ϵ'/ϵ on & off the Lattice

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Outline

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I. Introduction

Recapitulate: $\Delta I = 1/2$ Rule

$\Gamma(K^+ \to \pi^+\pi^0) << \Gamma(K_s \to \pi^+\pi^-)$

$\frac{\tau_{K^+}}{\tau_{K_s}} \approx 400!!$ w/o QCD expect O(1)

This is the long-standing $\Delta I = 1/2$ PUZZLE

$K^+ \to \pi^+\pi^0$ ......$\Delta I = 3/2$ PURE

$K^0 \to \pi^+\pi^-$ ......$\Delta I = 1/2$ and $3/2$ MIXTURE

In the absence of QCD, these weak decays are governed in a simple manner by the weak Hamiltonian,

$$H_W = \frac{G_F}{\sqrt{2}} \sin\theta_C [\bar{u}\gamma_\mu(1-\gamma_5)s][\bar{d}\gamma_\mu(1-\gamma_5)u]$$

For DISCUSSION to FOLLOW, note that in modern notation,

$$H_W = \frac{G_F}{\sqrt{2}} \sin\theta_C Q_2$$

CP violation in $K \to \pi\pi$ decays

The long-lived and short-lived neutral kaons are **almost** CP eigenstates, but contain small <u>mixtures</u> of each other

$$\begin{aligned} |K_S\rangle &= |K^{\text{even}}\rangle + \epsilon |K^{\text{odd}}\rangle \\ |K_L\rangle &= |K^{\text{odd}}\rangle + \epsilon |K^{\text{even}}\rangle \\ |K^{\text{even,odd}}\rangle &= |K^0\rangle \pm |\bar{K}^0\rangle \end{aligned}$$

Indirect CP violation occurs when K_L oscillates into the K^{even} state and decays to the $\boxed{CP\text{-even state } |\pi\pi\rangle}$. Loosely speaking, the probability for this is parameterized by the observable $\boxed{\epsilon}$.

$$\epsilon_K = \frac{A(K_L \to \pi\pi)}{A(K_s \to \pi\pi)} \approx 2.3 \times 10^{-3}$$

Direct CP violation occurs when the K_L decays to $|\pi\pi\rangle$ directly from the K^{odd} component. Loosely speaking, the probability for this is parameterized by the observable $\boxed{\epsilon'}$.

$$Re\frac{\epsilon'}{\epsilon} = \frac{1}{6}[1 - |\frac{\eta_{00}}{\eta_{+-}}|^2]$$

$$\eta_{ij} = \frac{A(K_L \to \pi^i\pi^j)}{A(K_s \to \pi^i\pi^j)}$$

In the SM:

$$\frac{\epsilon'}{\epsilon} \approx \frac{\omega}{\sqrt{2}|\epsilon|}[\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}]$$

where

$$\omega = \frac{ReA_2}{ReA_0} \approx \frac{1}{20}$$

- Summary of CP **violation** found in nature

  - Indirect: ϵ

    1. Christensen, Cronin, Fitch, Turlay (BNL 1964)

    $$2.271(17) \times 10^{-3}$$

  - Direct: ϵ'/ϵ

    1. KTEV (FNAL 2001) $20.7 \pm 2.8 \times 10^{-4}$
    2. NA48 (CERN 2001) $15.3 \pm 2.6 \times 10^{-4}$

  - B factories measure $\sin{(2\beta)}$

    World Average (2002-3): 0.734 $\pm$ 0.055

Theory History (not exhaustive)

- Continuum
  - M. Kobayashi and T. Maskawa (1973)
  - G. Altarelli and L. Maiani (1974)
  - M. K. Gaillard and B. W. Lee (1974)
  - M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov (1977)
  - F. J. Gilman and M. B. Wise (1979)
  - C. Bernard, T. Draper, A. Soni, H. D. Politzer, and M. B. Wise (1985)
  - Munich Group (A. J. Buras) (1993)
  - Roma Group (G. Martinelli) (1994)
- Lattice
  - C. Bernard and A. Soni; G. Martinelli, *et al.* (Wilson fermions, circa 1985)
  - R. Gupta, G. Kilcup, and S. Sharpe (Kogut-Susskind, circa 1987)
  - G. Kilcup and D. Pekurovsky (Kogut-Susskind, 1999)
  - CP-PACS (Domain Wall Fermions, J. Noaki et al, circa 2001)
  - RBC (Domain Wall Fermions, T. Blum et al, circa 2001)
  - W. Y. Lee, S. Sharpe et al (Improved Kogut-Susskind, circa 2003)

Use theory + experiment to constrain the Standard Model

All CP violation in the Standard Model arises from a *single phase* in the (unitary) Cabibbo-Kobayashi-Maskawa mixing matrix

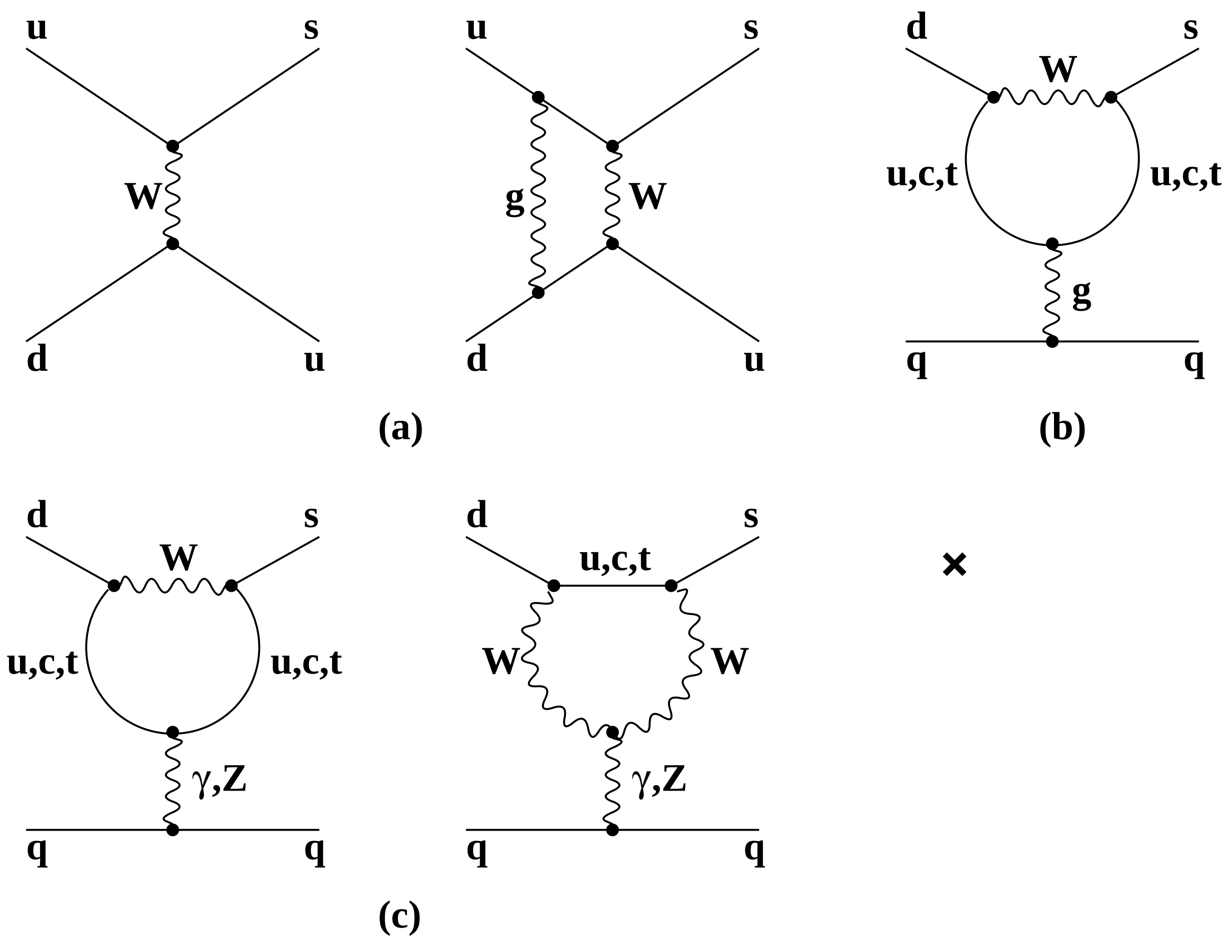
$$\mathcal{L}_{int} = \frac{g_2}{2\sqrt{2}}(\bar{u}, \bar{c}, \bar{t})\gamma_\mu(1-\gamma_5)V_{CKM}\begin{pmatrix} d \\ s \\ b \end{pmatrix} W^{+\mu}$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\approx \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\boxed{\eta \neq 0 \leftrightarrow CP \text{ violation}}$$

On the quark level, $K \to \pi\pi$ decays are mediated by strangeness changing $s \to d$ transitions ($\Delta S = 1$). In the Standard Model, typical Feynman diagrams are (a) current-current, (b) QCD penguin, and (c) Electroweak penguin:



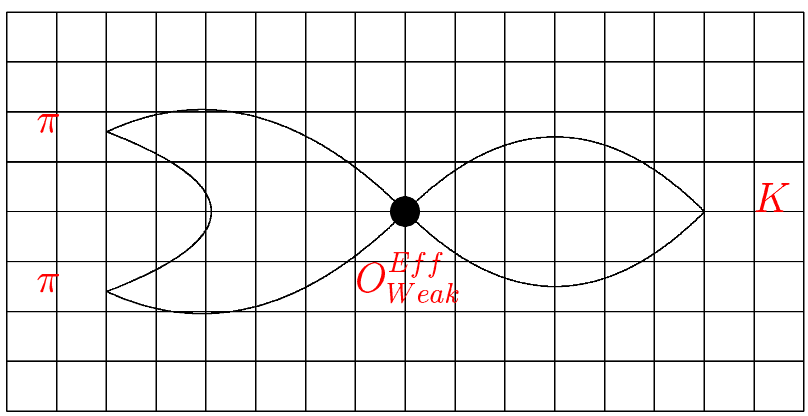
Buchalla, Buras, and Lautenbacher, Rev. Mod. Phys. 68 (1996)

Technical problem: vastly different energy (distance) scales.

weak interactions: $\mu \sim M_W \approx 80$ GeV

strong interactions: $\mu \sim 1$ GeV.

Solution: operator product expansion (OPE) and renormalization group (RG). Integrate out short distance part perturbatively $\rightarrow$ low energy effective theory. Effective Hamiltonian: linear combination of all local operators allowed by symmetries, coefficients given by underlying theory.



Expansion coefficients $\sim M_W^{-(d_i-4)}$, so only a few operators.

II. The $\Delta S = 1$ Effective Hamiltonian

The Standard Model Hamiltonian: a *short distance expansion* in terms of effective local **four-quark operators** $Q_i(\mu)$ (*c.f.* Fermi interaction) with **Wilson coefficients** $z_i(\mu)$ **and** $y_i(\mu)$.

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} y_i(\mu) \right] Q_i(\mu) \right\}$$

Both depend on an *arbitrary factorization scale* μ. Effective Hamiltonian is **independent** of this scale. Take μ low enough that non-perturbavtive calculations are practical ($1/a \ll M_W$), but high enough that continuum perturbation theory remains valid.

Key to the OPE is that the full amplitude is divided into low energy (hadronic matrix elements of Q_i) and high energy (Wilson coefficients) parts that can be computed separately.

$\Delta S = 1$ FOUR QUARK OPERATORS

$$Q_1(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \bar{u}_\beta \gamma_\mu (1-\gamma_5) u_\beta$$

$$Q_2(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \bar{u}_\beta \gamma_\mu (1-\gamma_5) u_\alpha$$

$$Q_{3,5}(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \sum_{u,d,s,\ldots} \bar{q}_\beta \gamma_\mu (1 \mp \gamma_5) q_\beta$$

$$Q_{4,6}(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \sum_{u,d,s,\ldots} \bar{q}_\beta \gamma_\mu (1 \mp \gamma_5) q_\alpha$$

$$Q_{7,9}(\mu) = \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \sum_{u,d,s,\ldots} e_q \bar{q}_\beta \gamma_\mu (1 \pm \gamma_5) q_\beta$$

$$Q_{8,10}(\mu) = \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \sum_{u,d,s,\ldots} e_q \bar{q}_\beta \gamma_\mu (1 \pm \gamma_5) q_\alpha$$

$$Q_{1c}(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \bar{c}_\beta \gamma_\mu (1-\gamma_5) c_\beta$$

$$Q_{2c}(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \bar{c}_\beta \gamma_\mu (1-\gamma_5) c_\alpha$$

NOTE: *active* charm and charm *integrated out* cases are being investigated; however, only charm-out is finished (at a lattice spacing of 2 GeV).

Note also: $SU(3)_L \times SU(3)_R$ very useful for classification of these operators.

$$\epsilon' \sim \sum_i M_i \equiv \sum_i C_i(\mu)\langle\pi\pi|Q_i(\mu)|K\rangle$$

$$
\begin{aligned}
Q_i(\mu) &\equiv Q_i^{1/2} + Q_i^{3/2} \\
M_i^{I=0} &= \langle \pi\pi | C_i(\mu) Q_i^{\Delta I=1/2}(\mu) | K \rangle e^{-i\delta_0} \\
M_i^{I=2} &= \langle \pi\pi | C_i(\mu) Q_i^{\Delta I=3/2} | K \rangle e^{-i\delta_2}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Re}A_0 &\approx \frac{G_F}{\sqrt{2}} \mathrm{Re}(V_{us}^* V_{ud}) \sum_{j=1,2} \left(M_i^{I=0} - M_{i,c}^{I=0} \right) \\
\mathrm{Im}A_0 &= \frac{-G_F}{\sqrt{2}} \mathrm{Im}(V_{ts}^* V_{td}) \left(\sum_{j=1,2} M_{j,c}^{I=0} + \sum_{j=3\ldots 10} M_j^{I=0} \right) \\
\mathrm{Re}A_2 &\approx \frac{G_F}{\sqrt{2}} \mathrm{Re}(V_{us}^* V_{ud}) \sum_{j=1,2} M_j^{I=2} \\
\mathrm{Im}A_2 &= \frac{-G_F}{\sqrt{2}} \mathrm{Im}(V_{ts}^* V_{td}) \sum_{\ell=7,8,9} M_\ell^{I=2}
\end{aligned}
$$

$$
\epsilon' = \frac{i e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \frac{|A_2|}{|A_0|} \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right)
$$

$$
\begin{aligned}
\mathrm{Re}A_2^{\mathrm{expt}} &\simeq 1.25 \times 10^{-8}\ \mathrm{GeV} \\
\mathrm{Re}A_0^{\mathrm{expt}} &\simeq 2.73 \times 10^{-7}\ \mathrm{GeV}
\end{aligned}
$$

$\Delta S = 1$: 10 linearly *dependent* Q_i (12 if charm is active)

$SU_L(3) \times SU_R(3)$ chiral symmetry (L,R): (27,1), (8,1), (8,8) irr. rep's

Isospin symmetry: $\Delta I = 1/2$ and $3/2$ irr. rep's

Important examples:

$$
\begin{aligned}
current - current \quad & (\text{progenitor weak op.}) \\
Q_2 \; = \quad & \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \; \bar{u}_\beta \gamma_\mu (1-\gamma_5) u_\alpha \\
& \{(8,1)\,1/2\}\,\{(27,1)\,1/2\}\,\{(27,1)\,3/2\}
\end{aligned}
$$

$$
\begin{aligned}
QCD\;Penguin \quad & (\text{generated by QCD}) \\
Q_6 = \quad & \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \sum_{q=u,d,s} \bar{q}_\beta \gamma_\mu (1+\gamma_5) q_\alpha \\
& \{(8,1)\,1/2\}
\end{aligned}
$$

$$
\begin{aligned}
Electroweak\;Penguin \quad & (\propto \alpha,\, m_t^2) \\
Q_8 = \quad & \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \sum_{q=u,d,s} e_q \bar{q}_\beta \gamma_\mu (1+\gamma_5) q_\alpha \\
& \{(8,8)\,1/2\}\,\{(8,8)\,3/2\}
\end{aligned}
$$

NONPERTURBATIVE RENORMALIZATION

- OPE: operators get renormalized, depend on energy scale μ, $O^{ren}(\mu) = Z_{ij}(\mu) O_{ij}^{bare}$
- Operator Mixing: present in continuum and on the lattice **restricted by symmetries**, more is better: advantage for DWF
- Use the *Regularization Independent* Scheme (Martinelli et al NPB445,81,1995) require Green functions calculated on the lattice between *off-shell* quark and gluon states equal their tree level values.
  $Z^{-1}(\mu) \left\langle \bar{q}(p=\mu)|O^{Latt}|q(p=\mu)\right\rangle = \langle \bar{q}(p=\mu)|O^{tree}|q(p=\mu)\rangle$
- Construct amputated vertex from lattice fourier transformed propagator, project out desired spin, color, flavor: $Z^{-1}(\mu)\mathrm{tr}(P\Lambda) = 1$
- $p^2 = \mu^2 >> 0$ to match on to perturbation theory("window"). Procedure is *not gauge invariant*. *i.e.*, Z's depend on gauge and external states. After matching to continuum scheme ($\overline{\mathrm{MS}}$), $O^{ren}(\mu) = Z_{mat}^{-1}(\mu,\lambda) Z_{RI}^{-1}(a\mu,\lambda) O^{Latt}(a)$ is *gauge invariant*.

III. Chiral Perturbation theory

Significant technical difficulties associated with $|\pi\pi\rangle$, so use **lowest order** chiral perturbation theory to relate physical $K \to \pi\pi$ amplitudes to unphysical $K \to \pi$ and $K \to 0$ ones calculated on the lattice.

$$\langle 0|\Theta^{(8,1)}|K^0\rangle = \frac{16iv}{f^3}(m_s' - m_d')\alpha_2^{(8,1)}$$

$$\begin{aligned}
\langle \pi^+|\Theta^{(8,1)}|K^+\rangle &= \frac{4m_M^2}{f^2}\left(\alpha_1^{(8,1)} - \alpha_2^{(8,1)}\right) \\
\langle \pi^+|\Theta^{(27,1)}|K^+\rangle &= -\frac{4m_M^2}{f^2}\alpha^{(27,1)} \\
\langle \pi^+|\Theta^{(8,8)}|K^+\rangle &= \frac{12}{f^2}\alpha^{(8,8)}
\end{aligned}$$

$$\begin{aligned}
\langle \pi^+\pi^-|\Theta^{(8,1)}|K^0\rangle &= \frac{4i}{f^3}\left(m_{K^0}^2 - m_{\pi^+}^2\right)\alpha_1^{(8,1)} \\
\langle \pi^+\pi^-|\Theta^{(27,1)}|K^0\rangle &= -\frac{4i}{f^3}\left(m_{K^0}^2 - m_{\pi^+}^2\right)\alpha^{(27,1)} \\
\langle \pi^+\pi^-|\Theta^{(8,8)}|K^0\rangle &= \frac{-12i}{f^3}\alpha^{(8,8)}
\end{aligned}$$

Significant approximation, but can be improved by **one-loop** plus $\mathcal{O}(p^4)$ tree-level calculations (program is incomplete).

Also complicated by use of the quenched approximation.

Biggest problem with $K \to \pi$ is mixing with **lower dimensional operators**. Because the kaon and pion states are off-shell, $K \to \pi$ matrix elements receive non-vanishing contributions from $\bar{s}d$ with a **power divergent** coefficient, in our case $\sim (m_s + m_d)/a^2$.

We subtract this contribution by enforcing the condition

$$\begin{aligned} \langle 0|Q_{i,\mathrm{lat}}^{(1/2)}|K^0\rangle_{\mathrm{sub}} &\equiv \langle 0|Q_{i,\mathrm{lat}}^{(1/2)}|K^0\rangle + \eta_{1,i}(m_s - m_d)\langle 0|(\bar{s}\gamma_5 d)_{\mathrm{lat}}|K^0\rangle \\ &= 0 \end{aligned}$$

which determines the coefficient $\eta_{1,i}$ of the $\bar{s}d$ subtraction. Thus,

$$\begin{aligned} \langle \pi^+|Q_{i,\mathrm{lat}}^{(1/2)}|K^+\rangle_{\mathrm{sub}} \equiv\ & \langle \pi^+|Q_{i,\mathrm{lat}}^{(1/2)}|K^+\rangle \\ &+ \eta_{1,i}(m_s + m_d)\langle \pi^+|(\bar{s}d)_{\mathrm{lat}}|K^+\rangle \end{aligned}$$

We find that this subtraction is quite large, and thus it is crucial to have very good control of it.

WHY DOMAIN WALL FERMIONS?

Conventional discretizations explicitly break chiral symmetry of the continuum to remove **doublers**. Only restored in the continuum limit, $a \to 0$.

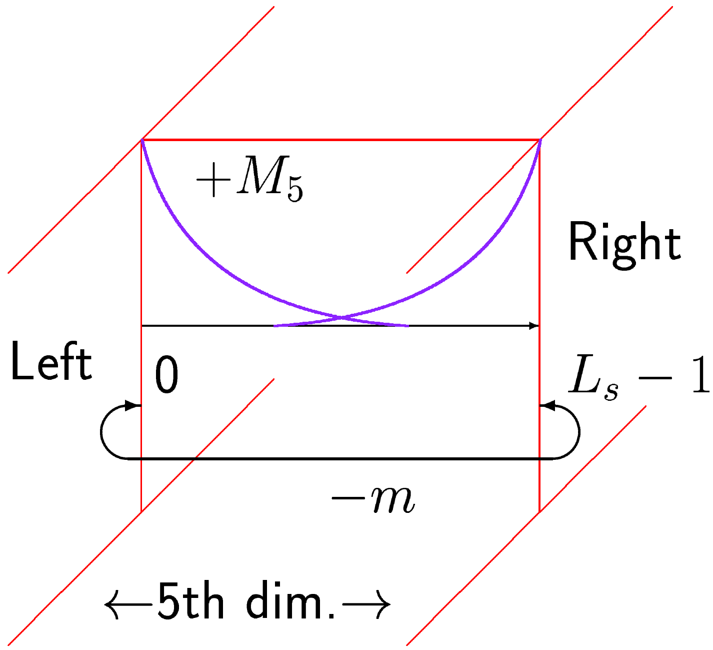
Kogut-Susskind. $SU(4)_L \times SU(4)_R \to U(1)_A$(non-singlet). Chiral limit is still $m_q \to 0$. But *flavor symmetry* is broken: only one light pion instead of $N_f^2 - 1$ and, operator flavor is complicated. Errors are $O(a^2)$.

Wilson fermions. $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. Chiral limit $\neq m_q \to 0$. Complicated **fine tuning** (operator mixing) of observables required for correct chiral behavior. Errors are $O(a)$.

Domain wall fermions. Remove the doublers while preserving $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry, **even at non-zero lattice spacing**. Combines best parts of KS and Wilson quarks. Errors are $O(a^2)(O(a)e^{-\alpha N_s})$ (initial results: improved scaling over conventional quarks).

IV. Domain Wall Fermions

Kaplan, Shamir, Narayanan and Neuberger



Chiral symmetry on the lattice, $a \neq 0$! Huge improvement

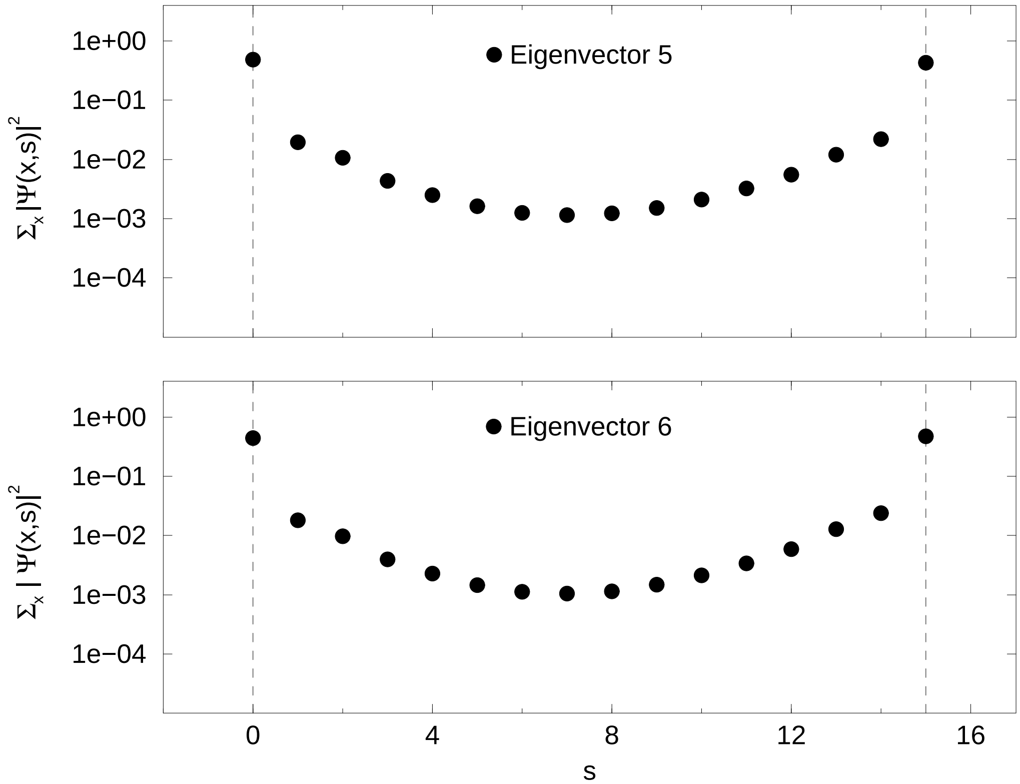
Low energy ($E \ll 1/a$) **chiral** states propagate on boundaries 0 and $L_s - 1$, but not into bulk

Chiral symmetry manifest: left/right separated in 5th dimension

Couple to $4d$ gauge fields → vector gauge theory (QCD)

Finite 5th dimension L_s: mixing of chiral modes softly breaks chiral symmetry.

Non-perturbative simulations confirm expectations. Explicit chiral symmetry breaking effects in **low energy** observables are under good control.



V. Details of the Simulation

Wilson gauge action, **quenched** $6/g^2 = 6.0 \to 1/a = 1.922$ GeV,

400 configurations

Lattice size $16^3 \times 32 \to (1.6\,\text{fm})^3 \times 3.2\,\text{fm}$

Domain wall fermions with $M_5 = 1.8$, $L_s = 16$, and quark masses

$\boxed{m_f = 0.01, 0.02, 0.03, 0.04, 0.05}$ (in units of lattice spacing).

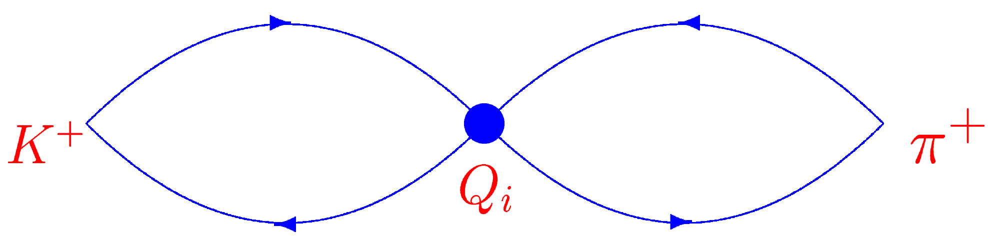
$m_{\text{strange}}^{\text{phys}}$ corresponds to $m_f \approx .02$. $\boxed{m_c = 0.1, 0.2, 0.3, 0.4}$

The Non-perturbative renormalization of operators was done with the same parameters as above, except $m_f = 0.04$ only, 490 configurations. Checked that results were not significantly affected by $m_f \to 0$ extrapolation on 100 configurations.

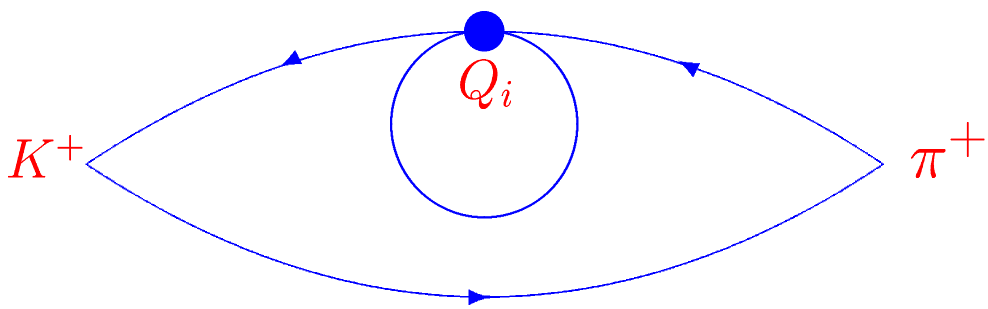
Simulations done on the QCDSP supercomputers at *Columbia University* and *RIKEN BNL Research Center*. Matrix element calculation took roughly 4 months x 0.8 TFlops (peak).

Lattice correlation functions

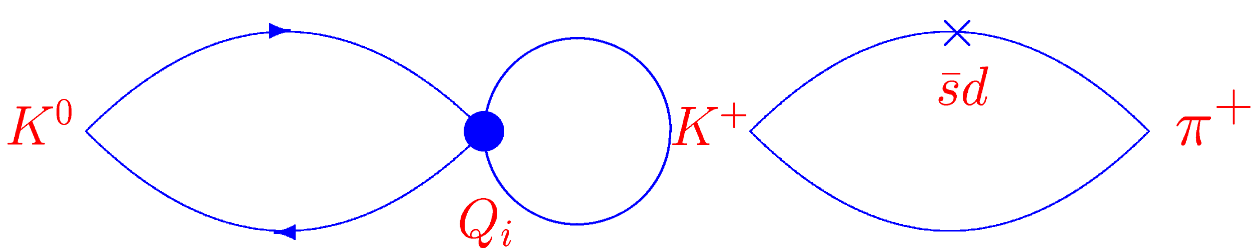
Figure 8



Eye



Subtraction



Euclidean time $\rightarrow$

V. $\Delta I = 1/2$ and 3/2 Low Energy Constants

Low energy constants $\alpha_{i,\text{lat}}^{(1/2,3/2)}$ are, up to known factors and to lowest order in χPT, the $K \to \pi\pi$ matrix elements in QCD of the effective weak operators. *These constants are the fundamental result of our lattice calculation.*

To obtain "full" $K \to \pi\pi$ matrix elements to 1-loop in χPT:

$$\langle \pi^+ | Q_i^{(1/2,3/2)} | K^+ \rangle \text{ (quenched)}$$

$$\downarrow 1-\text{Loop Q}\chi\text{PT}$$

$$\alpha_i^{(1/2,3/2)}$$

$$\downarrow \text{LO}\ \chi\text{PT}$$

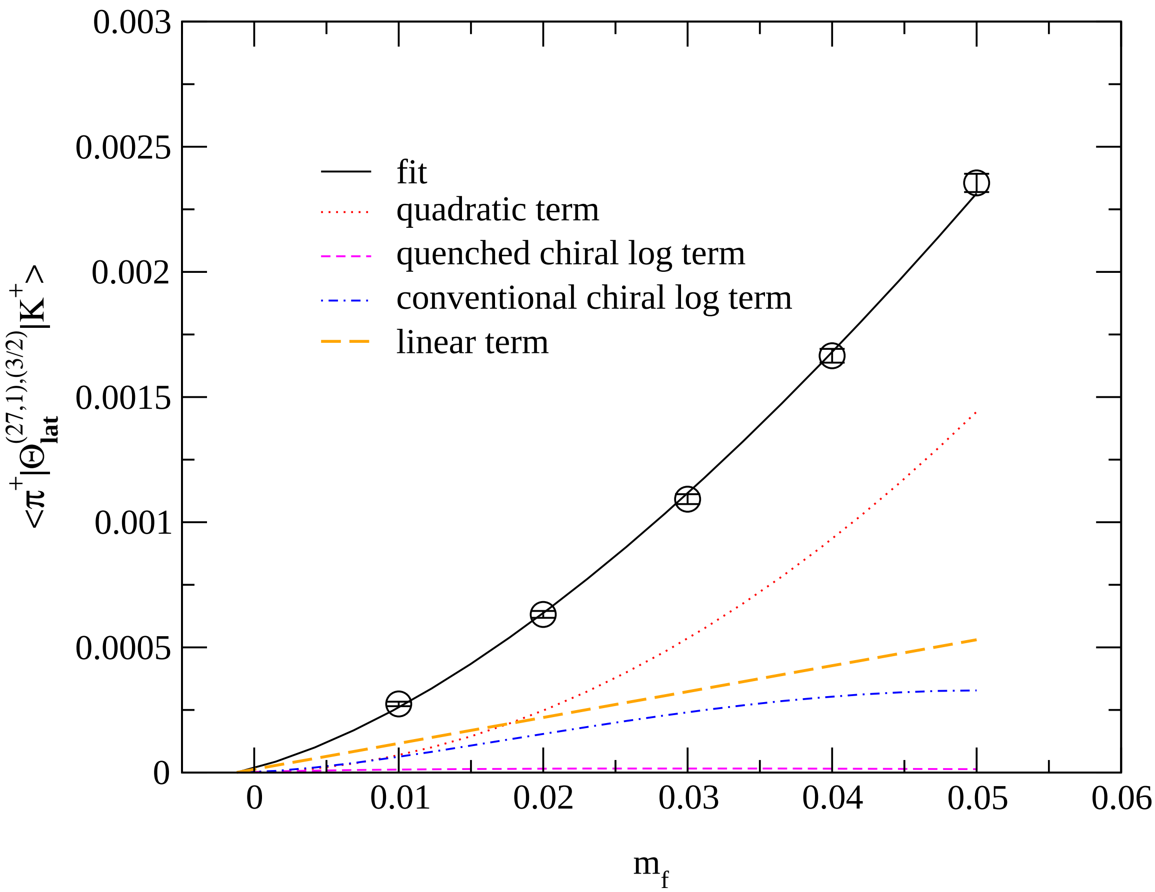
$$\langle \pi^+\pi^- | Q_i^{(1/2,3/2)} | K^+ \rangle \text{ (quenched} \to \text{full)}$$

$$\downarrow 1-\text{Loop}\ \chi\text{PT}$$

$$\langle \pi^+\pi^- | Q_i^{(1/2,3/2)} | K^+ \rangle \text{ (full, } 1-\text{loop}\ \chi\text{PT)}$$

$\langle \pi^+ | Q_{i,\mathrm{lat}}^{(27,1),3/2} | K^+ \rangle \propto \langle \pi^+ | \Theta_{\mathrm{lat}}^{(27,1),3/2} | K^+ \rangle (i = 1, 2, 9, 10)$

Higher order effects are **large**. Fit is forced to vanish at $m_f = -m_{res} = -.00124$. Quenched log is small. Conventional log is large and mimics linear term. *Known coefficients* calculated in QχPT.

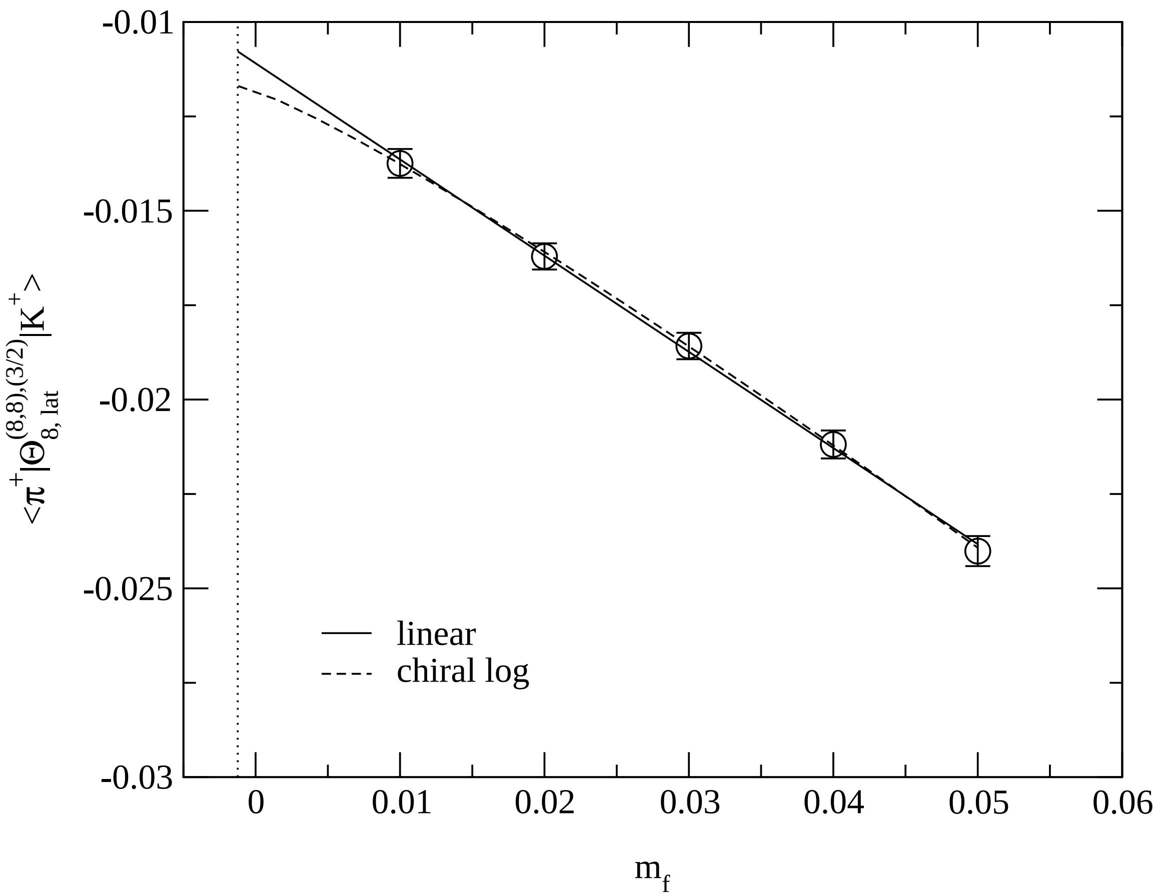


$$\alpha_i^{(27,1),3/2} = -\frac{f^2}{4} \times \text{slope} = (-4.13 \pm 0.18) \times 10^{-6}$$

and gives the lowest order contribution to Re$\mathcal{A}_2$

Electroweak penguin $\langle \pi^+ | Q_{8,\text{lat}}^{(3/2)} | K^+ \rangle$ (important to ϵ')

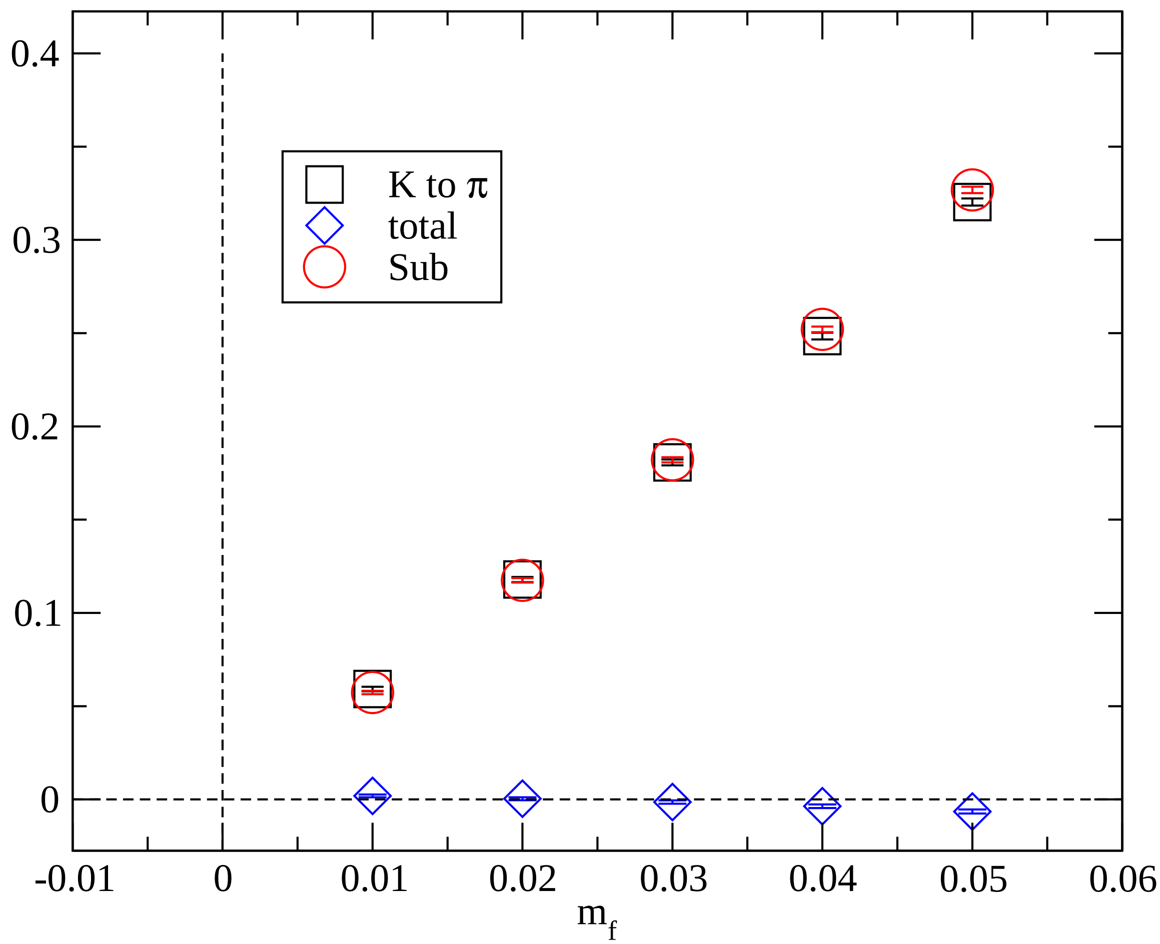
Lowest order contribution is a constant, higher order correction to *this constant* appears **small**. Extrapolate to $m_f = -m_{res} = -.00124$. Quenched log is unknown (ignore). Coefficient of conventional log is also unknown, but fit to it.



$$\alpha_8^{(3/2)} = \frac{f^2}{12} \times \text{intercept} = (-4.96 \pm 0.27) \times 10^{-6}$$

QCD Penguin $\langle \pi^+ | Q_{6,\mathrm{lat}} | K^+ \rangle$ and $2\, m_f\, \eta_6\, \langle \pi^+ | \bar{s} d_{\mathrm{lat}} | K^+ \rangle$

Divergent subtraction is almost complete. Physical slope is roughly 50 x smaller than the unsubtracted one

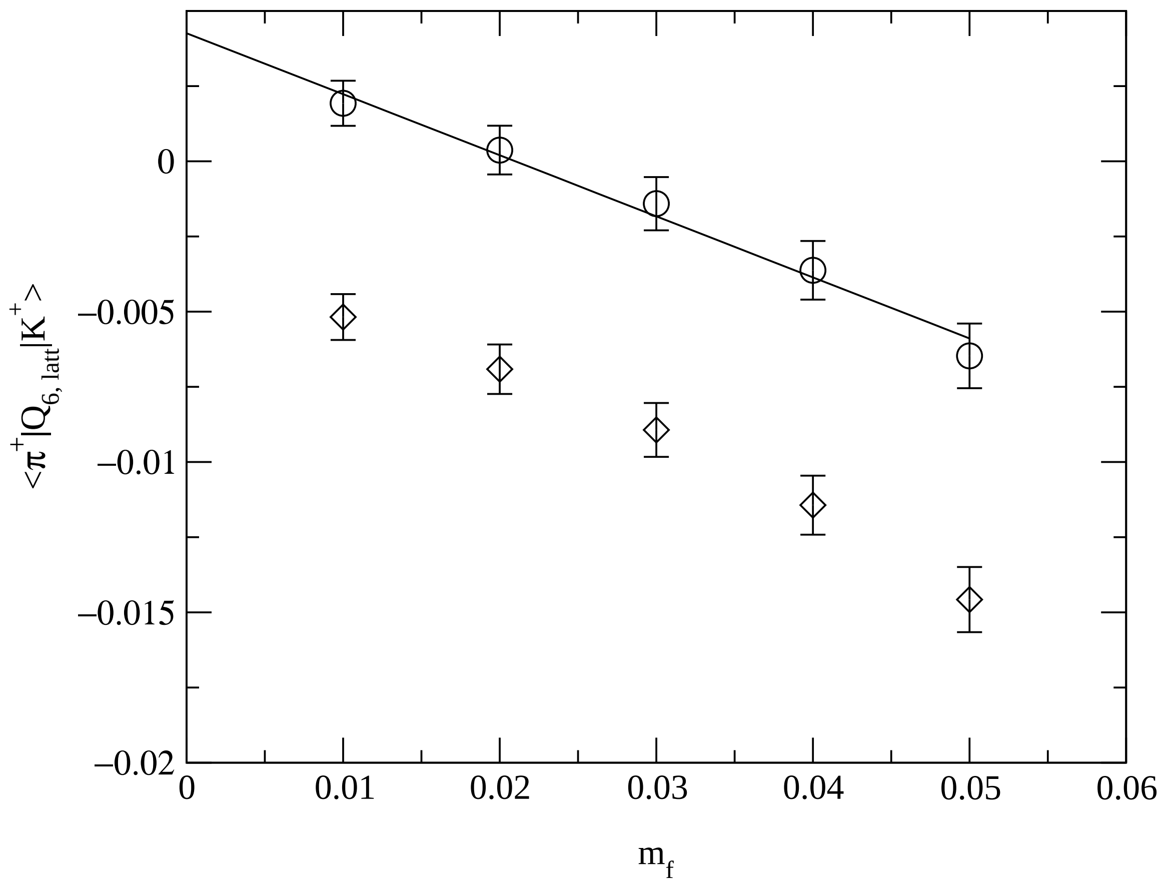


Data are highly correlated!

Subtracted QCD Penguin $\langle \pi^+ | Q_{6,\text{lat}} | K^+ \rangle$ (important to ϵ')

Does not vanish as $m_f \to -m_{res}$ because valence quark loop is sensitive to **high energy** chiral symmetry breaking effects (mixing between domain walls). Effect is an additive shift in the quark mass, which is eliminated by taking the slope.

$$\alpha_6 = \frac{f^2}{4} \times \text{slope} = (-8.12 \pm 0.98) \times 10^{-5}$$

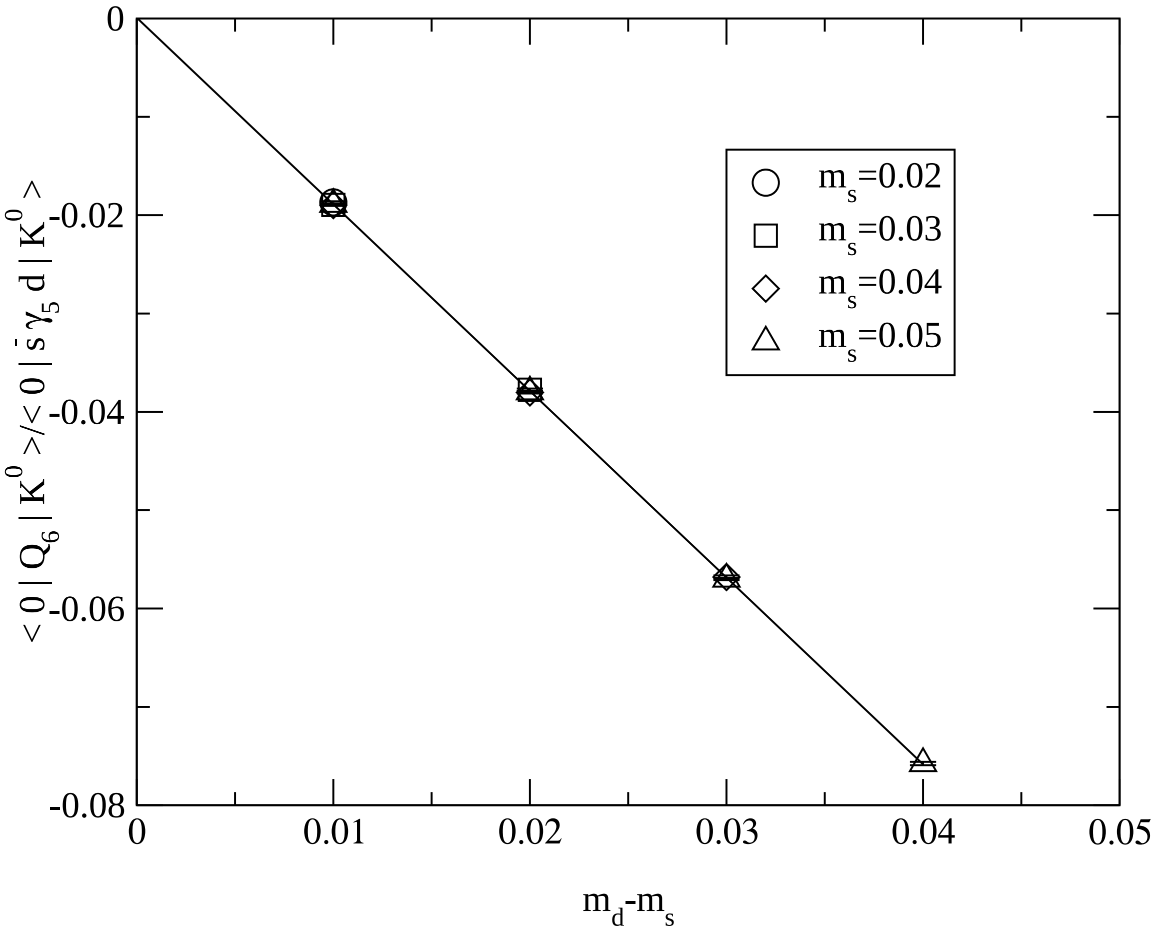


QCD Penguin $\langle 0|Q_{6,\mathrm{lat}}|K^0\rangle$

Must know η_6 very precisely. Data are quite linear.

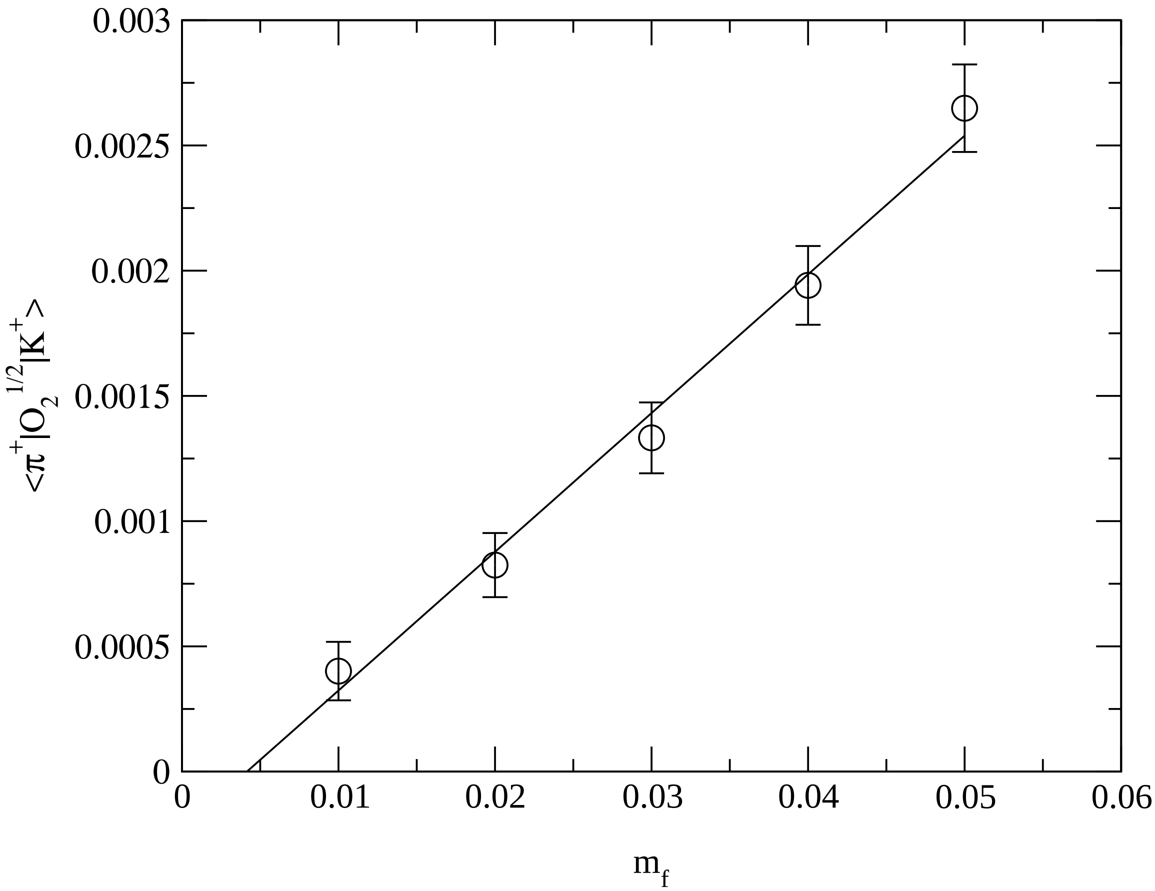
$$\eta_6 \;=\; \text{slope} \;=\; 1.8978 \pm 0.0036$$

$$\text{intercept} = (7.7 \pm 7.1) \times 10^{-5}$$



Subtracted $\langle\pi^+|Q_{2,\mathrm{lat}}^{1/2}|K^+\rangle$ (important to Re$\mathcal{A}_0$)

Explicit chiral symmetry breaking effects smaller than for Q_6.



$$\alpha_2 = \frac{f^2}{4} \times \text{slope} = (2.22 \pm 0.16) \times 10^{-5}$$

Final values for low energy constants

i	$\alpha_{i,\text{lat}}^{(1/2)}$	$\alpha_{i,\text{lat}}^{(3/2)}$
1	$-1.19(31) \times 10^{-5}$	$-1.38(6) \times 10^{-6}$
2	$2.22(16) \times 10^{-5}$	$-1.38(6) \times 10^{-6}$
3	$0.15(113) \times 10^{-5}$	0.0
4	$3.55(96) \times 10^{-5}$	0.0
5	$-2.97(100) \times 10^{-5}$	0.0
6	$-8.12(98) \times 10^{-5}$	0.0
7	$-3.22(16) \times 10^{-6}$	$-1.61(8) \times 10^{-6}$
8	$-9.92(54) \times 10^{-6}$	$-4.96(27) \times 10^{-6}$
9	$-1.85(16) \times 10^{-5}$	$-2.07(9) \times 10^{-6}$
10	$1.55(31) \times 10^{-5}$	$-2.07(9) \times 10^{-6}$

Statistical errors only, for now; Work in progress to quantify various systematic error.

Note $< \pi|Q_2^{1/2}|K > / < \pi|Q_2^{3/2}|K > \approx 15$ and not O(1)!

$\Rightarrow$ large fraction of the observed enhancement

[Recall $Q_2 \equiv \bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma_\mu(1-\gamma_5)d$, the aboroginal 4-Fermi operator.]

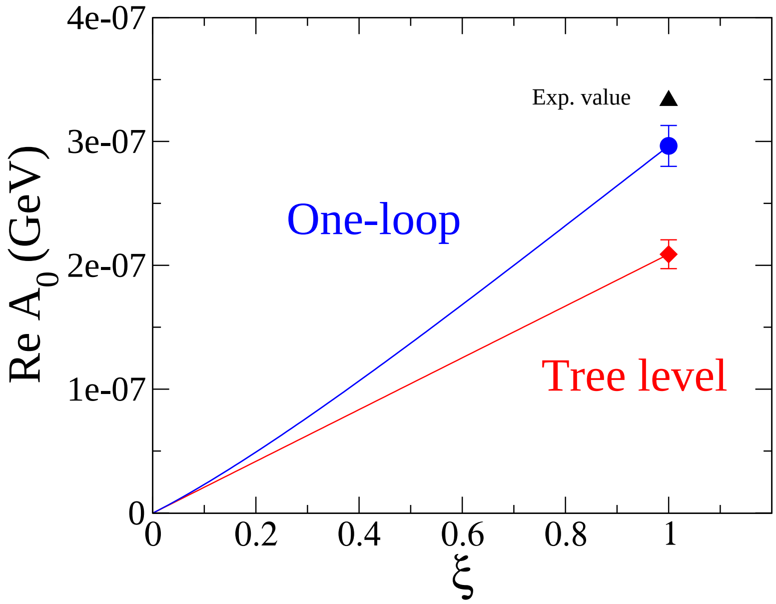
Real part of A_0

Study "fictional" world with masses

$$m^2_{K^0},\, m^2_{\pi^+} \to \xi \times (m^2_{K^0},\, m^2_{\pi^+})$$

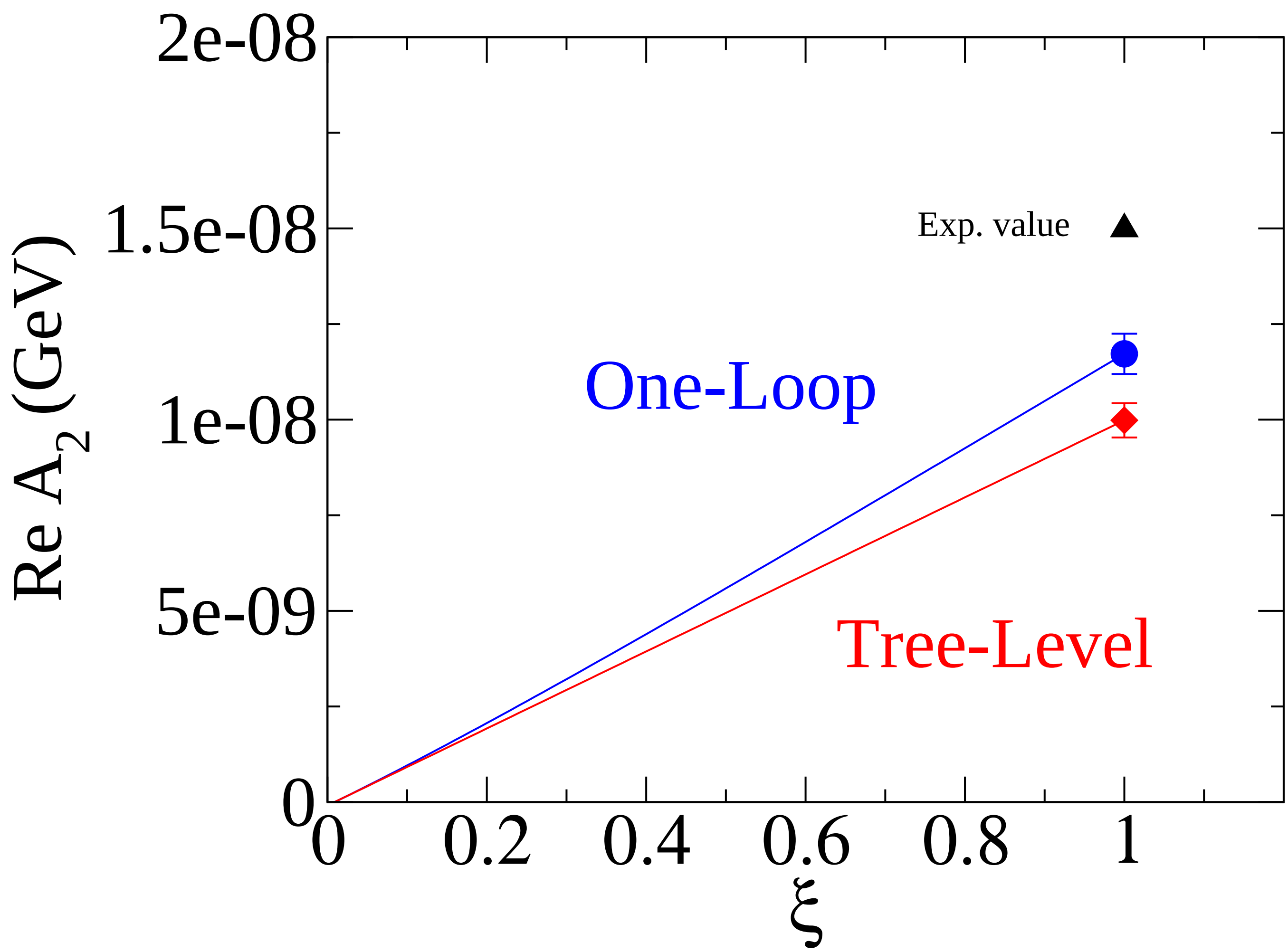
"Tree-Level" $=$ Lowest order chiral perturbation theory

"One-Loop" $=$ Tree-Level $+$ full QCD chiral logs ($K \to \pi\pi$)



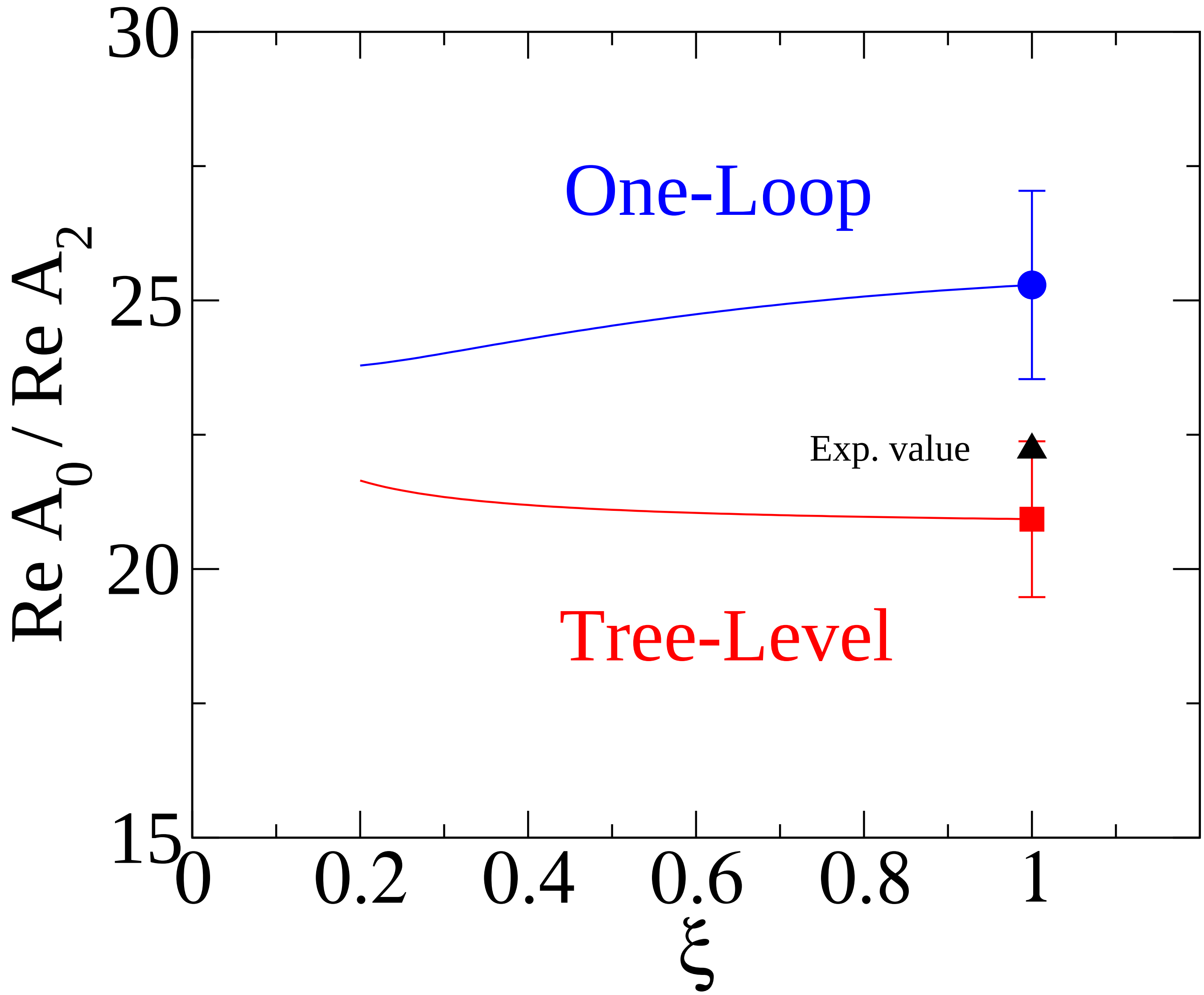
($\mu = 2.13$ GeV)

Real part of A_2



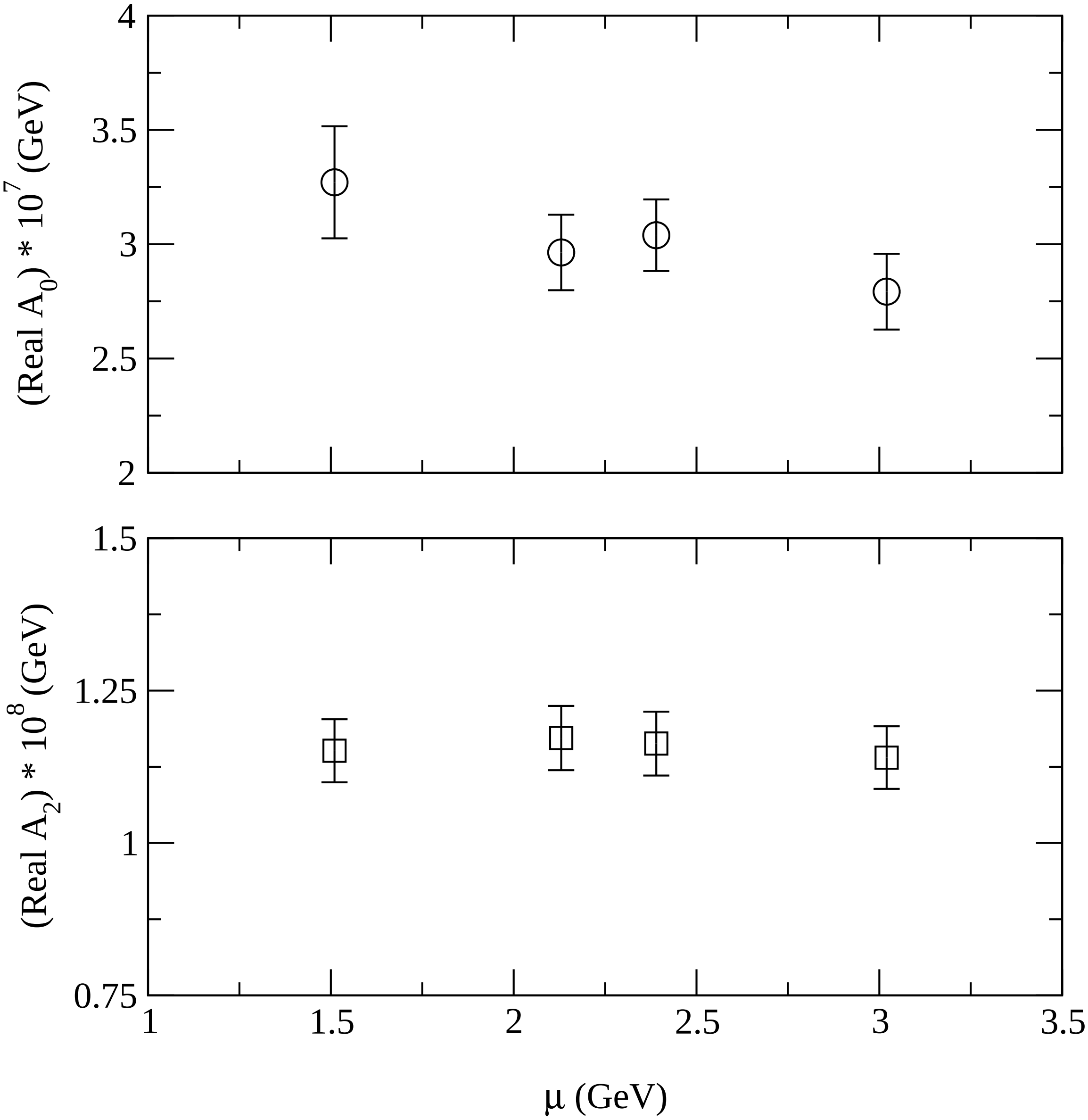
($\mu = 2.13$ GeV)

$\Delta I = 1/2$ Rule

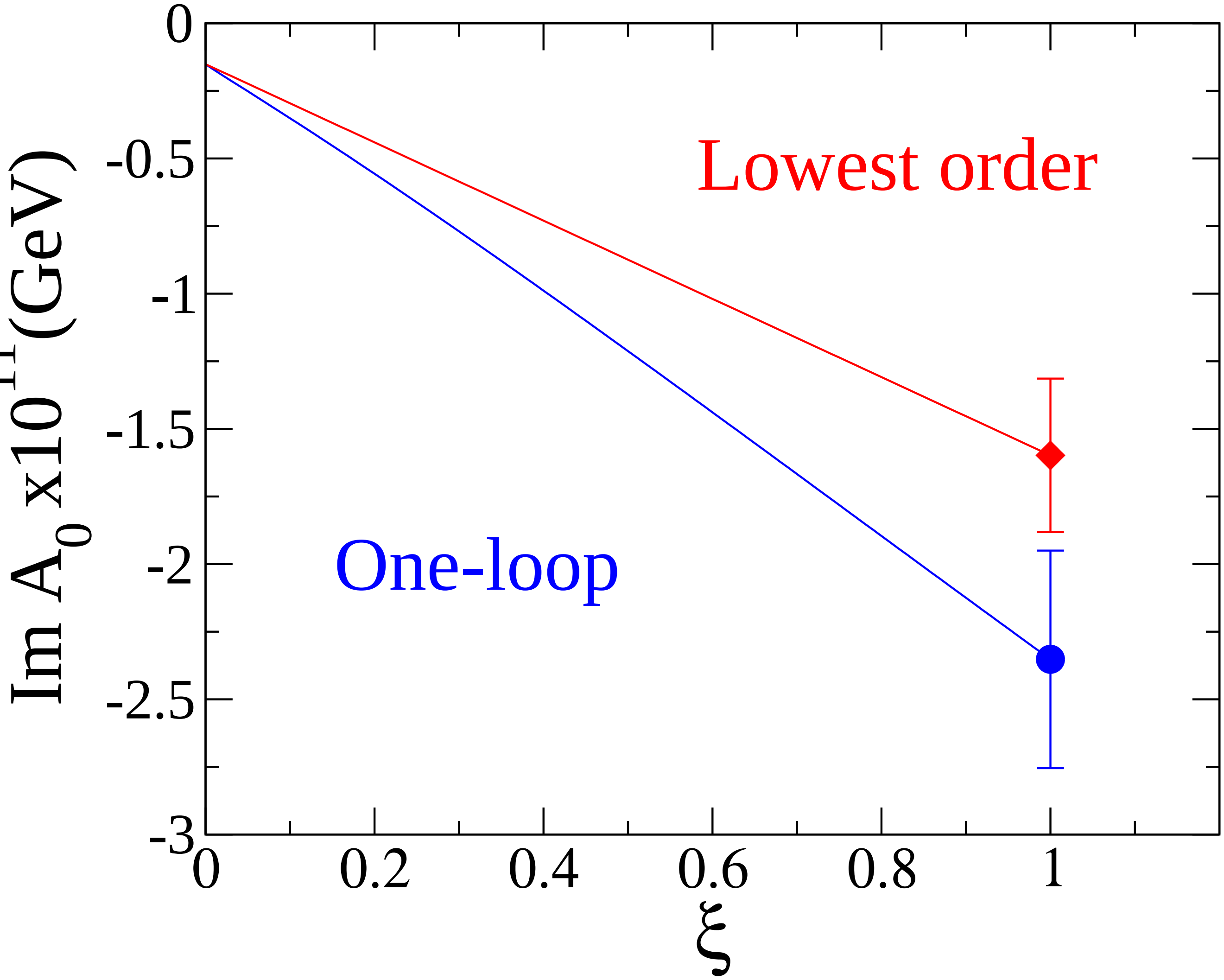


$(\mu = 2.13$ GeV$)$

Residual Scale Dependence in Real Amplitudes

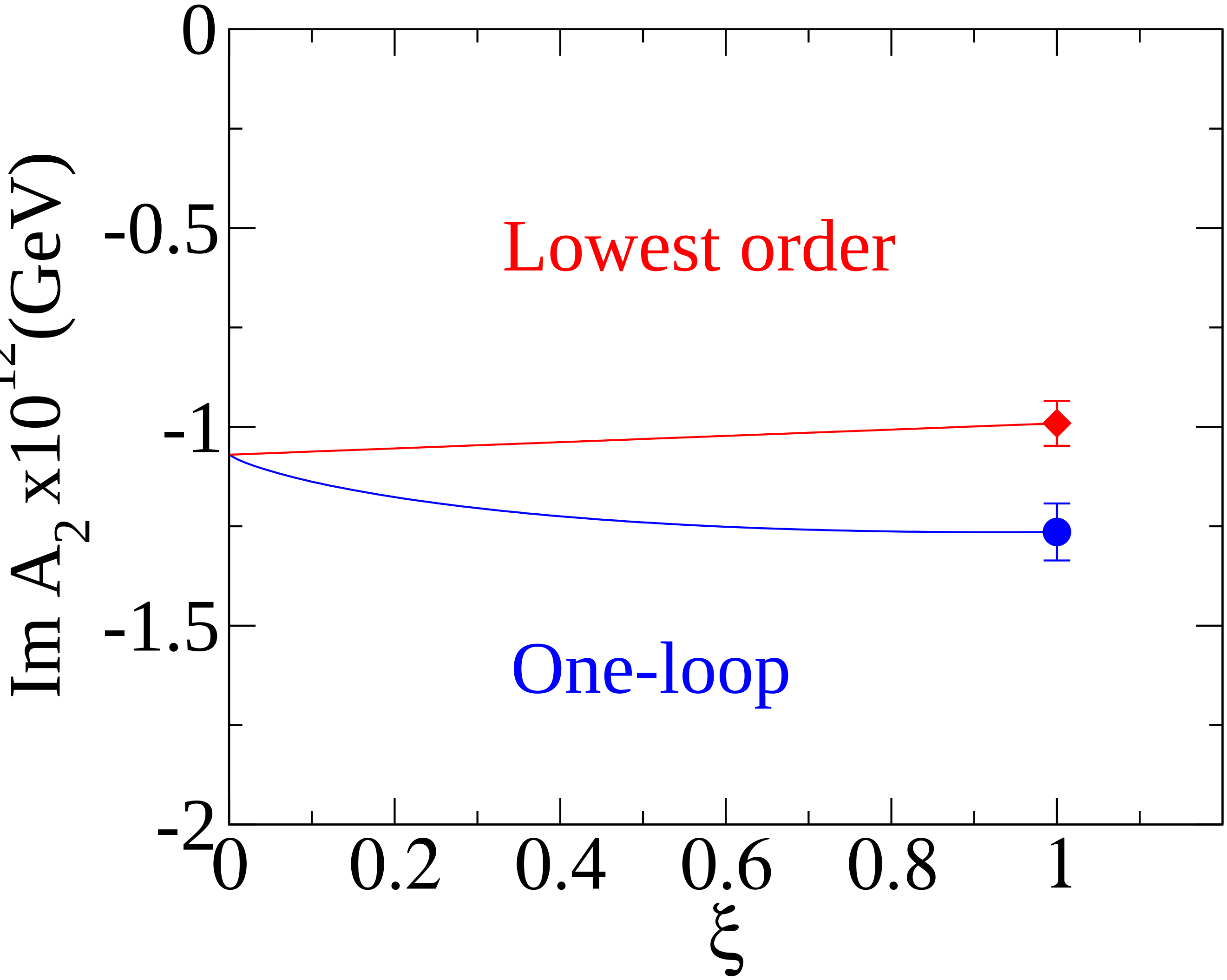


Imaginary part of A_0



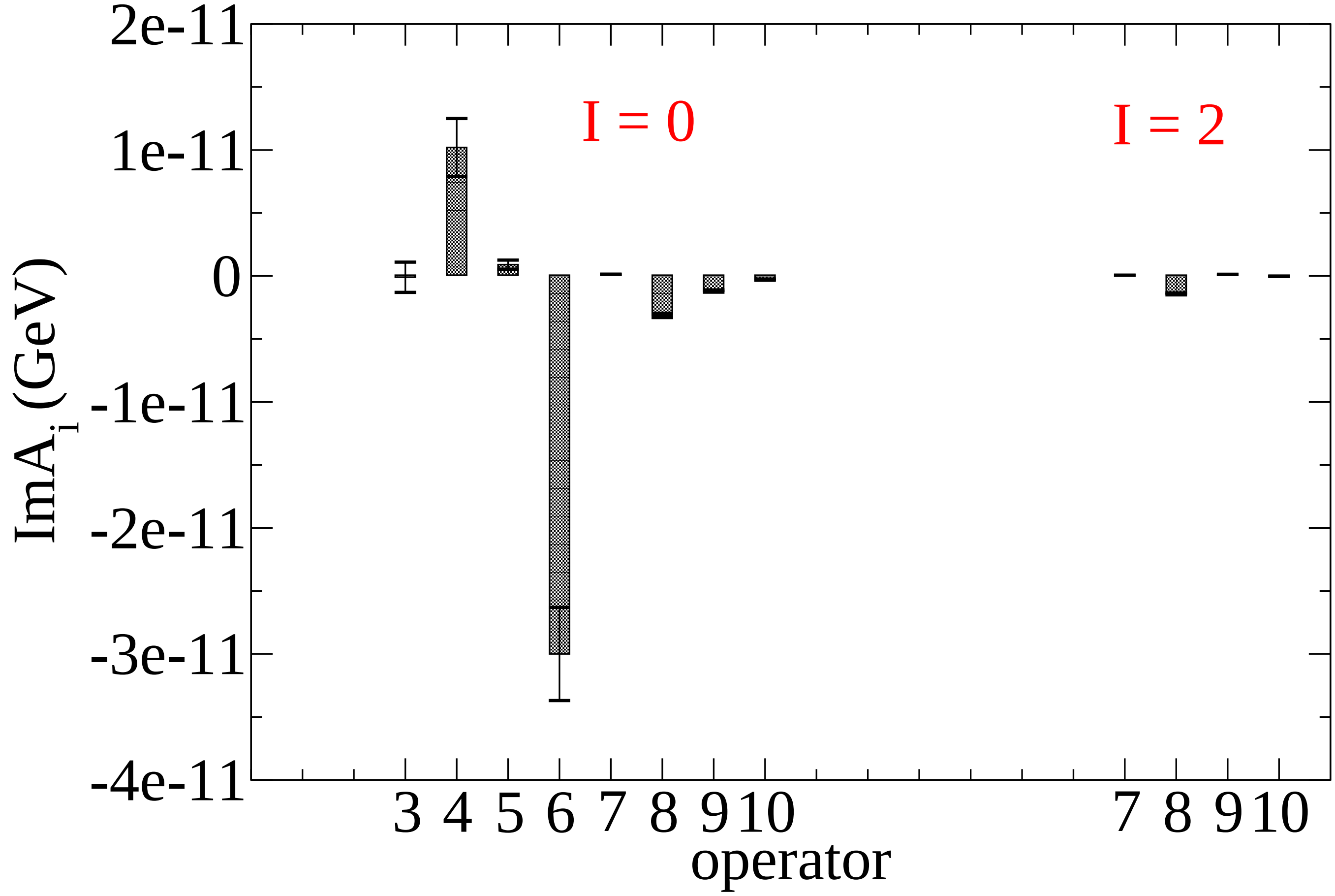
($\mu = 2.13$ GeV)

Imaginary part of A_2



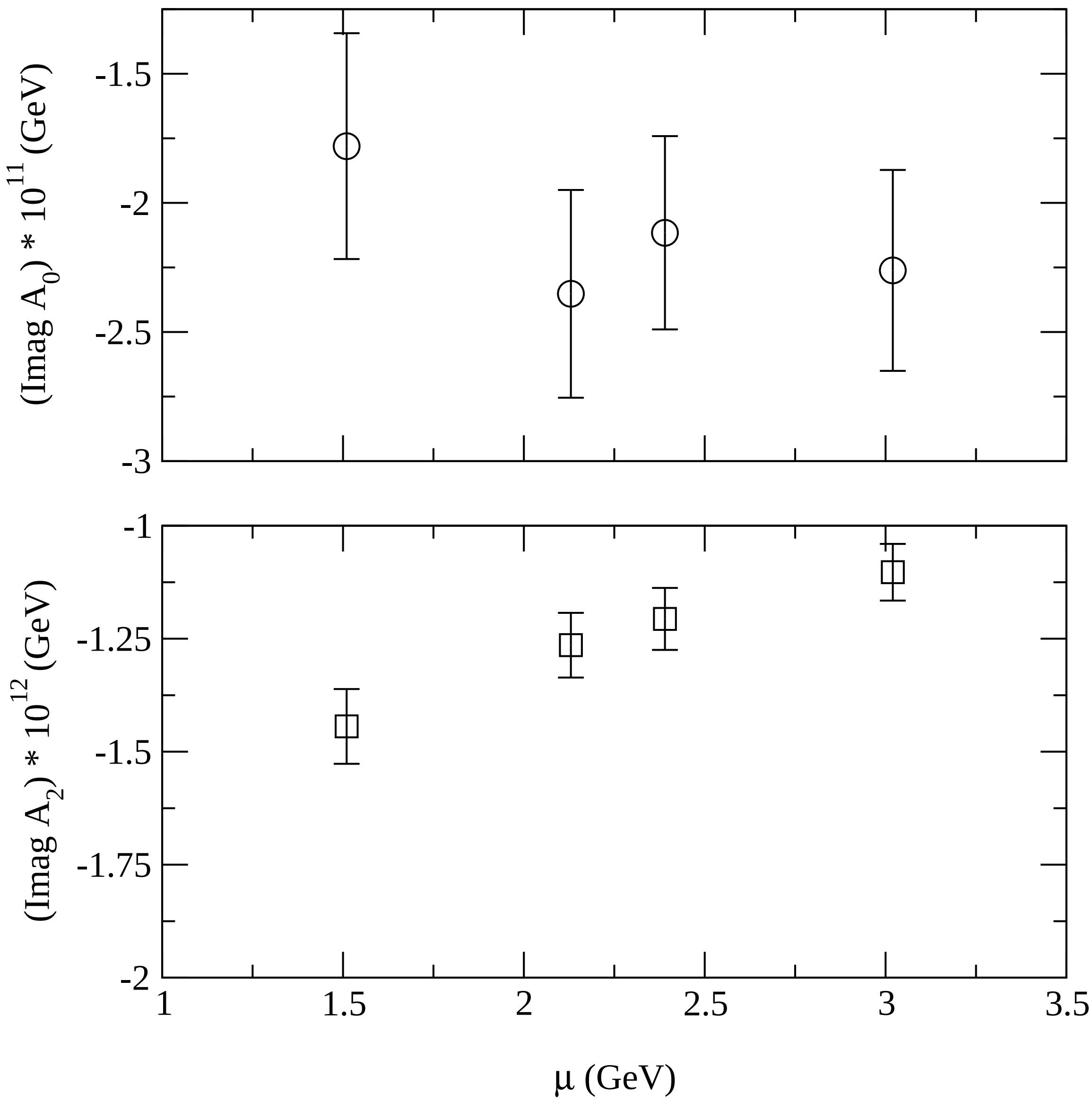
$(\mu = 2.13$ GeV$)$

Individual Contributions to Im$A_{0,2}$



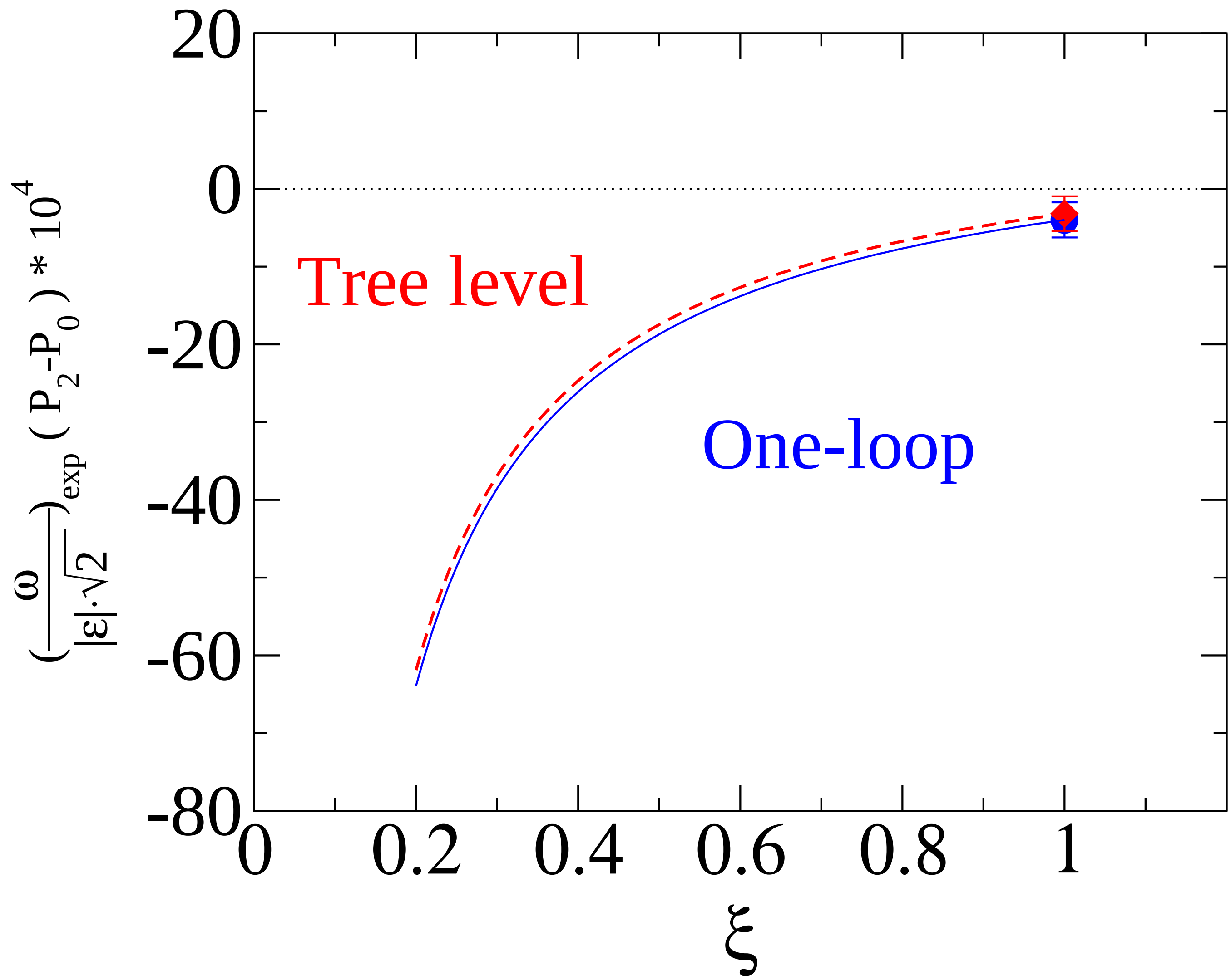
$\mu = 2.13$ GeV, One-loop χPT logs ($\xi = 1$)

Residual Scale Dependence in Imag. Amplitudes



ϵ'/ϵ

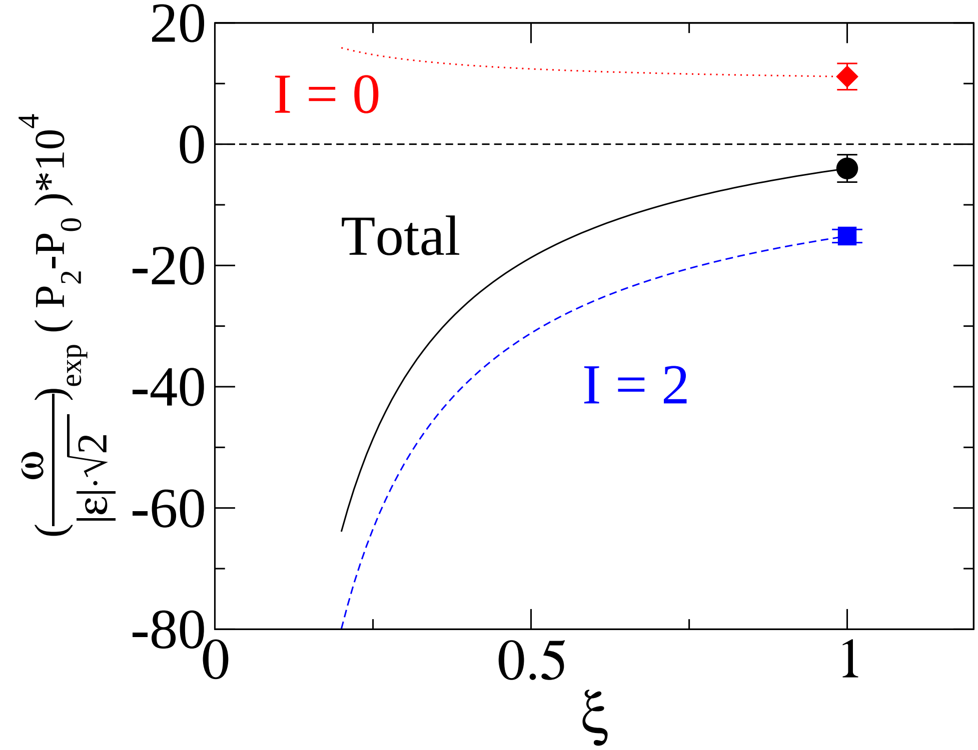
$$\frac{\epsilon'}{\epsilon} = \left(\frac{\omega}{\sqrt{2}|\epsilon|}\right)_{\text{exp}} \left(\frac{\text{Im}\,A_2}{\text{Re}\,A_2} - \frac{\text{Im}\,A_0}{\text{Re}\,A_0}\right) \tag{1}$$



($\mu = 2.13$ GeV)

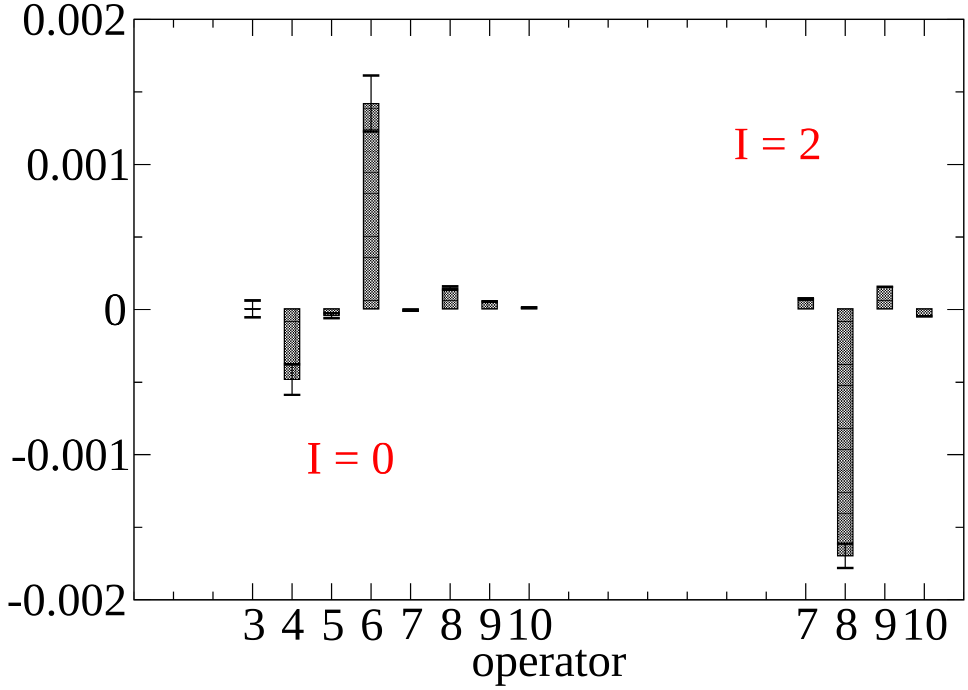
Isospin Breakdown P_0 and P_2

$$P_{0,2} \equiv \frac{\text{Im } A_{0,2}}{\text{Re } A_{0,2}}$$



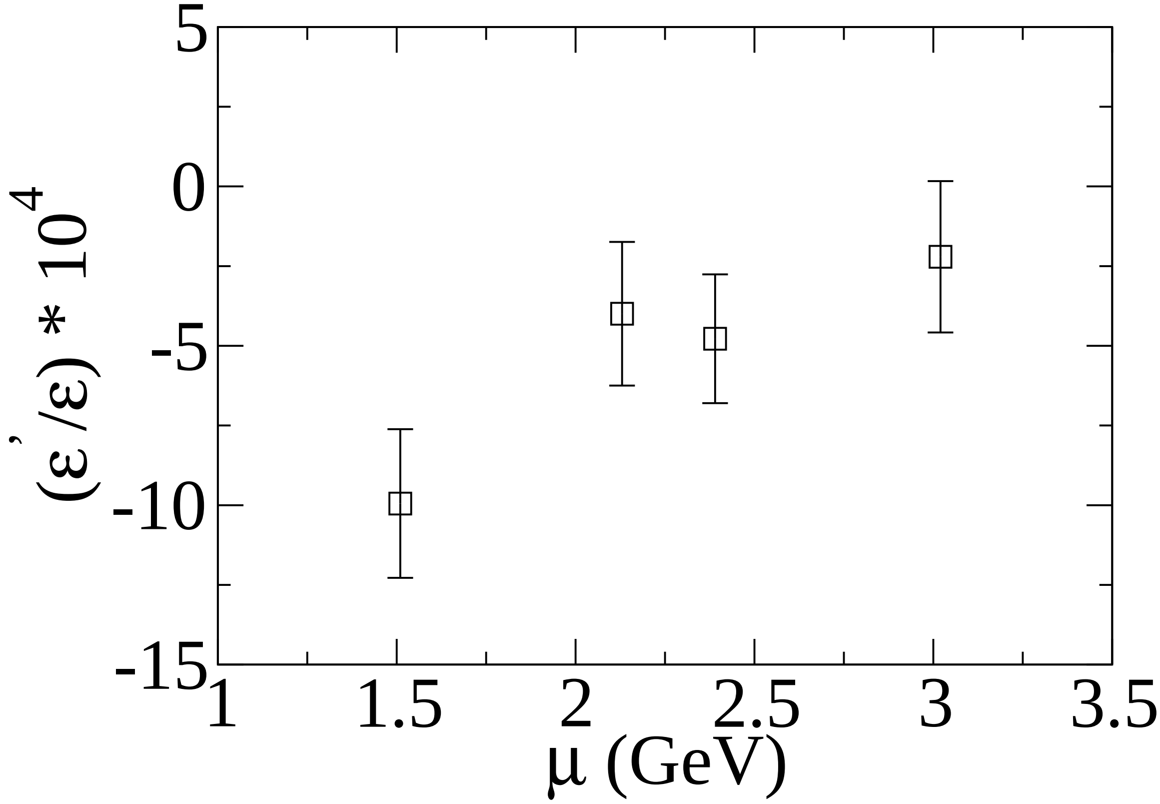
$\mu = 2.13$ GeV, One-loop χPT logs ($\xi = 1$)

Individual Contributions to ϵ'/ϵ



$\mu = 2.13$ GeV, One-loop χPT logs ($\xi = 1$)

Residual Scale Dependence in ϵ'/ϵ



Final Results

QUANTITY	Experiment	This calculation (statistical errors only)
Re A_0(GeV)	3.33×10^{-7}	$2.96 \pm 0.17 \times 10^{-7}$
Re A_2(GeV)	1.50×10^{-8}	$1.17 \pm 0.05 \times 10^{-8}$
Re A_0/Re A_2	22.2	25.3 ± 1.8
Re (ϵ'/ϵ)	$17 \pm 2 \times 10^{-4}$ (WA)	$-4.0 \pm 2.3 \times 10^{-4}$

($\mu = 2.13$ GeV, One-loop chiral logs)

- Results show clear $\Delta I = 1/2$ enhancement!

  $\Rightarrow$ (Although further scrutiny and confirmation is desirable and will be coming shortly) it certainly seems that we are very very close to putting a complete end to some 4-decades of speculation on the origin of this enhancement.

- Much more work is needed to improve our calculation of ϵ'/ϵ; discrepancy with experiment due to uncontrollable systematic errors (see later).

WHEN THE DUST SETTLES

$$\left[\frac{\epsilon'}{\epsilon} = \left.\frac{\epsilon'}{\epsilon}\right|_{I=0} + \left.\frac{\epsilon'}{\epsilon}\right|_{I=2}\right] \qquad ; \qquad \text{units } 10^{-4}$$

EXPT $\sim (17 \pm 2)$

OPERATOR	$I = 0$	$I = 2$
Q_4	4.8 ± 1.1	
Q_6	$\boxed{14.2 \pm 1.9}$	
Q_8	$1.48 \pm .12$	$\boxed{-16.97 \pm .84}$
Q_9		$1.56 \pm .00$

$\Rightarrow$ Although Q_6 and Q_8 are the dominant players, due to the significant cancellations:

- other ops. (e.g. Q_4 & Q_9) are also important and cannot be ignored.
- Accuracy need to be improved before impact on SM can be reliably extracted.
- The experimental value of $\frac{\epsilon'}{\epsilon}$ is comparable to $\left.\frac{\epsilon'}{\epsilon}\right|_{I=0}$ and $\left.\frac{\epsilon'}{\epsilon}\right|_{I=2}$ and not much smaller

$\Rightarrow$ Cancellation NOT between "large numbers"

$\Rightarrow$ To the extent that $\left.\frac{\epsilon'}{\epsilon}\right|_{I=2} < 0$, there is a useful bound,

$$\left.\frac{\epsilon'}{\epsilon}\right|_{I=0} > \left.\frac{\epsilon'}{\epsilon}\right|_{expt}$$

with which to test the SM rendering the cancellations somewhat irrelevant.

$\Rightarrow$ Prospects for BETTER test of the SM with improvement in accuraccy appear promising.

Regarding the $\Delta I = 1/2$ Rule

	ReA_0 (Gev)	ReA_2 (GeV)
Q_1	$(3.48 \pm .77) \times 10^{-8}$	$(-.363 \pm .016) \times 10^{-8}$
Q_2	$(24.5 \pm 1.6) \times 10^{-8}$	$(1.520 \pm .068) \times 10^{-8}$
Q_6	$(0.050 \pm .006) \times 10^{-8}$	

$$\Rightarrow Q_1/Q_2 \sim .14$$

$$Q_6/Q_2 < .01$$

$\Rightarrow$ For the $\Delta I = 1/2$ rule Q_2 **[the aboriginal 4-Fermi operator]** is the most important; in particular, Q_6 is completely negligible

$\Rightarrow$ CLEARLY Rules out SVZ conjecture, at least at our renormalization scale.

$\Rightarrow$ Repercussions for phenomenological calculations ofϵ'/ϵ

Concerns and Limitations

1. Quenched Approx. may well be much worse for the contribution of the QCD Penguin to ϵ'/ϵ than in other applications....emphasized in particular by Golterman and Pallante. (See more below).

2. Use of Lowest Order chiral perturbation theory

   - Unknown values for (8,1) and (8,8) conventional chiral logarithms ($m_M^2 \ln(m_M^2)$) in quenched $K^+ \to \pi^+$ matrix elements.
   - Unknown $\mathcal{O}(p^4)$ counter terms in full and quenched QCD (See more below)

3. Charm is integrated out assuming it is very heavy....This is unlikely to be a good approximation as the charm mass ($\approx$ 1.3 GeV) is not very heavy. Specifically this causes two types of problems.

   - Corrections to H_{eff} due higher dimensional operators which are suppressed only by powers of $1/m_c$ can be significant.

- Reliance on continuum perturbation theory down to $\mu = 1.3$ GeV in the calculation of the Wilson coefficients.

A 25 % error in each, “added linearly”, could easily produce a much different final result of ϵ'/ϵ especially given the significant cancellation between QCD pengins and EWP contributions to ϵ'/ϵ.

Significant Quenched Systematics?

Embedding of the LR Penguin operators (e.g. Q_6), which makes the dominant contribution to ϵ'/ϵ is ambiguous in the quenched approximation.

Emphasized 1st by Golterman and Pallante (hep-lat/0108..., hep-lat/0212008). See also, Laiho and Soni, (hep-lat/0306035)

Root of the problem can be understood in terms of the figures given below for $< \pi|Q_6|K >$ to serve as a concrete illustration;see L&Sfor details

Eye-Contractions for $< \pi|Q_6|K >$

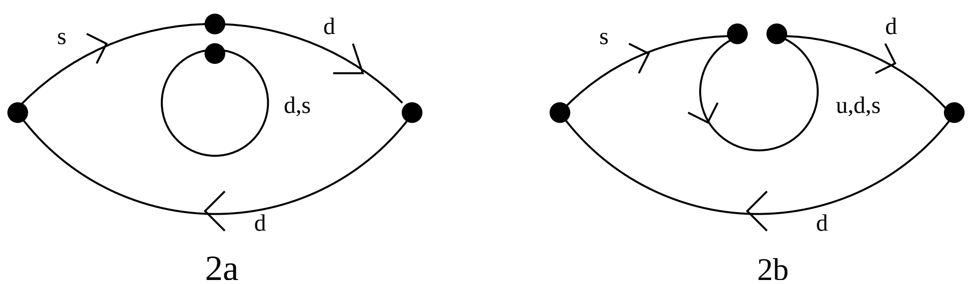


Fig. 2a represents $Tr \times Tr$ whereas b is a single color Trace

Magnified View of Eye-Contractions for $< \pi|Q_6|K >$

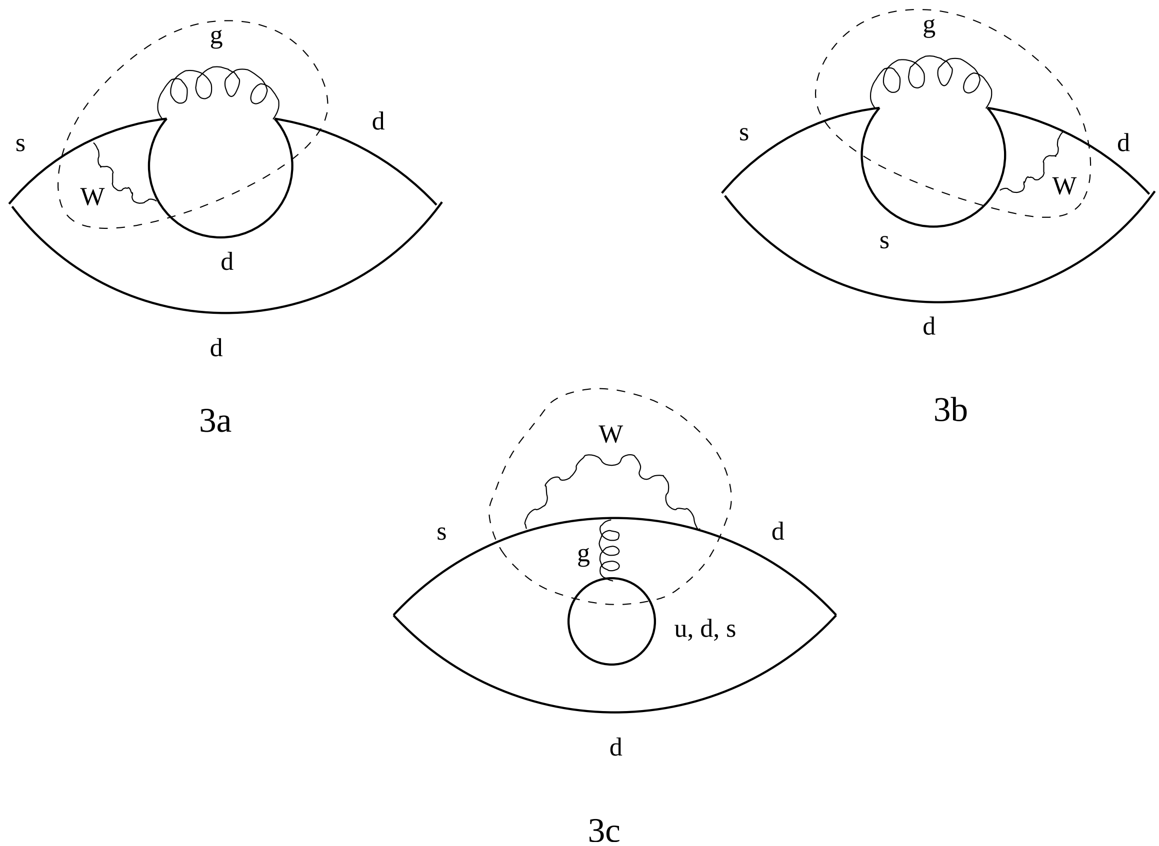


Fig 2b (single color Trace) originates from Fig.3c in which gluon connects to a $q\bar{q}$ vacuum-bubble...precisely the type of loop that in the propagtion of the gluon are typically left out in the Quenched Approximation.... Therefore consistent inclusion of Fig.2 b in quenched lattice calculation of $< \pi|Q_6|K >$ is not possible.

Inclusion of Fig.2b or not makes easily a factor of two or more in the $\epsilon'/\epsilon(I=0)$ contribution as preliminary numerical studies by RBC (DWQ) as well as W. Y. Lee et al (Staggered) show.

$\Rightarrow$ According to our current-level of understanding, the Quenched Approximation causes much larger systematic error than in most other (simpler) applications.

Feasibilty of $K \to \pi\pi$ and ϵ'/ϵ at NLO in ChPT

See Laiho (student) and AS, PRD 65,114020,'02; hep-lat/0306035

All existing lattice calculations of $K \to \pi\pi$ use LOχPT following BDSPW'85... This is a very serious shortcoming and must be addressed as: $\Rightarrow$ Loss of FS phases...Phseas need Loops i.e. NLO treatment

Especially in $I = 0$ Large FS corrections are anticipated

But χPT is LEET, that is non-renormalizable. $\#$ of arbitrary coefficients (LEC $=$ low energy constants) proliferate as you go to higher order as many operators are allowed by the SU(3)$\times$SU(3) chiral symm. e.g. AT LO (8,1) $\Delta I = 1/2$ needs 2 LEC's whereas @ NLO it needs 8 etc.

Past investigations (see Bijnens et al hep-ph/9801326; Golterman and Pallante, hep-lat/0110206) show that $K \to \pi$ & $K \to vac.$ not enough for getting $K \to \pi\pi$ to NLO.

Laiho & AS Explicitly

1. Demonstrate $K \to 2\pi$ @ O(p^4) on the lattice is feasible
2. Provide all finite log correctons emanating from NLO loop contributions to all the necessary processes needed to be studied.

   The following 4 processes (all computable on the lattice) are needed:

1. $K \to \pi$ with momentum
2. $K \to 0$
3. $K \to \bar{K}$
4. Crucial additional ingredient to get the job done, is $K \to \pi\pi$ at *unphysical kinematics* points ("UKX") such that $m_K > m_\pi$. [Note UKX bypass Maini-Testa restrictions]

Additional Remarks:

- Actually we are able to show that all of the operators of interest can be obtained w/o using any 3-momentum insertion; nondegenerate meson masses (i.e. $m_K \neq m_\pi$) suffice.

- Since sequence of quark masses are needed on the lattice anyway, this recipe means NO EXTRA SIMULATION COST in computing the needed $K \to \pi$ for NLO calculation.

- Even when working at LO, with $m_K \neq m_\pi$, a very useful alternate method for extracting $K \to \pi\pi$ becomes available. (See L&S for details):

  $< \pi^+\pi^-|O^{(8,1)}|K^0 >= 4i\frac{\alpha_1}{f^3}[m_K^2 - m_\pi^2]$

  $< \pi^+|O^{(8,1)}|K^+ >= \frac{4}{f^2}\alpha_1 m_K m_\pi$

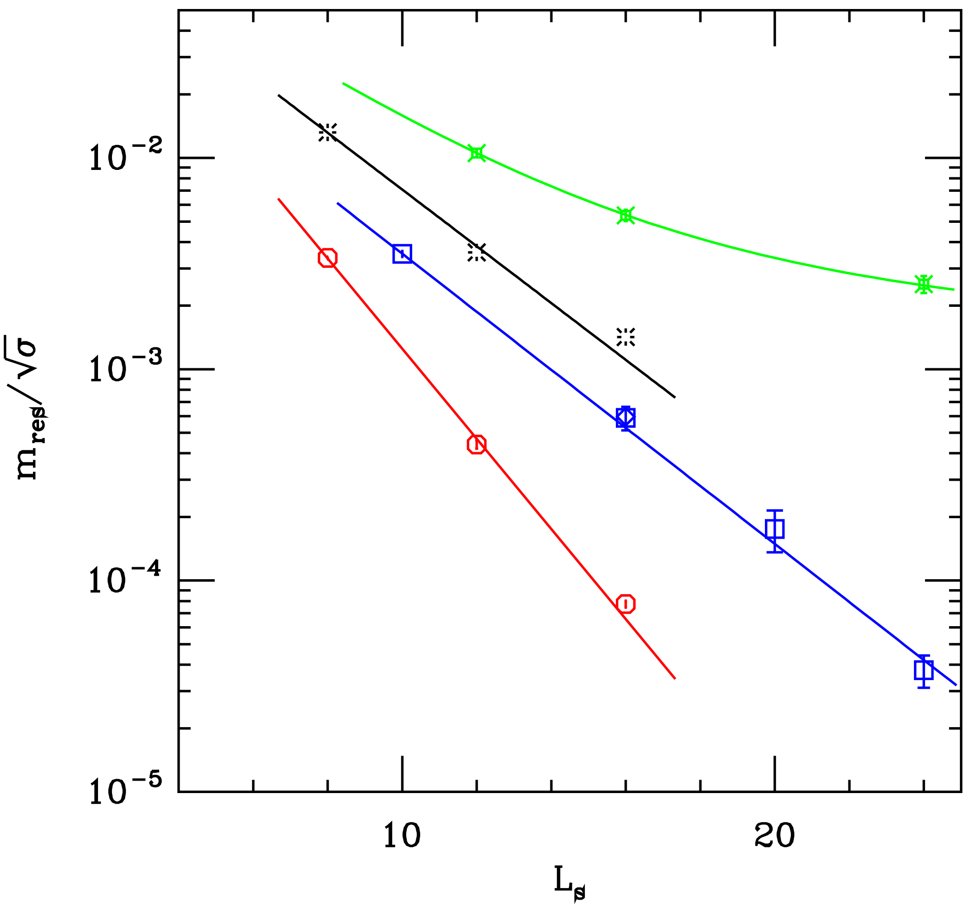
  Thus the needed LEC, α_1 can be obtained by measuring the slope of $K \to \pi$ as a function of m_π holding m_K fixed.

  $K \to vac$ subtraction, that entails humungous cancellation is not needed and is completely avoided in this method

- For the EWP operator, L&S are able to show that with the PQ simulation, $K \to \pi$ with $m_K = m\pi$ suffice to give $< \pi\pi|O^{8,8}|K >$ to NLO !

Steps Underway to Address Major Concerns

1. Improved Gauge Action (DBW2) is being used for the past two years in WME calculations. Contrast red(now) with green(old) in fig. below..This further improves chiral symmetry (which was already very good), with DWQ by over an order of magnitude [See RBC,hep-lat/0211023] and fig. below



2. Simulation at weaker coupling ($a^{-1} = 3GeV$) is well underway (DBW2, 24^3). This should allow better treatment of a propagating charm quark.[See J.Noaki, for RBC, LAT02-03]

3. Dynamical DW $N_f = 2$ simulation with 3 values of the dynamical mass (0.02,0.03,0.04, where 0.02 is about $m_{strange}/2$), $a^{-1} \approx 1.6GeV$, 16^3 ongoing for about 2 years. Should allow 1st glimpse of ϵ'/ϵ in Partially Quenched QCD (only two light dynamical masses rather than three in the real world....) [See T. Izubuchi, for RBC, LAT02-03]

Unitarity Triangle from K-Decays

SM-CKM phase $\Rightarrow$ large CP violation in B; very small in K

Can BSM-phase do the opposite?

For this reason and others Kaons, shouldn't be counted out yet

Construction of UT purely from K-physics ["KUT"]' will provide a critical test of the CKM-Paradigm.

This becomes possible with (at least) 3 of the following 4 processes:

1. $K^+ \to \pi^+\nu\bar{\nu}$...clean theory can give $|V_{td}|$ as improved expt. measurements become available at BNL, KEK.
2. $K_L \to \pi^0\nu\bar{\nu}$ ..super-clean theory, Rate is CP violating! $\propto \eta$
3. $K_L \to \pi\pi$ Indirect CP, $\epsilon_K \propto B_K$ needs theory (lattice), known now to about 15-20% accuraccy, most of the error comes from quenching...significant improvement expected in the next few years
4. $K_l \to \pi^+\pi^-$ versus $\pi^0\pi^0$ DIRECT CP-violation... Experimental

situation now under very good control. Theory still extremely unreliable.....Major efforts on the lattice underway...should improve appreciably in the next five years

Efforts in Expts. and on the lattice could yield construction of the KUT in the next several years that would open an important new avenue for testing the SM.

Summary and Outlook

- Theoretical Developments of Exact Chiral Symm. on the Lattice has opened the way for continued Progress in the ancient problem, $K \to \pi\pi$
- Significant Progress by the lattice in getting ReA_0, ReA_2 and the $\Delta I = 1/2$ Rule. In particular, enhancement all originates from "tree" operator, Q_2...and penguin operators are negligible for ReA_0.
- For ϵ'/ϵ within the approximations used (Quenched, LOChPT, large m_{charm}) see substantial cancellation between EWP and QCDP. As a result expect very large systematic errors...Disagreement between central value of lattice calculation and experiment for now is merely a reflection of the approximations.
- Efforts now underway to improve the calculation by using: 1.) even better chiral symmetry with DBW2; 2.) dynamical domain wall quarks; 3.) weaker coupling for better treatement of the

charm quark; 4.) NLO in ChPT and 5) direct $K \to \pi\pi$ using Lellouch-Luscher approach of finite volume correlation functions.

- Getting a separate UT from K-decays is an important goal. These lattice efforts along with experiments on very rare K decays ($\pi\nu\nu$) could allow another avenue for searching for New Physics in the next several years.