Parity-Violating Spin Asymmetries at RHIC
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Global Analysis of Fragmentation Functions and Their Uncertainties

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in collaboration with
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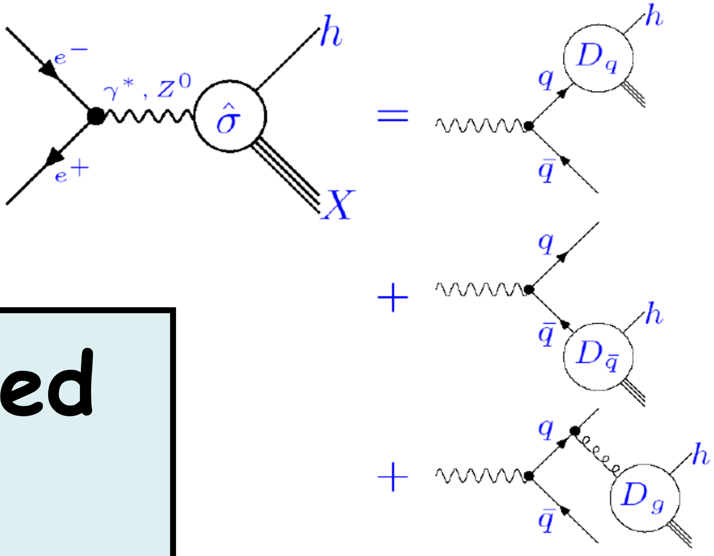
hep-ph/0703242 (PRD)

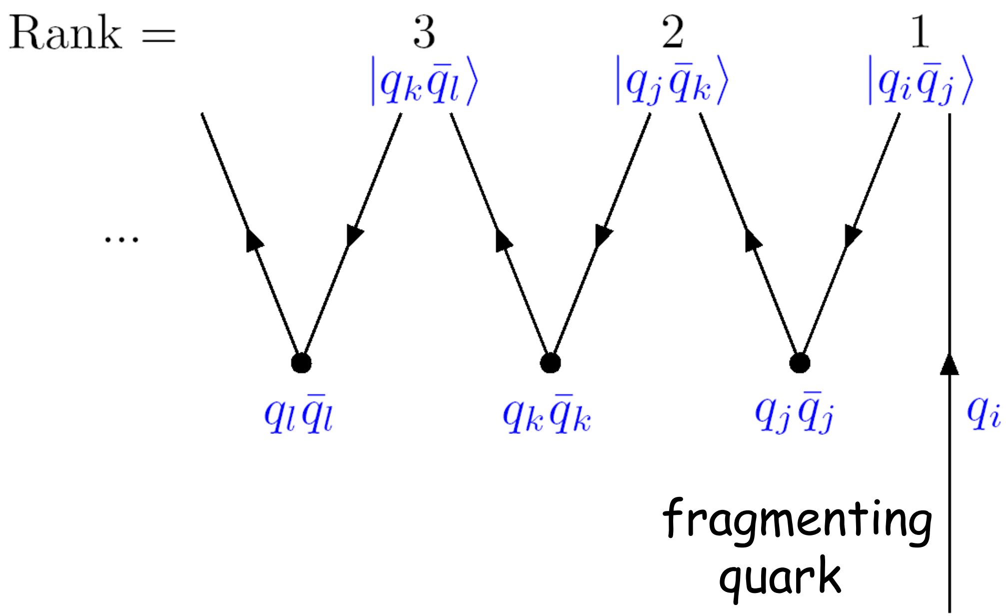


questions to be addressed in this talk:

- Is it possible to arrive at a *unified* description of all e^+e^-, ep, and pp data in terms of a *universal set of fragmentation functions* ?
- If so, what are the *uncertainties* ?

analyses so far: overview

the very beginning (before data) : "cascade fragmentation" model
Feynman, Field '78



rank = 1: "valence", e.g., $u \to \pi^+$

rank $\geq$ 2: "sea", e.g., $u \to \pi^-$

quote (R.D. Field):
"not meant to be a theory"

now we mainly use two well-known fits, e.g., to analyse RHIC data

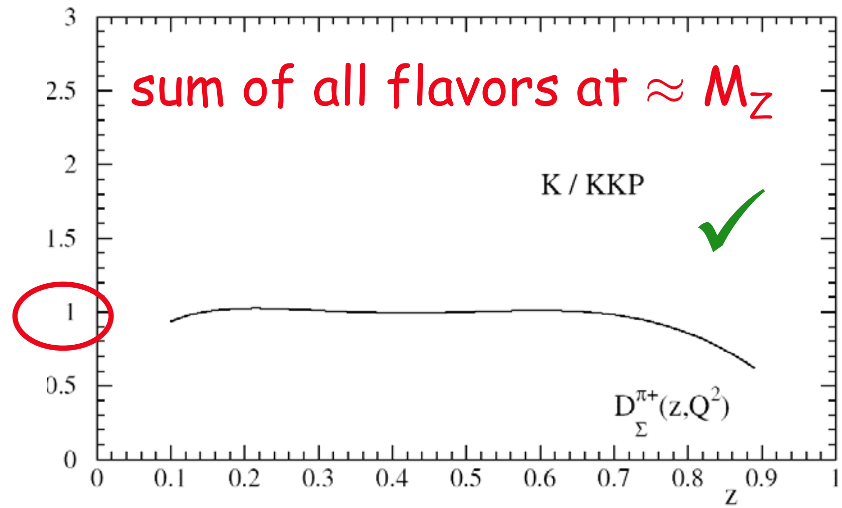
"Kretzer" and **"xKK" family of fits**

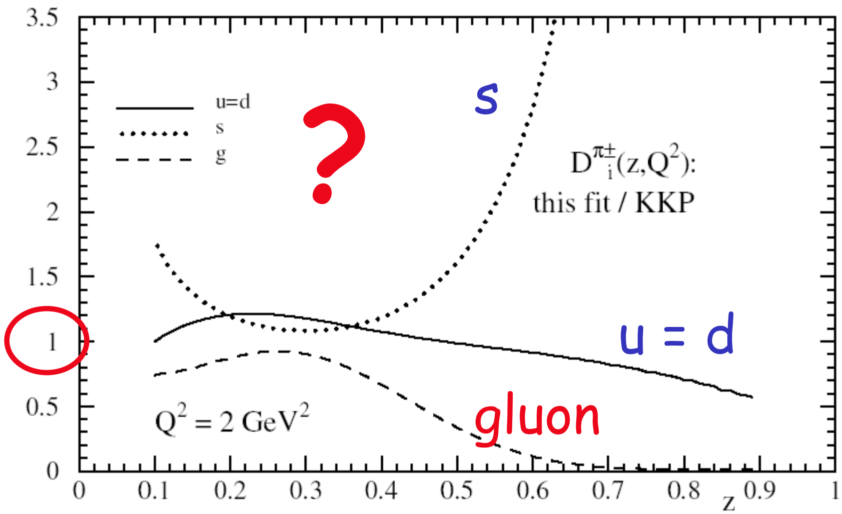
S. Kretzer Binnewies/Potter/Albino, Kniehl, Kramer (BKK,KKP,AKK)

analyses so far: problems & limitations

all fits are based on e^+e^- data only, no error estimates

they nicely agree on the flavor singlet at M_Z where the bulk of the fitted data is





figs. by S. Kretzer

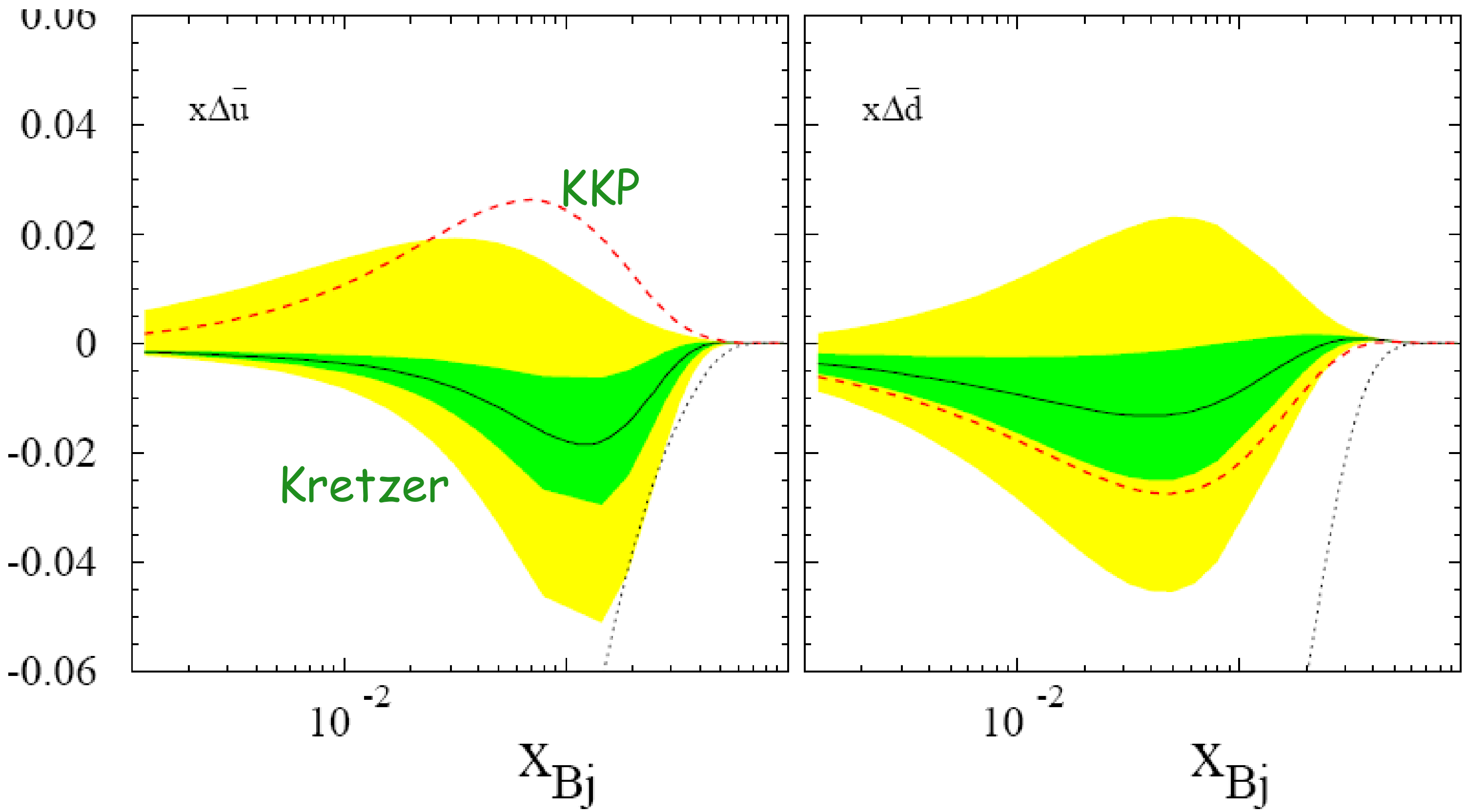
at lower scales (relevant for most ep and pp data), individual flavors and, in particular, the gluon can be very different

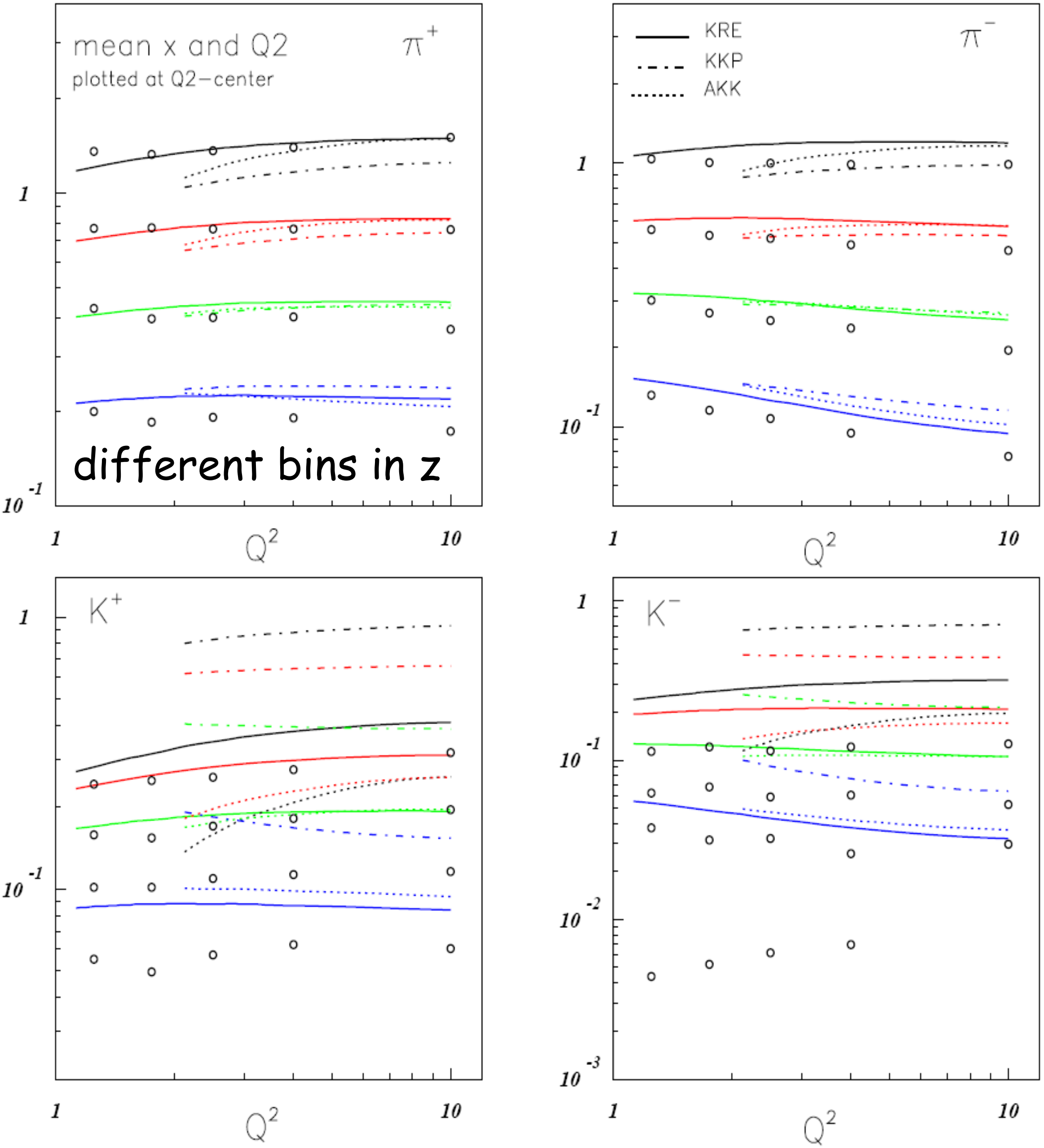
similar results for BKK '95 and AKK '05

lack of flavor separation has profound impact on the extraction of pol. sea pdfs from SIDIS data

$$\frac{d\Delta\sigma^h}{dx\,dQ^2\,dz^h} \propto \sum_{f=q,\bar{q}} e_f^2\,\Delta f(x,\mu_f)\,D_f^h(z^h,\mu'_f) + \mathcal{O}(\alpha_s)$$

de Florian, Navarro, Sassot





multiplicities (HERMES)

- Kretzer *assumes*

  lin. suppression of sea

  $$D_{\bar{u}}^{\pi^+}/D_u^{\pi^+} = (1-z)$$

  works well for π^+, not π^-

  quadr. strangeness suppr.

  $$D_u^{K^+}/D_{\bar{s}}^{K^+} = (1-z)^2$$

  does not work at all

- xKK

  should not be used for charge separated data !!

  needs illegal modification

  e.g. $D_u^{\pi^\pm}/D_u^{\pi^0} = (1 \pm z)$

[data taken from A. Hillenbrand's (HERMES) thesis]

precise knowledge of fragmentation functions also crucial for interpretation of RHIC results

unpolarized pp cross sections are an important baseline for

- studies of saturation effects in dAu and AuAu collisions
- our understanding of A_{LL} and the extraction of, e.g., Δg

RHIC also puts fundamental ideas (factorization, universality) to the test

- apparently pQCD works amazingly well for $pp \rightarrow \pi^0 X$ even at large rapidities η and small p_T
- xKK works better than Kretzer (blamed on too small D_g)

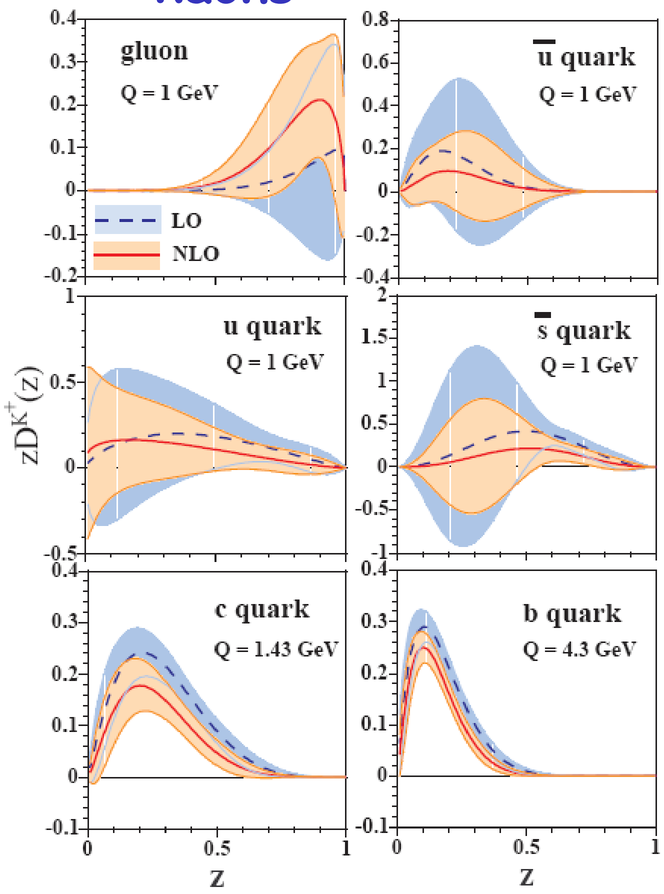
But how sure are we ?

■ recent progress

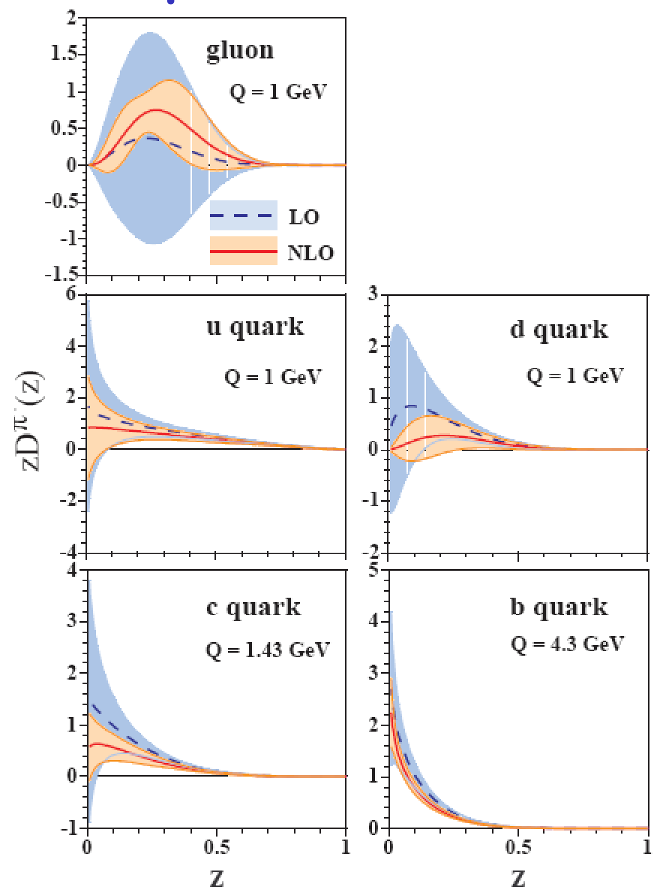
Hirai, Kumano, Nagai, Sudoh
hep-ph/0612009

again only based on e^+e^- data; kaons and pions analyzed

kaons



pions



important step:

- quantifies uncertainty known from comparing xKK with "Kretzer"
- **but** errors extend to unphysical region → neg. pp cross sec.??
- D_g^π looks small → RHIC pp data ??
- $D_g^\pi < D_g^K$ as $z \to 1$??
- *assumes* sym. sea D^π_{sea}

upshot:

we can only make real progress on

fragmentation functions by performing

a global QCD analysis like CTEQ does for pdfs

global QCD analysis

plenty of data from ep and pp – **so, why has it not been done already?**

- NLO expressions for ep and, in particular, for pp are very lengthy & difficult to handle in fits (CPU time)
- many sources of uncertainties: experimental (syst., stat., normalization) & theory (scale variation, bias from chosen ansatz, framework, …)

analysis collaboration formed:

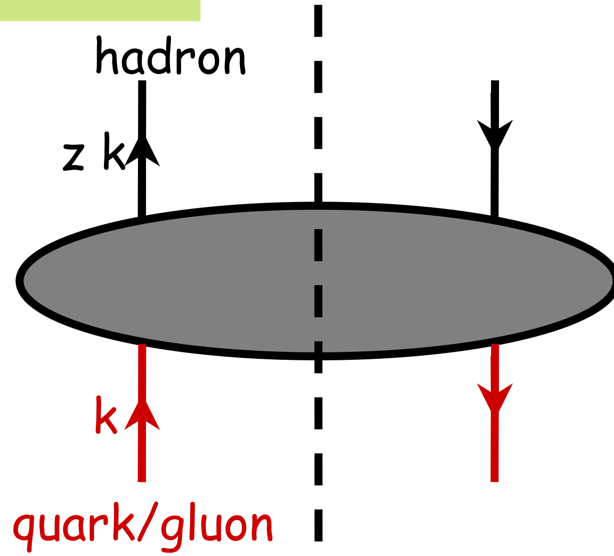
D. de Florian, R. Sassot (Buenos Aires)
MS (RIKEN)

uses technology spin-offs & experience from global pdf analyses:

- "Mellin technique" to handle exact NLO expressions in fits
- "Lagrange multiplier" technique to estimate uncertainties
- fast & well-tested DGLAP evolution codes
- vast array of NLO calculations for ep and pp at hand

■ theory: properties of $D(z,\mu)$

$D_i^h(z,\mu)$



- non-perturbative ***universal*** objects to be extracted from data (like pdfs); scale μ-dep. predicted by pQCD
- describe the collinear transition of a massless parton "i" into a massless hadron "h" carrying fractional momentum z
- fragmentation independent of other colored particles
- needed to consistently absorb final-state collinear singularities like, e.g., in $pp \rightarrow \pi^0 X$ ("***factorization***")

these properties are very much at variance with hadronization models in PYTHIA et al.

(colored “strings”, “soft physics”, non-collinear)

pQCD framework not applicable to all processes at least a “hard scale” is required

theory: constraints & limitations

- bi-local operator: $D(z) \simeq \int dy^- e^{iP^+/z y^-} \mathrm{Tr}\gamma^+ \langle 0|\,\psi(y^-)\,|hX\rangle\,\langle hX|\,\bar{\psi}(0)\,|0\rangle$

  Collins, Soper '81, '83

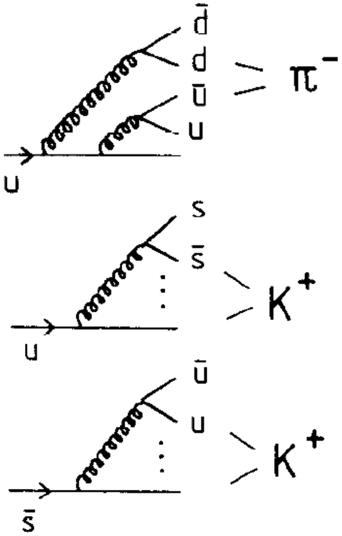
  no inclusive final-state
  $\rightarrow$ no local OPE $\rightarrow$ no lattice formulation

- isospin & charge conjugation invariance:

  expect, e.g., $D_u^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+}$

- "valence enhancement", e.g., $D_u^{\pi^+} > D_d^{\pi^+} \simeq D_s^{\pi^+}$

- "strangeness suppression", e.g., $D_{\bar{s}}^{K^+} > D_u^{K^+}$



basis for "leading particle"/"flavor tag" assumptions:

highest momentum particle in jet resembles flavor of parent quark

- **but: "leading particle" picture incompatible with definition of $D_i^h(z)$**

> can compute *incl. distributions* of hadrons with momentum fractions z
> but *not* a cross section for a "leading hadron"

(under certain kin. conditions it might be a good approximation though)

- "energy-momentum conservation": $\sum_h \int_0^1 z D_i^h(z,\mu) = 1$

(a parton fragments with 100% probability into *something* preserving its momentum)

of very limited practical use in fits because

- "mass effects" completely spoil framework for $D_i^h(z)$ for $z \lesssim 0.05 \div 0.1$

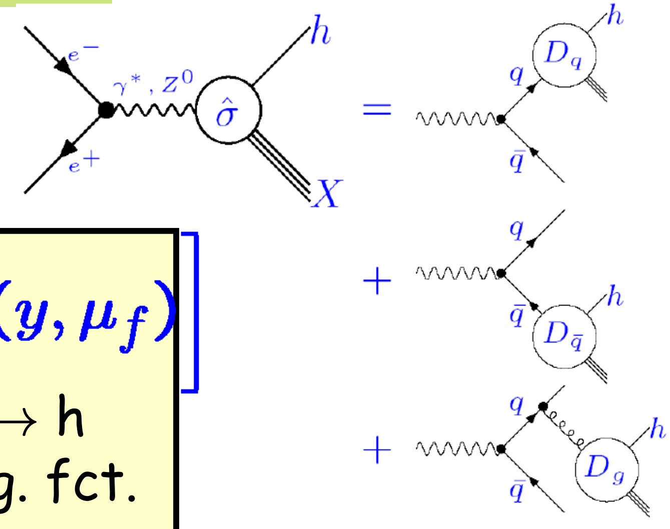
also, μ-evolution "goes crazy" as $z \to 0$ (soft gluons)

no systematic way to include mass/higher twist effects

rough measure: O.K. as long as $\frac{p_h}{E_h} = \sqrt{1 - \frac{m_h^2}{E_h^2}} \simeq 1$

■ e^+e^- annihilation: theory

relevant: "normalized distribution" $\dfrac{1}{\sigma_{tot}}\dfrac{d\sigma^h}{dz}$



$$= \frac{1}{\sigma_{tot}} \sum_{i=q,\bar{q},g} \left[\int_z^1 \frac{dy}{y}\, C_i\!\left(\frac{z}{y}, Q, \mu_r, \mu_f\right) D_i^h(y, \mu_f) \right]$$

$C_i(\frac{z}{y}, Q, \mu_r, \mu_f)$	$D_i^h(y, \mu_f)$
"coeff. fct." calculable in pQCD	$i \to h$ frag. fct.

↓ total hadronic cross section

$$\sum_q \frac{4\pi\alpha^2}{s}\hat{e}_q^2 \left(1 + \frac{\alpha_s}{\pi} + \ldots\right)$$

$\equiv \sigma_0$

LO: $C_q = \delta(1-y)\,\sigma_0$; $C_g = 0$

$O(\alpha_s)$ NLO: Altarelli, Ellis, Martinelli, Pi '79; Furmanski, Petronzio '82

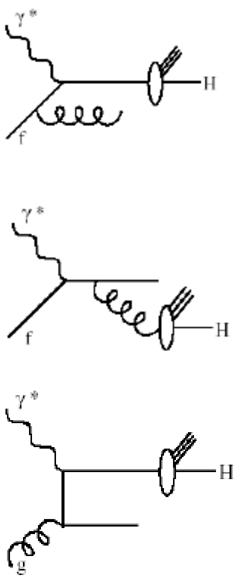
$O(\alpha_s^2)$ NNLO: Rijken, van Neerven '96,'97

"scaling" variable $z \equiv \dfrac{2P^h \cdot q}{Q^2} = \dfrac{2E^h}{Q}$

where

$$s = q^2 = Q^2$$
$$P_{e\pm} = (Q/2, 0, 0, \pm Q/2)$$
$$q = P_{e+} + P_{e-}$$

semi-inclusive DIS: theory



SIDIS = DIS plus one identified hadron with $x_F > 0$

$$\frac{d\sigma^h}{dx dQ^2 dz^h} \approx \sum_{f=q,\bar{q}} e_f^2 f(x,\mu_f) D_f^h(z^h,\mu'_f) + \mathcal{O}(\alpha_s)$$

"scaling" variable $z^h \equiv \frac{P^h \cdot P^N}{P^N \cdot q}$

Altarelli et al. '79;
Furmanski, Petronzio '82;
de Florian, MS, Vogelsang '98

why important?

- charge separated data for π^+, π^-, K^+, K^- from HERMES

  → **valuable handle on flavor separation!!**

  LO analysis: $D_d^{\pi^+} \simeq (1-z) D_u^{\pi^+}$ Christova, Kretzer, Leader

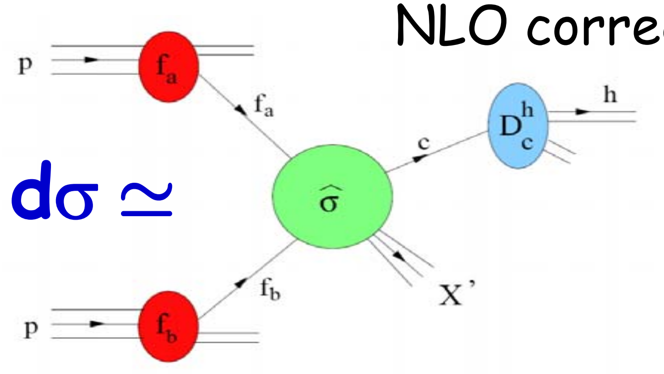
■ pp → hX: theory

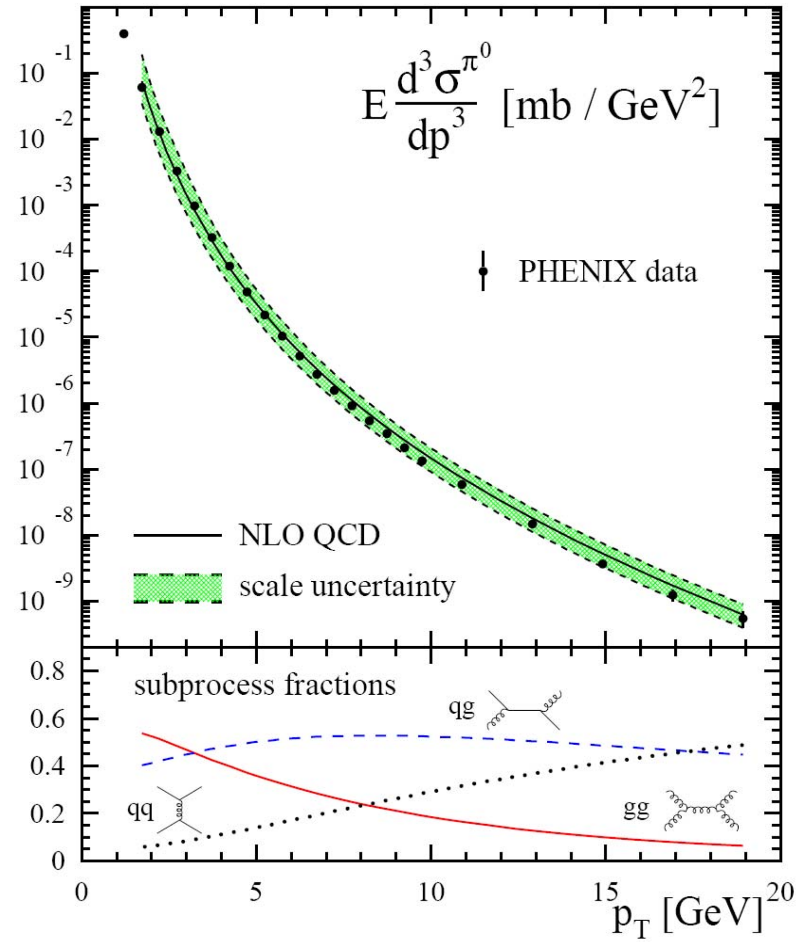
Aversa et al.; Jäger,Schäfer, MS, Vogelsang; de Florian

well-know framework: factorization, NLO corrections

nice data from RHIC:

$d\sigma \simeq$





long-distance

from exp.; μ-dep.: $d\sigma/d\mu = 0$ (pQCD)

$$\frac{d\Delta\sigma^{\vec{p}\vec{p}\to\pi X}}{dp_T d\eta} = \sum_{abc} \int dx_a\, dx_b\, dz_c\, \Delta f_a(x_a,\mu_f)\, \Delta f_b(x_b,\mu_f)\, D_c^\pi(z_c,\mu'_f) \times \frac{d\Delta\hat{\sigma}^{ab\to cX'}}{dp_T d\eta}(x_a P_a, x_b P_b, P^\pi/z_c, \mu_f, \mu'_f, \mu_r) + \mathcal{O}\left(\frac{\lambda}{p_T}\right)^n$$

short-distance

calculable in pQCD: power series in α_s

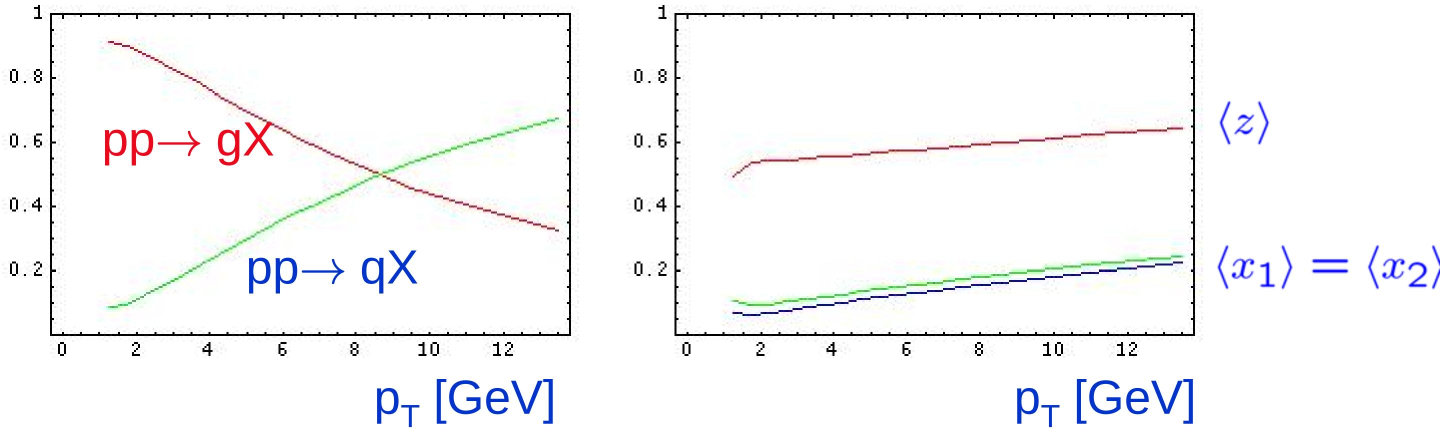
power corrections

neglected

plus STAR and BRAHMS

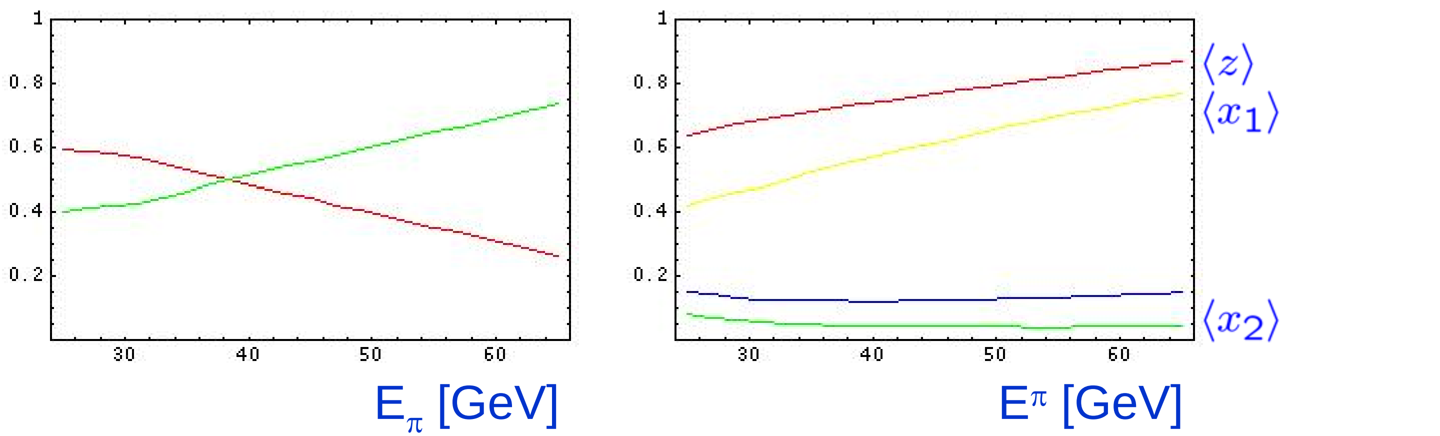
■ pp → hX data: why important?

central rapidity (PHENIX data)



→ low p_T data probe **gluon fragmentation**

forward rapidity (STAR data)



→ probe **gluon and quark fragmentation at large z**

BRAHMS $\pi^\pm$, $K^\pm$ data ($\eta \simeq 3$) → **flavor separation from pp data !!**

global analysis – setup & goals

goal: provide NLO (and LO) D_i^h for $\pi^\pm$ ✓, $K^\pm$ ✓, p, η, Λ, ... in progress

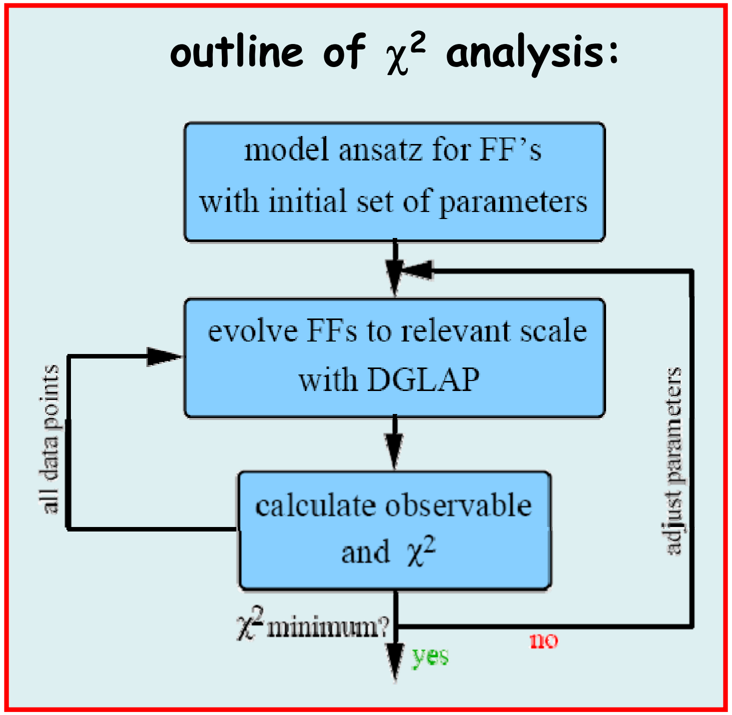
and estimate their uncertainties

analysis method:

- input scale for evolution: μ = 1 GeV
- $D_i(z,\mu) = N z^\alpha (1-z)^\beta [1 + \gamma (1-z)^\delta]$
  for i = u,d,s; i = g, c, b w/o [...],
  allowing for small isospin violations

Σ: 23 parameters

- 7 parameters for exp. normalizations
- standard χ^2 minimization (MINUIT);
  "Lagrange multiplier technique"
  for studies of uncertainties



comments on data selection

included: all LEP, SLD, TASSO, TPC e^+e^- data w/o "flavor tagging"
[other (= older e^+e^-) data have no impact on the fit]

HERMES SIDIS ep data

BRAHMS, PHENIX, and STAR pp data

reluctantly included: e^+e^- "flavor tagged" data

only constraint on c, b $\rightarrow$ light hadrons

many conceptual problems: strong model dependence;
leading hadron assumptions;
possible contamination from weak decays; …

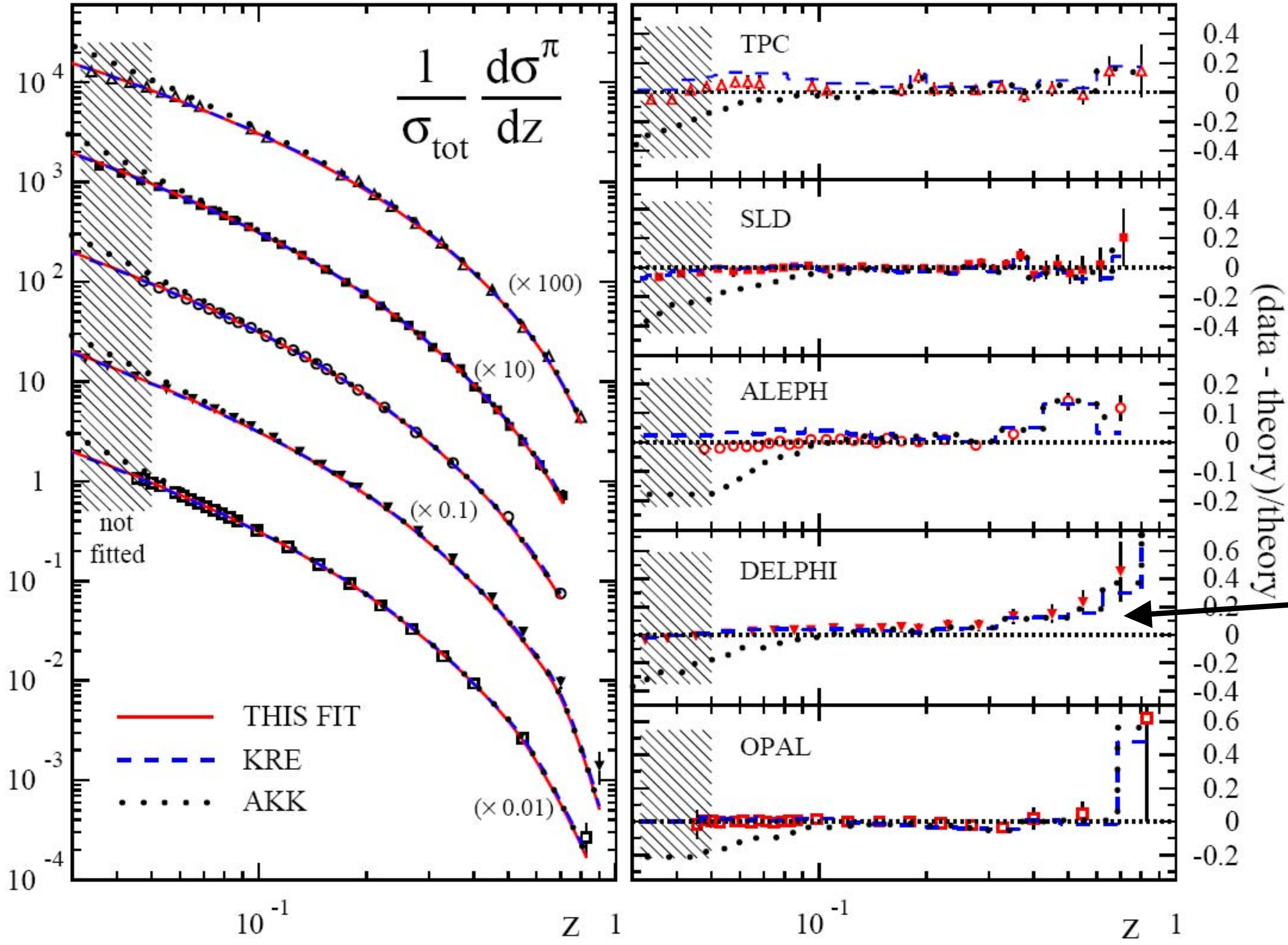
excluded: e^+e^- unidentified charged hadron data (work in progress!)

ep photoproduction from H1, ZEUS
[large uncertainties from photon structure;
but known to be consistent with e^+e^- data (xKK)]

UA1 pp data for kaons [inconsistent with STAR]

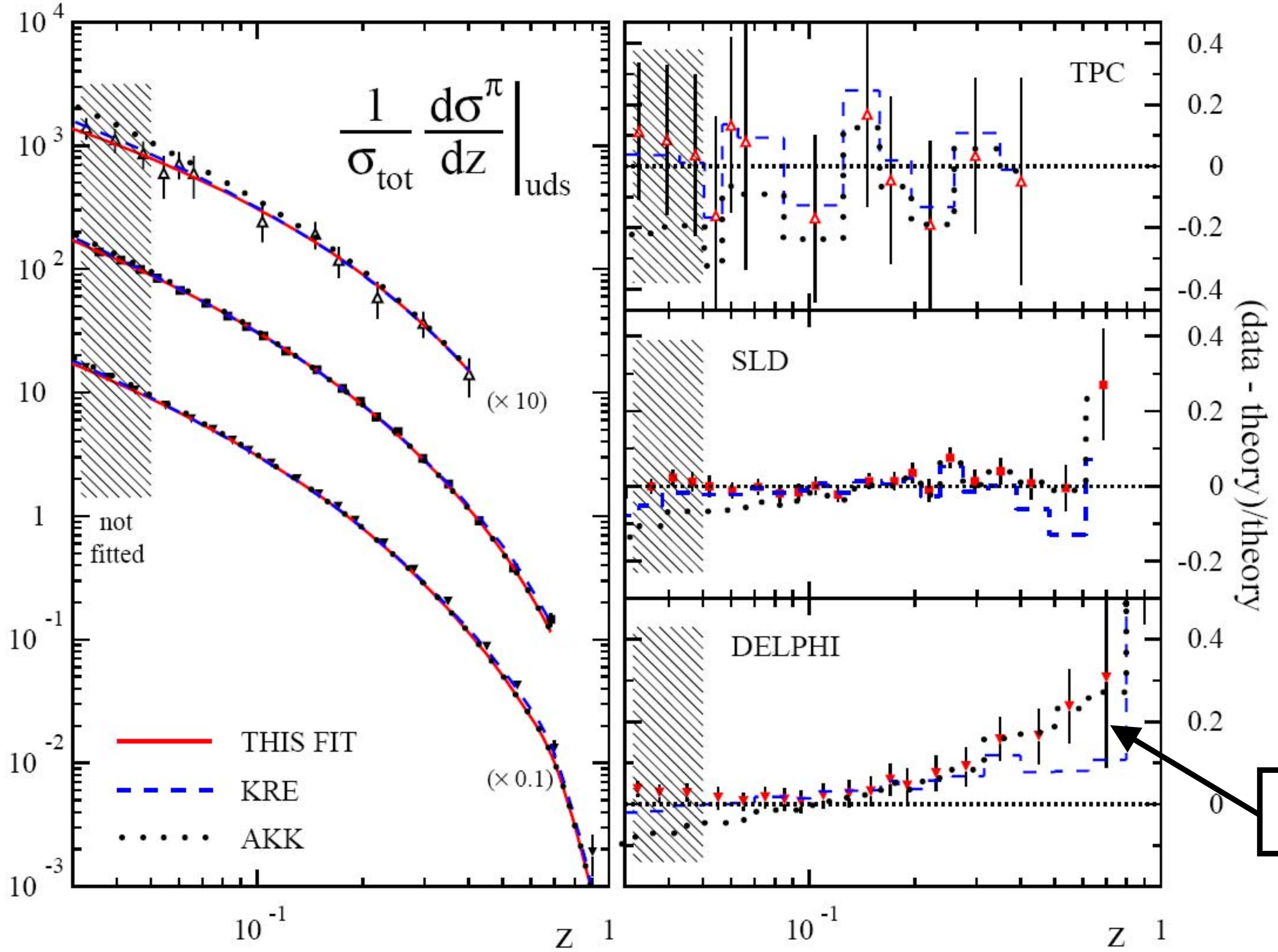
global analysis: results e^+e^- ("untagged")

excellent description of all data for $e^+e^- \to (\pi^+ + \pi^-)\, X$ for $z \geq 0.05$



some tension for large-z DELPHI data

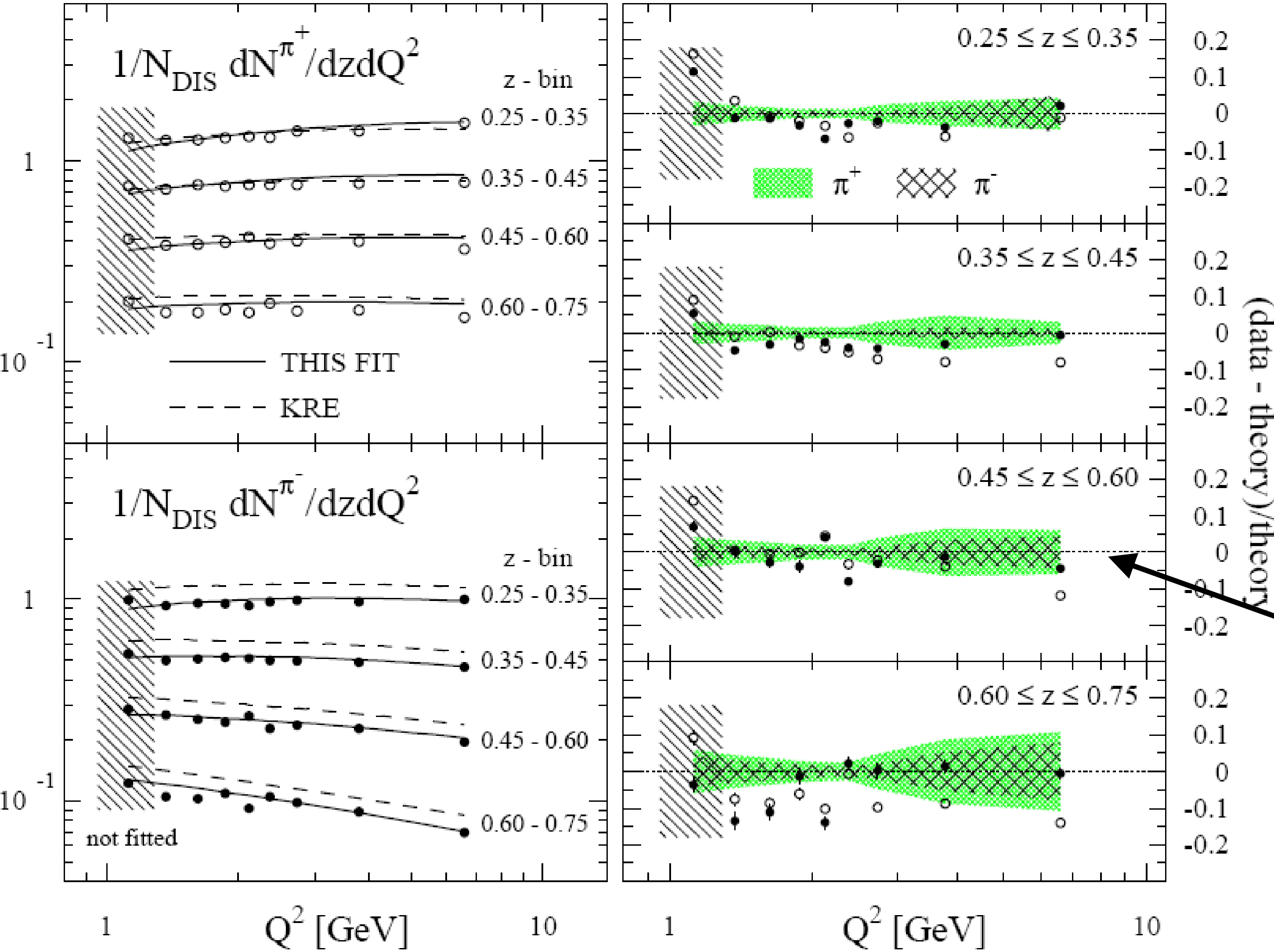
despite conceptual difficulties, also “uds-flavor-tagged” data work well



DELPHI again …

… similar for charm and bottom-tagged data

global analysis: results ep semi-incl. DIS



nice description of HERMES $\pi^\pm$ multiplicities

shaded bands: our estimate of "Q^2-bin effects"

recall: xKK of no use for $\pi^\pm$ data

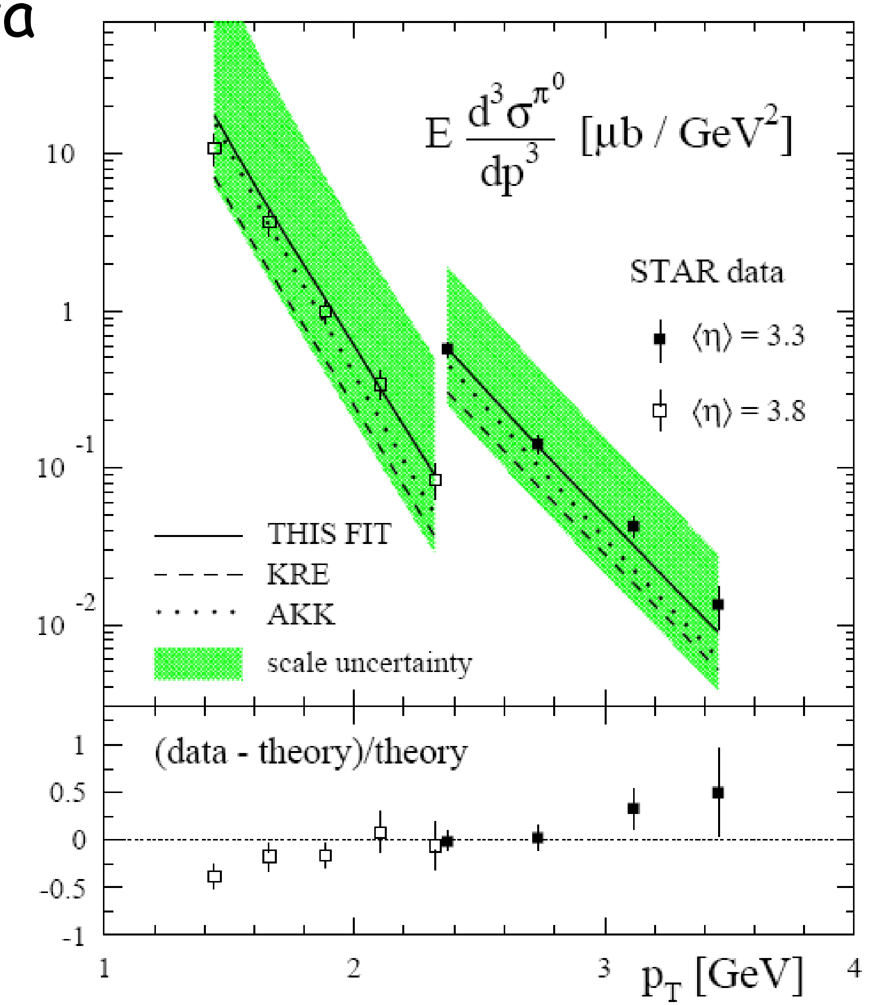
results $pp \to \pi^{0,\pm} X$

example:
STAR forward pions

good description of all RHIC data with scale choice $\mu = p_T$

scale uncertainties sizable

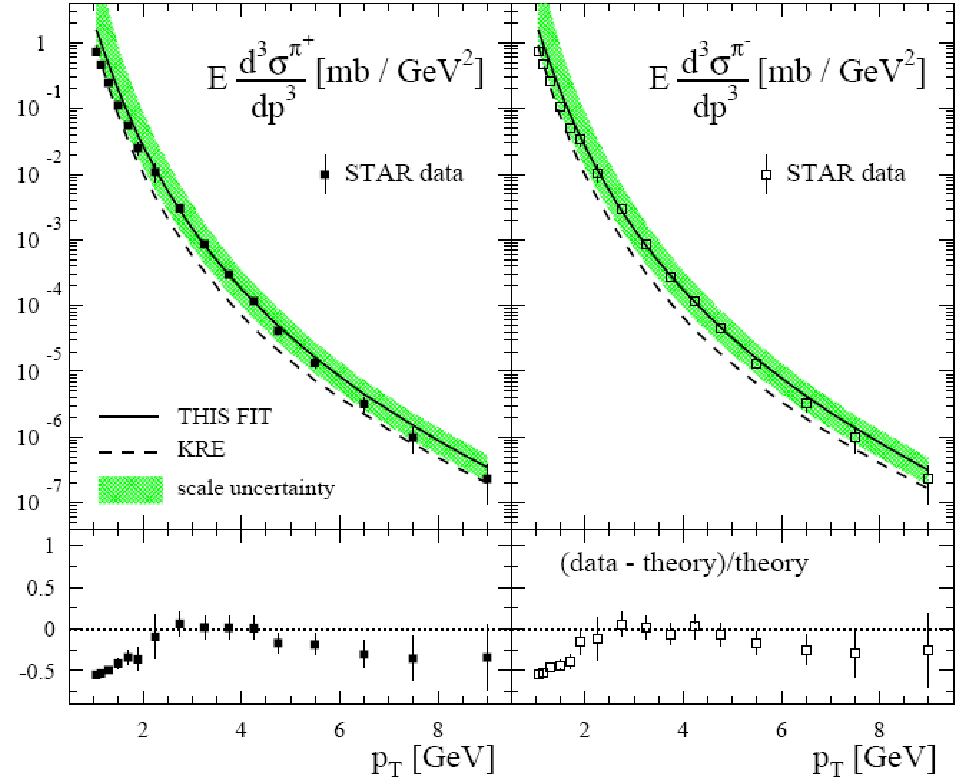
"Kretzer" always much too low for $\mu = p_T$

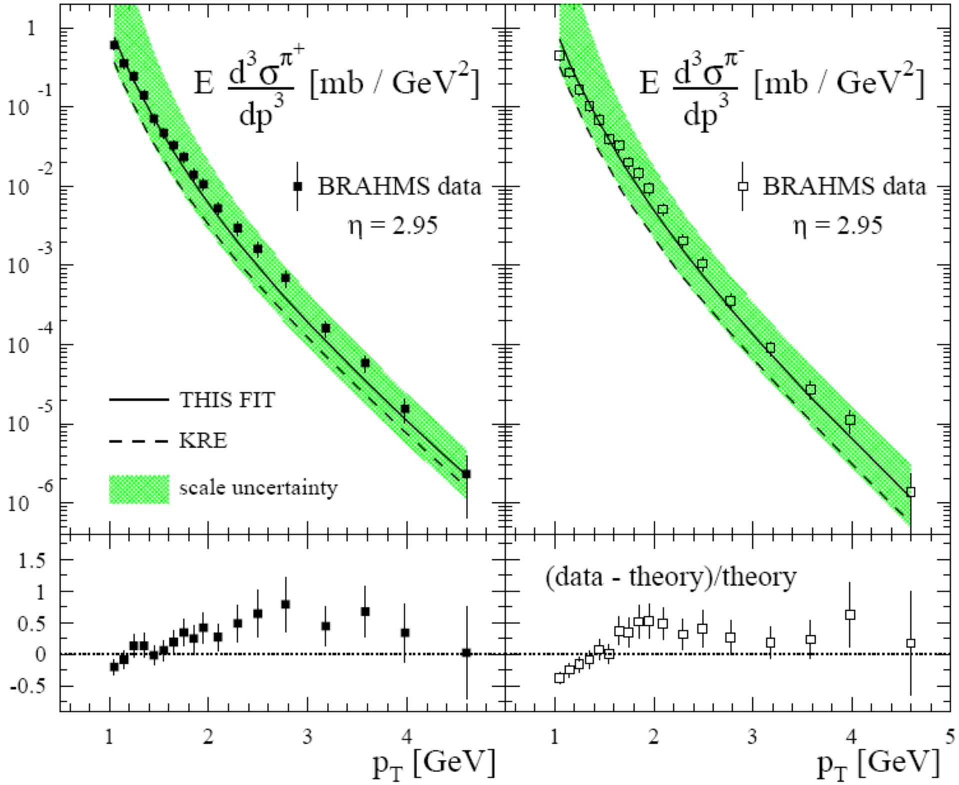


charge separated data

STAR

(upps, we missed them in the fit)





BRAHMS

uncertainties: method

basically two methods explored for pdf analyses:

pioneering work by CTEQ collaboration

- **"Hessian method"** $$[\delta\Delta f]^2 = \Delta\chi^2 \sum_{ij} \frac{\partial \Delta f}{\partial a_i} H_{ij}^{-1} \frac{\partial \Delta f}{\partial a_j}$$

  assumes that χ^2-profile is quadratic in all parameters a_i;
  uncertainties propagate linearly to observables

- **"Lagrange multiplier method"** $$\Phi(\lambda_i, a_j) = \chi^2(a_j) + \sum_i \lambda_i O_i(a_j)$$

  probes uncertainties in observables O_i *directly*, straightforward to implement

  steps: minimize for $\lambda_i = 0 \to$ "best fit" with χ_0^2, parameters $a_j^0, O_i(a_j^0)$

  explore χ^2-profile for various fixed $\lambda_i \neq 0$ (=force values of O_i)

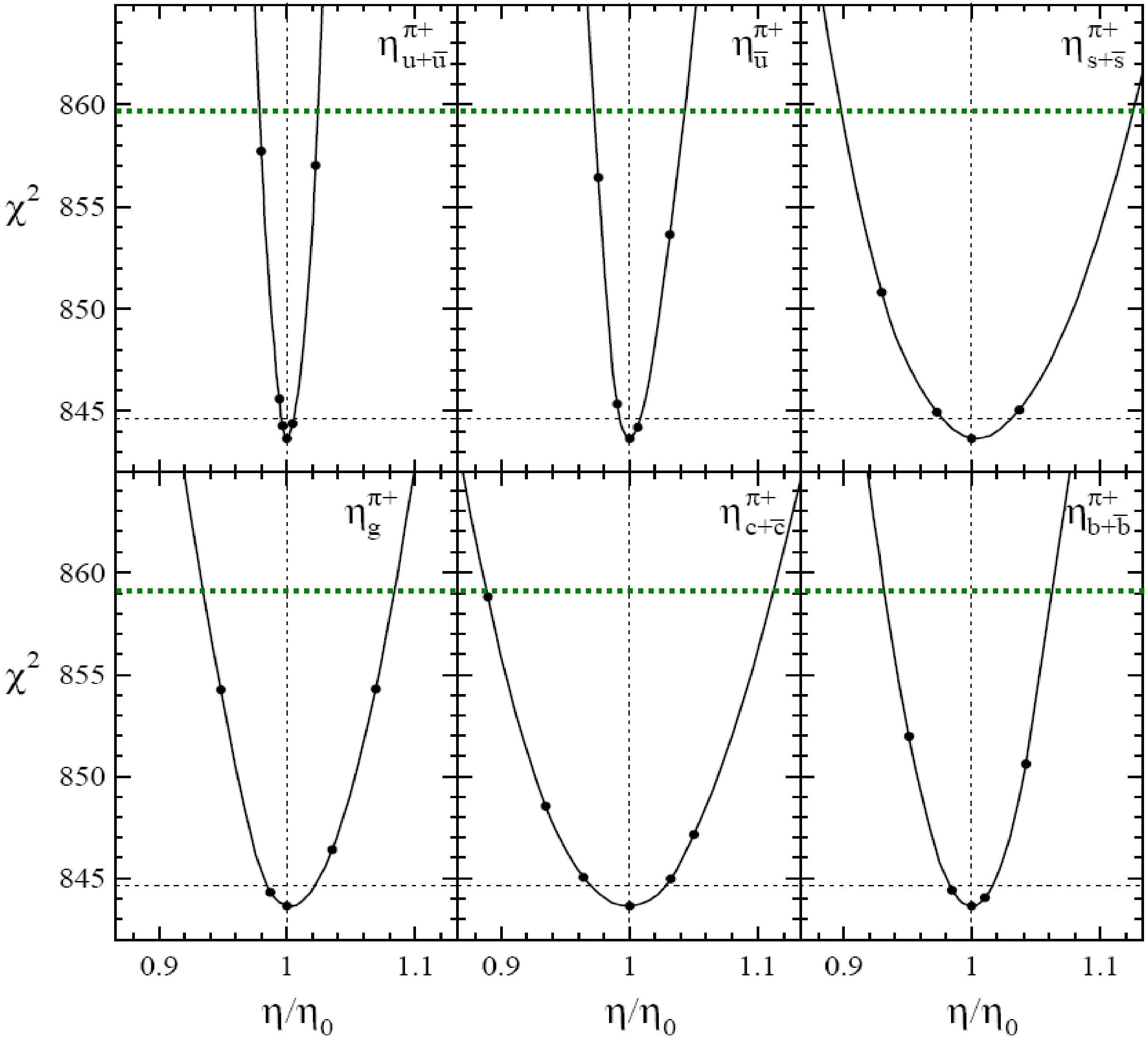
  ideal case: parabolic profiles, $\Delta\chi^2=1$ for 1σ errors

  in practice: unaccounted errors, $\Delta\chi^2 = 1 - 2\%$ more appropriate

our choice: use, e.g., $\eta_i = \int_{0.2}^{1} z\, D_i(z)\, dz$ as "observables"

uncertainties: χ^2-profiles

$\eta_i = \int_{0.2}^{1} z\, D_i(z)\, dz$
$Q = 5$ GeV



overall:
almost parabolic χ^2 - profiles

assuming $\Delta\chi^2 \approx$ **15**

$u_{tot} \to \pi^+$ constrained best (3% variation)

$u_{sea} \to \pi^+$: $\approx$ 5%

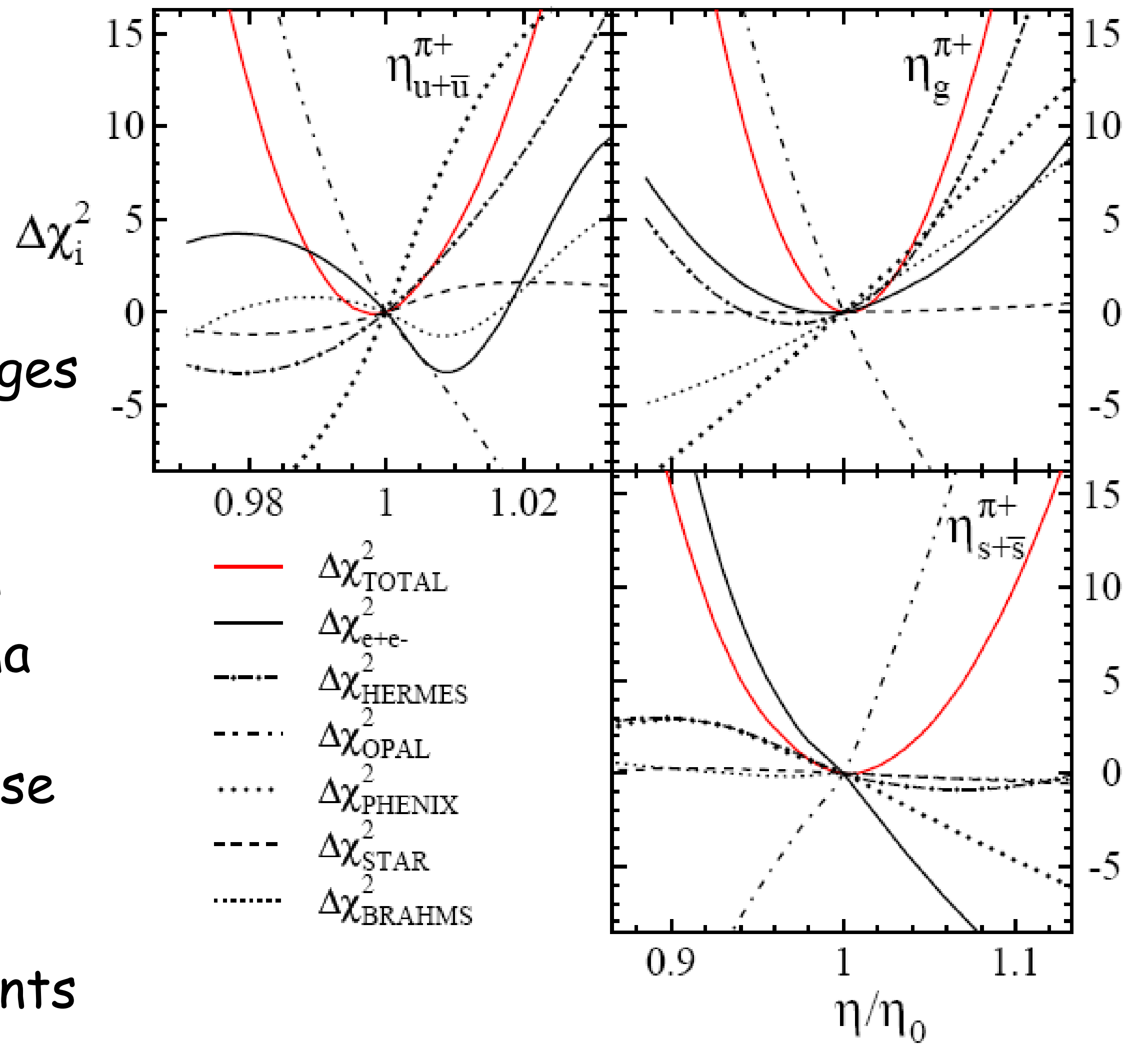
$s \to \pi^+$: $\approx$ 10%

$g \to \pi^+$: $\approx$ 8%

role of different data sets

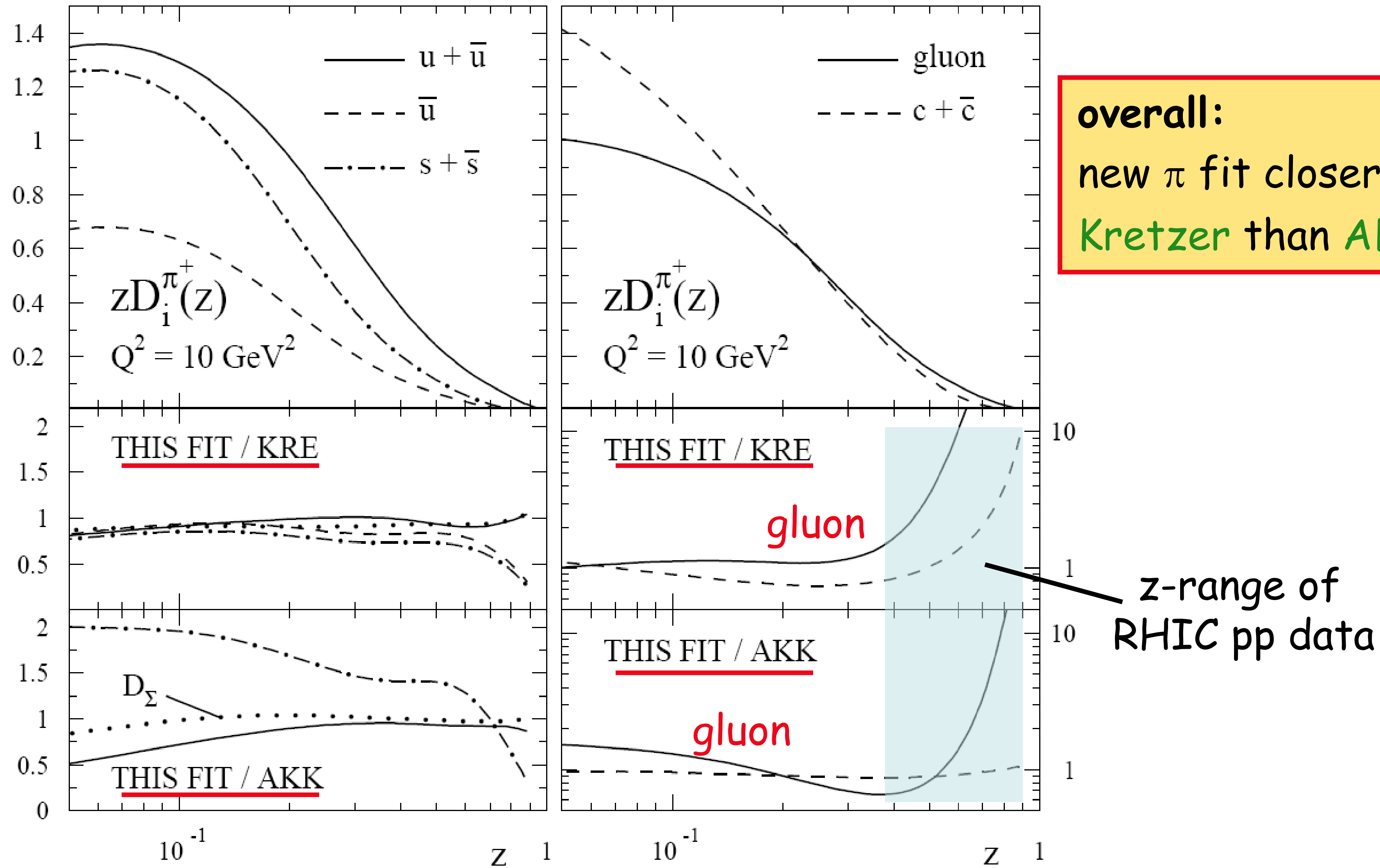
$\eta_i = \int_{0.2}^{1} z\, D_i(z)\, dz$
$Q = 5$ GeV

study partial contributions to χ^2 from different data sets:



- data sets probe diff. z-ranges and aspects of $D_i(z)$
- complementary information leads to well defined minima
- result is always a compromise
- e^+e^- data alone would lead to different or no constraints

comparison with KRE & AKK



overall:
new π fit closer to
Kretzer than AKK

final remarks: LO & kaon fits

main results of global analysis of kaon data:

- can achieve a good global description of all e^+e^-, ep, and pp data
- quality of available data not as good as for pions
- STAR and old UA1 pp data cannot be described both (we use STAR)
- **overall:** uncertainties are considerably larger; χ^2 profiles less parabolic

main results of LO fits:

- quality of fit much worse: 25 ÷ 30% increase in χ^2 w.r.t. NLO
- quarks fairly stable; LO D_g larger (make up for large pp K-factors)
- LO sets should be only used for rough estimates

summary & outlook

- 1^{st} global analysis of fragmentation function
  - good description of e^+e^-, ep, and pp data
  - pion fragmentation well determined; kaons less
  - new data sets can be straightforwardly included
  - important stress test of Mellin technique for global anal.
- outlook
  - additional sets (protons, chg. hadrons, …) in preparation
  - more detailed studies of uncertainties