

“All the World’s a Stage” (almost a) Critical Point

And all the men and women, merely players...

“As You Like It”, Shakespeare.

Phase transitions:

liquid => gas: bubbles, *critical point*.

membranes & vesicles: ~zero surface tension => ~ critical point.

nuclei: deconfining “spins”: ~ *critical point* for Quark-Gluon Plasma

Quark-Gluon Plasma & the “Unicorn”

New State of Matter in Heavy Ion Collisions? “Jets” *get eaten!*

Exact critical point for QGP? GSI.

Phase Transitions

Boil some water: change in phase from *liquid* => *gas*

“Order parameter”: some #, changes between phases

Here: density. Liquid dense, gas dilute.

Liquid “obviously” different from gas.

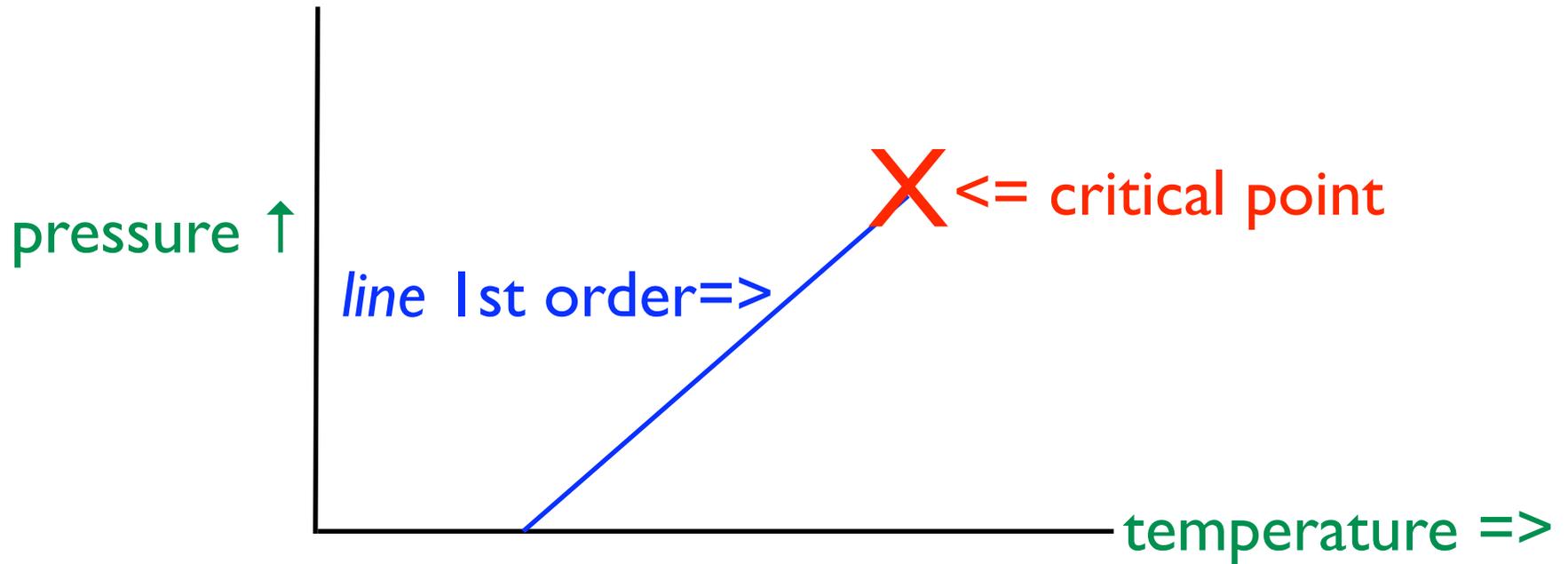
“First order transition”: *there are bubbles!*

And: bubbles have a given *size*.

Bubbles => metastable states! supercooling, superheating...

“Critical Point”: *BIG* Bubbles

OK, now boil water at *much* higher pressures:



Along line of first order transitions, finite size bubbles.

As $\Rightarrow X = \text{critical point}$, bubble size $\Rightarrow \text{infinity}$. Critical opalescence.
= second order phase transition.

Water: $X = 218 \text{ Atm}$, 374° C .

Have to hunt for the critical (end) point. No *symmetry*.

Two state model

Consider model with two states, up  or down 

Assume *symmetry* = interchange up and down.

Low temperature: “liquid” = (mostly) up **OR** (mostly) down

Symmetry => *domains* in liquid phase, either up or down.

High temperature: “gas” = mixture, up + down.

Find: at transition, NO bubbles. *Correlations over ~infinite sizes.*

“Critical fluctuations”

Can “see” in numerical (Monte Carlo) simulations.

Critical Point: with Symmetry!

All critical points have symmetry.

Symmetry not enough to guarantee critical point.

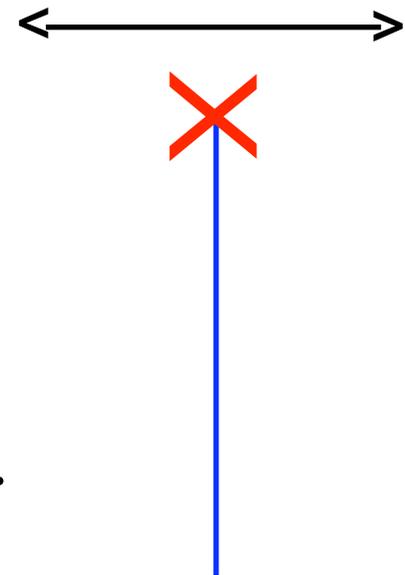
“Universality”: at critical point, depend *only* upon symmetry (& dimension) At large distances, *everything* unique. Wilson, '71.

Symmetry for liquid-gas? “Hidden” two state.

Rotate first order line in plane of P & T:

Symmetry = perpendicular to first order line.

Symmetries can show up in surprising places...



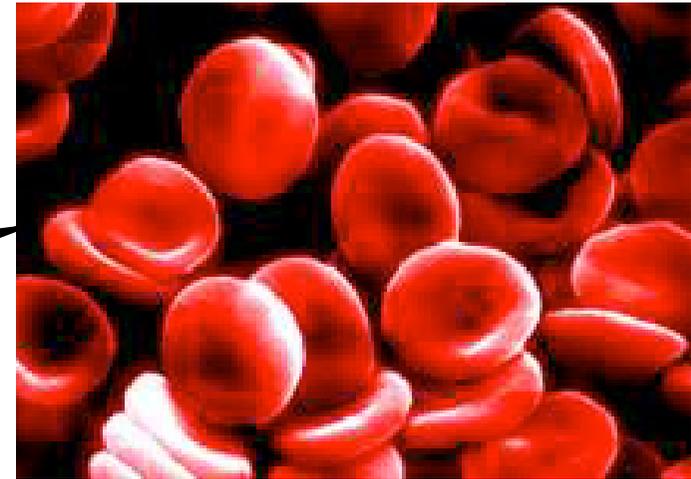
Another critical point: red blood cells

Red blood cell = *membrane*.

“Flicker” under microscope.

Equilibrium shape *not* spherical

very bendable

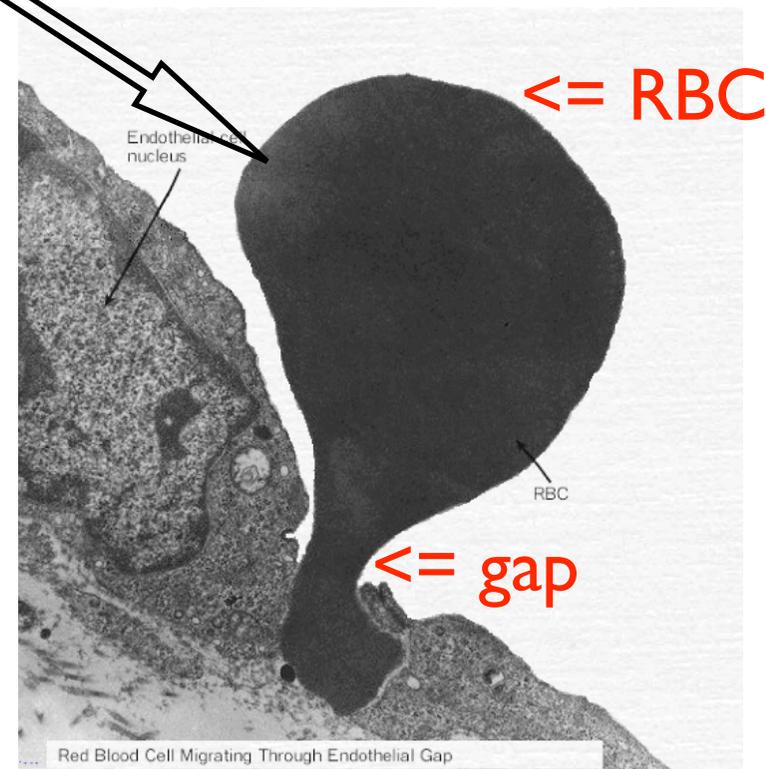


Exp.'y: minimize with respect with volume
and area *independently*

=> (renormalized) surface tension *vanishes*
=> = critical point.

Critical fluctuations => flickering.

Shape: determined by curvature terms.



Squeezing through gap ↑

Surfaces as critical point at *short distances*

Critical point in three dimensions: very hard to compute.

Correlation functions fall off as powers (“critical exponents”)

In four dimensions, easy to compute: coupling constant

vanishes $\sim 1/\log(r)$ as $r \rightarrow \infty =$ *large distances*

surfaces: free energy = sum of area + curvature².

Coupling for curvature term *asymptotically free*:

vanishes $\sim 1/\log(r)$ as $r \rightarrow 0 =$ *short distances*

flat surfaces: RDP ‘83; Peliti & Leibler ‘85, Polyakov ‘86

membranes in large # dim.’s: RDP ‘87 (“smooth” strings)

Another example (*spin models in two dim.’s* \sim *gluons in four*)

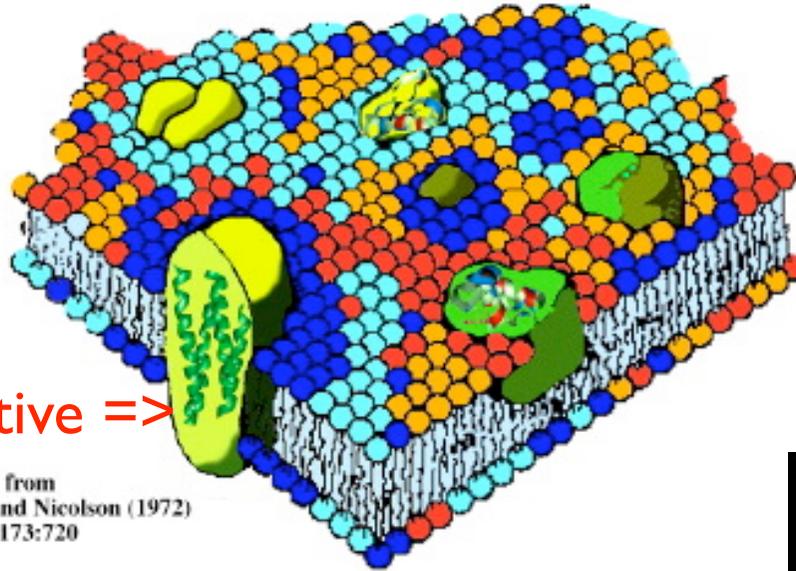
(Quarks) & gluons = strong interactions = *asymptotically free*.

Membranes & Giant Vesicles

Biological membranes = *proteins* embedded in lipid bilayer:

Proteins bind on/in surface. Diffuse freely in plane, **biologically active** =>

adapted from
Singer and Nicolson (1972)
Science 173:720



<= Singer & Nelson
“Fluid mosaic”
model

From **just** lipid bilayers, make **giant vesicles** =>
(W. Webb + ...)

Understand “patches”? = **domain formation.**

(Proteins may bind preferentially to domains.)

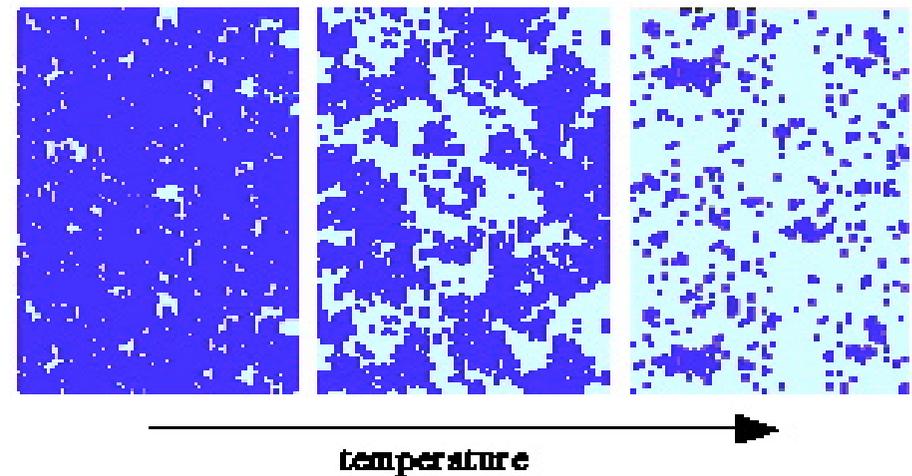


Melting of Lipid Bilayers

T. Heimburg: NBI & Gottingen

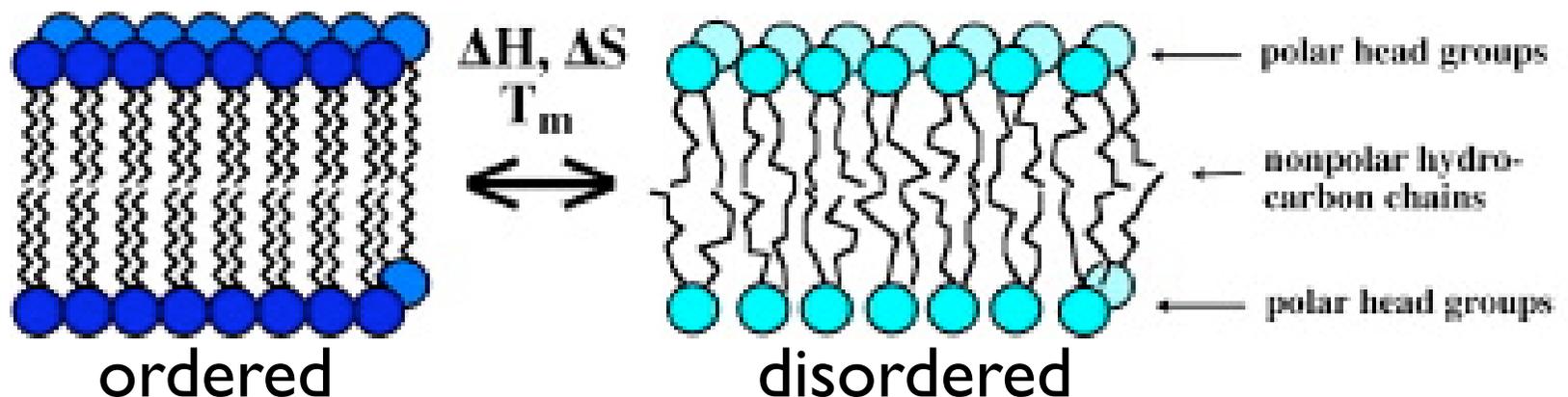
Transitions in lipid bilayers:
domains “melt”.

Monte Carlo simulation =>
~ like two states?



Lipids = polar head + (long) nonpolar chain

Transition: both (hexatic) ordering of polar heads *and*
“wiggling” of long chains



Giant Vesicles & Two Critical Points

T. Heimburg + ... *two* order parameters:

Hexatic ordering of heads *and* wiggling of tails.

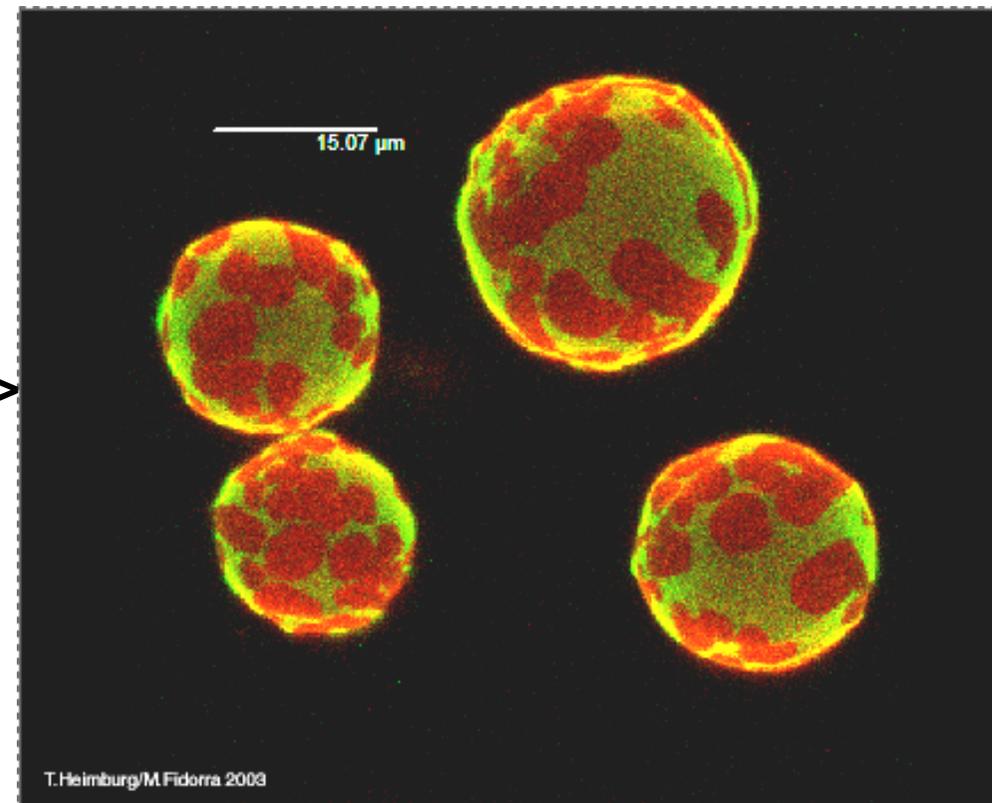
Describe by *two* two state models, *each* with a critical point, whose critical temperatures are close, but different.

Study: 1. Mean field theory.
2. Monte Carlo simulations.
3. Experiment.

Obtain good description of data: =>

(Almost) critical points are very important for biology!

And quarks & gluons, too.



Quark Model

Nucleons=neutrons & protons (N&P): strongly interacting particles.

From high energy experiments, discover each is made of

three quarks: N, P = qqq, q = quark.

Also: “mesons” = quark + “anti”-quark (pions, kaons...)

Scales: mass proton = $\sim 1 \text{ GeV} = 10^9 \text{ eV}$.

All scales $\sim 10^9$ times usual biological scales; T \sim trillion degrees.

Quarks “confined” by gluons:

quarks & gluons “asymptotically free” =

critical point at short distances (like membranes)

(’t Hooft) Gross, Politzer, Wilczek ‘72

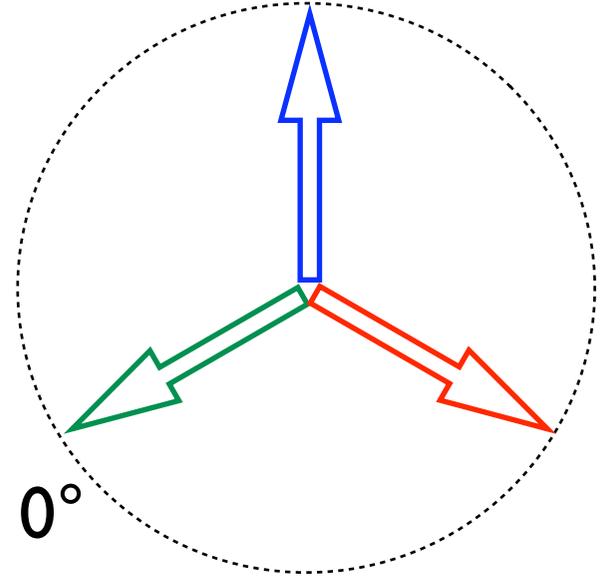
things simple at short distances = high energy.

Gluons and a three state model

Confinement: gluons \Rightarrow individual quarks don't exist, only nucleons & mesons.
 $=$ type of *three state model*.

For a given state, assume a transformation:
for *each* quark, rotate by 120° .

With three quarks, rotate by $3 \times 120^\circ = 360^\circ = 0^\circ$
 \Rightarrow nucleons *invariant* under transformation.



Anti-quark \Rightarrow rotation by -120° .

Mesons: rotate by $+120^\circ - 120^\circ = 0^\circ \Rightarrow$ *invariant* under transf

Confinement: (at zero temperature) only *invariant* states exist.
i.e., don't rotate. 't Hooft '79.

Confinement and a Surface Tension

Meson = quark plus an anti-quark.

If you try to stretch them apart, gluons => *constant force*
=> mesons form flux tube:



At *one* instant in time, flux tube = “*string*”
(as in: “String theories” a.k.a. “Theory of Everything”)

In *space-time*, string sweeps out *sheet = membrane*.

Surface tension ~ 14 tons (= string tension)

Confinement & Ordering

Confinement \Rightarrow all states *invariant* under 3-state rotations.

Valid at zero temperature, for some range of temp.'s

“Asymptotic freedom”: coupling vanishes as $T \Rightarrow \infty$,
 \Rightarrow ideal gas at infinitely high temperatures \Rightarrow *NO* confinement.

Transition to deconfined phase at $T_d =$ deconfining trans. temp.
Above T_d , ok to have system with 1, 2, 3... quarks (anti-quarks)

In terms of 3-state model, “deconfinement” =

Ordering at *high*, instead of *low*, temperature!

‘t Hooft ‘79. Ordering not obvious: “duality”?

Quark-Gluon Plasma =
Deconfined Quarks and Gluons

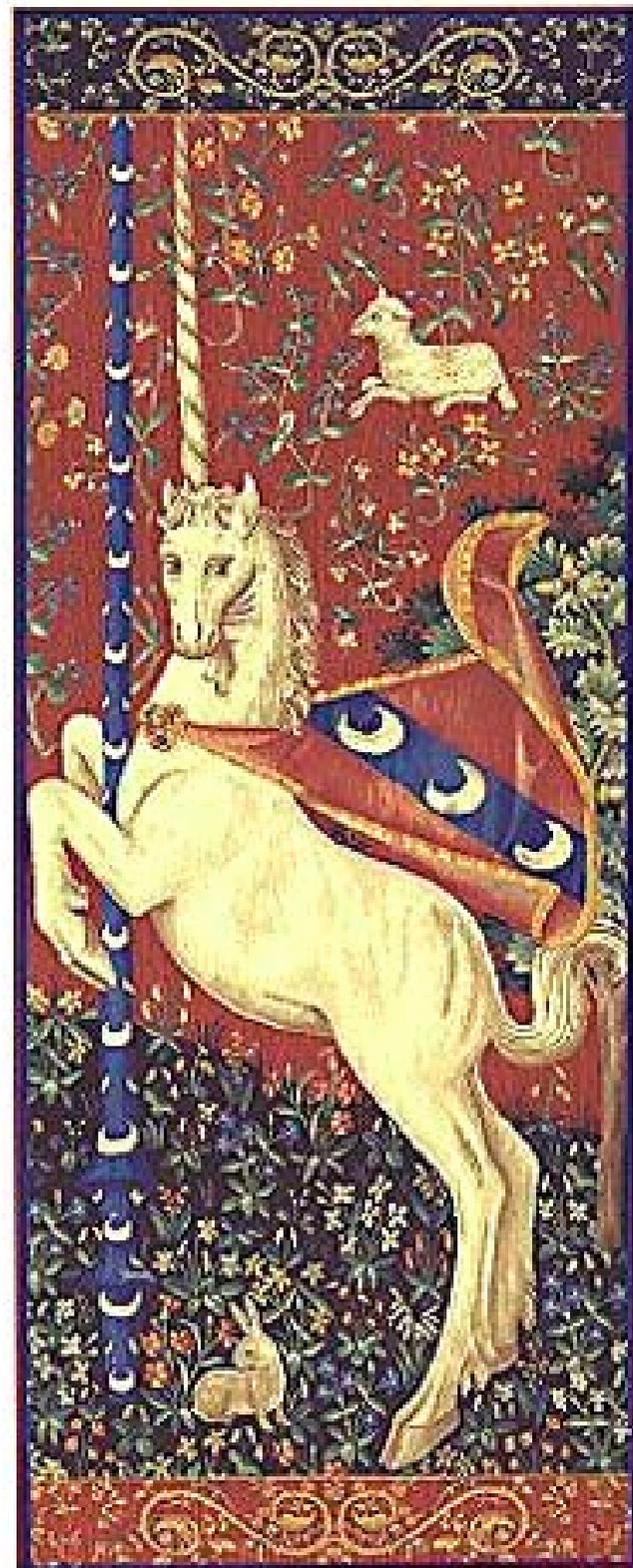
To compute properties of the QGP:

Mean field theory

Monte Carlo simulations

(Experiment)

The QGP & the “Unicorn”:
mythical creature(s)?



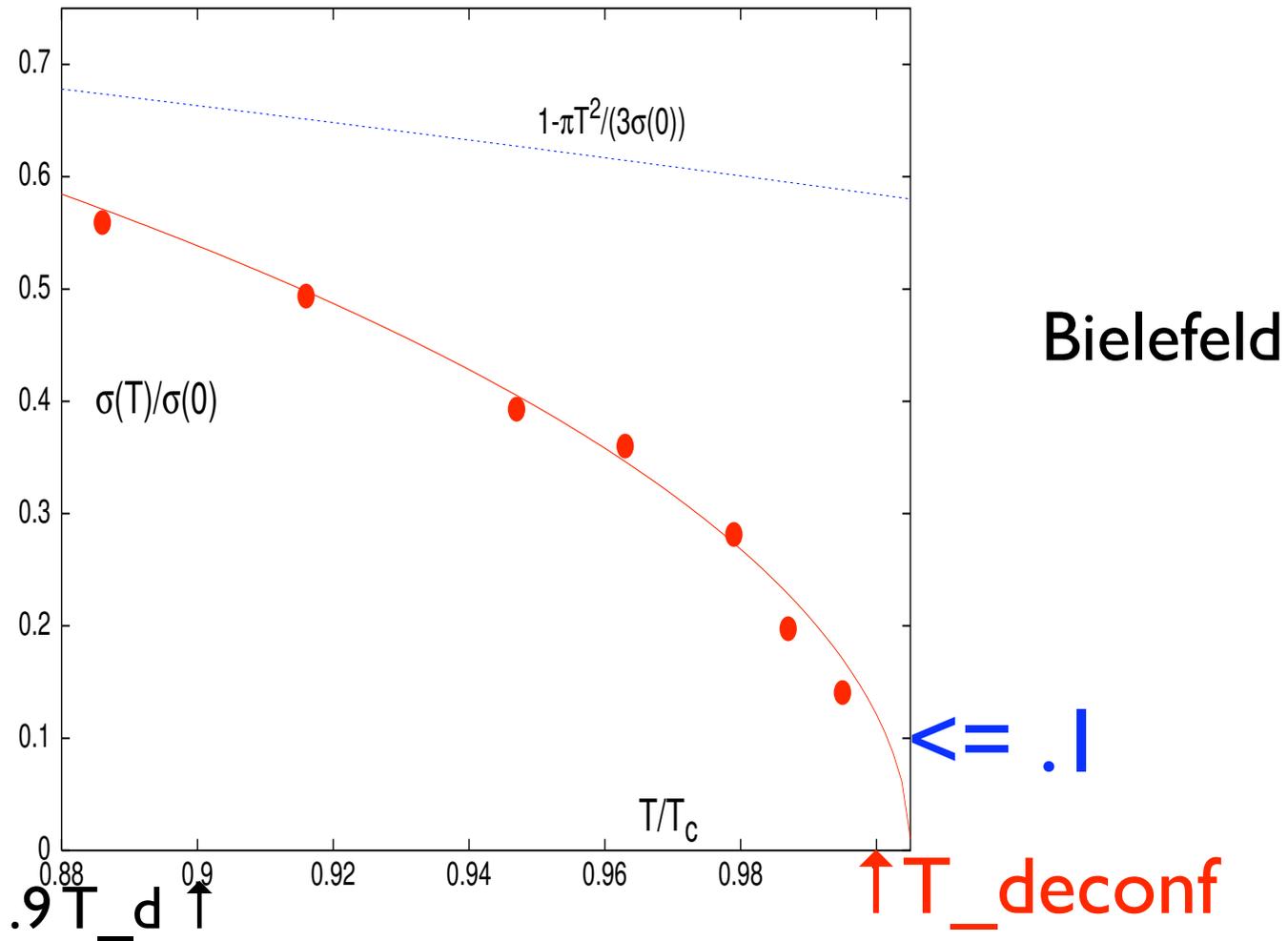
Monte Carlo for Gluons

Numerical simulations for three “colors” of gluons:

Deconfinement at $T_d \sim .27$ mass proton ($\sim 10^{12}$ K)

“Surface tension” for confinement decreases by ~ 10 by deconfining transition \Rightarrow *almost a critical point*:

(Confining)
surface tension
at temperature
 T /value at $T=0$ \uparrow



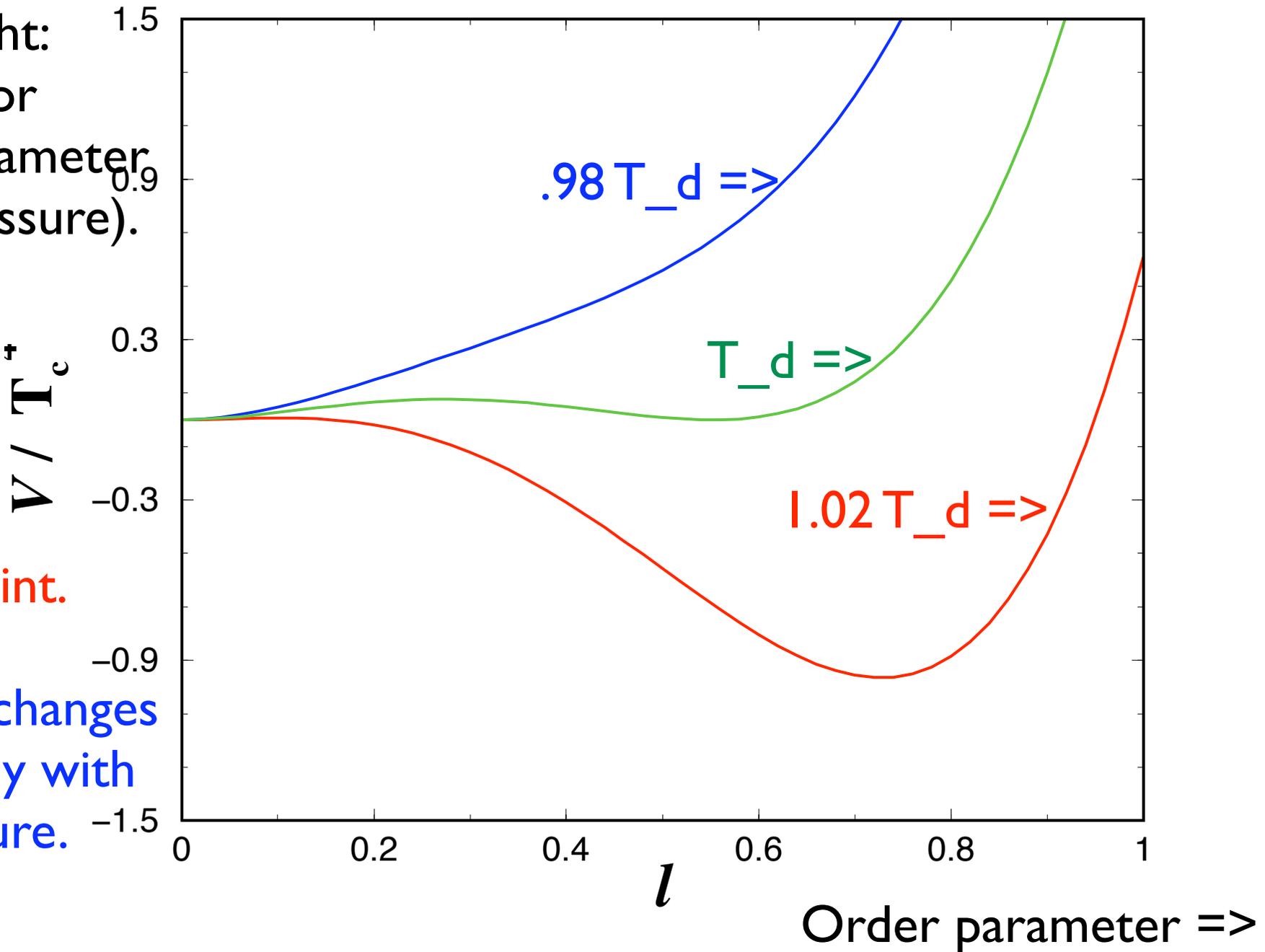
Deconfinement is almost a critical point

A. Dumitru & RDP '01

To the right:
potential for
order parameter
(fit to pressure).

Potential
very flat
at T_d
 \Rightarrow *almost*
critical point.

Potential changes
very rapidly with
temperature.



Monte Carlo vs mean field for three state order

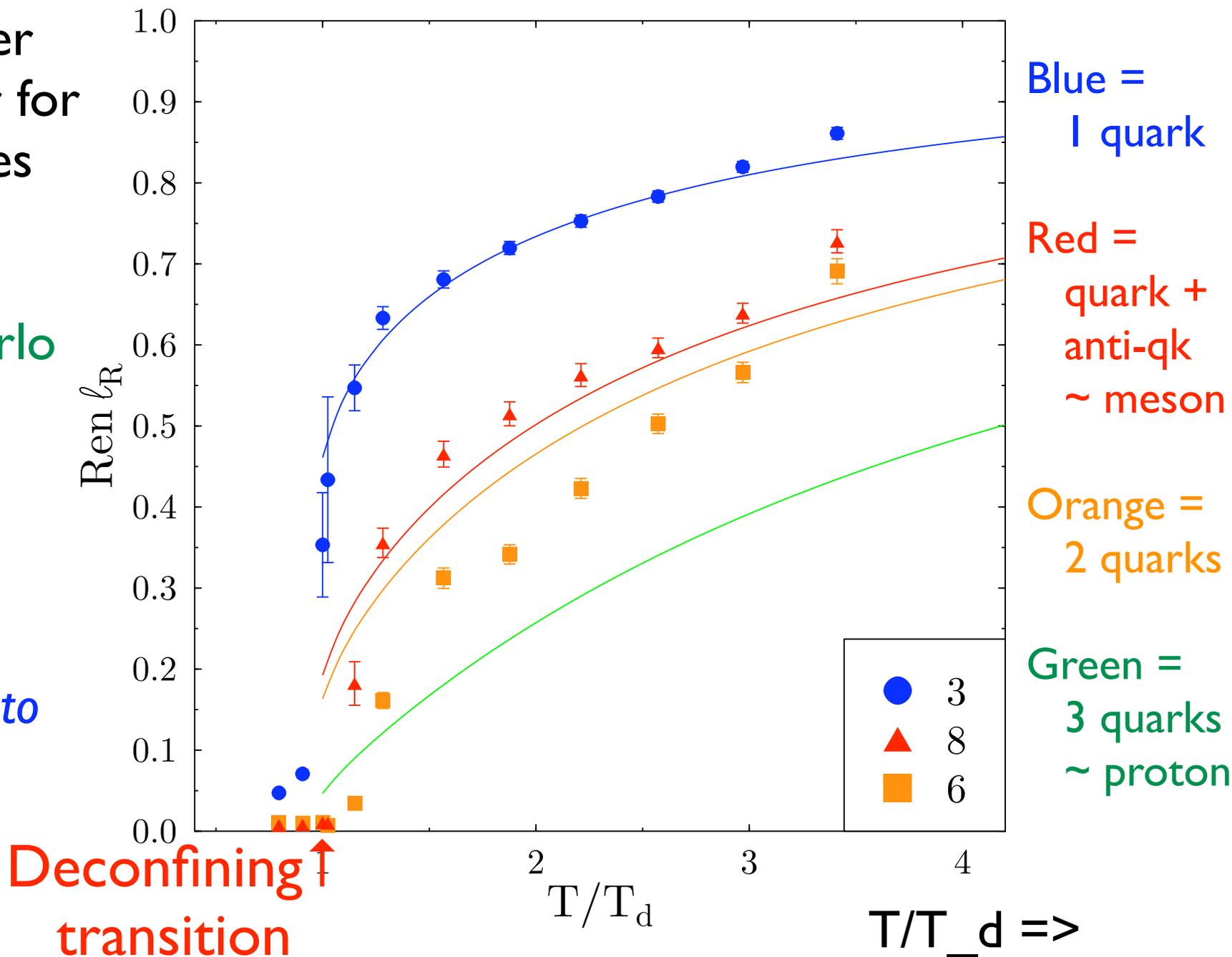
A. Dumitru + ... + RDP '03

Right: order parameter for three states

Points = Monte Carlo

Lines = Mean field

Not bad!
 $N=3$ close to
 $N = \infty$



The QGP Exists!
Hunting for the “Unicorn”
in Experiment: Heavy Ion Collisions



“Unicorn” & the QGP: Scott, Stock, Gyulassy...

Why collide big nuclei at high energies?

One can collide:

pp: protons on protons. Benchmark for “ordinary” strong int.’s

AA: nucleus with **atomic number A** on same.

dA: deuteron (N+P) on nucleus. Serves as another check.

Why AA? Nucleons (N’s & P’s) are like hard spheres,

so nuclear size $r_A \sim A^{1/3}$

Biggest: **Pb** (lead) or **Au** (gold), $A \sim 200 \Rightarrow r_A \sim 7$.

Transverse radius of nucleus $\sim A^{2/3} \Rightarrow$ trans. size $\sim 50 \times$ proton.

$A \sim 200$ close to $A \rightarrow \infty =$ *infinite nuclear matter?*

Colliders: Energy, Machines

Particles accelerated in rings. Either: one ring against **fixed target**, or **collider**, two rings, with particles travelling in opposite directions.

| Machines | Total energy/nucleon | Type |
|-----------------|----------------------|------------------------|
| SPS @ CERN | 5 => 17 GeV | (fixed target) |
| **** RHIC @ BNL | 20, 130, 200 GeV | (collider, > 2000) |
| LHC @ CERN | 5500 GeV = 5.5 TeV | (collider, > 2007) |
| SIS200 @ GSI | 2 => 6 GeV | (fixed target, > 2010) |

SPS = Super Proton Synchrotron: CERN @ Geneva, Switzerland.

RHIC = Relativistic Heavy Ion Collider: BNL @ Long Island, NY.

LHC = Large Hadron Collider.

SIS = SchwerionenSynchrotron: GSI @ Darmstadt, Germany.

Essentials of AA collisions

At high energies, nuclei *slam* through each other (no “blob”).
Hence physics very different along beam, vs. transverse to beam.
Measure momenta of particles \sim relativistic measure of velocity.

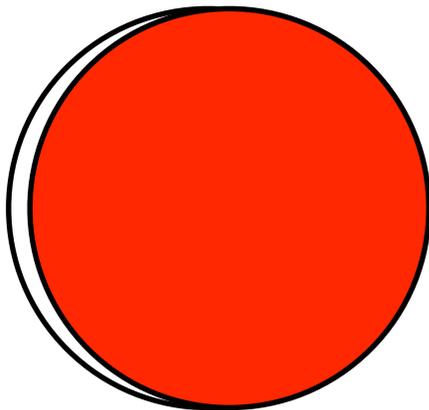
Look just at 90° to beam in collider = “central” rapidity.

(rapidity = relativistic inv. for mom. along beam.)

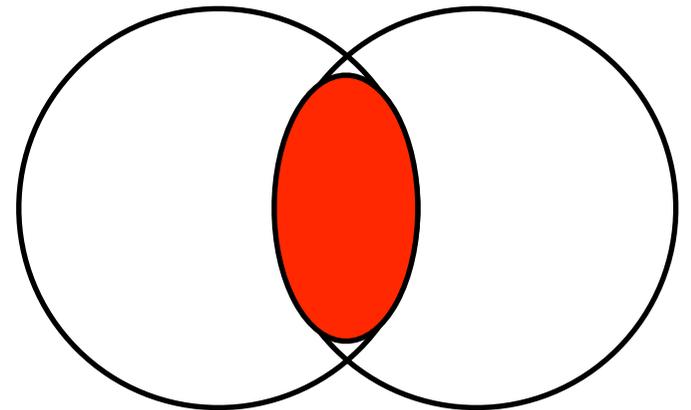
$90^\circ \sim$ free of incident nucl.'s \Rightarrow most likely to see nonzero temp.

Bulk of particles at 90° , peaked about transverse momentum.

Central:
Maximum
Overlap



Peripheral \Rightarrow
“Almond” of
overlap region



Typical Heavy Ion Event @ RHIC

Experiments @ RHIC:

Total # particles(/unit rapidity)
~ 900↓

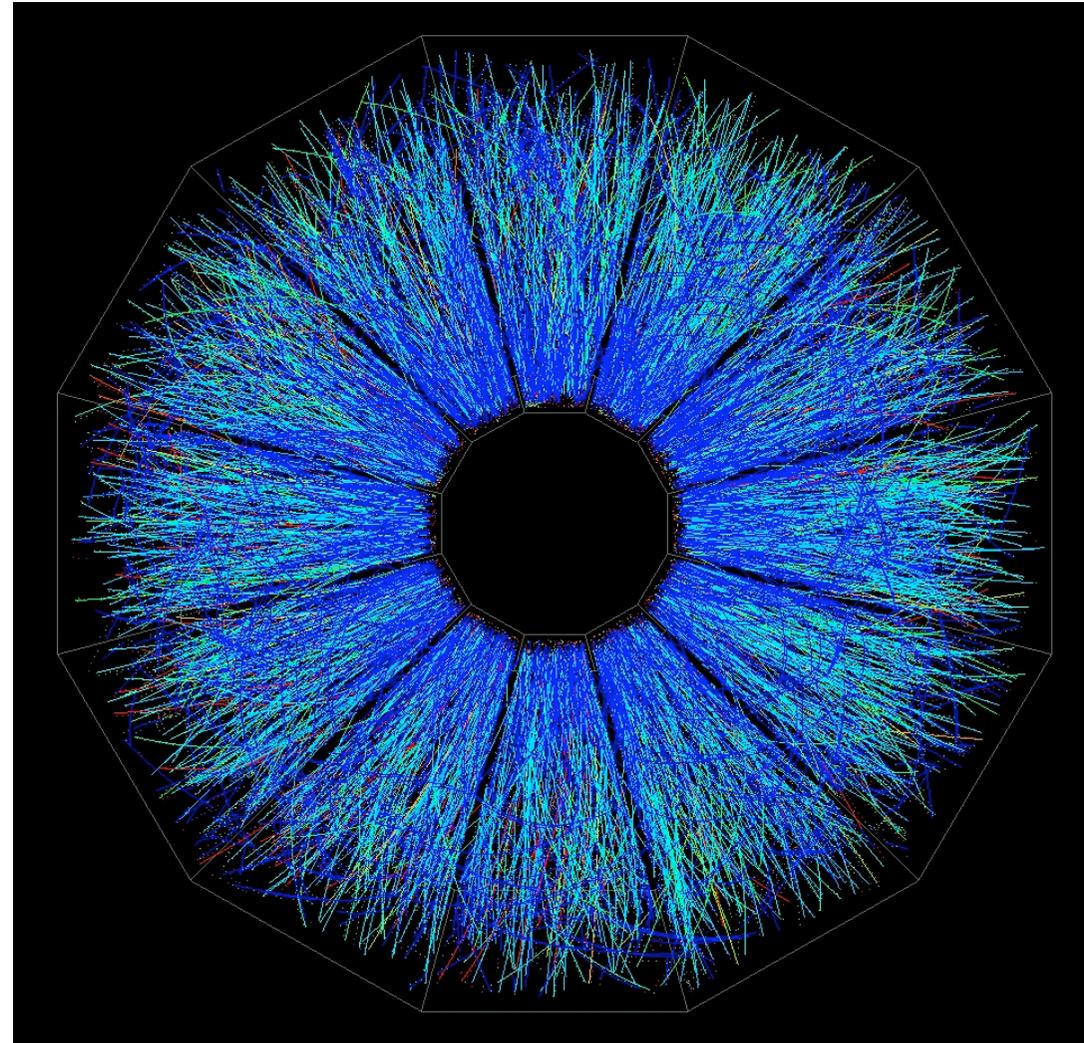
“Big” expts: ~ 400 people
STAR & PHENIX

“Small” expts.: ~ 50 people
PHOBOS & BRAHMS

Note: total # particles ~
total # experimentalists
~ $\log(\text{total energy})$

theorists
~ $\log(\log(\text{total energy}))$.

Need hunters more than dogs...



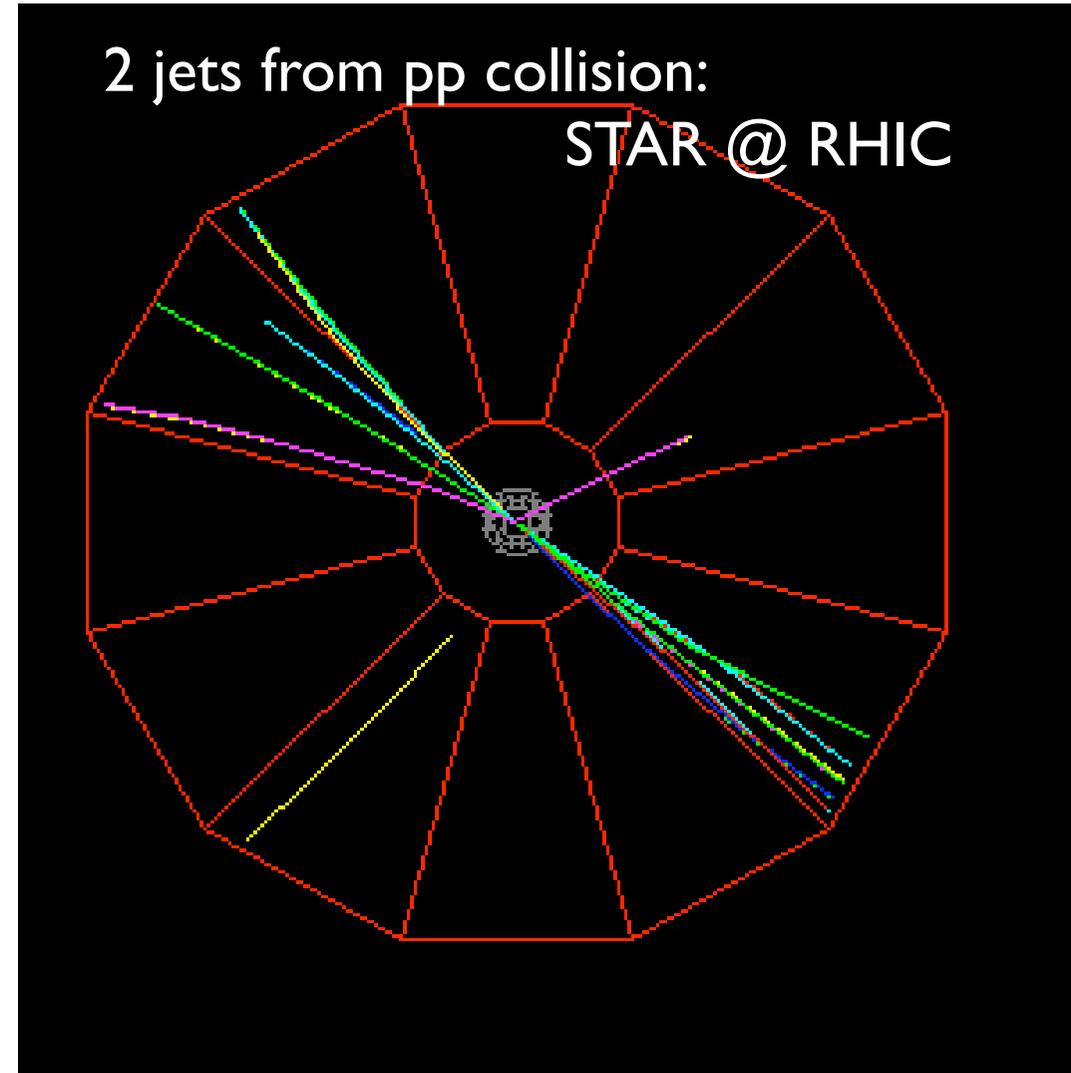
Jets: “seeing” quarks and gluons in QCD

Quarks & Gluons are *asymptotically free* => simple at *short* distances.

At high energies, “see” quarks & gluons as jets: sprays of particles (see right=>) quark or gluon => mesons & nucleons.

Not typical of most collisions, but very striking. Can compute properties in weak coupling.

Note: also why Monte Carlo gives *unique* answer as lattice spacing => 0: “dimensional transmutation”.



“Jets” in central AA collisions

pp collisions: ~ 4 particles/unit rapidity, vs 900 in central AA.
Hence cannot see individual jets in AA.

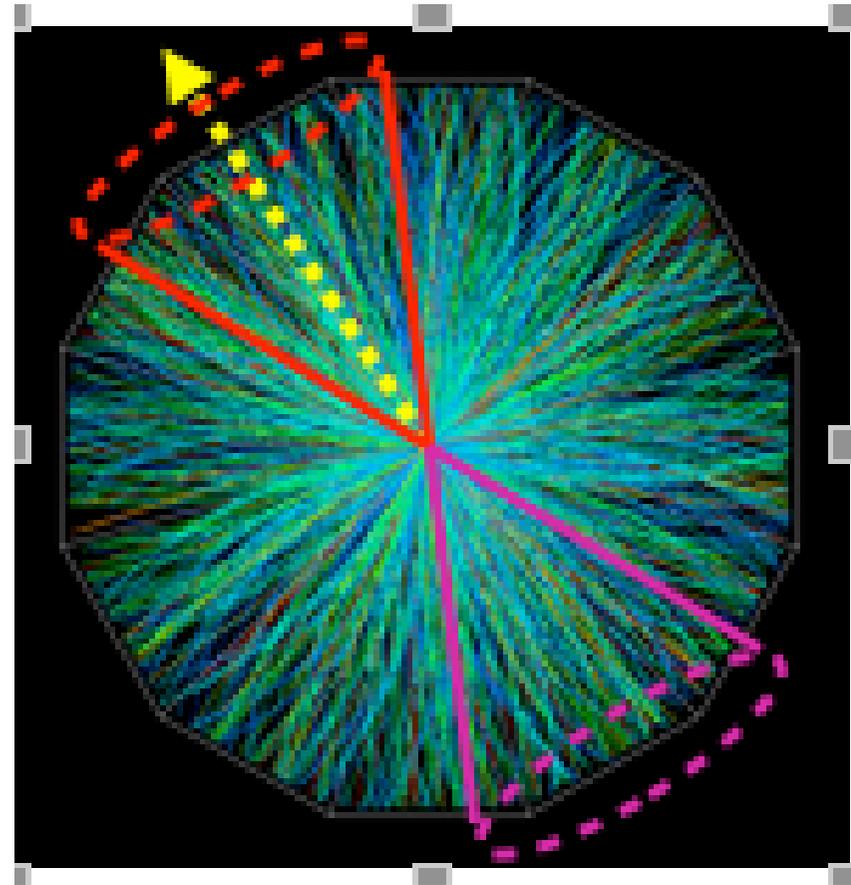
Can construct statistical measures.

p_t = momentum transverse to beam

Trigger on “hard” particle,
 $p_t: 4 \Rightarrow 6$ GeV

Given a jet in one direction,
there *must* be *something* in the
opposite direction.

Look for the “away” side jet, $p_t > 2$ GeV. (mass proton ~ 1 GeV)



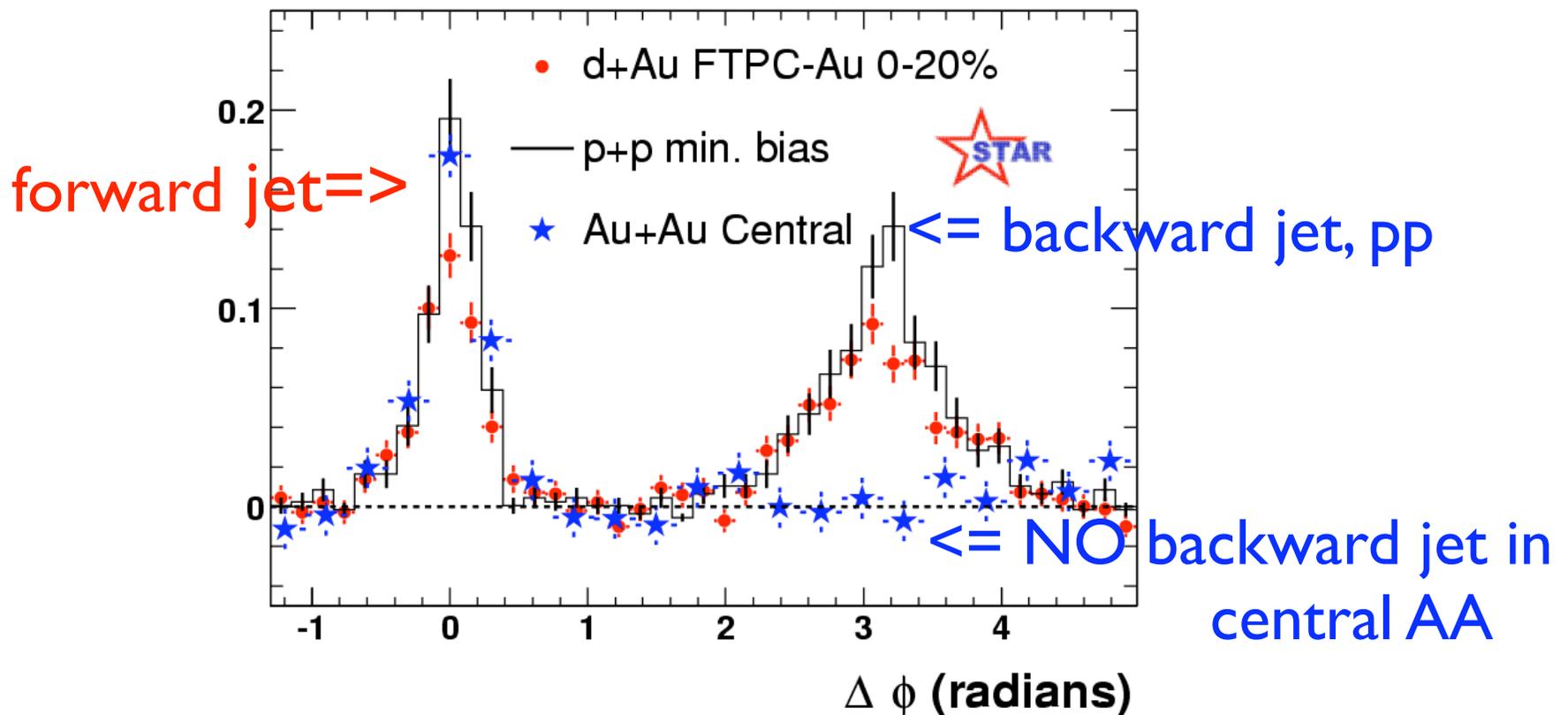
Central AA collisions “eat” jets!

In pp or dAu collisions, *clearly* see away side jet.

In central Au-Au, away side jet gone: “stuff” in central AA “eats” jets!

Fast jet tends to lose energy by many soft scatterings off “stuff”.

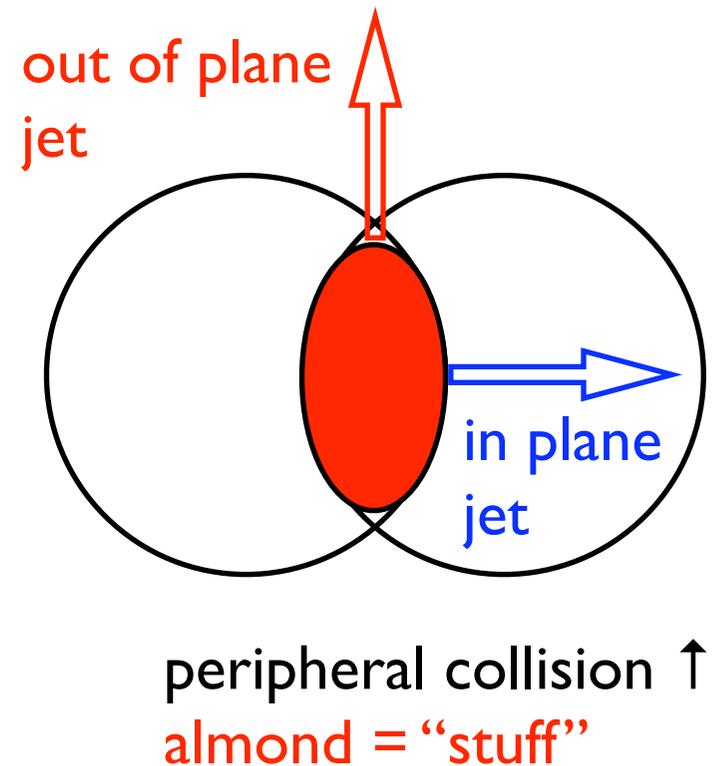
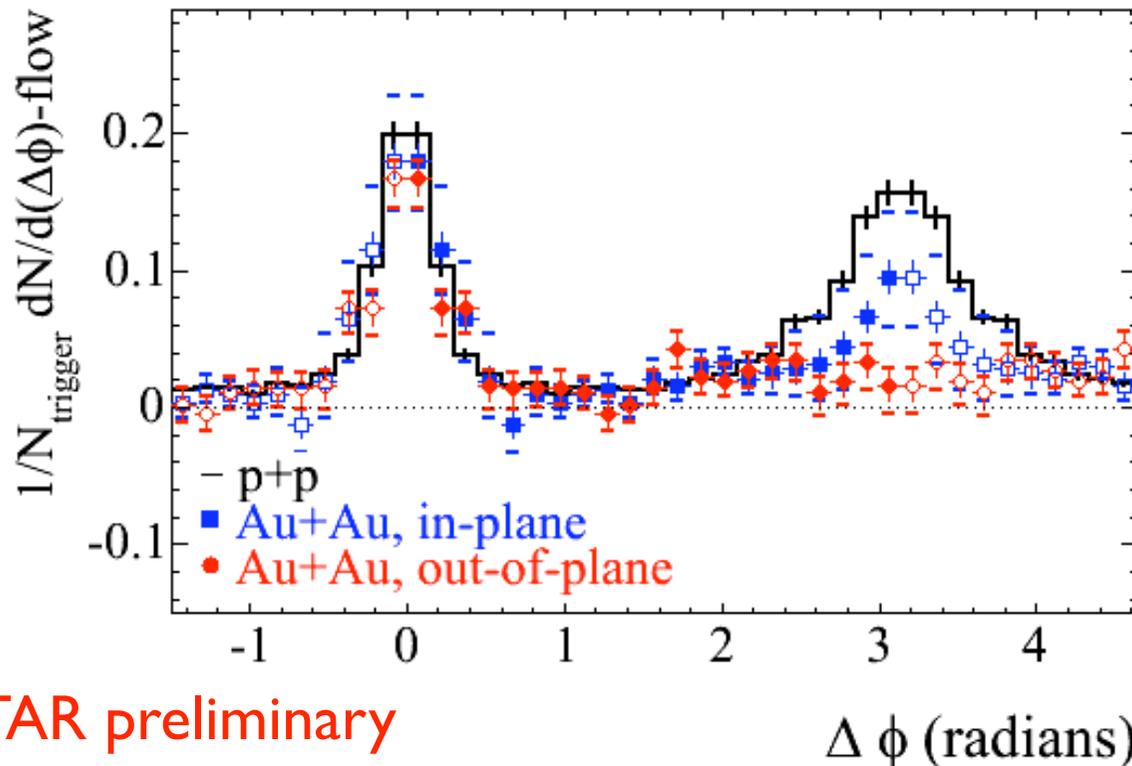
Adams *et al.*, Phys. Rev. Let. 91 (2003)



Peripheral Coll.'s: Geometrical Test that AA Eats Jets

Peripheral collisions, “stuff” forms “almond”: a jet travels farther through the almond, **out** of the reaction plane, than **in** the plane.

Exp.'y: backward jet more strongly suppressed **out** of plane than **in** plane => **geometrical** test that central AA “eats” jets



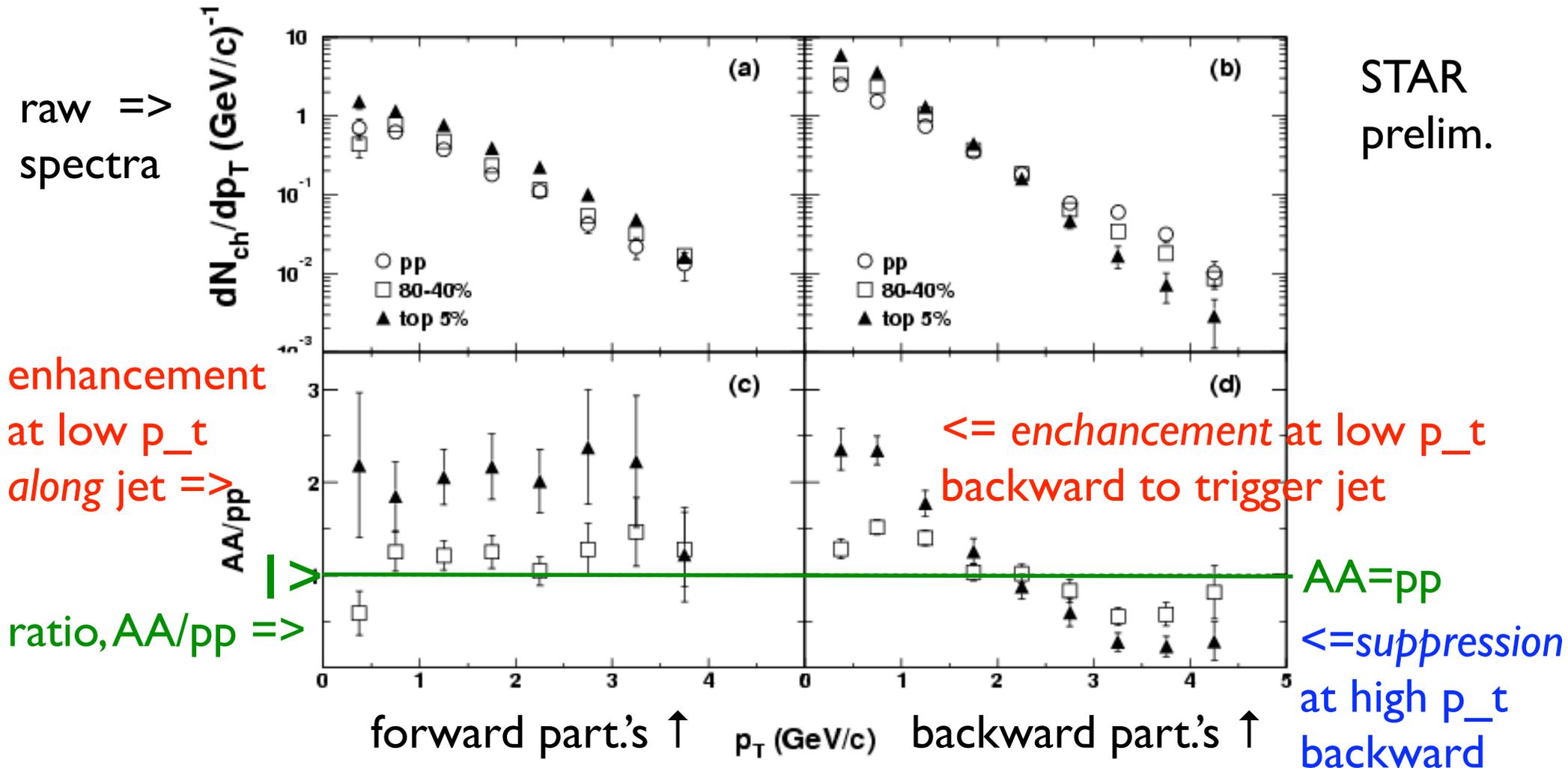
Jets in central AA: to low momentum!

Trigger on all particles, $p_{t} > .15$ GeV.

Backward jet: high p_{t} suppressed, low p_{t} enhanced.

“Stuff” in central AA slows fast particle down.

Forward jet: enhanced at low momentum: “stuff” dragged along!



Has RHIC found (tamed) the “Unicorn” = QGP?

New final state effects:

Suppression of backward jets

Clear exp. signal \neq SPS: R_AA

Has the Quark-Gluon Plasma been found in central AA?

Certainly: there is “stuff” which suppresses jet. *Not* clear if it is equilibrated at nonzero temp.

Relationship to order parameter?

Perhaps: it is a different beast....

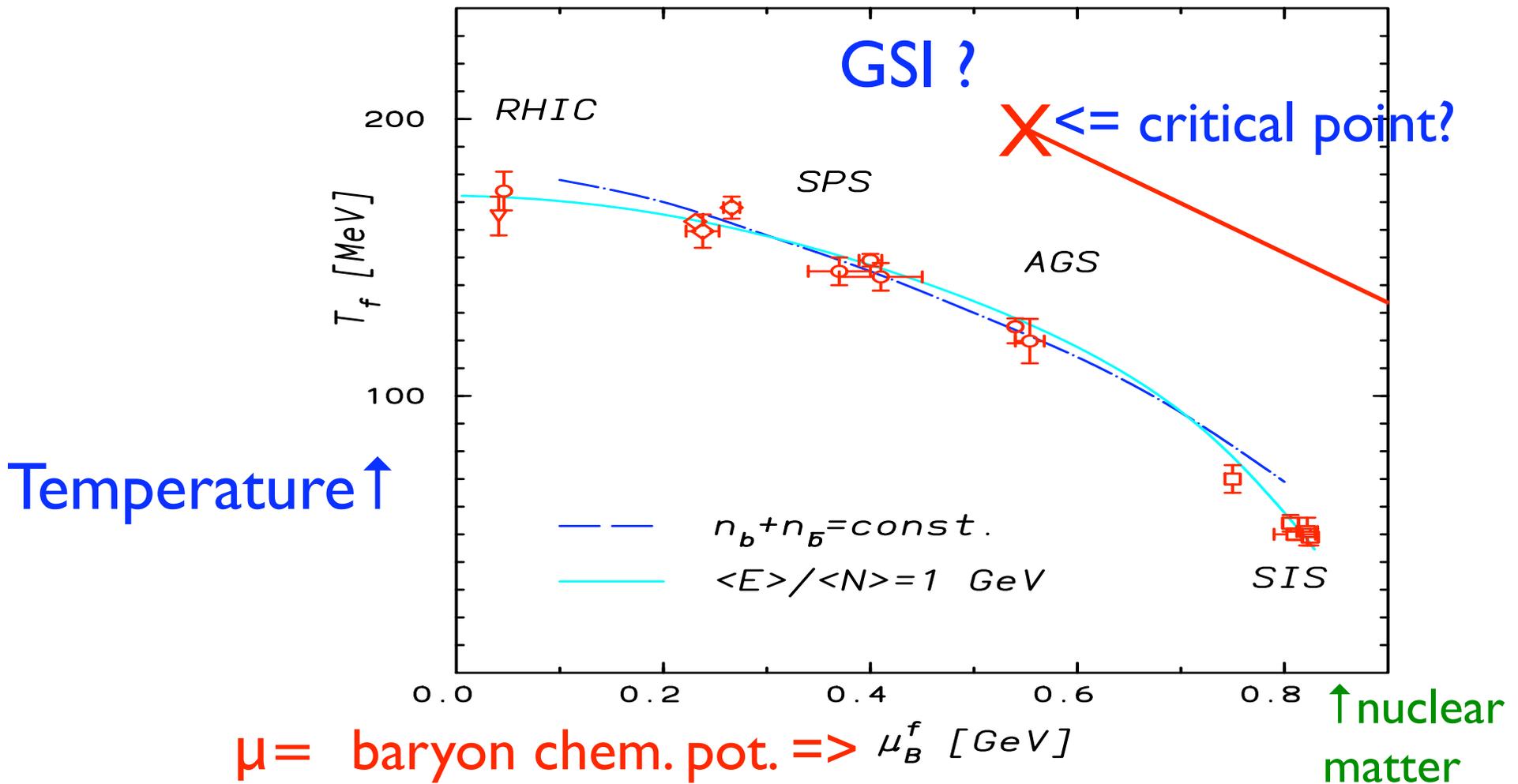
But its still a *NEW* beast!



Exact critical point in plane of T & μ 90° at RHIC probes ~ nucleon free region.

By going to *lower* energies (SIS200 @ GSI) will study region with baryon density: **expect exact critical (end) point.**

All the world's a....





"A possible eureka."