A triple point in the QCD phase diagram?  
From SPS, to RHIC, & (down) to FAIR

1. Large \( N_c \), small \( N_f \):  
   Quark-yonic matter - quark- Fermi sea *plus* bar-yonic Fermi surface  
   *Triple point*. Deconfining critical end point at *large* \( \mu_{qk} \sim N_c^{1/2} \)

2. New phase diagram for QCD

3. “Purely pionic” effective Lagrangians and nuclear matter:  
   The unbearable lightness of being (nuclear matter)?

Strange “MatterHorn” \( \approx \) Triple Point?

McLerran & RDP, 0706.2191. Hidaka, McLerran, & RDP 0803.0279  
McLerran, Redlich & Sasaki 0812.3585  
Hidaka, Kojo, McLerran, & RDP 09......  
Blaizot, Nowak, McLerran & RDP 09......

Blaschke, Braun-Munzinger, Cleymans, Fukushima, Oeschler,  
RDP, McLerran, Redlich, Sasaki, & Stachel (BBMCFOPMRSS) ’09....
So what *is* Quarkyonic matter?

*Dense* nuclear matter
QCD at large $N_c$ (small $N_f$)

In SU($N_c$), gluons matrices, $N_c \times N_c$, quarks column vectors. Denote fund. rep. by a line: quarks have one line, gluons have two.

‘t Hooft ’74: let $N_c = \# \text{colors} \to \infty$, $\lambda = g^2 N_c$ fixed. Keep $N_f = \# \text{flavors finite}$.

Consider gluon self energy at 1 loop order. For any $N_c$, color structure in all diagrams (3 gluon & 4 gluon vertices) reduces to (Hidaka & RDP 0906.1751)

\[
\begin{align*}
\sim g^2 N_c \sim \lambda \\
\sim g^2 \sim \frac{\lambda}{N_c}
\end{align*}
\]

First diagram is “planar”. Second, involving trace, is not, is down by $1/N_c$.

At large $N_c$ and small $N_f$, planar diagrams dominate.
Large $N_c$ and small $N_f$: *glue* dominates

Contribution of the quarks to the gluon self energy at 1 loop order, any $N_c$:

\[
\sim g^2 N_f \sim \frac{1}{N_c} N_f \lambda
\]

If $N_f/N_c \to 0$ as $N_c \to \infty$, loops *dominated* by gluons, *blind* to quarks.

Quarks act *something* like external sources, not quite.

N.B.: limit of large $N_c$, small $N_f$ is *free* of the pathologies of $N_f = 0$ (quenched)

*No* problems considering nonzero quark density, $\mu_{qk}$:

quarks do *not* affect gluons when $\mu_{qk} \sim 1!$
Phases at large $N_c$: \textit{pressure} as an order parameter

$T = \mu_{qk} = 0$: \textit{confined}, only color singlets. Glueballs, meson masses $\sim 1$. Baryons \textit{very} heavy, masses $\sim N_c$, so no virtual baryon anti-baryon pairs.

$T \neq 0, \mu_{qk} = 0$: 

$T < T_c$: \textit{Hadrons}. $T_c \sim \text{mass} \sim 1$. \# hadrons $\sim 1$, so pressure $= p \sim 1$: \textit{small}.

$T > T_c$: \textit{Quark-Gluon Plasma}. Deconfined gluons & quarks. \# gluons $\sim N_c^2$, so $p \sim N_c^2$: \textit{big}. Dominated by gluons.

$T \neq 0, \mu_{qk} \neq 0$: usual mass threshold, baryons only when $\mu_{qk} > M_N/N_c = m_{qk} \sim 1$.

$T < T_c, \mu_{qk} < m_{qk}$: Hadronic \textit{“box”} in $T$-$\mu_{qk}$ plane: \textit{no} baryons.

$T > T_c$ any $\mu_{qk}$: \textit{Quark-Gluon Plasma}. Some quarks, so what, $p_{qk} \sim N_c$.

$T < T_c, \mu_{qk} > m_{qk}$: \# quarks $\sim N_c$, so $p \sim N_c$: \textit{dense} nuclear matter (\textit{not} dilute) \textit{Confined} phase! But Fermi sea of quarks? \textit{“Quark-yonic”}
Phase diagram at large $N_c$ and small $N_f$

Lattice (Teper, 0812.0085): deconfining transition 1st order at $T \neq 0$, $\mu_{qk} = 0$. Must remain so when $\mu_{qk} \neq 0$. *Straight* line in $T - \mu_{qk}$ plane.

Hadronic/Quarkyonic transition: energy density jumps by $N_c$, 1st order? Chiral transition: in Quarkyonic phase? True triple point!

**Diagram:**
- **Quark-Gluon Plasma**
- **Triple Point**
- **Hadronic**
- **Quarkyonic**
- $T \uparrow$
- $T_c$
- $p \sim 1$
- $p \sim N_c^2$
- $p \sim N_c$
- $\mu_{qk}$
Lattice: (pure glue) SU(3) close to SU($\infty$)

Bringoltz & Teper, hep-lat/0506034 & 0508021:

SU($N_c$), no quarks, $N_c = 3, 4, 6, 8, 10, 12$.

Deconfining transition first order, latent heat $\sim N_c^2$.

Hagedorn temperature $T_H \sim 1.116(9) T_c$ for $N_c = \infty$

\[
\frac{e - 3p}{N^2 T^4} \sim \text{const.}
\]
Triple point for water

Triple point where three lines of first order transitions meet.
E.g., for ice/water/steam, in plane of temperature and pressure.
(Generalizes: four lines of first order transitions meeting is a quadruple point.)
Generically, distinct from critical (end) point, where one first order line ends.

\[
\begin{align*}
T_{\text{triple}} &= 0.01^\circ \text{C} \\
P_{\text{triple}} &= 0.006 \text{ atm}
\end{align*}
\]
Quarkyonic phase at large $N_c$, large $\mu$?

Let $\mu >> \Lambda_{QCD}$ but $\sim N_c^0$. Coupling runs with $\mu$, so pressure $\sim N_c$ is close to perturbative! 

How can the pressure be (nearly) perturbative in a confined theory?

Pressure: dominated by quarks far from Fermi surf.: 

$\rho_{qk} \sim N_c \mu^4 (1 + g^2(\mu) + g^4(\mu) \log(\mu) + ....)$

Within $\Lambda_{QCD}$ of Fermi surface: 

$\rho_{qk} \sim N_c \mu^4 (\Lambda_{QCD}/\mu)^2$, non-perturbative.

Within skin, only confined states contribute.

Fermi sea of quarks + Fermi surface of bar-yons = “quark-yonic”. $N=3$?

Pressure dominated by quarks. 
But transport properties dominated by confined states near Fermi surface!

For QCD: what is (cold) nuclear matter like at high density? 
Just a quark NJL model?
Deconfining critical end point at (large) \( \mu_{qk} \sim N_c^{1/2} \)

Semi-QGP theory of deconfinement: Hidaka & RDP 0803.0453

\[
A_0 = \frac{T}{g} Q
\]

For large \( \mu \): compute one loop determinant in background field.
Korthals-Altes, Sinkovics, & RDP hep-ph/9904305

\[
S_{qk} = \text{tr} (\mu + i T Q)^4, \quad T^2 \text{tr} (\mu + i T Q)^2, \quad N_c^2 T^4 V(Q)
\]

RDP '09: for large \( \mu \), expand:

\[
S_{qk}^{\mu \sim \sqrt{N_c}, T \sim 1} \sim N_c \mu^4 - 6 \mu^2 T^2 \text{tr} Q^2 + \ldots \sim N_c^3, \quad N_c^2 (\text{tr} Q^2 / N_c)
\]

Consider \( \mu \sim N_c^{1/2}, T \sim 1 \): gluons do feel quarks.

Term \( \mu^4 \sim N_c^3 \) dominates, but \textit{independent} of Q \textit{and} temperature.

Term \( \mu^2 \sim N_c^2 \) Q-dependent. Breaks \( Z(N_c) \) symmetry, so washes out 1st order deconfining transition: Deconfining Critical End Point (CEP)
Phase diagram at large $N_c$ and small $N_f$, II

About deconfining Critical End Point (CEP), smooth transition between deconfined and quarkyonic phases.

Since gluons are sensitive to quarks for such large $\mu$, expect curvature in line. Triple point still well defined, as coincidence of three 1st order lines.

*Chiral transition?*

**Quark-Gluon Plasma**

**Quadruple Point**

**Dilute Nucl. Mat.**

$T_{c}$

$T \uparrow$

$\mu_{qk} \sim 1$

$\mu_{qk} \sim N_{c}^{1/2}$

**Hadronic**

**Quarkyonic**

**Deconf.g CEP**
So what does this have to do with experiment?

Strange “MatterHorn” ≈ Triple Point?
Wonderous utility of statistical/hadron resonance gas models

Chemical equilibration at SIS, AGS, SPS, RHIC, and onto NICA and FAIR: Braun-Munzinger, Cleymans, Oeschler, Redlich, Stachel plus: Bialas, Biro, Broniowski, Florkowski, Levai, Ko, Satz + ...

\[ T \uparrow \]
Smooth evolution in $T$, $\mu_{\text{Baryon}}$ with $\sqrt{s_{\text{NN}}}$
Devil is in the details: RDP, Review of Quark Matter 2004:

Is this related to the narrow peak in K+/pi+ @ SPS? The “MatterHorn” of NA49?

Peak not confirmed by other groups, not seen in other ratios...
Strange MatterHorn: peak in $K^+/\pi^+$, not $K^-/\pi^-$
Strange MatterHorn: also in baryons

Natural to have peaks in $K^+/\pi^+$, strange baryons: start with (s s-bar) pairs. At $\mu \neq 0$, strange quarks combine into baryons, anti-strange into pions. For different baryons, peaks do not occur at same energy, but nearby, so not true phase transition, but approximate.
Strange MatterHorn and the triple point?

Usual explanation of MatterHorn: transition from baryons to mesons at freezeout.

Or: changing from Hadronic/Quarkyonic boundary to Hadronic/QGP boundary: i.e., (approximate) triple point.
HBT radii: minimum near strange MatterHorn?

freeze out
volume ↑

\[ \sqrt{s_{NN}} \]
HBT radii: flat from NA49.
Triple point versus critical end point

Critical endpoint: correlation lengths *diverge*.

Hence: **HBT radii should *increase***.

Effects should be greatest on the lightest particles, *not* the heaviest: $K^+/\pi^+$ should *decrease*, not *increase*. *Neither* is seen in the data.

Assume that at triple point, chiral transition splits from deconfining.

Leading operator which couples the two transitions is

Mocsy, Sannino, & Tuominen, hep-ph/0301229, 0306069, 0308135, 0403160:

$$c_1 \ell \text{ tr } \Phi^\dagger \Phi \sim c_1 \ell (\pi^2 + K^2 \ldots)$$

If this coupling $c_1$ flips sign, transitions diverge. Hence $c_1 = 0$ at triple point?

If so, leading coupling then becomes

$$c_2 \ell \text{ tr } M \Phi \sim c_2 \ell (m_\pi^2 \pi^2 + m_K^2 K^2 \ldots)$$

This coupling is proportional to mass squared: *bigger* for kaons than pions!

Enhancement of $K^+/\pi^+$, strange baryons due to dense environment.

Implicitly: line for chiral transition crossover, not 1st order.
(Lunatic) ideas about nuclear matter:

“The unbearable lightness of being (nuclear matter)”
Nucleon-nucleon potentials from the lattice

Ishii, Aoki & Hatsuda, PACS-CS, 0903.5497
Nucleon-nucleon potentials from quenched and 2+1 flavors.
Pions heavy: 700 MeV (left) and 300 MeV (right)
Standard lore: delicate cancellation. *So why independent of pion mass?*
Essentially *zero* potential plus strong hard core repulsion

\[ m_\pi = 702 \text{ MeV} \]

\[ m_\pi = 296 \text{ MeV} \]
Purely pionic nuclear matter

J.-P. Blaizot, L. McLerran, M. Nowak, & RDP ’09....

At infinite $N_c$, integrate out all degrees of freedom except pions:

Lagrangian power series in $U = e^{i\pi/f_\pi}$, $V_\mu = U^\dagger \partial_\mu U$

Infinite # couplings: Skyrme plus complete Gasser-Leutwyler expansion,

$$\mathcal{L}_\pi = f_\pi^2 V_\mu^2 + \kappa [V_\mu, V_\nu]^2 + c_1 (V_\mu^2)^2 + c_2 (V_\mu^2)^3 + \ldots$$

All couplings $\sim N_c$, every mass scale $\sim$ typical hadronic.

Need infinite series, but nothing (special) depends upon exact values

Valid for momenta $< f_\pi$, masses of sigma, omega, rho...

Useful in (entire?) phase with chiral symmetry breaking?

Higher time derivatives, but no acausality at low momenta.
Purely pions give free baryons

From purely pionic Lagrangian, take baryon as stationary point.

Find baryon mass \( \sim N_c \), some function of couplings.

Couplings of baryon dictated by chiral symmetry:

\[
\bar{\psi} \left( i\partial + M_B \, e^{i\tau \cdot \gamma_5 / f_\pi} \right) \psi
\]

By chiral rotation,

\[
W = \exp(-i\pi\gamma_5 / 2 f_\pi)
\]

\[
\mathcal{L}_B = \bar{\psi} (iW^\dagger \partial W + M_B) \psi \sim \frac{1}{f_\pi} \bar{\psi} \gamma_5 \partial \pi \psi + \ldots
\]

At large \( \sim N_c \), \( f_\pi \sim N_c^{1/2} \) is big. Thus for momenta \( k < \) hadronic, interactions are small, \( \sim 1/f_\pi^2 \sim 1/N_c \).

Thus: baryons from chiral Lag. free at large \( N_c \), down to distances \( 1/f_\pi \).

Manifestly special to chiral baryons. True for u, d, s, but not charm?
The Unbearable Lightness of Being (Nuclear Matter)

Use purely pionic Lagrangian for all of nuclear matter?

Then pressure $\sim 1$, and not $N_c$. Like hadronic phase, not quarkyonic.

Unlike standard lore, where pressure (nucl mat) grows quickly, $\sim N_c$


IF chiral transition 1st order, etc, Quadruple Point where four phases coexist.

Quark-Gluon Plasma, $P \sim N_c^2$

Quadruple Point

Hadronic $P \sim 1$

Quarkyonic $P \sim N_c$

Dilute Nucleons, $P \sim 1$

$\mu_{qk} \sim 1$

$\mu_{qk} \sim N_c^{1/2}$

Deconf.g CEP
An unbearably light phase diagram for QCD

Quadruple Point

Chiral?
Quarkyonic matter for *two* colors?

i.e., are large $N_c$ arguments even ok for $N_c = 2$?
Quarkyonic matter for two colors?

*Hands*, Ilgenfritz, Kenny, Kim, Mueller-Preussker, Schubert, Sitch, & Skullerud, HIKKMPSSS 09....


Mesons and baryons are both bosons. Baryon $\mu_{qk}$ like isospin $\mu_{qk}$ for 3 colors.

$\mu_{qk}$ only matters when $> \mu_0 = m_\pi/2$ (not $> m_{\text{Baryon}}$).

Expect Bose-Einstein condensate for $\mu_{qk} > \mu_0$, compare to Chiral Pert. Theory.

Find: $\mu_0 \ a = 0.2$: BEC turns on, good agreement with CPT only *very* near $\mu_0$.

$\mu_t \ a = 0.4$: *big* jump in energy density - ?

$\mu_d \ a = 0.65$ : Polyakov loop nonzero, deconfined quarks, only at high density

Suggests: Quarkyonic matter for a large range, between $\mu_0$ and $\mu_d$, for $N_c = 2$!
Superfluid condensate scaling \( \approx \) BCS for \( \mu_Q \lesssim \mu \lesssim \mu_d \)
Polyakov loop \( \approx 0 \) for \( \mu < \mu_d \), but then rises from zero
\( \Rightarrow \) Deconfinement at \( \mu \approx 900 \text{MeV}, n_q \approx 35 \text{fm}^{-3} \)
In contrast to $\chi$PT prediction, (unrenormalised) quark energy density $\varepsilon_q$ greatly exceeds SB value as $\mu \downarrow \mu_{o+}$

$\Rightarrow$

Energy per quark $\varepsilon_q/n_q$ has shallow minimum for $\mu > \mu_Q$