Deconfinement and the Gross-Witten point

A. Dumitru, Y. Hatta, J. Lenaghan, K. Orginos, & R.D.P. = DHLOP '03, hep-th/0311223
Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk = AMMPR '03, hep-th/0310285
Furuuchi, Schridber, & Semenoff: hep-th/0310286; Schnitzer: hep-th/0402219
A. Dumitru, J. Lenaghan, R.D.P., & K. Splittorff = DLPS '04.

Consider a "pure" SU(N) gauge theory, *no* dynamical quarks.
Rigorously, a deconfining phase transition at a temperature T.

1. SU(∞) matrix models: Gross-Witten model as a tri-critical point.
   AMMPR: spinoidal temp. = Hagedorn temp.

2. SU(3) renormalized Polyakov loops from the lattice.

3. Lattice: SU(3) deconfining transition close to the SU(∞) Gross-Witten point.

But *unnatural* to be close to a tri-critical point.

Mean field theory: Z(N) vector

Consider a scalar field invariant under a global U(1) symmetry: $\phi \to e^{i\theta}\,\phi$

Look for spontaneous breaking of U(1) symmetry through $\langle\phi\rangle \neq 0$

Start with the most general potential invariant under U(1):

$$\mathcal{V}_{U(1)} = m^2|\phi|^2 + \lambda_4(|\phi|^2)^2 + \lambda_6(|\phi|^2)^3 + \ldots$$

Can *always* fit $\langle\phi\rangle$, versus the temperature T, as the minimum of a potential.

But how do the mass and couplings depend upon T?

Mean field theory: mass^2 linear in T, coupling constants don't change with T

If only Z(N) symmetry, $\phi \to e^{2\pi i/N}\phi$ the potential also includes

$$\mathcal{V}_{Z(N)} = \lambda_N(\phi^N + (\phi^*)^N)$$

Mean field phase diagram

For Z(3), cubic term => transition always first order.

When N≠3, all phase diagrams look alike:

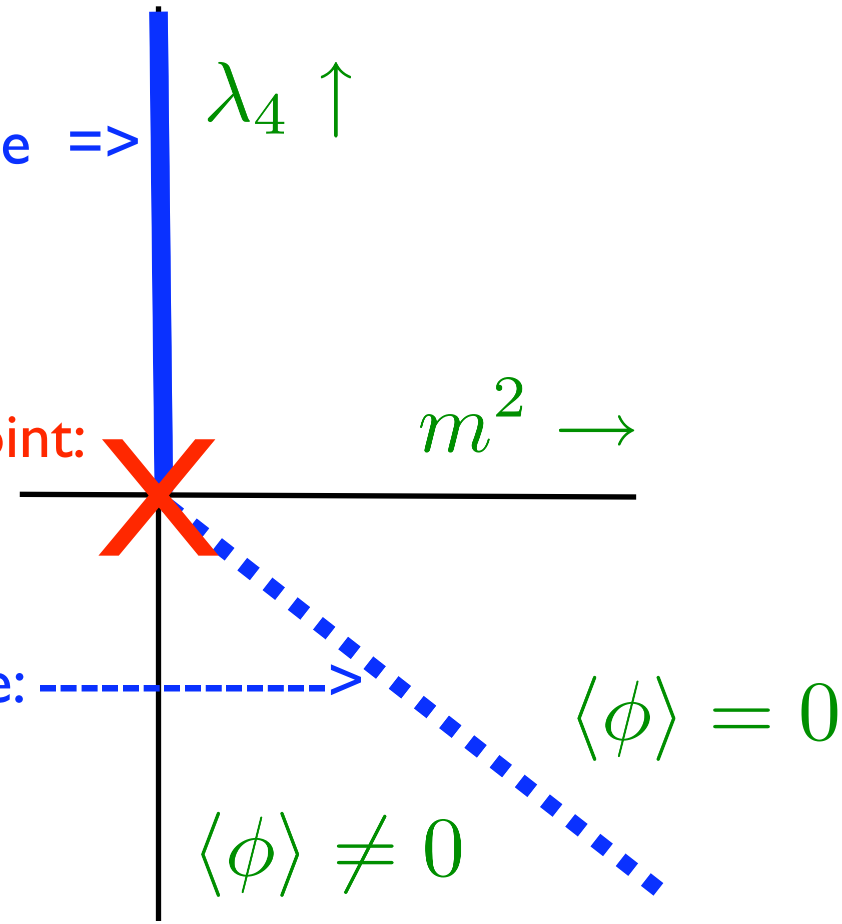
Lines of 1st and 2nd order transitions meet at a tri-critical point

$$\mathcal{V}_{U(1)} = m^2|\phi|^2 + \lambda_4(|\phi|^2)^2 + \lambda_6(|\phi|^2)^3 + \ldots$$

$m^2 = 0\ ,\ \lambda_4 > 0$ 2nd order line =>

$m^2 = \lambda_4 = 0$ Tri-critical point:

$m^2 > 0\ ,\ \lambda_4 < 0$ 1st order line: ---------------->



Mean field theory: SU(N) matrix

Consider the matrix for a Wilson loop $\mathbf{L} = \mathcal{P} e^{ig \oint A_\mu dx^\mu}$

In the fundamental representation, $\mathbf{L}$ is a SU(N) matrix: $\mathbf{L}^\dagger \mathbf{L} = \mathbf{1} \; , \; \det \mathbf{L} = 1$

Transforms under local SU(N) transf.'s, Ω, as: $\mathbf{L} \rightarrow \Omega^\dagger \, \mathbf{L} \, \Omega$

Can have a global Z(N) symmetry ("topological") which breaks with*out* breaking SU(N). $\mathbf{L} \rightarrow e^{2\pi i/N} \, \mathbf{L}$

Usual order parameter: loop in fundamental representation: $\ell = \dfrac{1}{N} \, \mathrm{tr} \, \mathbf{L}$

T ≠ 0: thermal Wilson line => Polyakov loop. Phases:

Z(N) symmetric = confined: $\langle \ell \rangle = 0 \; , \; T < T_d$

Z(N) sym. broken = deconfined: $\langle \ell \rangle \neq 0 \; , \; T > T_d$

Loop ~ (trace) "test" quark propagator. Deconfining transition at T_d.

Matrix mean field theory

Matrix model = a matrix in the measure: $\mathcal{Z} = \int d\mathbf{L} \; \exp(-\mathcal{V})$

Naturally obtain loops in all representations (unlike Potts model). Adjoint loop:

$$\ell_{adj} = \frac{1}{N^2 - 1} \left(|\mathrm{tr}\mathbf{L}|^2 - 1 \right)$$

Loop in rep. R $\ell_R = \mathrm{tr}\; \mathbf{L}_R$/dim. R. Z(N) charge, mod N: $e_{fund} = 1 \;,\; e_{adj} = 0$

Most general potential sum of Z(N) *neutral* loops:

$$\mathcal{V} = m^2 \; \ell_{adj} + \Sigma_j \; \lambda_j \; \ell_j \;,\; e_j = 0$$

Naively: adjoint loop is a mass term, other Z(N) neutral loops higher couplings.

In the deconfined phase, all loops condense. How are they related?

In the confined phase, Z(N) chg'd loops = 0: how big are Z(N) neutral loops?

Large N matrix models

Enormous simplifications at large N: e.g., $\ell_{adj} \approx |\ell|^2 + 1/N^2$

As N=>∞, "factorization" => potential merely powers of the fundamental loop:

$$\mathcal{V}_{U(1)}/N^2 = m^2|\ell|^2 + \lambda_4(|\ell|^2)^2 + \lambda_6(|\ell|^2)^3 + \ldots$$

The global symmetry is reduced from U(1) to Z(N) by the term

$$\mathcal{V}_{Z(N)}/N^2 = +\widetilde{\lambda}_N(\ell^N + (\ell^*)^N) + \ldots$$

At large N, vary mass and couplings to reach:

confined phase: $\langle \ell \rangle = 0 \; , \; \langle \mathcal{V} \rangle / N^2 = 0$

deconfined phase: $\langle \ell \rangle \neq 0 \; , \; \langle \mathcal{V} \rangle / N^2 \sim 1$

Free energy of deconfined phase ~N^2 from # gluons.

Large N: van der Monde potential

Brezin, Ityzkson, Parisi & Zuber = BIPZ '78; Gross & Witten = G&W '81
Kogut, Snow & Stone = KSS '82; Green & Karsch '84; Damgaard '87, D & Hasenbusch = D&H '94
Aharony + ... = AMMPR '03 Dumitru + ... = DHLOP '03; Dumitru + ... = DLPS '04

Do U(1) rotation so the fundamental loop, $\ell = \mathrm{tr}\,\mathbf{L}/N$, is real & positive.

BIPZ +...AMMPR: minimize with respect to eigenvalues of $\mathbf{L}$
=> "potential" from the van der Monde determinant

$$\mathcal{V}_{vdM}/N^2 = +\,\ell^2\;,\;\ell < 1/2$$

$$\mathcal{V}_{vdM}/N^2 = -\log(2(1-\ell))/2 + 1/4\;,\;\ell > 1/2$$

G&W: the vdM potential is discontinuous, of *third* order, at $\ell = 1/2$

$\ell < 1/2$: only mass term - no $\ell^4\;,\;\ell^6\ldots$ to ℓ^N

$\ell > 1/2$: eigenvalue repulsion from vdM det. => $\ell < 1$

Matrix models & the Gross-Witten point

Solutions minima of effective potential, $\mathcal{V}_{eff} = \mathcal{V}_{U(1)} + \mathcal{V}_{vdM}$

Introduce new mass parameter $\widetilde{m}^2 = m^2 + 1$

For $\ell < 1/2$

$$\mathcal{V}_{eff}/N^2 = +\widetilde{m}^2 \, \ell^2$$

$\widetilde{m}^2 > 0$: confined

$\widetilde{m}^2 < 0$: deconfined

Gross-Witten point: $\widetilde{m}^2 = \lambda_4 = \lambda_6 = \ldots = 0$

AMMPR: take space = very small sphere, so gauge coupling small.

spinoidal point, $\widetilde{m}^2 = 0$, = Hagedorn temperature T_{Hag}

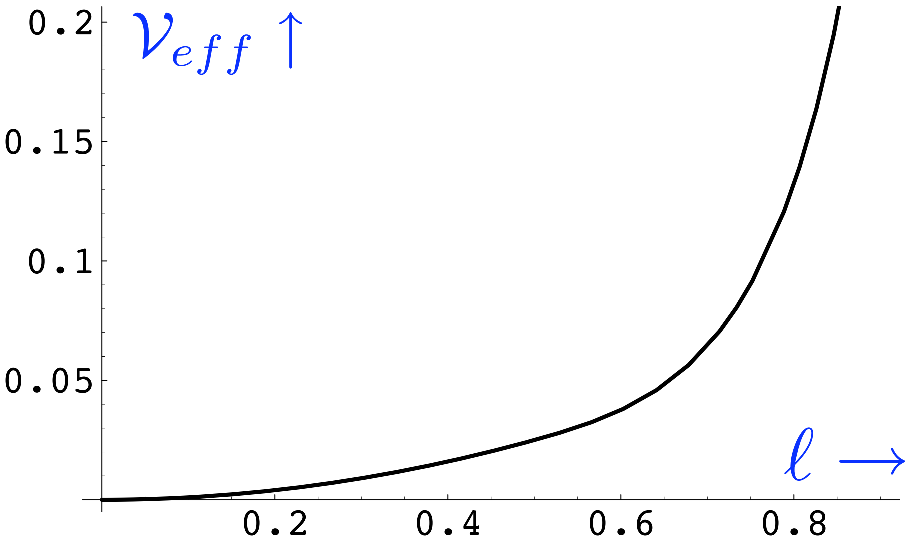
Hagedorn exponential growth in density states, *not* limiting temp.

=> at Gross-Witten point, Hagedorn temp. = deconfining trans. temp.

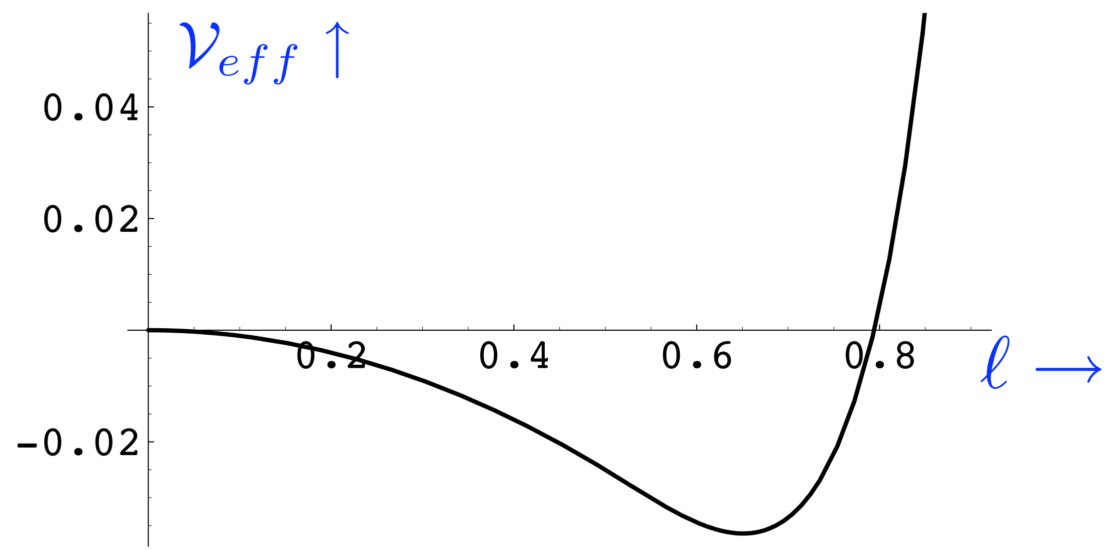
away from Gross-Witten point, Hagedorn ≠ deconfinement

Near the Gross-Witten point

All potentials have 3rd order discontinuity at $\ell = 1/2$



<= confined: $\widetilde{m}^2 = +.1$



deconfined=>

$\widetilde{m}^2 = -.1$

At the Gross-Witten point: "critical" 1st order

Potential completely flat from 0 to 1/2.

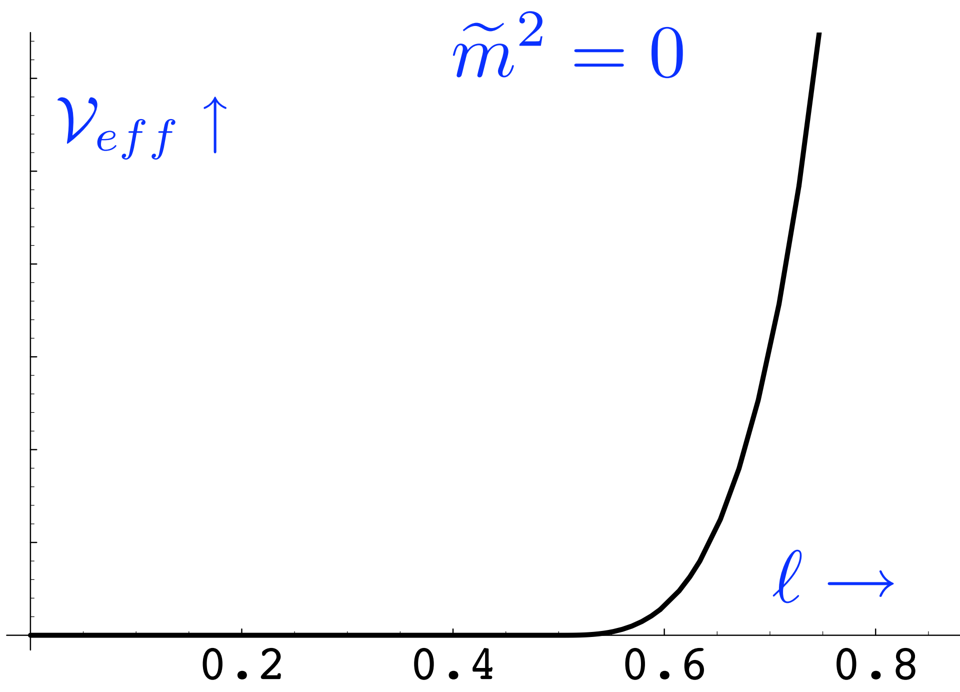
Order parameter jumps at transition:

$$\langle \ell \rangle : 0 \rightarrow 1/2 \;, \; \widetilde{m}^2 = 0$$

Non-analytic point in potential coincides with new minimum.

KSS '82, AMMPR '03, DHLOP '03

Mean field: varying m^2 ~ temperature.



Transition first order: latent heat nonzero, $\partial \mathcal{V}_{eff}(\langle \ell \rangle)/\partial m^2 = 1/4$

But "critical": (physical) masses => 0, asymmetrically, at transition:

$$m^2_{phys} = \left. \frac{\partial^2 \mathcal{V}_{eff}}{\partial \ell^2} \right|_{\ell = \langle \ell \rangle}$$

$$m^2_{phys} \sim \widetilde{m}^2 \;, \; \widetilde{m}^2 \rightarrow 0^+$$

$$m^2_{phys} \sim \sqrt{-\widetilde{m}^2} \;, \; \widetilde{m}^2 \rightarrow 0^-$$

Gross-Witten = tri-critical point

At large N,

$$\mathcal{V}_{eff}/N^2 = \widetilde{m}^2\,\ell^2 + \lambda_4\,\ell^4 + \lambda_6\,\ell^6 + \ldots \qquad \ell < 1/2$$

Phase diagram looks the same, but: tri-critical point is the Gross-Witten point

AMMPR: $\lambda_4 \neq 0\;,\;\lambda_6 = 0$

Away from G-W pt along $\lambda_4 < 0$

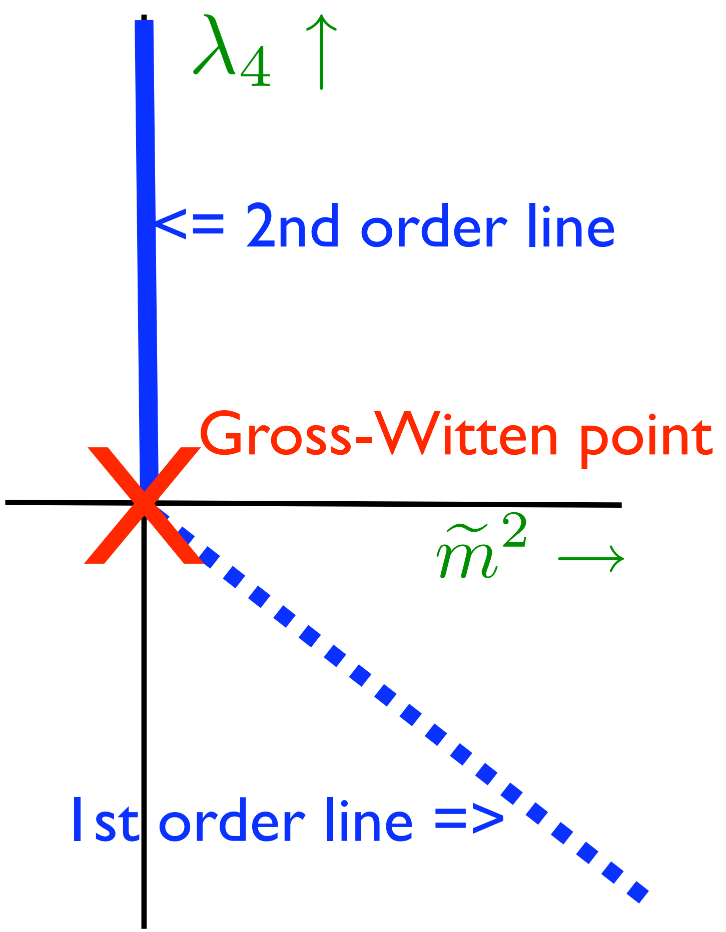
~ ordinary 1st order trans.: masses ≠ 0

Jump in $\langle \ell \rangle \geq 1/2$

DLPS: $\lambda_4\;,\;\lambda_6 \neq 0$ Away from G-W point,

ordinary 1st order trans.'s: masses ≠ 0

Jump in $\langle \ell \rangle$ arbitrary



Only G-W point "critical" 1st order:

masses = 0 and jump in $\langle \ell \rangle$ (to 1/2)

But must tune λ_4 = 0 to reach G-W point.

(Other λ's marginal or irrelevant)

Matrix models, N < ∞

Matrix model: 4 ≤ N < ∞, Gross-Witten pt = ordinary 1st order (masses ≠ 0)

N=3: (adjoint = octet) + decuplet: $\mathcal{V}/9 = m^2\,\ell_8 + \lambda_{10}\,\ell_{10} + \ldots$

$\ell_{10} = (\mathrm{tr}\,\mathbf{L}\;\mathrm{tr}\,\mathbf{L}^2 + 1)/10$ ~ cubic invariant, transition always 1st order

Svetitsky & Yaffe '82

KSS: at G-W pt, $\langle \ell \rangle$: 0 => .485 ± .001 (~ 1/2!)

DLPS: with λ_{10} ≠ 0, at transition jump in $\langle \ell \rangle$ can decrease from 1/2

N=2: G-W pt 2nd order transition (higher loops => Z(2) critical point)

D&H '94, DLPS: in a matrix model, N ≥ 2, all loops vanish in the confined phase

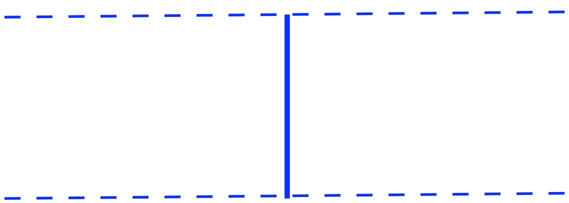
$$\langle \ell_R \rangle = \langle \mathcal{V} \rangle = 0\;,\; T < T_d\;,\; \forall\, R$$

=> expectation values of Z(N) neutral loops in the confined phase, $\langle \ell_{adj} \rangle$ ≠ 0, due to fluctuations about matrix model.

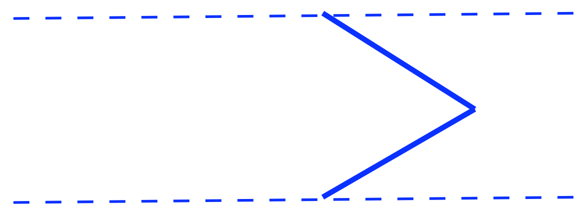
Renormalized Polyakov Loops

Gervais & Neveu '80. Polyakov '80. Dotsenko & Vergeles '80. Brandt, Gocksch, Neri, Sato '81, '82
Ivanov, Korchemsky & Radyushkin '86. K & R '87, '92. Belitksy, Gorsky & K '03.
Kaczmarek, Karsch, Petreczsky & Zantow = KKPZ '02. DHLOP '03.

Straight Polyakov loop:



Polyakov loop with two cusps, 0 & 1/2T



τ↑: imaginary time,
0 => 1/T

In d+1 spacetime dim.'s, T ≠0, ultraviolet divergences of straight loop ~ $\int d^d p/p^2$
propagating field in d space dim.'s. With cusp, $\sim 1/\sqrt{p^2}$

Renormalized loop after "mass" ren.,
R = irreducible representation:

$$\widetilde{\ell}_R = \mathcal{Z}_R \, \ell_R \; , \; \mathcal{Z}_R = e^{-m_R^{div}/T}$$

3+1 dim.'s: $a \, m_R^{div} = + C_R \, g^2 (1 + \# g^2 + \ldots)$ (a=lattice spacing, C_R= Casimir)

Straight loops have no logarithm. Loops with cusps do.

2+1 dim.'s: Straight loops have logarithm, no new log with cusps.

No bound on ren.'d loops

Bare loop is a normalized trace => $|\ell_R| \leq 1$

For renormalized loop: $\widetilde{\ell}_R = \ell_R / \mathcal{Z}_R \Rightarrow |\langle \widetilde{\ell}_R \rangle| \leq 1/\mathcal{Z}_R$

=> IF $m_R^{div} > 0 \; \forall \; T$, $\mathcal{Z}_R = e^{-a m_R^{div}} \rightarrow 0$ in the continuum limit, a → 0,
bare loops vanish & there is no bound on ren'd loops.

Numerically: we find that all divergent masses always positive.
E.g.: as T → ∞, ren'd loops approach 1 from *above*: (Gava & Jengo '81)

$$\langle \widetilde{\ell}_R \rangle - 1 \sim - \left(\frac{1}{T} \right) \mathcal{C}_R g^2 \int \frac{d^3 k}{k^2 + m^2_{Debye}} \sim (-) C_R g^2 (-) (m^2_{Debye})^{1/2}$$

$$\langle \widetilde{\ell}_R \rangle \approx \exp \left(+ \frac{C_R}{N} \frac{(g^2 N)^{3/2}}{8\pi\sqrt{3}} \right)$$

=> *negative* "free energy"
McLerran & Svetitsky '82

Smooth large N limit: $C_R \approx \# N + O(1)$, $N \rightarrow \infty$

Ren.'d Polyakov loops on the lattice

Basic idea: compare two lattices, *same* temperature, *different* lattice spacing.

If a << 1/T, ren'd quantities the same.

N_t = # time steps = 1/(aT) changes between the two lattices: get Z_R

$$\log\left(|\langle \ell_R \rangle|\right) = -f_R^{div} N_t + f_R^{cont} + f_R^{lat} \frac{1}{N_t} + \ldots$$

$$\langle \widetilde{\ell}_R \rangle = \exp(-f_R^{cont})$$

Numerically, we find $f_R^{lat} \approx 0$

SU(3) Wilson action, N_t = 4,6,8,10; # spatial steps = 3 N_t

Lattice coupling constant β = 6/g^2: related to temperature by Non-Pert. Ren.

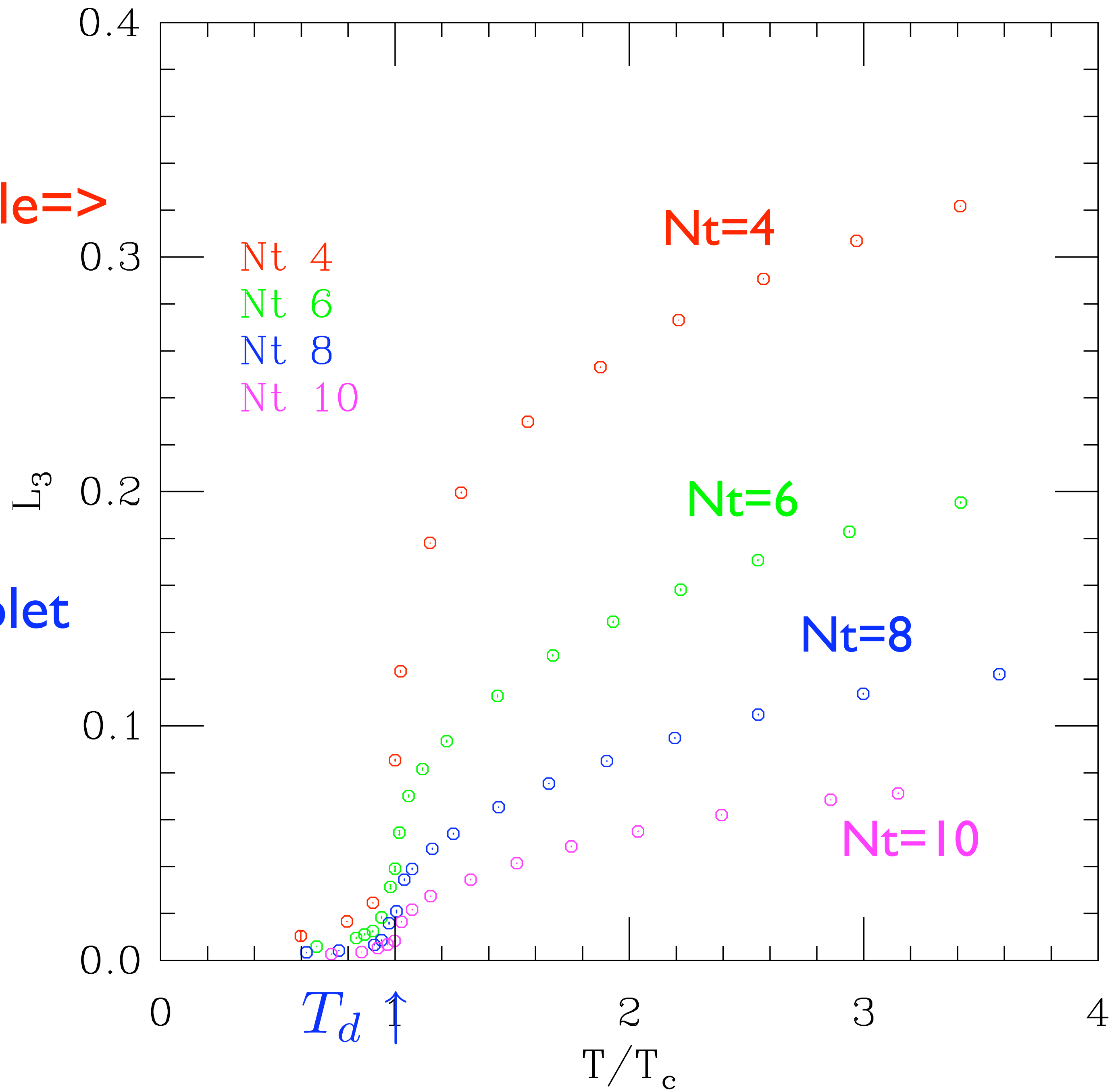
Coupling for transition changes with N_t, $\beta_{deconf}(N_t)$

=> to obtain the same T at different N_t, must compute at different β

Doable, not trivial.

Bare triplet loop vs T, Nt

Note scale=>
~ .3



Bare triplet loop↑

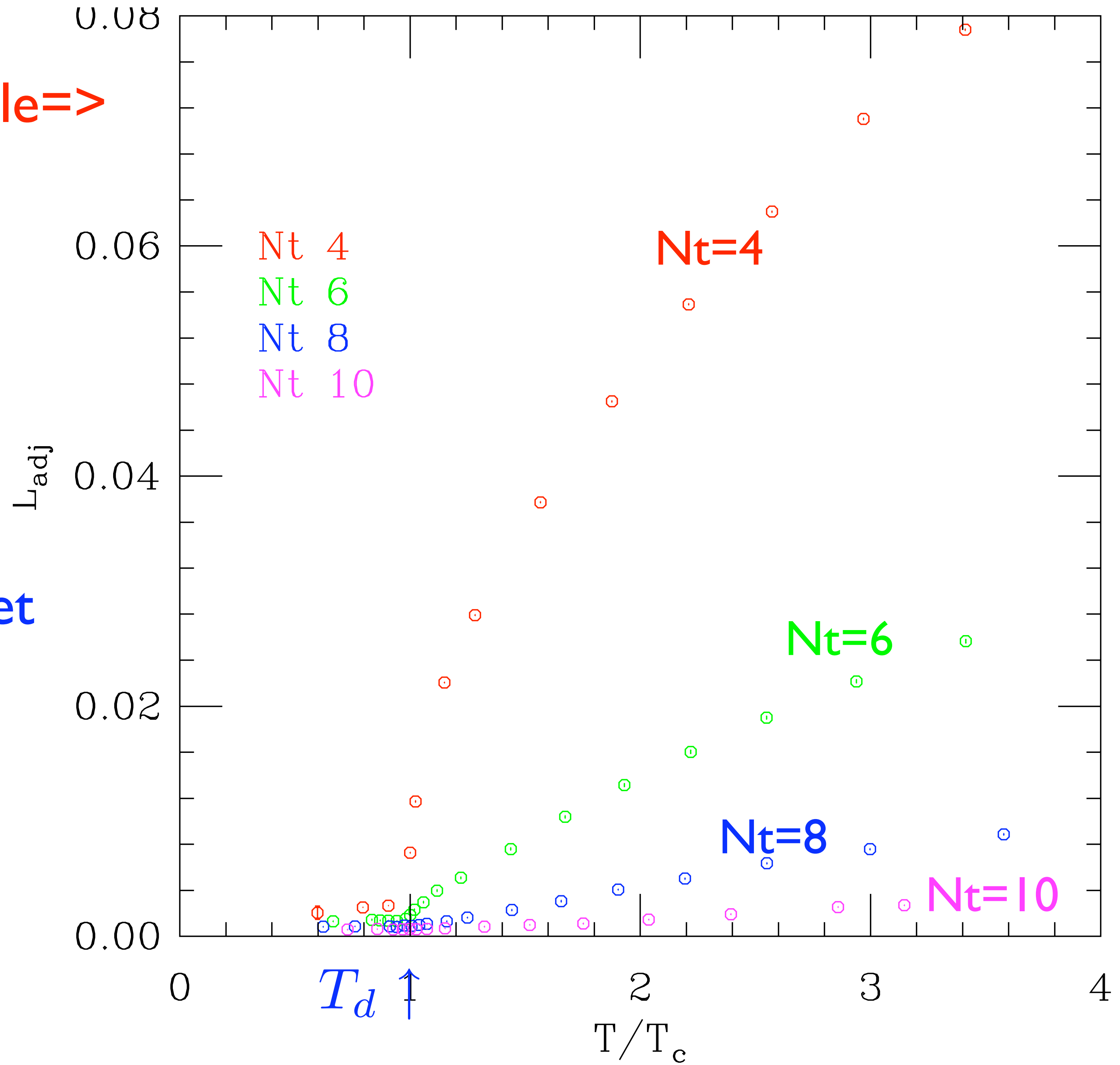
Nt = # time steps.

Bare loop vanishes as Nt → ∞

$T/T_d \rightarrow$

Bare octet loop vs T, Nt

Note scale=>
~ .06



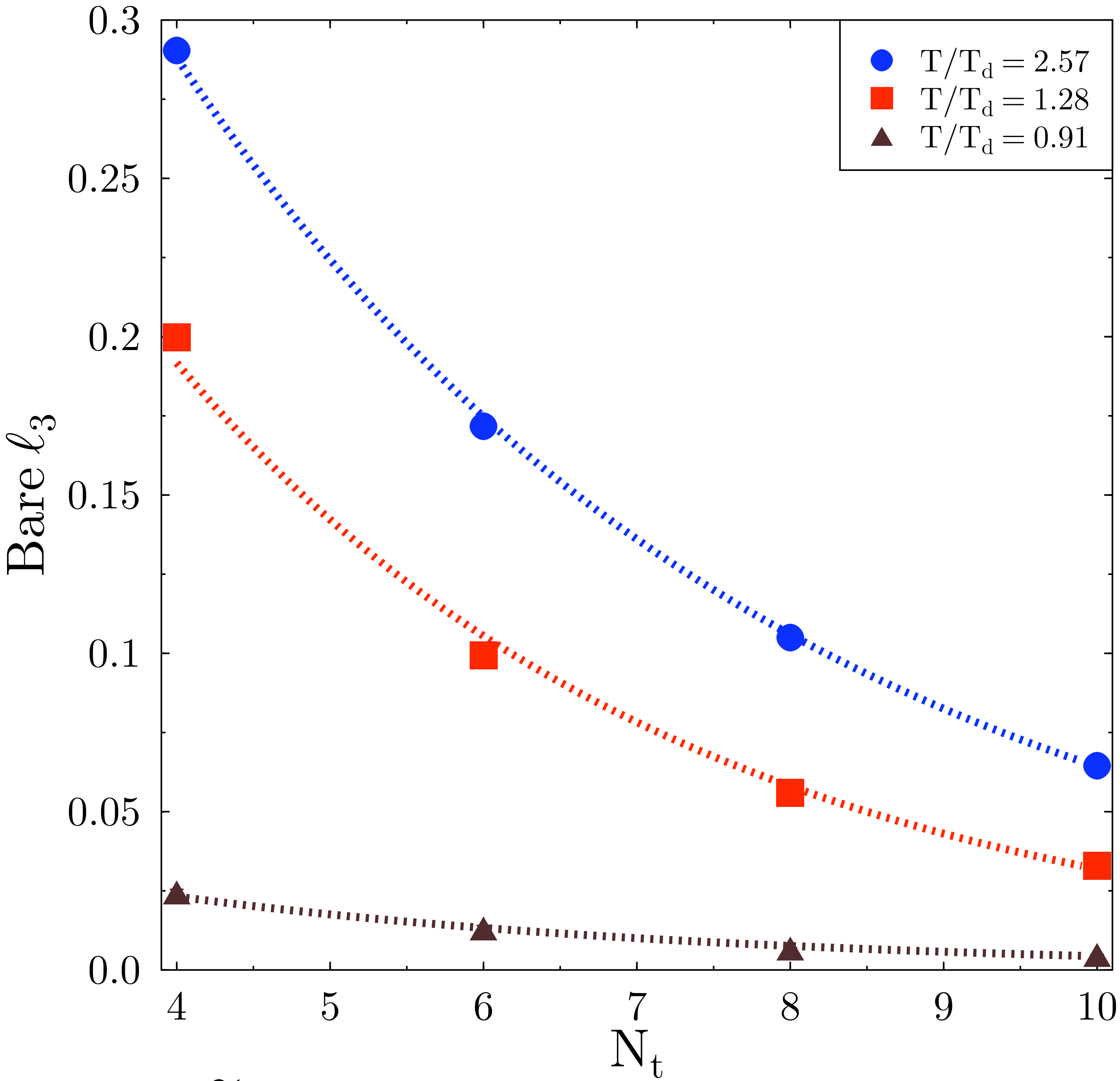
Bare octet loop↑

Sextet loop very similar

Decuplet loop only measurable at Nt=4

$T/T_d \rightarrow$

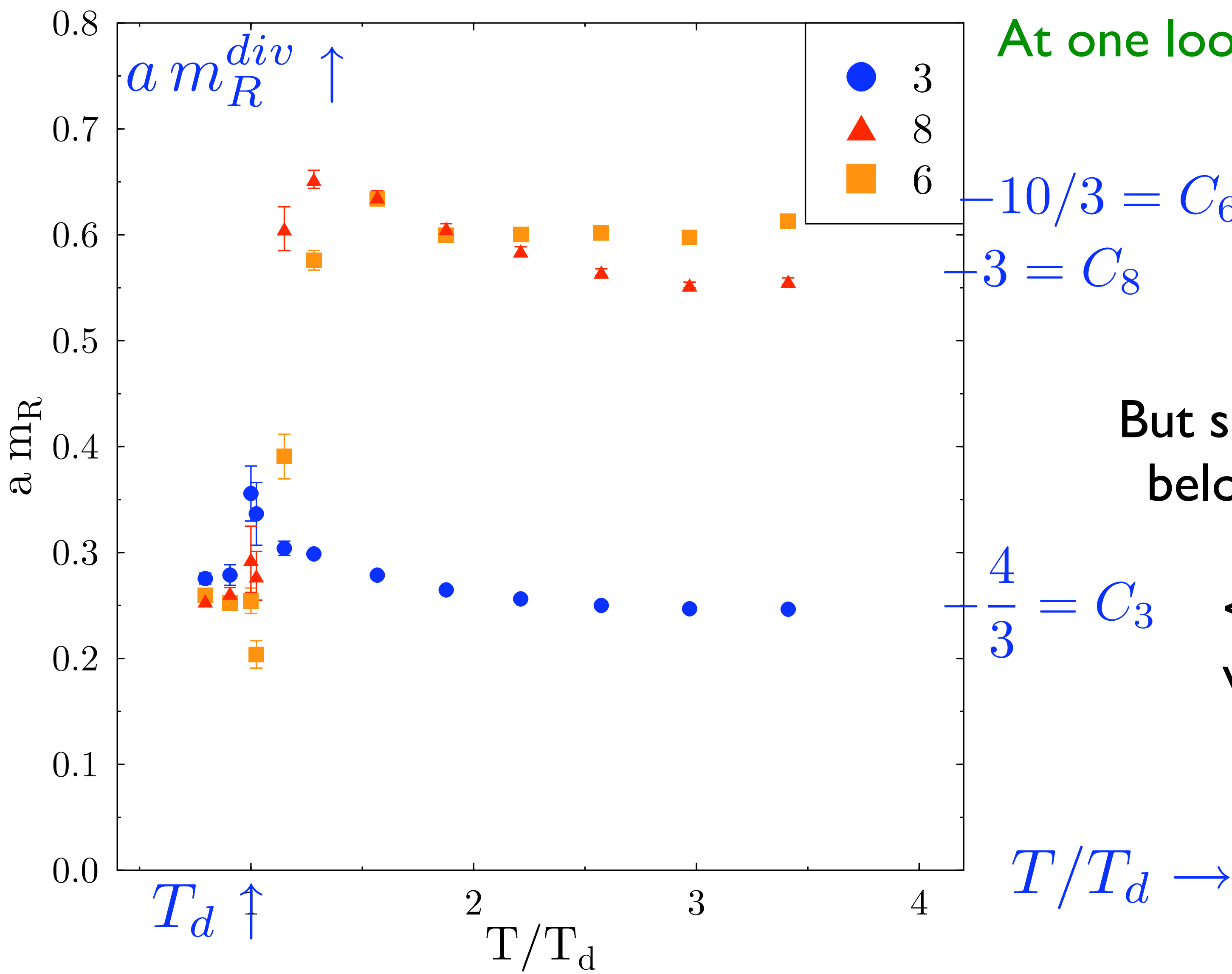
Bare $|\ell_3|$ vs N_t



$$|\langle \ell_3 \rangle| \equiv \exp(-m_3/T) |\langle \widetilde{\ell_3} \rangle|$$

Lattice SU(3): divergent "masses"

DHLOP: Z_R from same T, different lattice spacing. Triplet, sextet, octet loops.
KKPZ: Z_R from short distance behavior of two-point functions. Triplet loop.



At one loop, divergent masses ~ Casimir.

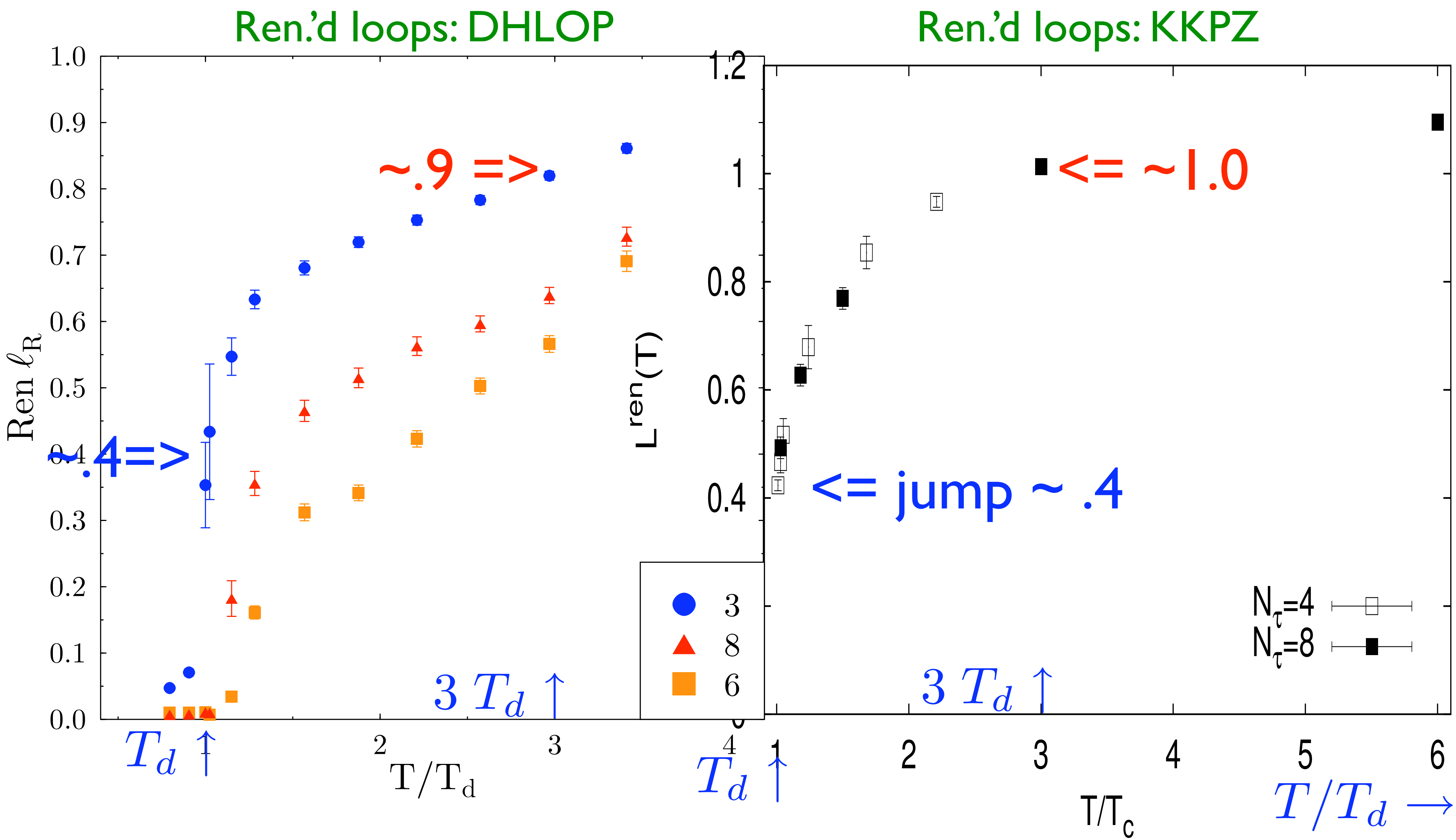
<= approximately valid, ~ 3 T_d

But sextet mass above octet below 1.5 T_d

<= Weak temperature variation in triplet mass

Lattice SU(3): renormalized Polyakov loops

At transition, jump in $\langle \ell \rangle$ to ~.4 (± 10%)



Triplet loop ≠ 1 => non-perturbative gluon plasma for T_d => 3 T_d

Lattice: SU(3) close to SU(∞), ~25%

At large N, “factorization” => all loops product of fundamental (& anti-fund.)

Migdal & Makeenko ‘80, Eguchi & Kawai ‘82, Damgaard ‘87, D&H ‘94

$$\delta\ell_6 \equiv \langle \ell_6 \rangle - \langle \ell_3 \rangle^2 \sim 1/N$$

$$\delta\ell_8 \equiv \langle \ell_8 \rangle - |\langle \ell_3 \rangle|^2 \sim 1/N^2$$

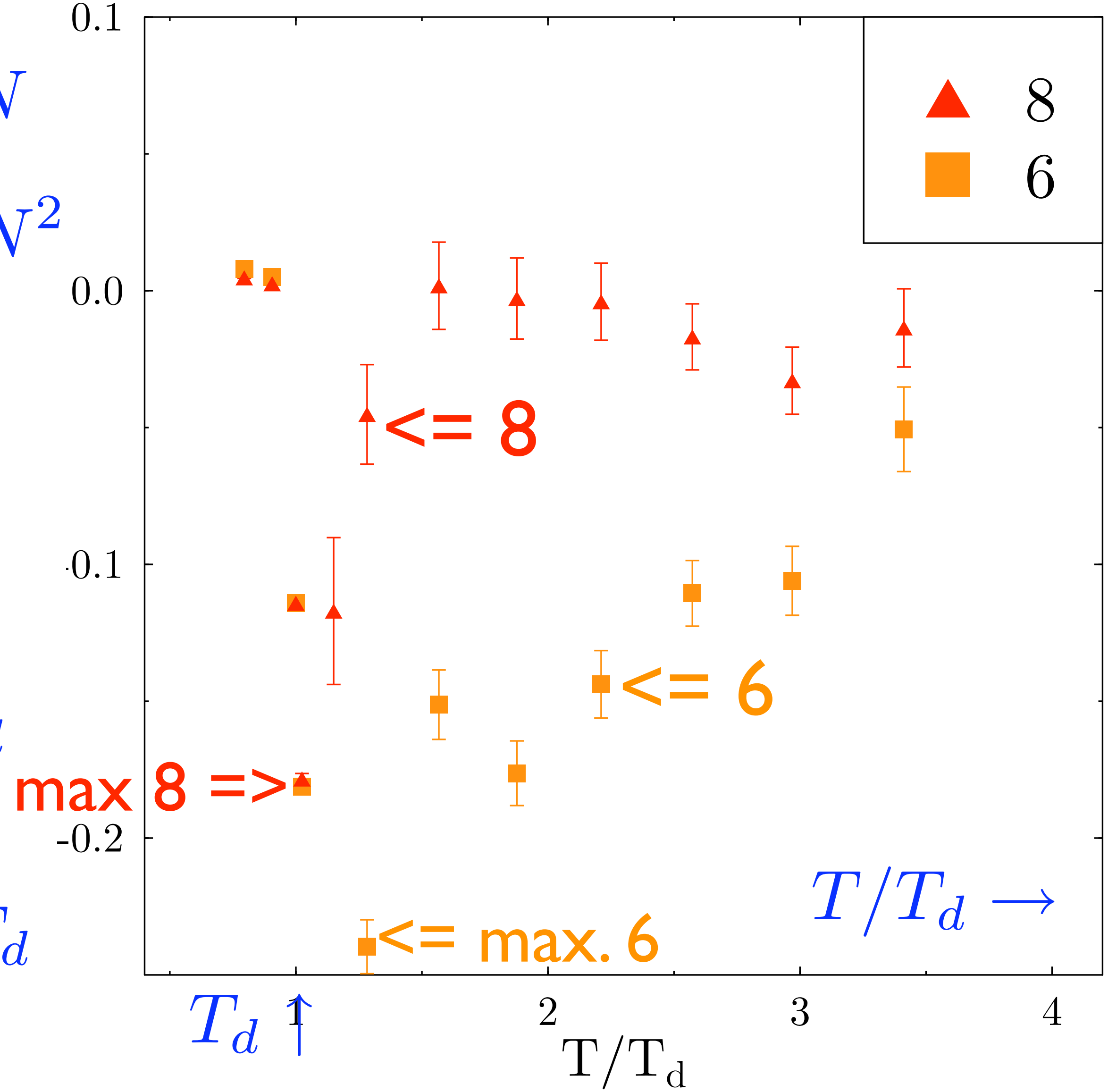
DHLOP: “spikes” in diff. loops

Corrections to factorization

small except just above T_d

$$\max |\delta\ell_8| \sim .2 \ @ \ 1.1 \ T_d$$

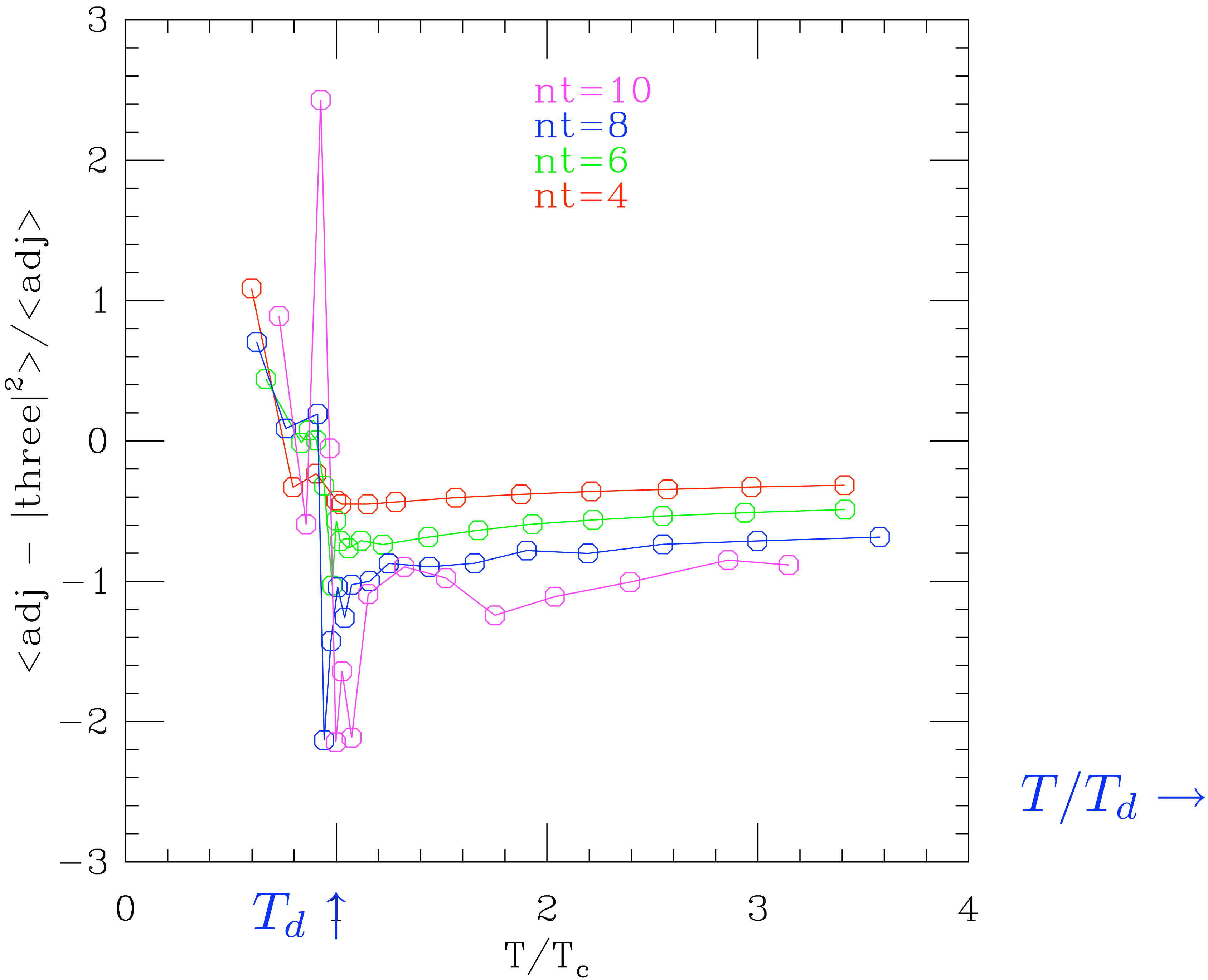
$$\max |\delta\ell_6| \sim .25 \ @ \ 1.3 \ T_d$$



Max. sextet ~ spinoidal point = Hagedorn temp. AMMPR: max. adjoint “spike”?

Bare loops don't factorize

Bare octet difference loop/bare octet loop: violations of factor. 50% @ Nt =4 200% @ Nt = 10.



Lattice SU(3): masses ~ 0 at T_d: near Gross-Witten

APE, Columbia ‘89, Bielefeld ‘93 + ...

In confined phase, string tension
from 2-pt function of Polyakov loops
at large distances:

$$\langle \ell^*(x)\ell(0)\rangle - |\langle \ell \rangle|^2 \sim e^{-\sigma(T)x/T}$$

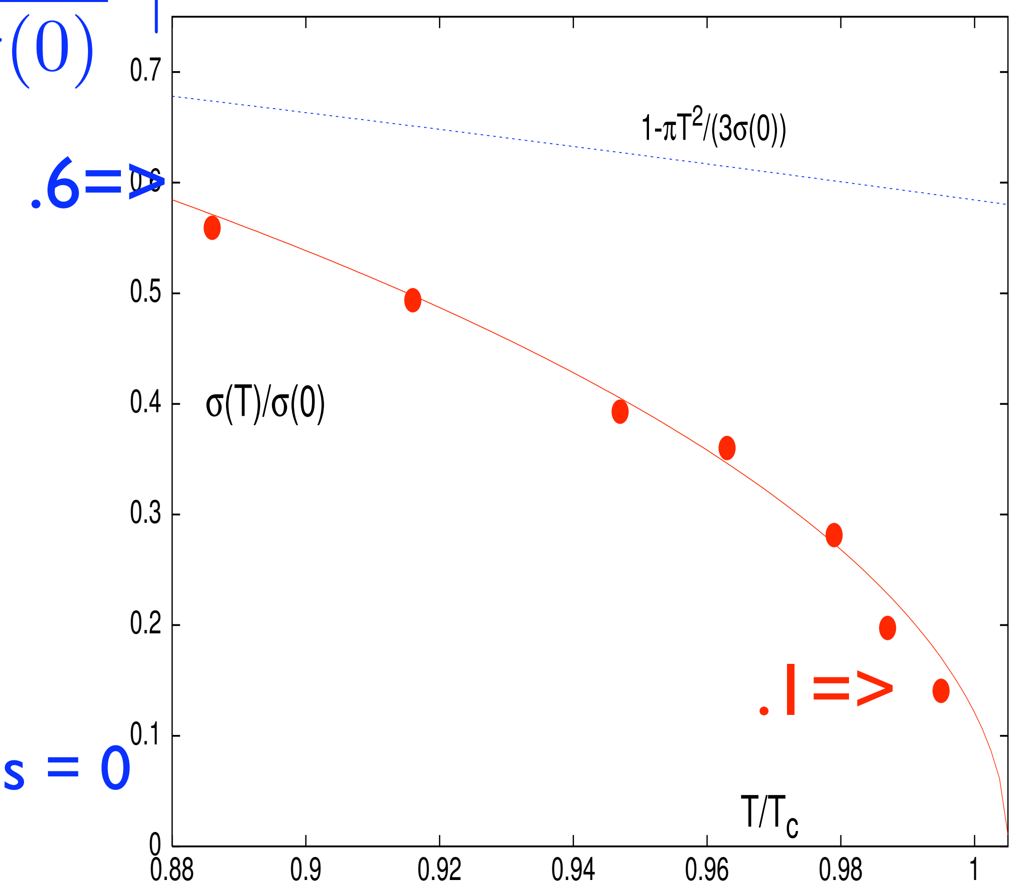
SU(3): versus zero temperature, at T_d
the string tension is smaller by ~ 10.

Gross-Witten, SU(∞): jump to 1/2, masses = 0

Lattice SU(3): jump to ~.4, masses ~ 0

Use matrix models to quantify how close SU(3)
is to the Gross-Witten point of SU(∞).

$\frac{\sigma(T)}{\sigma(0)} \uparrow$



$.9\ T_d \uparrow$ $T_d \uparrow$

FIG. 1:

$T/T_d \rightarrow$

Fluctuations in a matrix model

Up to now, only potential:

$$\mathcal{V} = m^2 \, \ell_{adj} + \Sigma_j \, \lambda_j \, \ell_j \; , \; e_j = 0$$

Lots of kinetic terms:

for Wilson line

$$\mathcal{W}_{el} = \mathcal{Z}_{el} \, \mathrm{tr} \, |D_i \mathbf{L}|^2$$

"electric loop" + induced:

$$\mathcal{Z}_{el} = g_{el}^{-2} + g_{adj} \, \ell_{adj} + \ldots$$

For loops alone,

$$\mathcal{W}_{loop} = \Sigma_j \, h_j \, \ell_{j'} \, \partial_i \ell_{j''} \, \partial_i \ell_{j'''} \; , \; \Sigma \, e_j = 0$$

and even (N=3):

$$\widetilde{\mathcal{W}} = \mathrm{tr} \, (D_i \mathbf{L})(D_i \mathbf{L})(\, \widetilde{g}_3 \, \ell_3 + \ldots)$$

g's & h's = couplings. Plus magnetic...

Fluctuations modify mean field equations.

Start with simplest case: just adjoint loop in potential, ~ "mass" term.

Matrix models & ren'd loops

OK fit with just adjoint loop, m^2 ~ temperature: $m^2 = .46 + .33\; T/T_d$

Fails near T_d: jump in $\langle \ell_3 \rangle$ to .485, not ~.4 from lattice. Need to add:

(1) decuplet loop in potential, λ_{10} big

(2) triplet-sextet kinetic, h small

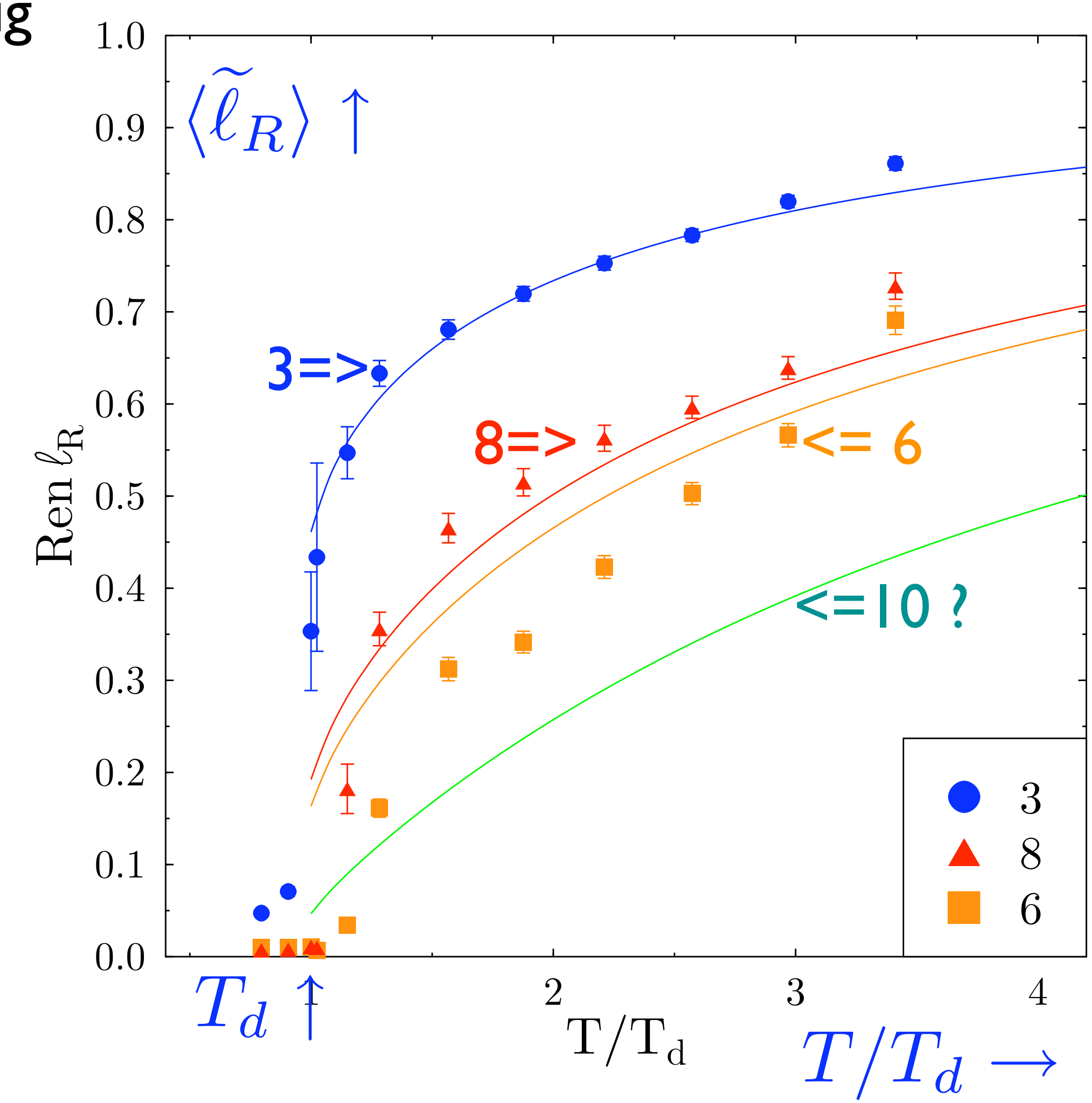
Use this m^2, plot all loops vs T =>

Deviations greatest for sextet loop, but sextet very sensitive to (1)&(2)

Fit "spike" in sextet difference loop?

"Spike" in octet difference loop:
=> "spike" in octet coupling?

Mean field predicts decuplet loop!



Deconfinement in SU(3) close to Gross-Witten?

SU(N) matrix models agree with lattice results for the deconfining transition:

SU(2) second order: Redlich, Satz & Seixas '88...Engels & Scheidler '98

SU(N), N≥4, first order: N = 4, 6, 8: Lucini, Teper, Wegner, '02, '03

N = 4: Batrouni & Svetitsky '84; Gocksch & Okawa '84 ....Ohta & Wingate '00; Gavai '01

SU(3) first order, but weakly:

closer to 2nd order point of SU(2) or Gross-Witten point of SU(∞)?

But need to tune two parameters reach a tri-critical point, $m^2 = \lambda_4 = 0$

(e.g., temperature and concentration of two-component systems)

=> *unnatural* for SU(3) to be near the Gross-Witten (tri-critical) point

Accident? Is deconfinement in SU(4), SU(5)... close to the Gross-Witten point?

Look for (large) decrease in the string tension just below T_d