

The Unbearable Lightness of Being (Nuclear Matter)

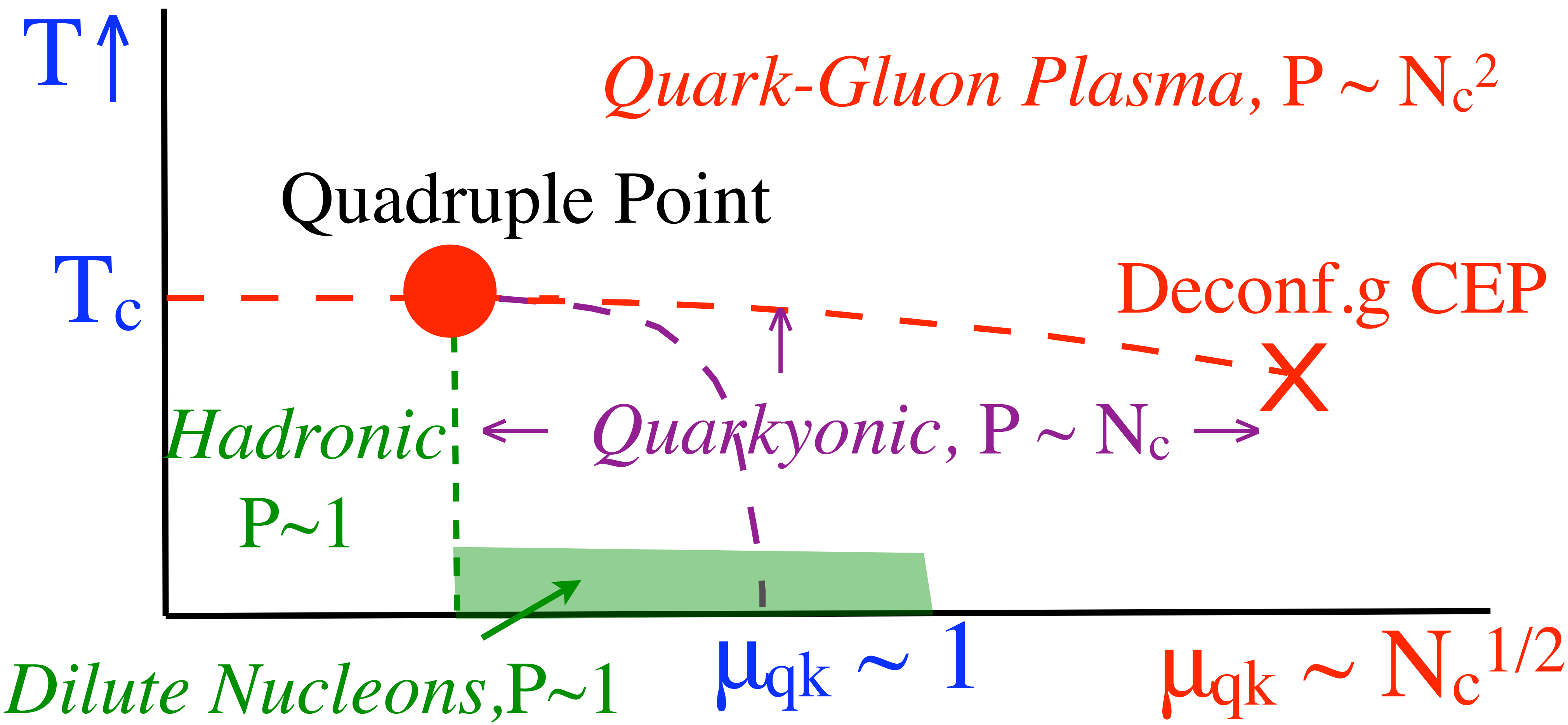
Use purely pionic Lagrangian for all of nuclear matter?

Then pressure ~ 1, and *not* N_c. Like hadronic phase, *not* quarkyonic.

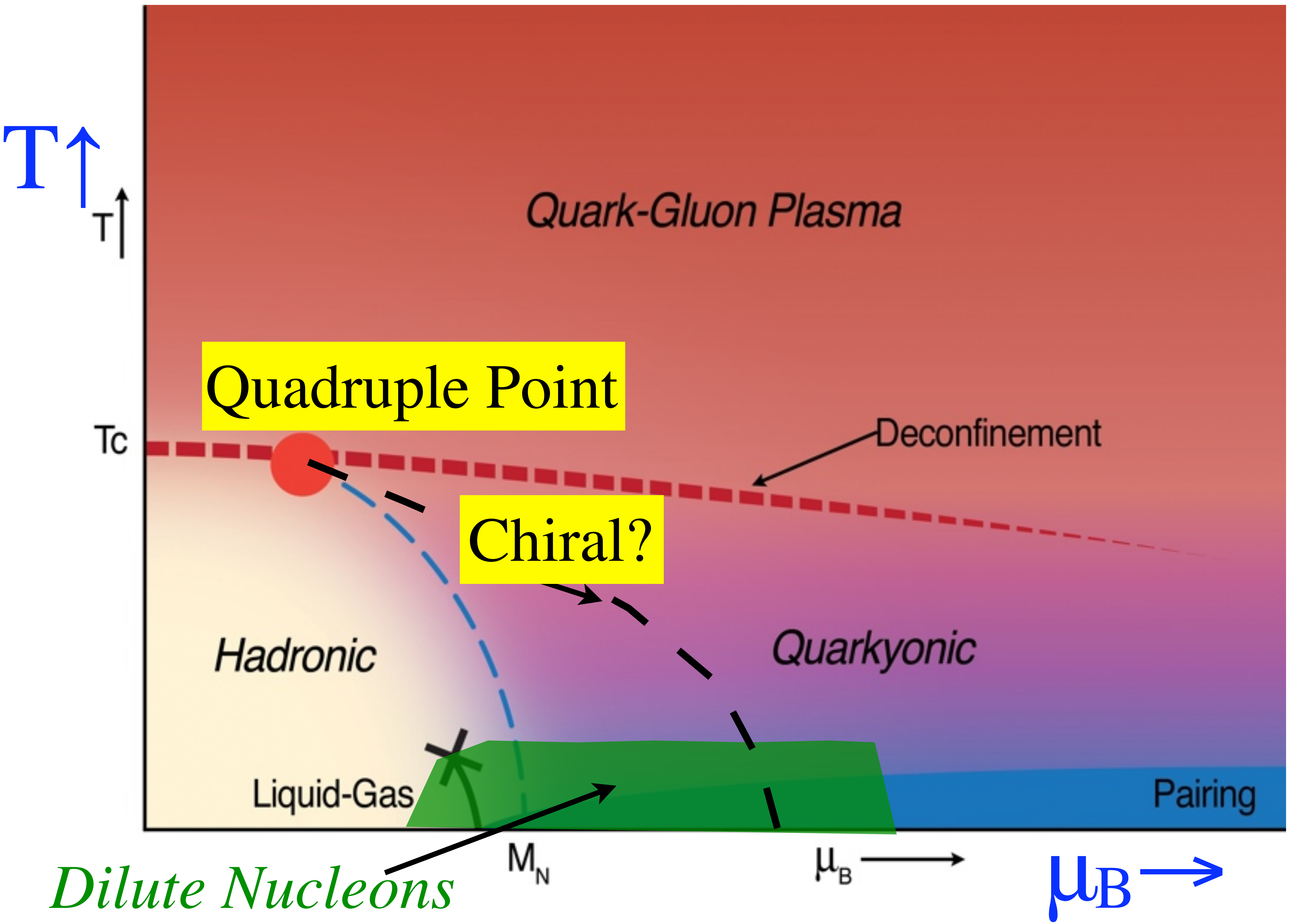
*Un*like standard lore, where pressure(nucl mat) grows quickly, ~ N_c

Red line: 1st order. Green line: Baryons condense. Purple: chiral trans.

IF chiral transition 1st order, etc, Quadruple Point where *four* phases coexist.



An unbearably light phase diagram for QCD



Quarkyonic matter for *two* colors?

i.e., are large N_c arguments even ok for $N_c = 2$?

Quarkyonic matter for two colors?

Hands, Ilgenfritz, Kenny, Kim, Mueller-Preussker, Schubert, Sitch, & Skullerud, HIKKMPSSS 09....

$N_c = N_f = 2$, Wilson fermions. *No* sign problem, measure real.

Mesons and baryons are both bosons. Baryon μ_{qk} like isospin μ_{qk} for 3 colors.

μ_{qk} only matters when $> \mu_0 = m_\pi/2$ (not $> m_{Baryon}$).

Expect Bose-Einstein condensate for $\mu_{qk} > \mu_0$, compare to Chiral Pert. Theory.

Find: $\mu_0\ a = 0.2$: BEC turns on, good agreement with CPT only *very* near μ_0 .

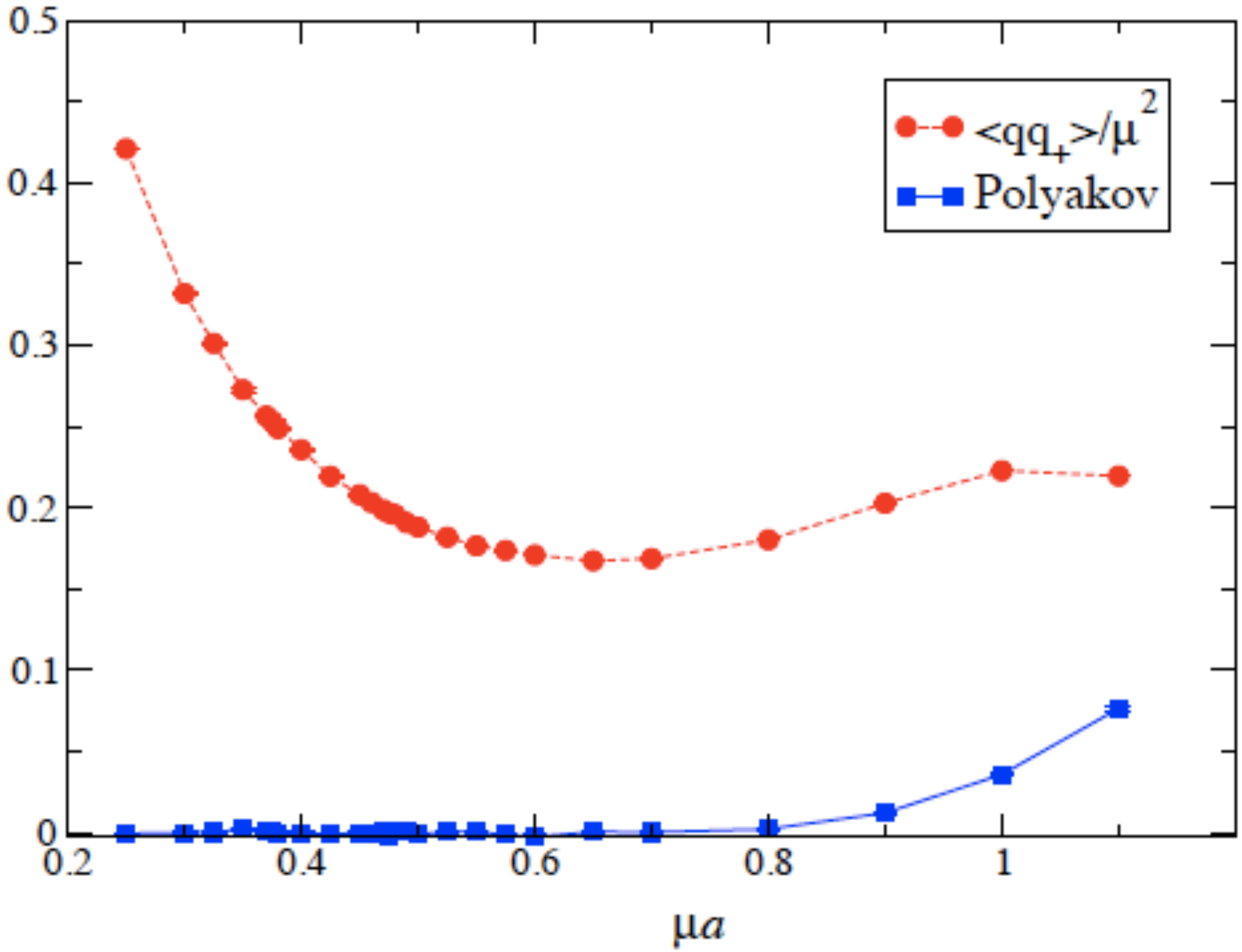
$\mu_t\ a = 0.4$: *big* jump in energy density - ?

$\mu_d\ a = 0.65$: Polyakov loop nonzero, deconfined quarks,
only at high density

Suggests: Quarkyonic matter for a large range, between μ_0 and μ_d , for $N_c = 2$!

Order Parameters

HIKKMPSSS 09...



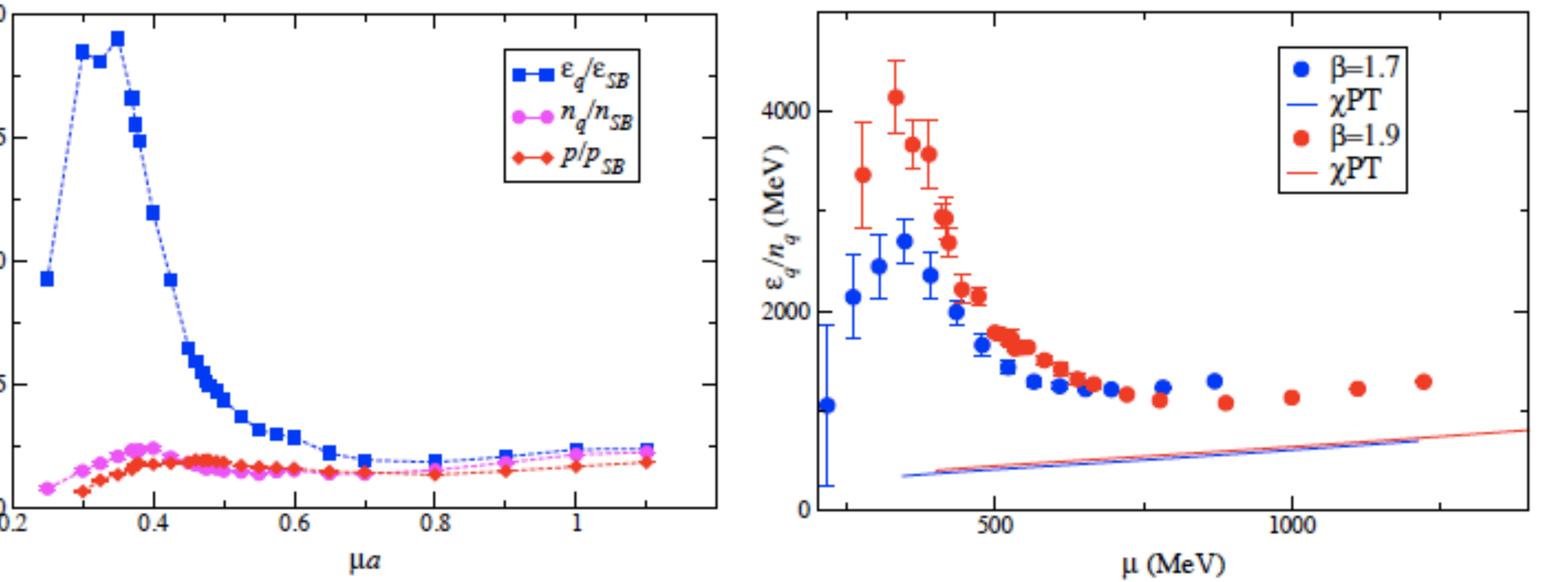
Superfluid condensate scaling ≈ BCS for $\mu_Q \lesssim \mu \lesssim \mu_d$

Polyakov loop ≈ 0 for $\mu < \mu_d$, but then rises from zero

$\Rightarrow$ Deconfinement at $\mu \approx 900$MeV, $n_q \approx 35\,\text{fm}^{-3}$

Quark Energy Density

HIKKMPSSS 09....



In contrast to χPT prediction, (unrenormalised) quark energy density ε_q greatly exceeds SB value as $\mu \searrow \mu_{o+}$

$\Rightarrow$

Energy per quark ε_q/n_q has shallow minimum for $\mu > \mu_Q$