Calculations of T-odd distribution and fragmentation functions

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Outline

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- T-odd distribution functions
  - general ideas
  - model results
  - conclusions

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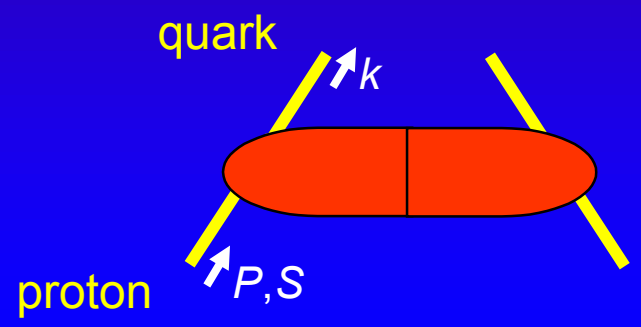
Why models?

- Make predictions
- Check against observed effects
- Discard wrong ideas
- Start again...

T-odd distribution functions

Decomposition of the correlation function



$$\Phi(x,k_T) \simeq \frac{1}{2}\left(f_1 + \frac{S_T \cdot k_T}{M} g_{1T}\gamma_5 + h_1 \gamma_5 \not{S}_T + \right.$$

$$\left. + f_{1T}^{\perp} \frac{\varepsilon_T^{\rho\sigma} S_{T\rho} k_{T\sigma}}{M} + i\, h_1^{\perp} \frac{\not{k}_T}{M} \right) \gamma^-$$

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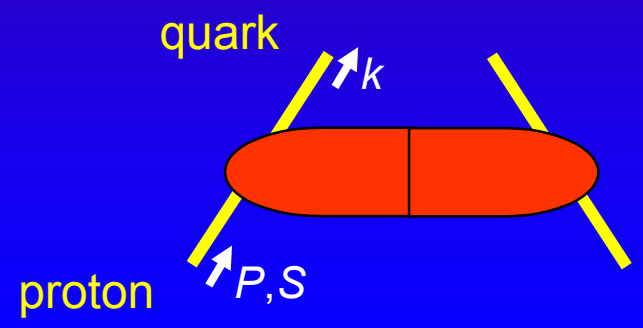

Leading twist only,
transverse spin only

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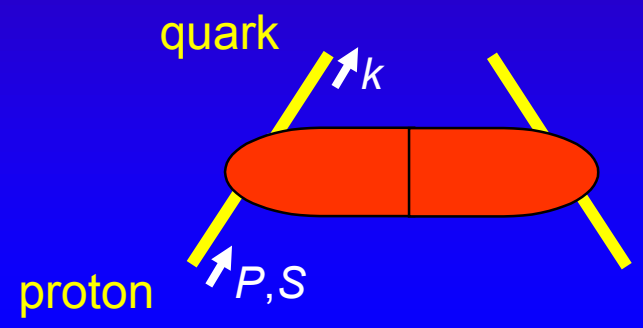

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Sivers

Boer-Mulders

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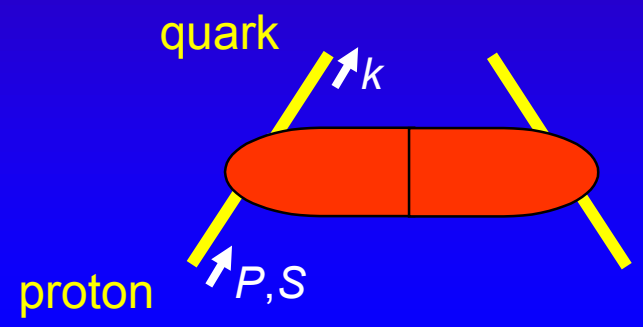

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Sivers

Boer-Mulders

Sivers, PRD 43 (91)

Boer, Mulders, PRD57 (98)

T-odd structures

$$\Phi^*_{\text{T-odd}}(p,P,S) = -i\,\gamma^1\gamma^3\,\Phi_{\text{T-odd}}(\tilde{p},\tilde{P},\tilde{S})\,i\gamma^1\gamma^3$$

where $\tilde{p}^\mu = \delta^{\mu\nu} p_\mu$

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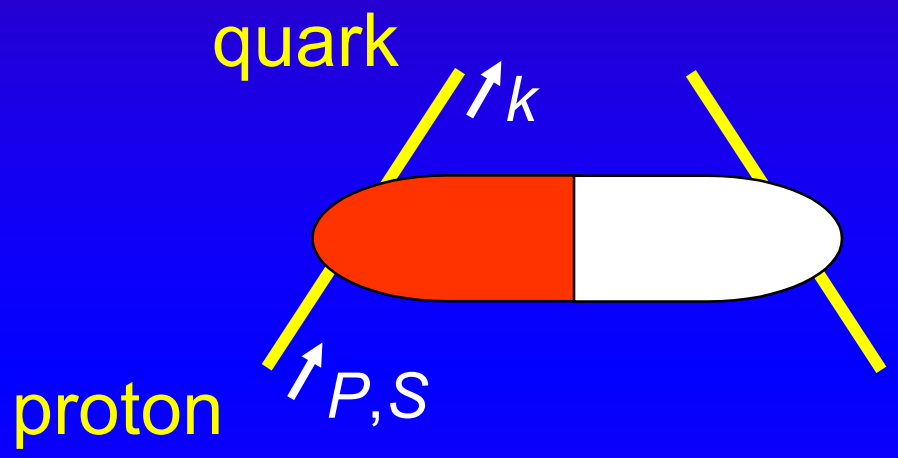
For instance...

$$\left(f_{1T}^{\perp}\,\frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M}\,\gamma^- \right)^* = -i\,\gamma^1\gamma^3 \left(f_{1T}^{\perp}\,\frac{-\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M}\,\gamma^- \right) i\gamma^1\gamma^3$$

Sivers function

The role of the imaginary

$$\Phi \sim M\bar{M}$$



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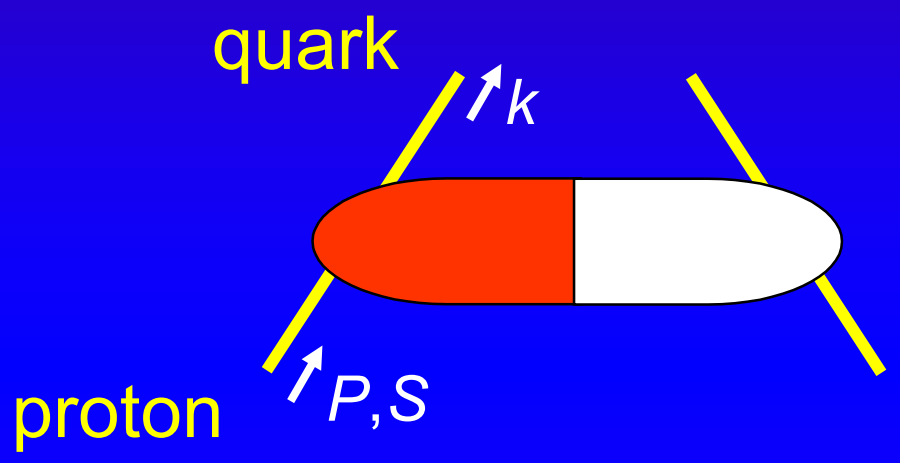
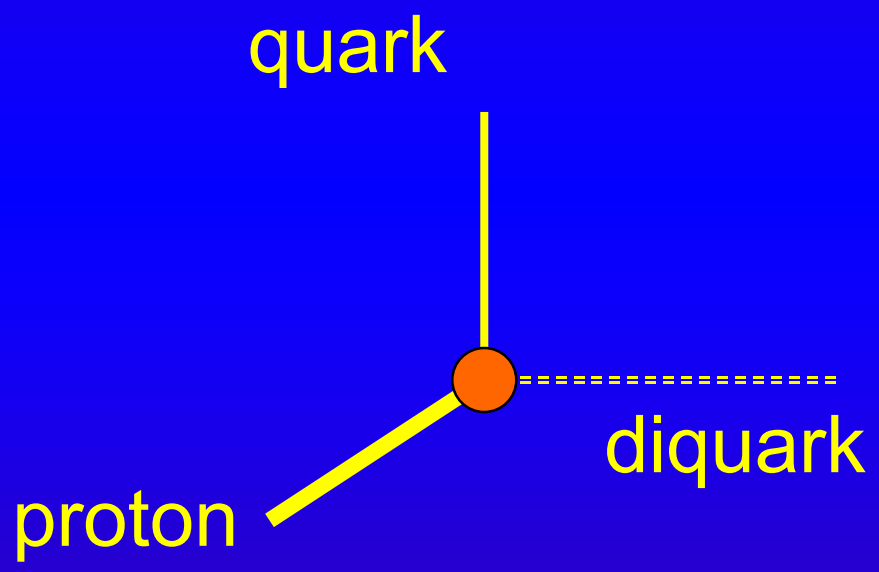


To have T-odd functions we need the interference between different amplitudes with different imaginary parts

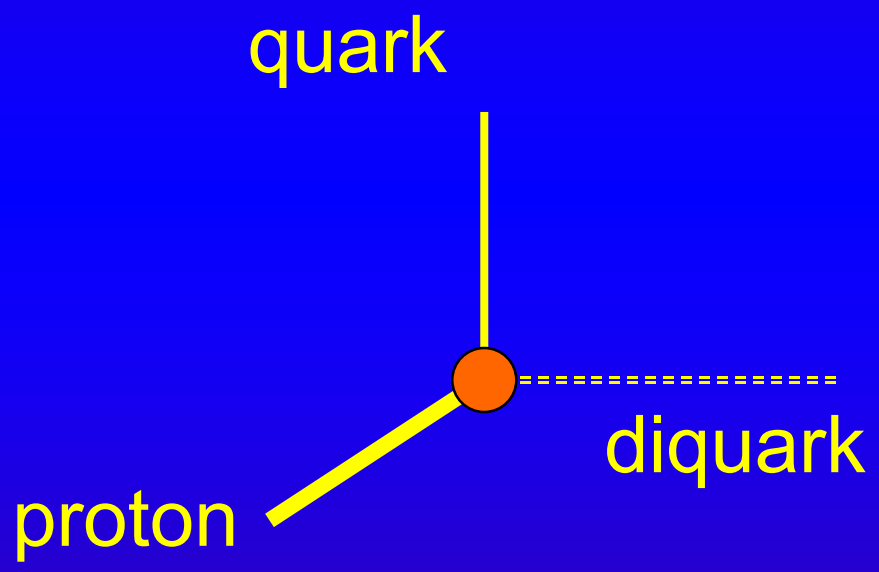
T-odd functions in diquark model



Brodsky, Hwang, Schmidt PLB530 (02)

T-odd functions in diquark model

Tree-level amplitude



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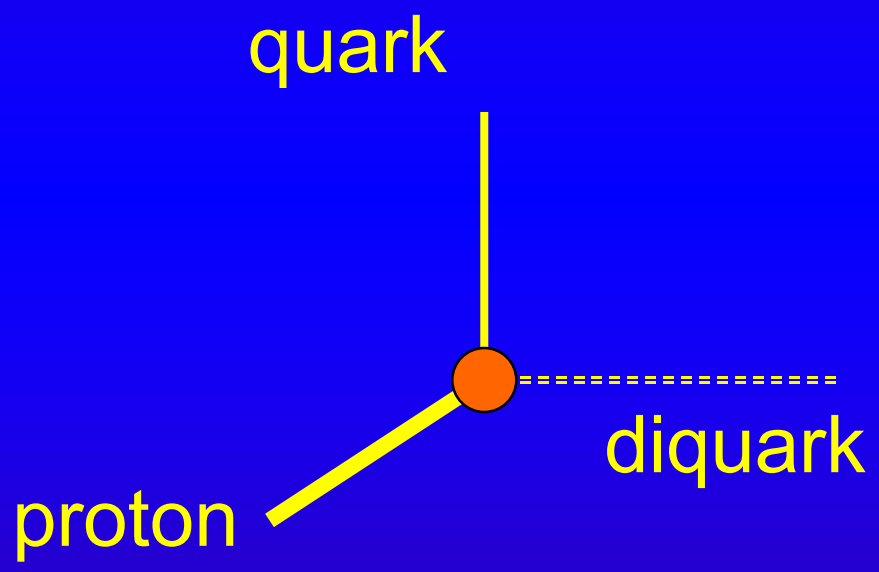

No imaginary parts.

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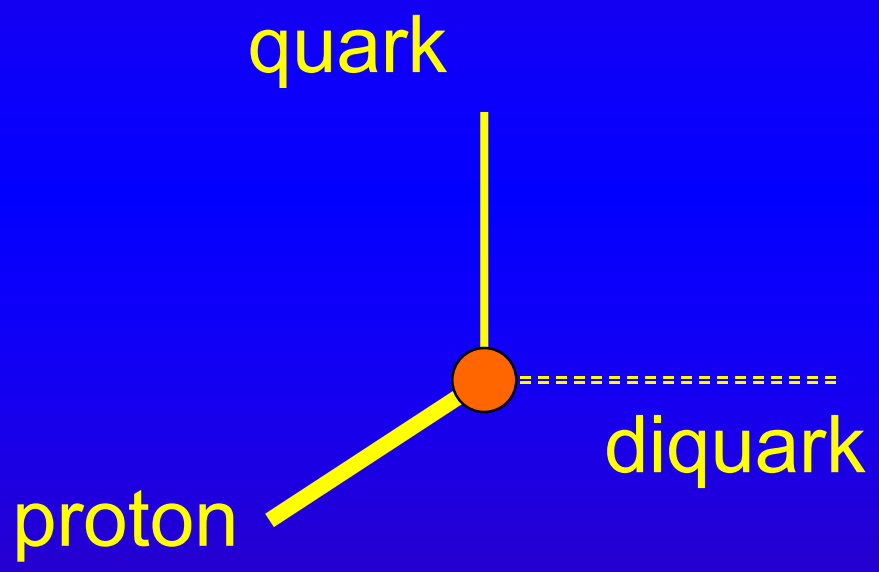

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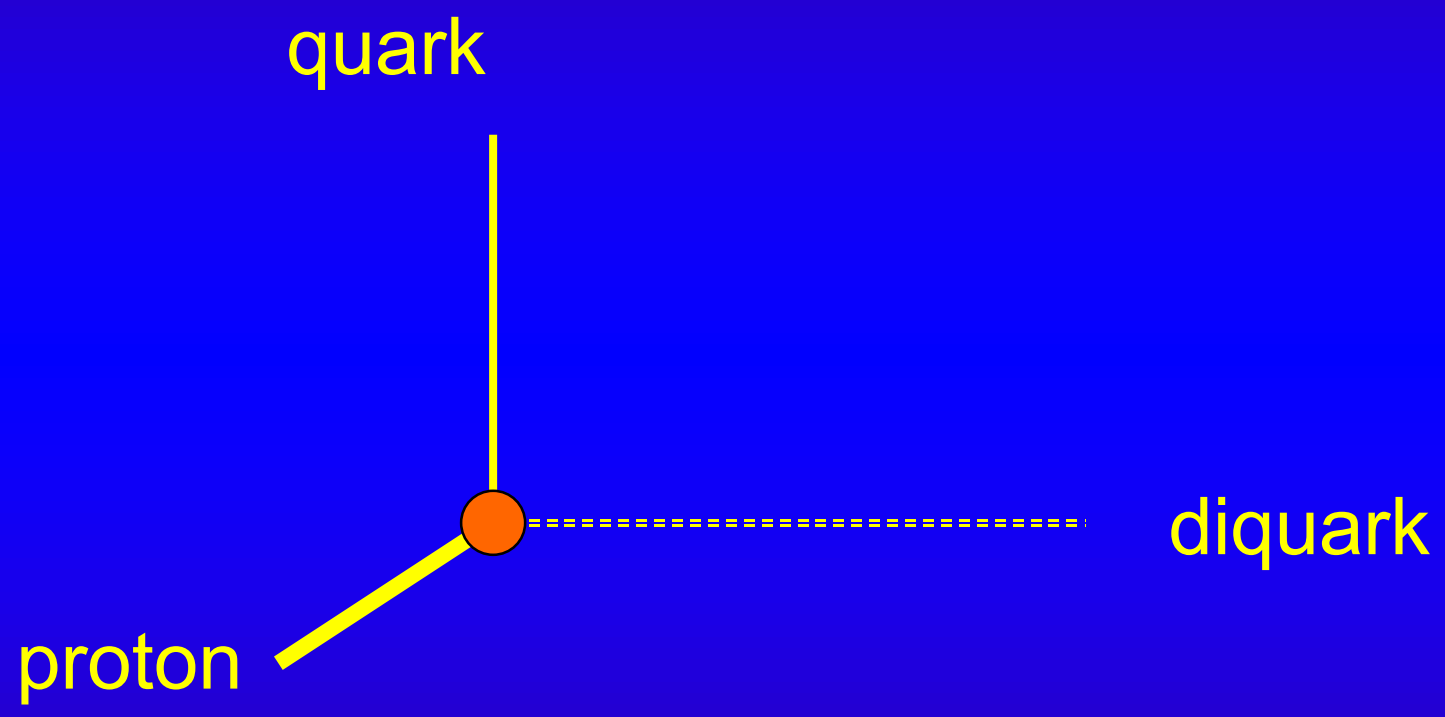
Single gluon loop corrections



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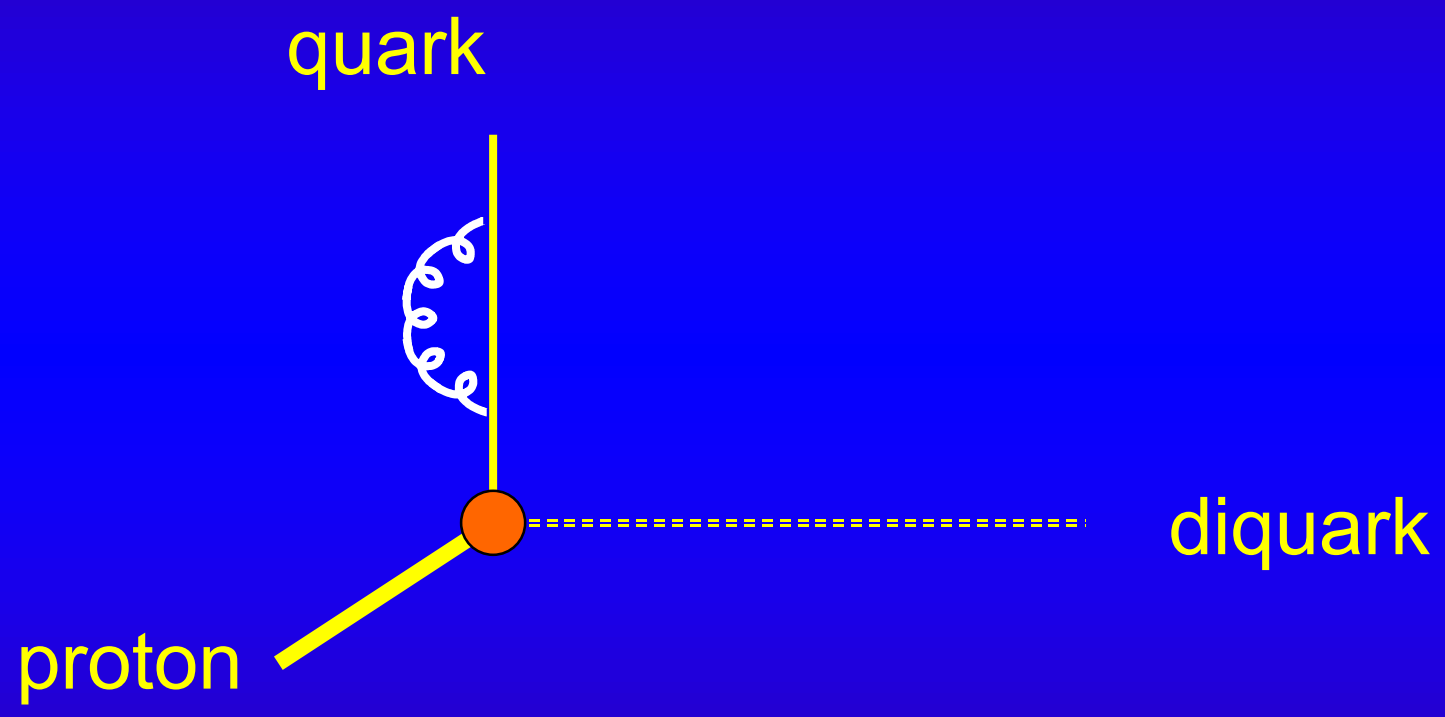

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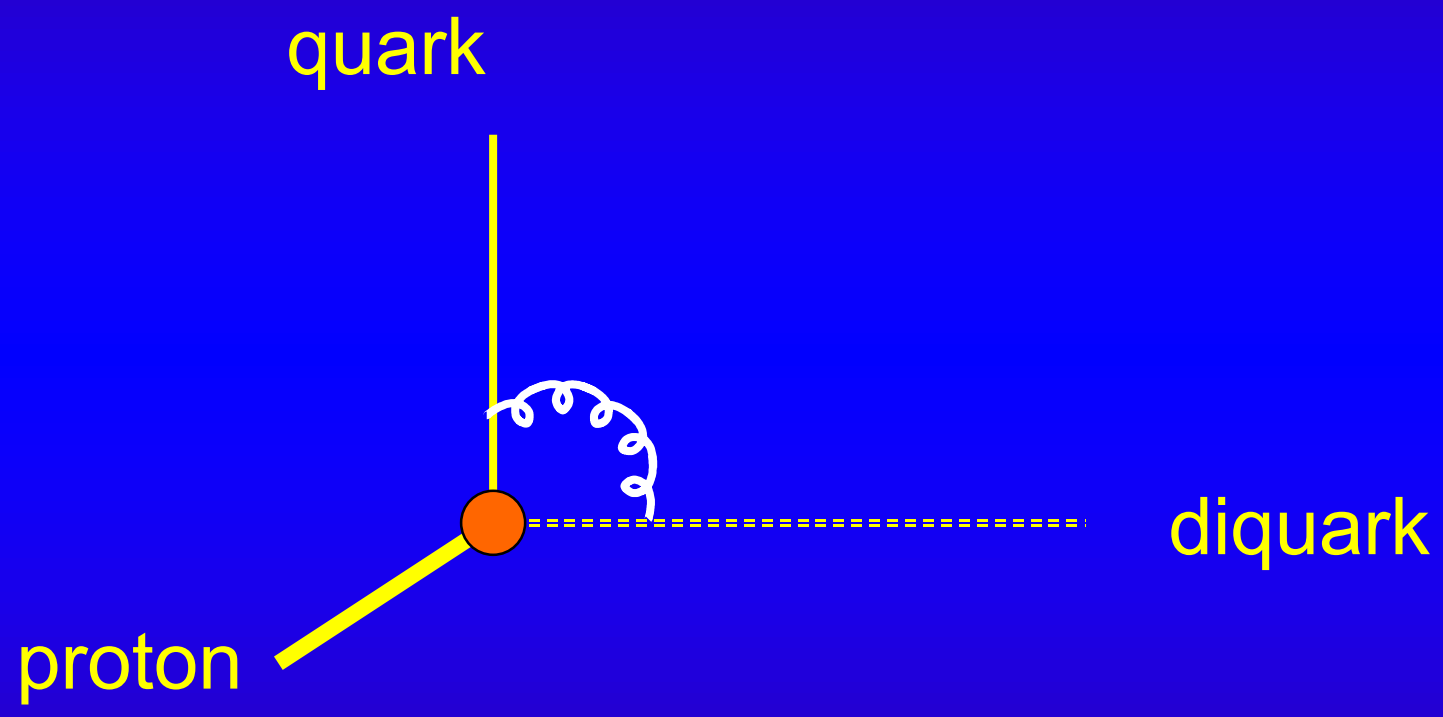

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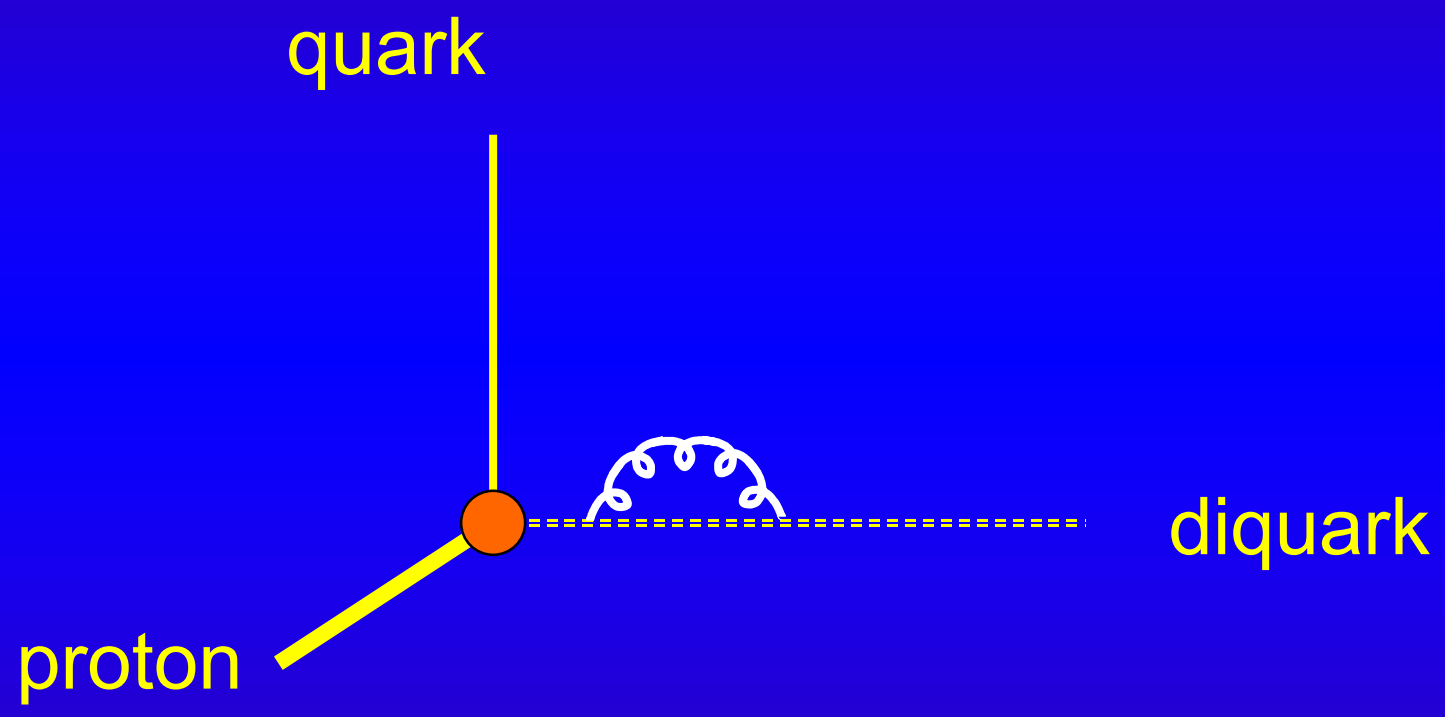

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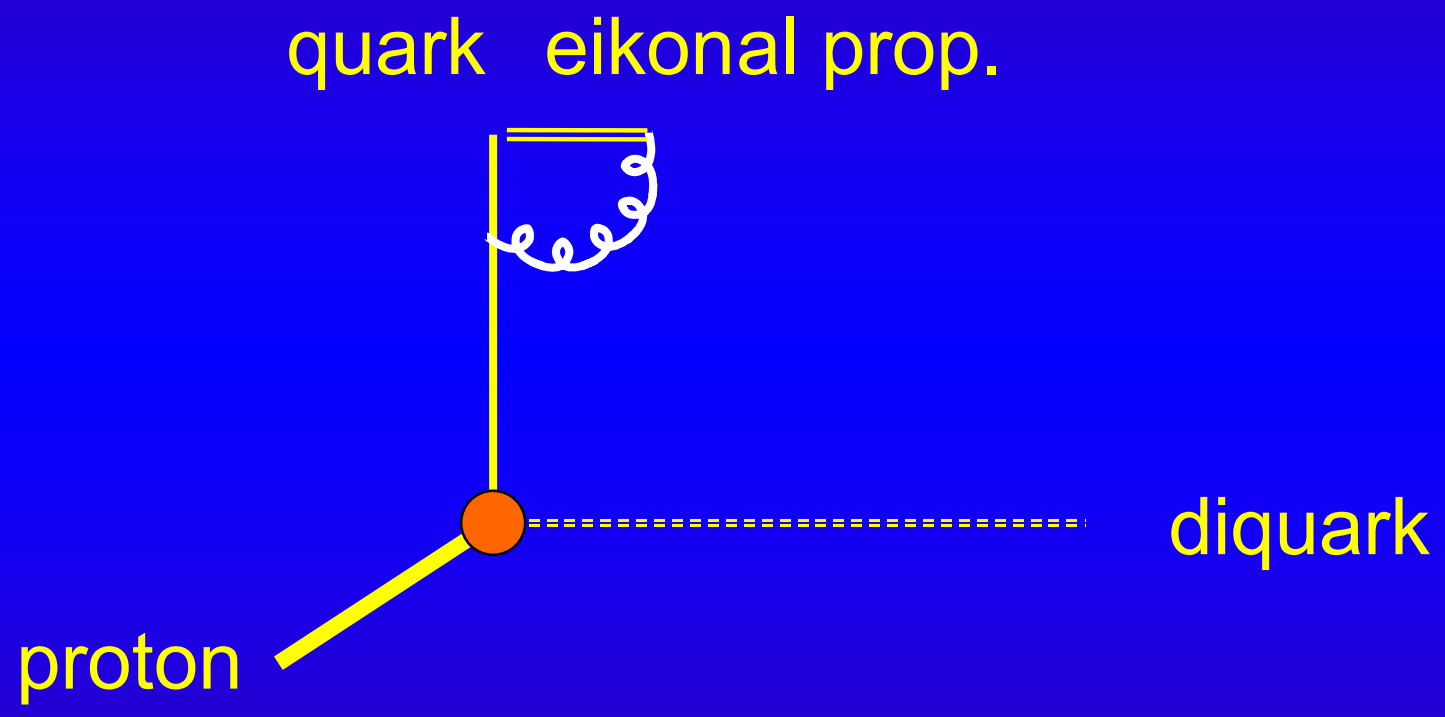

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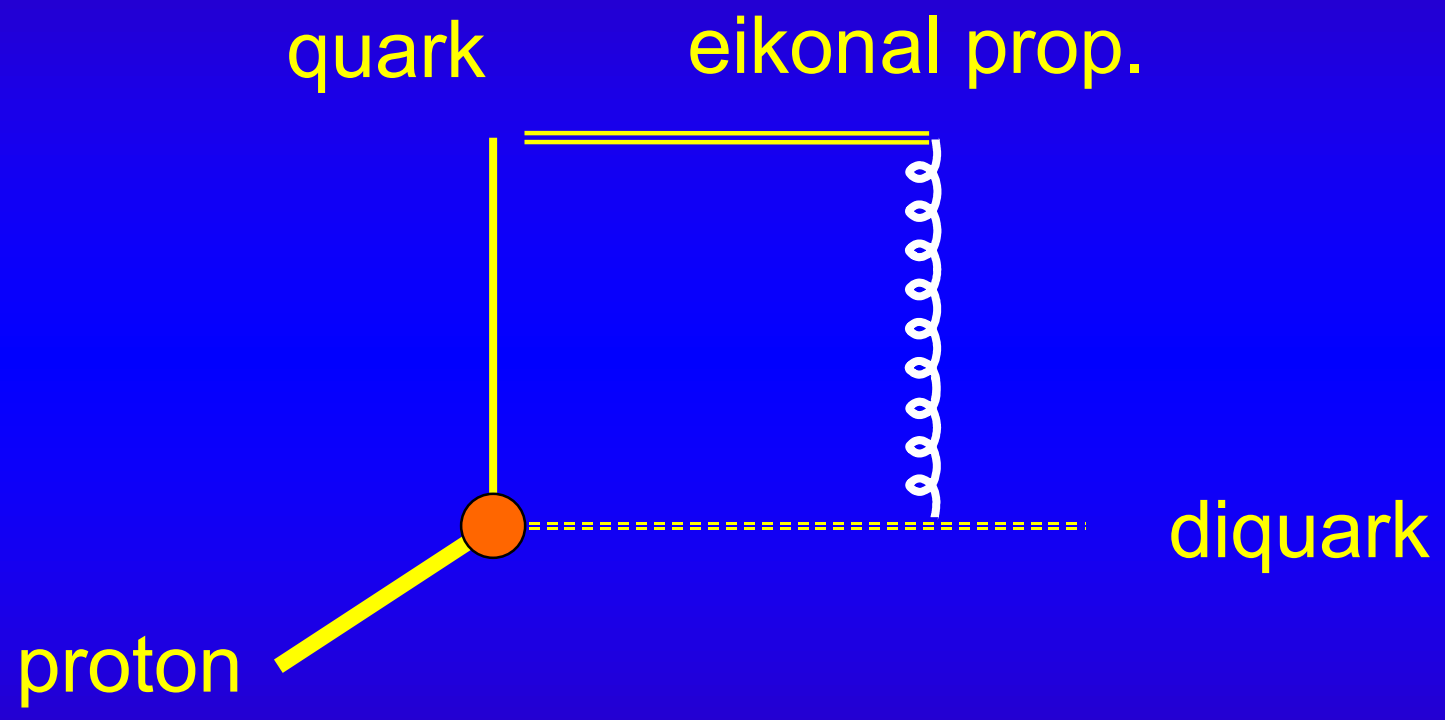
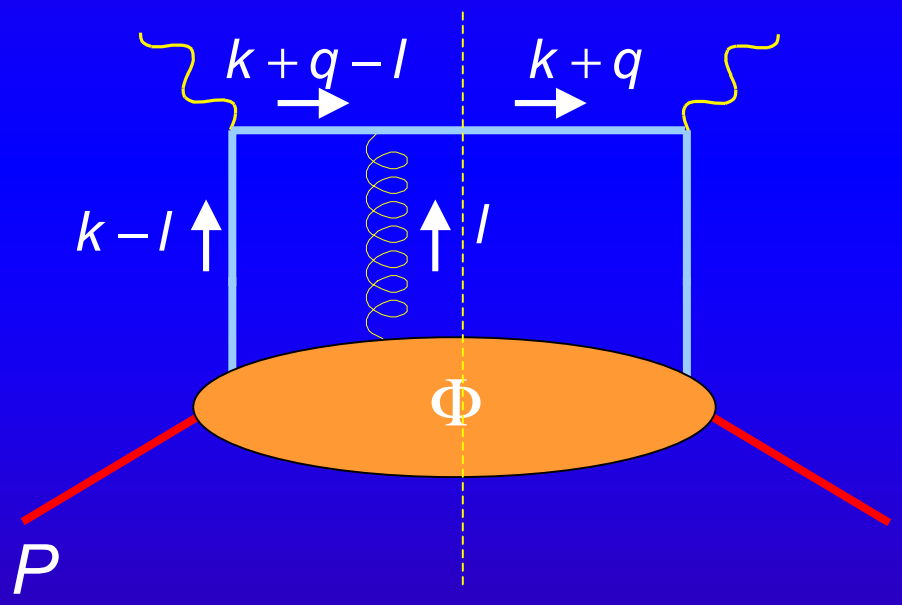


One-gluon contributions
(Feynman gauge)



Ji, Yuan, PLB 543 (02)
Belitsky, Ji, Yuan, NPB656 (03)

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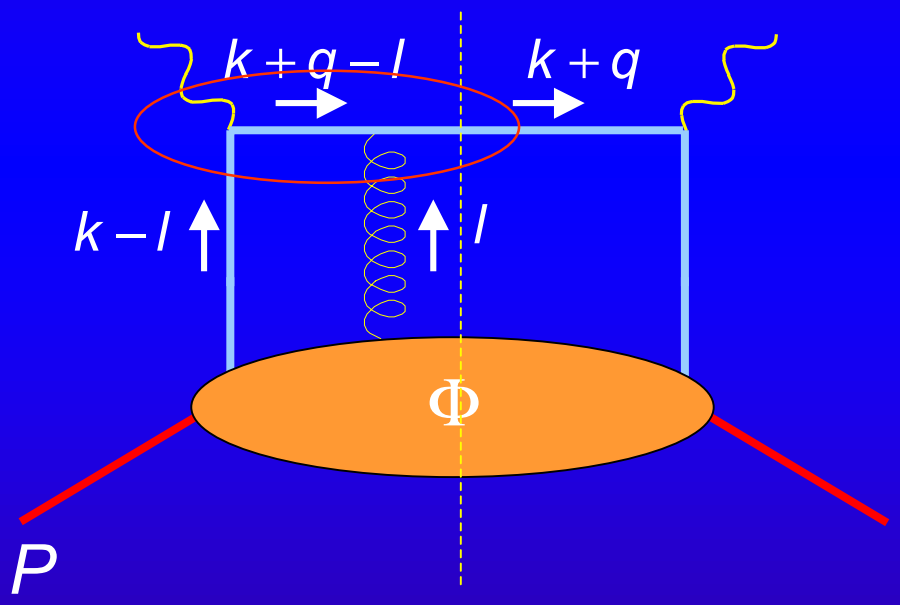

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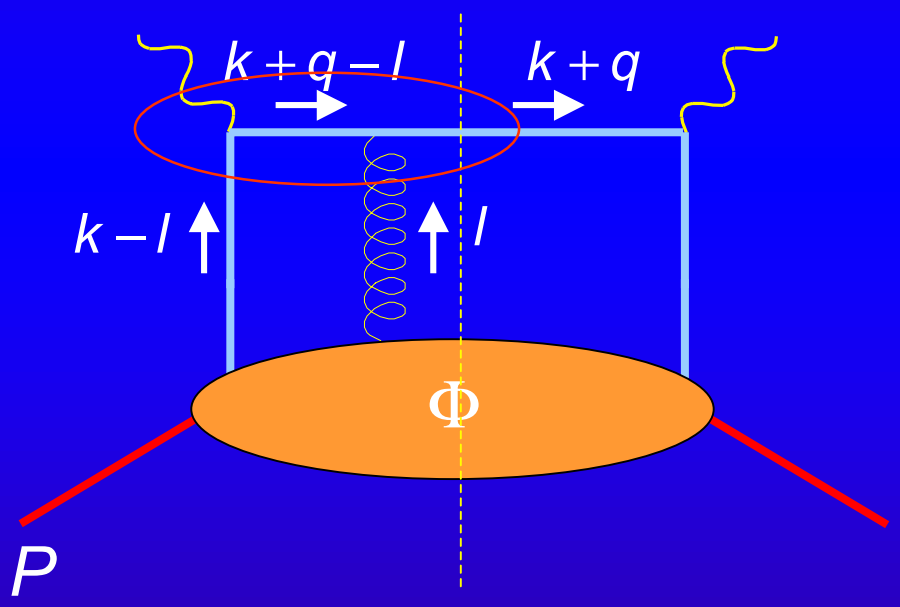
$$(\not{k}+\not{q})\not{A}\frac{(\not{k}+\not{q}-\not{l})}{\left[(k+q-l)^2+i\varepsilon\right]}\gamma^\mu$$



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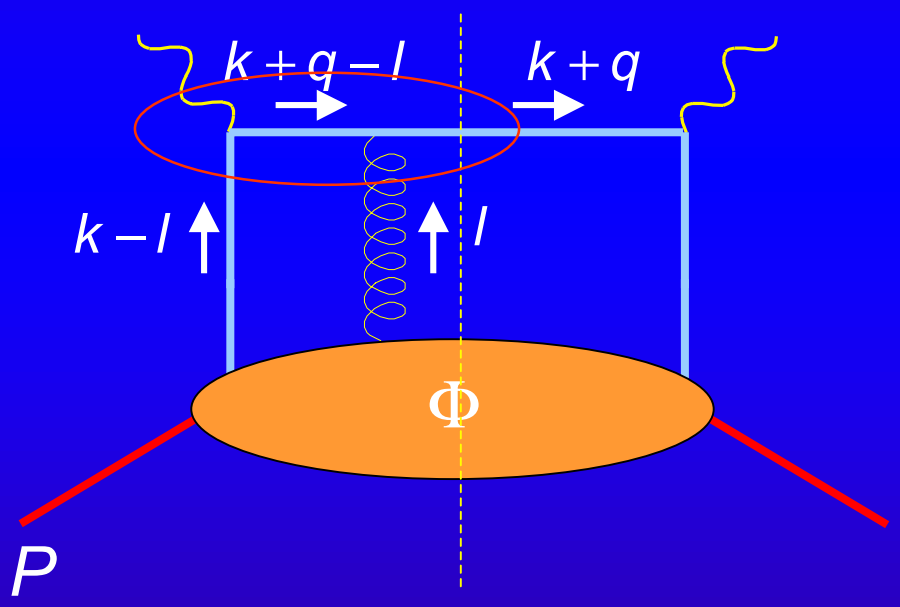

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eikonal approx.

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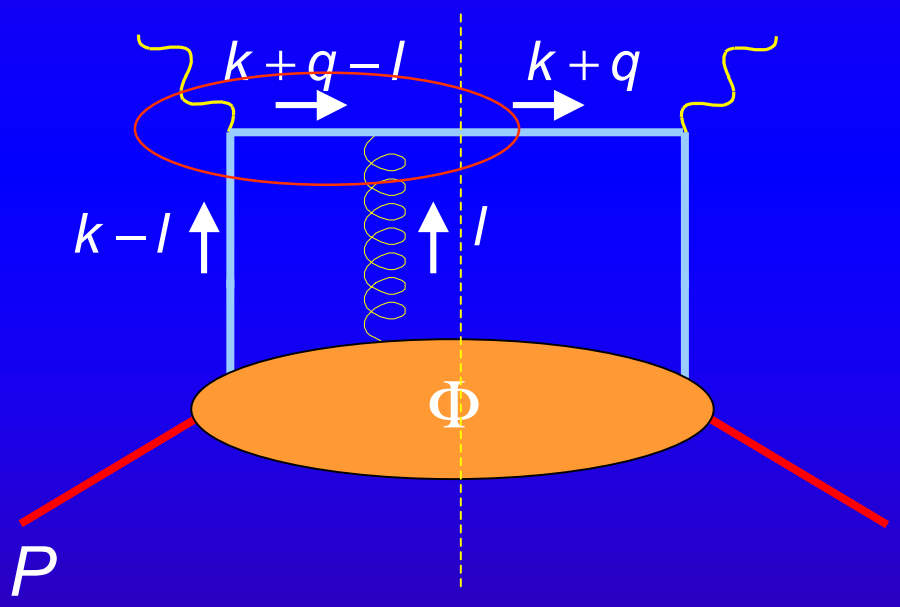

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eikonal approx.

$$\approx(\not{k}+\not{q})\gamma^\mu\frac{-A^+}{\left[l^+-i\varepsilon\right]}$$

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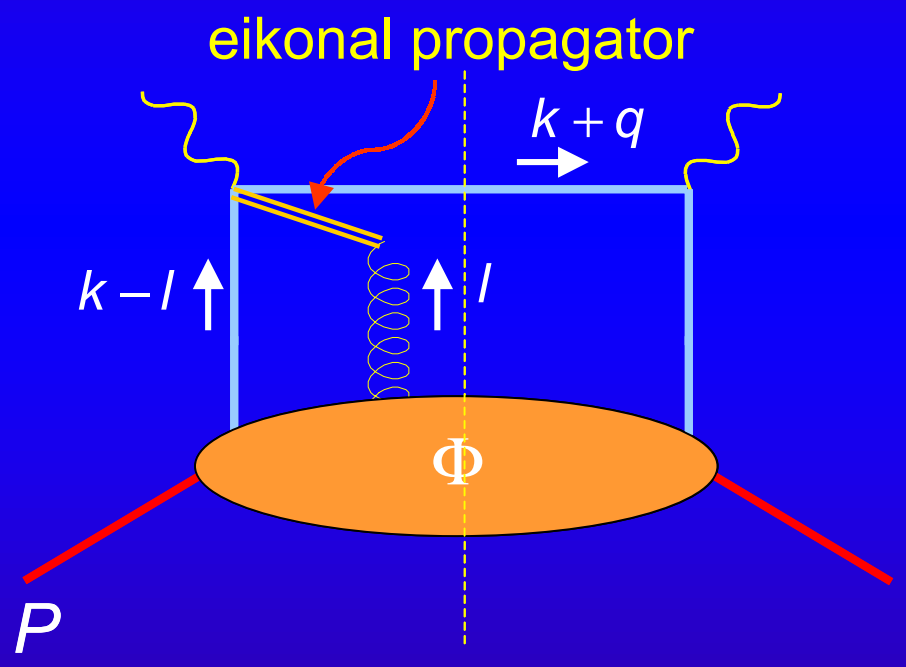
Eikonal propagator

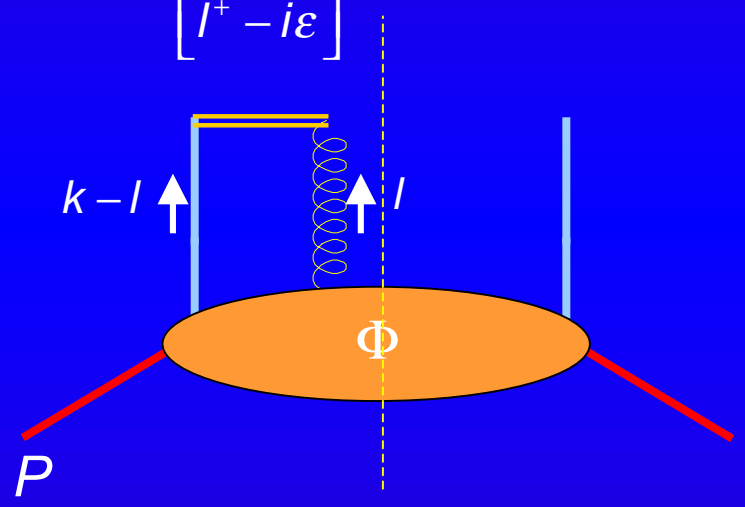


$$(\not{k} + \not{q})\gamma^{\mu} \frac{-A^{+}}{\left[l^{+} - i\varepsilon\right]}$$

Gauge link

eikonal propagator

$$-\frac{1}{\left[l^{+} - i\varepsilon\right]}$$



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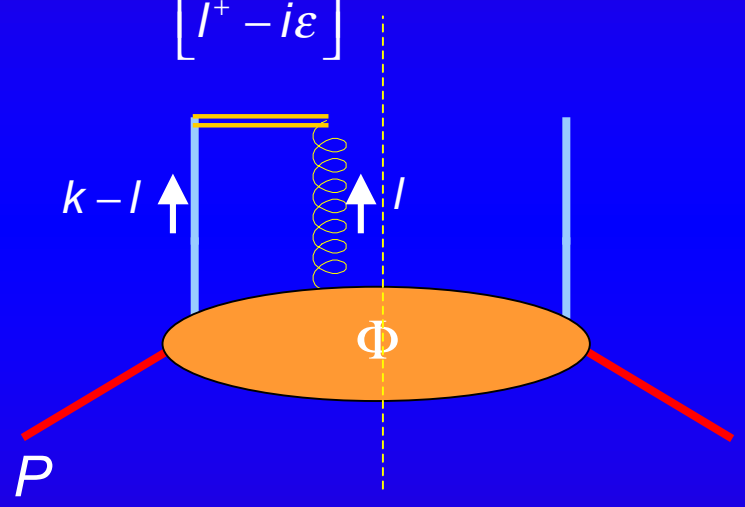

$$-\int \frac{d^4 l}{(2\pi)^4} \frac{gA^+(l)}{\left[l^+ - i\varepsilon\right]} = -ig\int \frac{d^2 l_\perp dl^-}{(2\pi)^3} A^+(l)\Big|_{l^+=0}$$

$$= -ig\int_0^\infty d\zeta^- \, A^+(\zeta)\Big|_{\zeta^+=\zeta_\perp=0}$$

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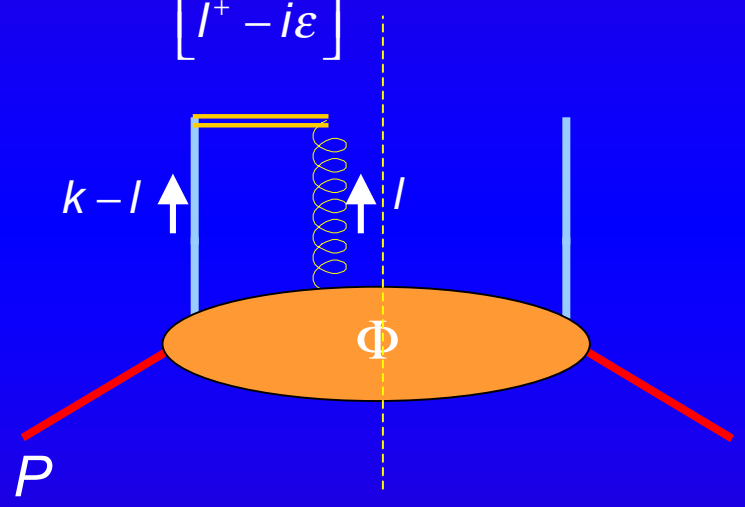

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$O(g)$ contribution to gauge link!

$$\mathcal{U}^-_{[0,\infty]} \equiv \mathcal{P}\exp\left(-ig\int_0^\infty d\zeta^- A^+(\zeta^-)\right)$$

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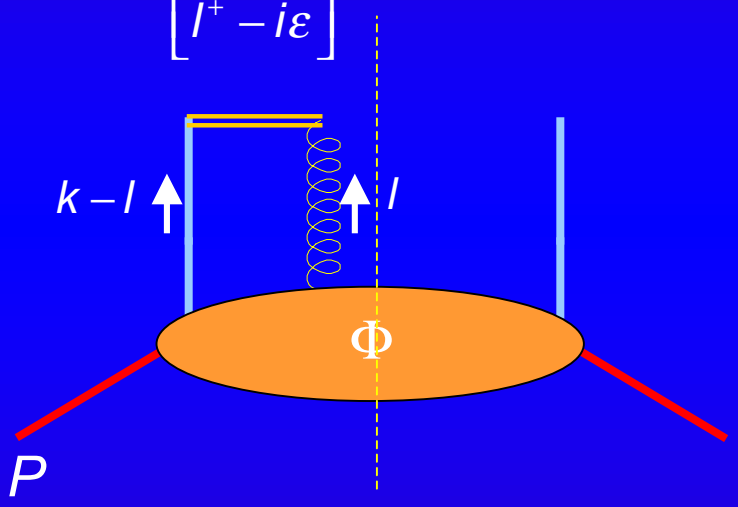

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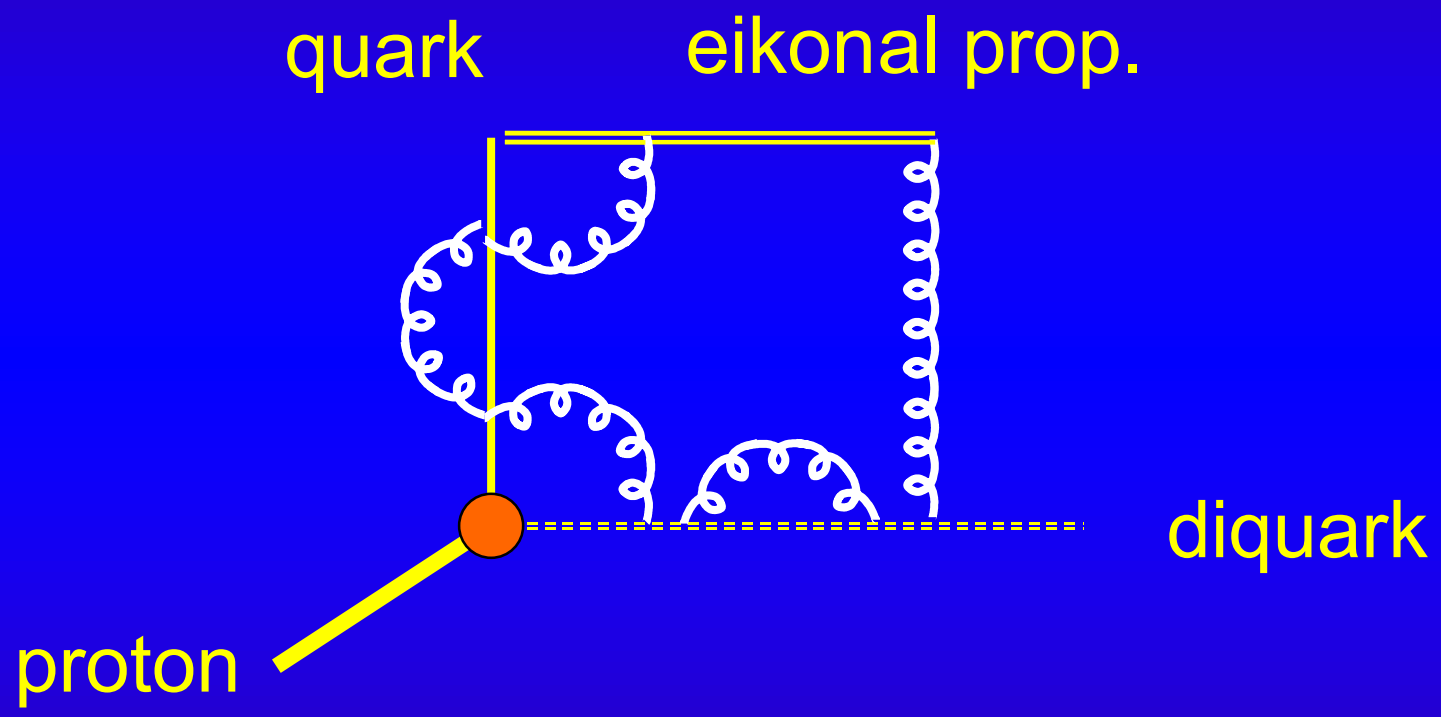
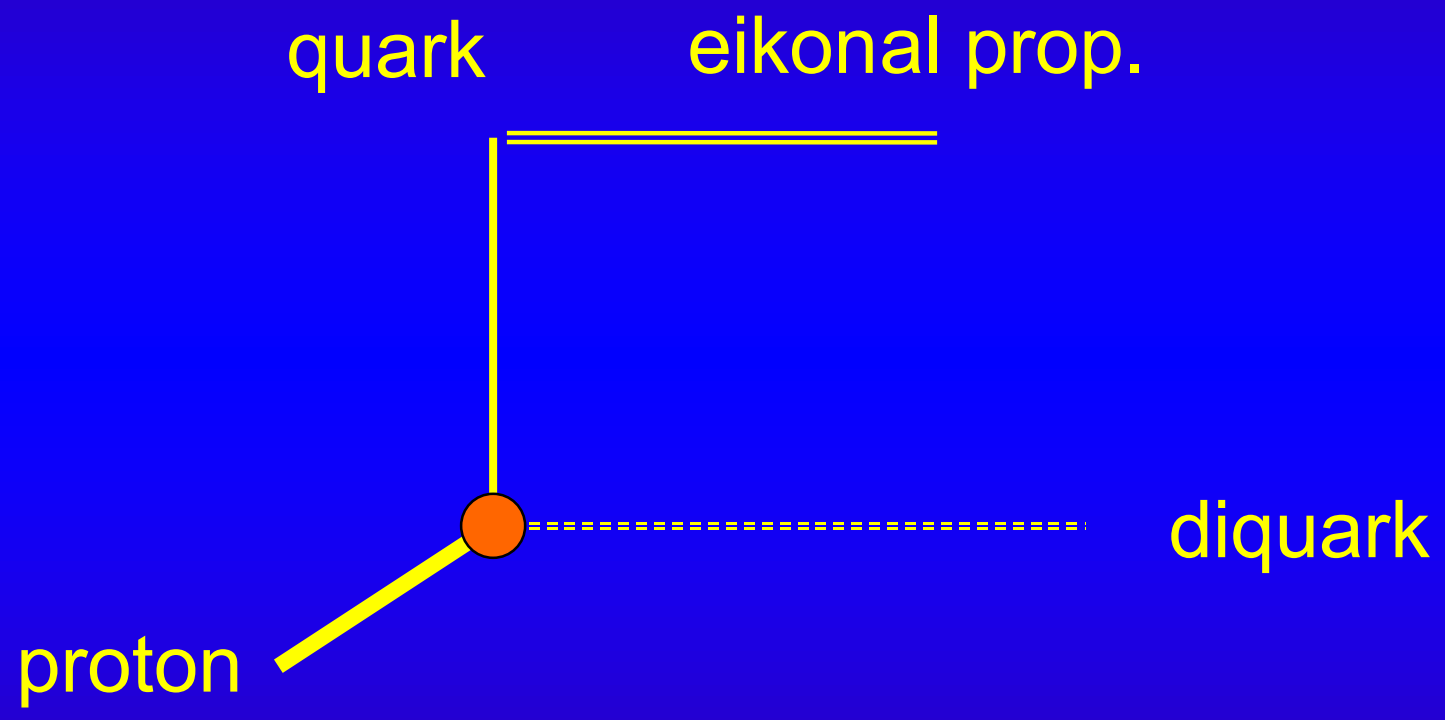
Gauge links represent factorizable final state interactions of the quark with the spectators

Single gluon loop corrections



Real gluon corrections



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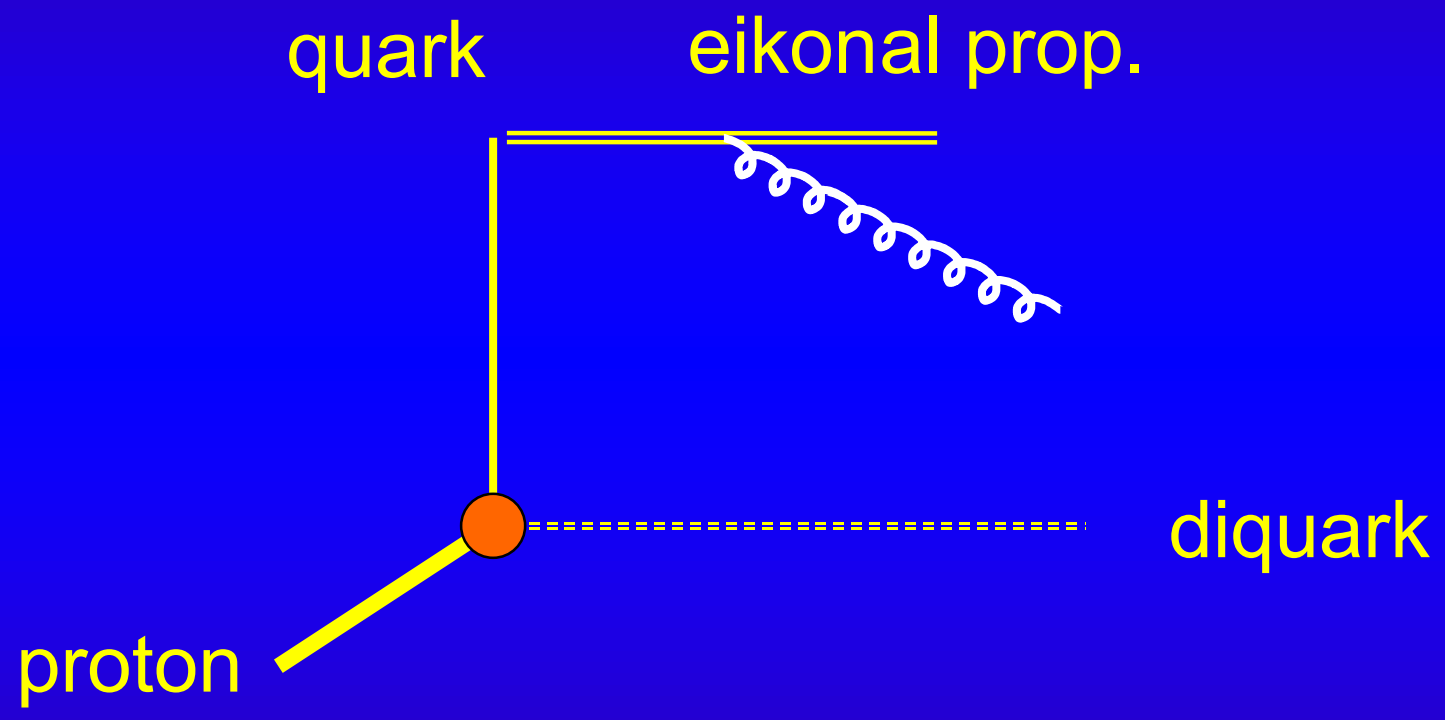

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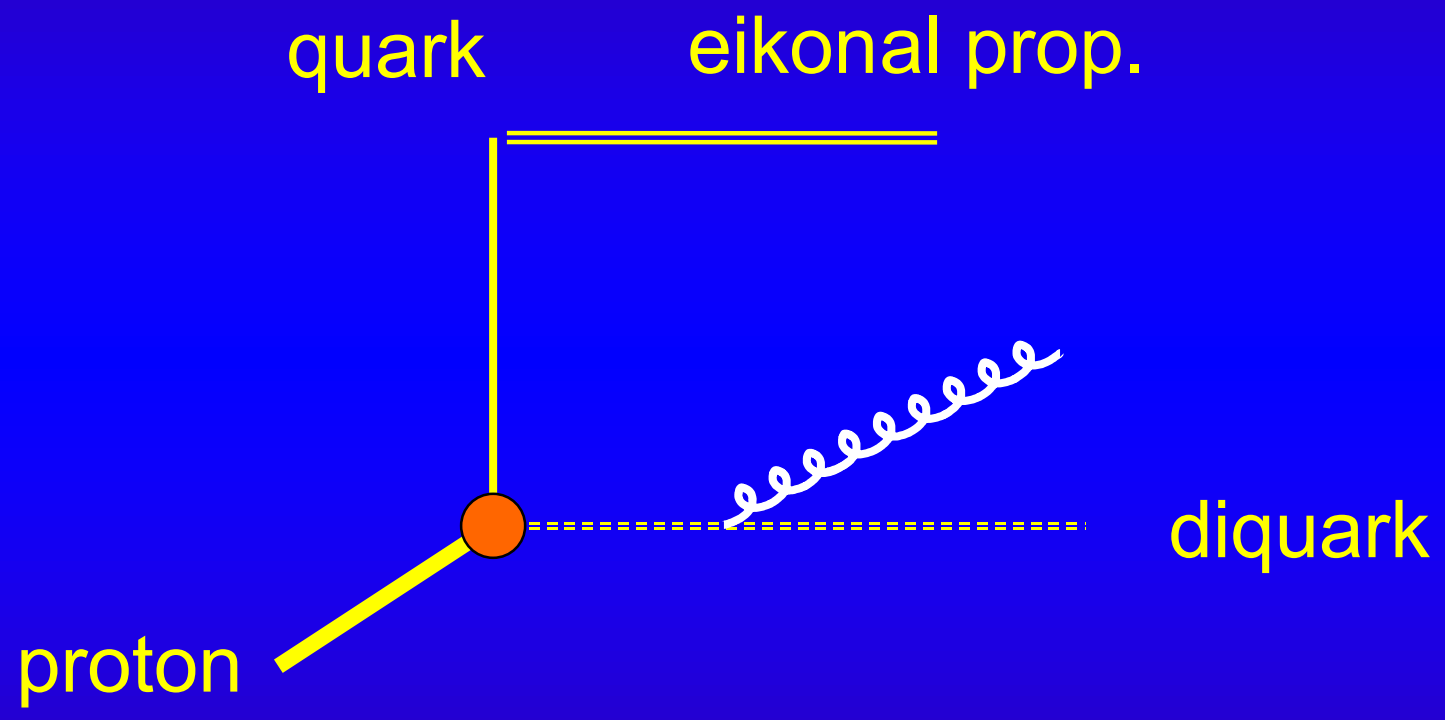

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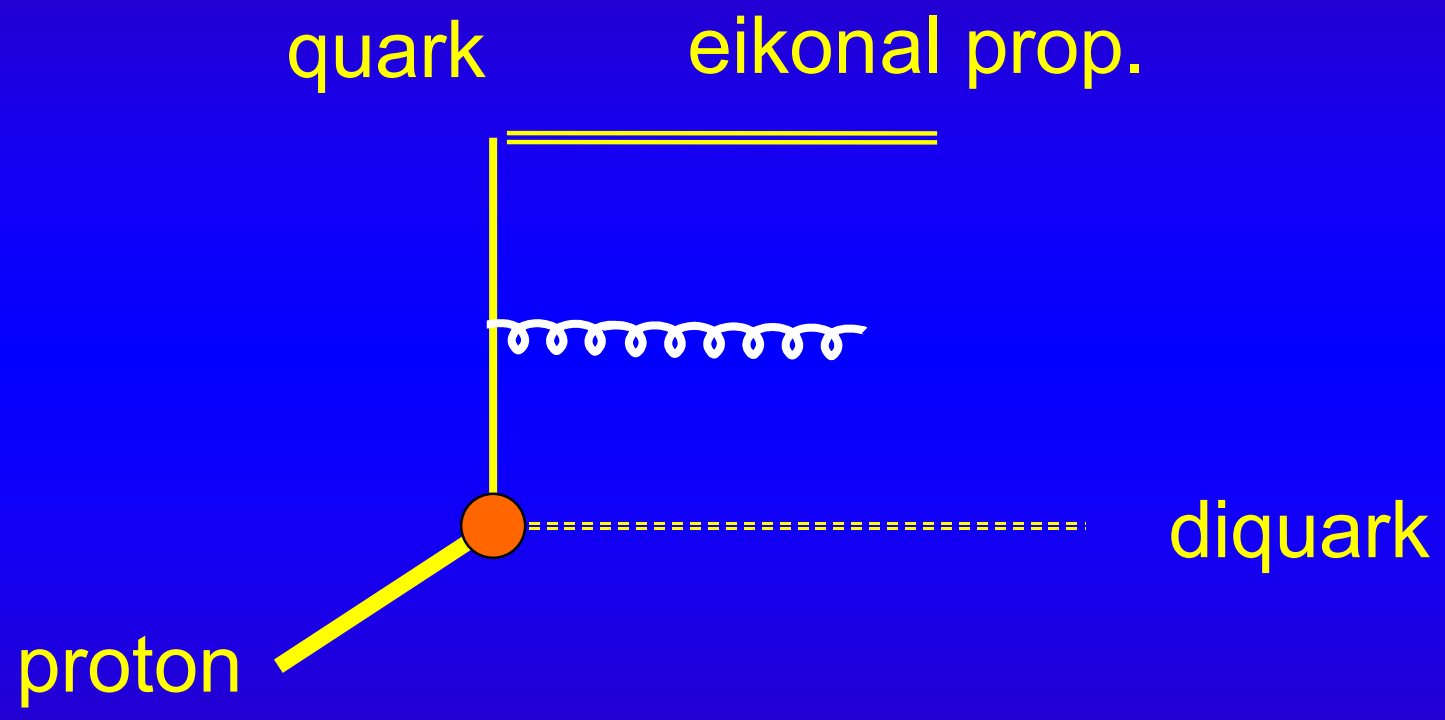

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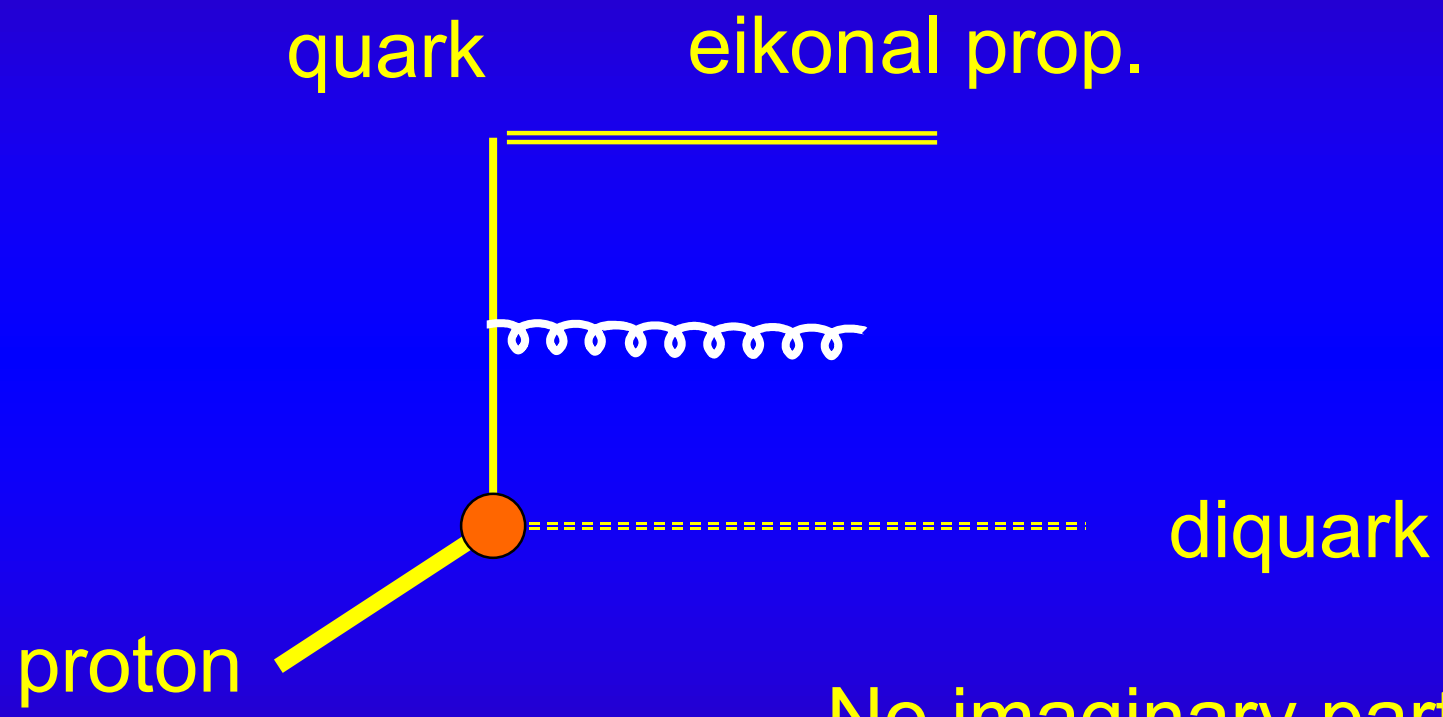
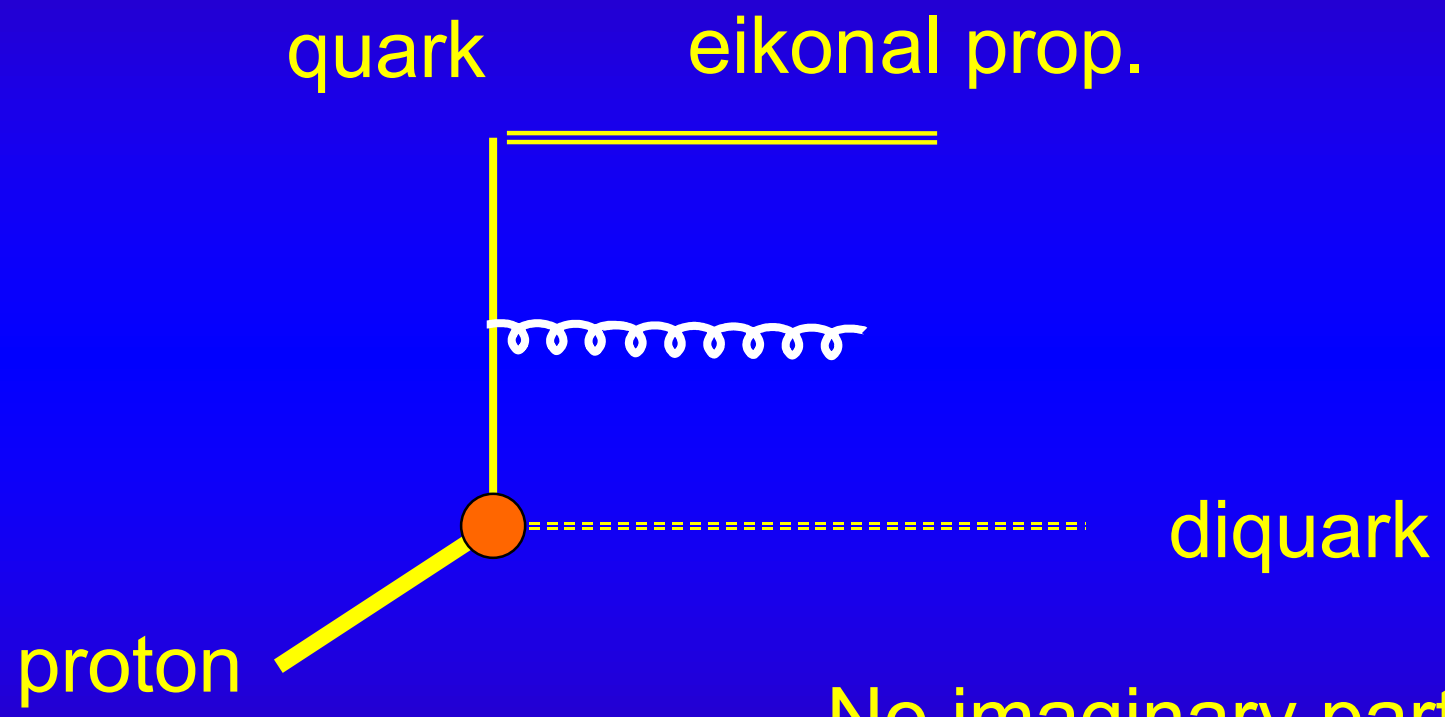


No imaginary parts.

Real gluon corrections

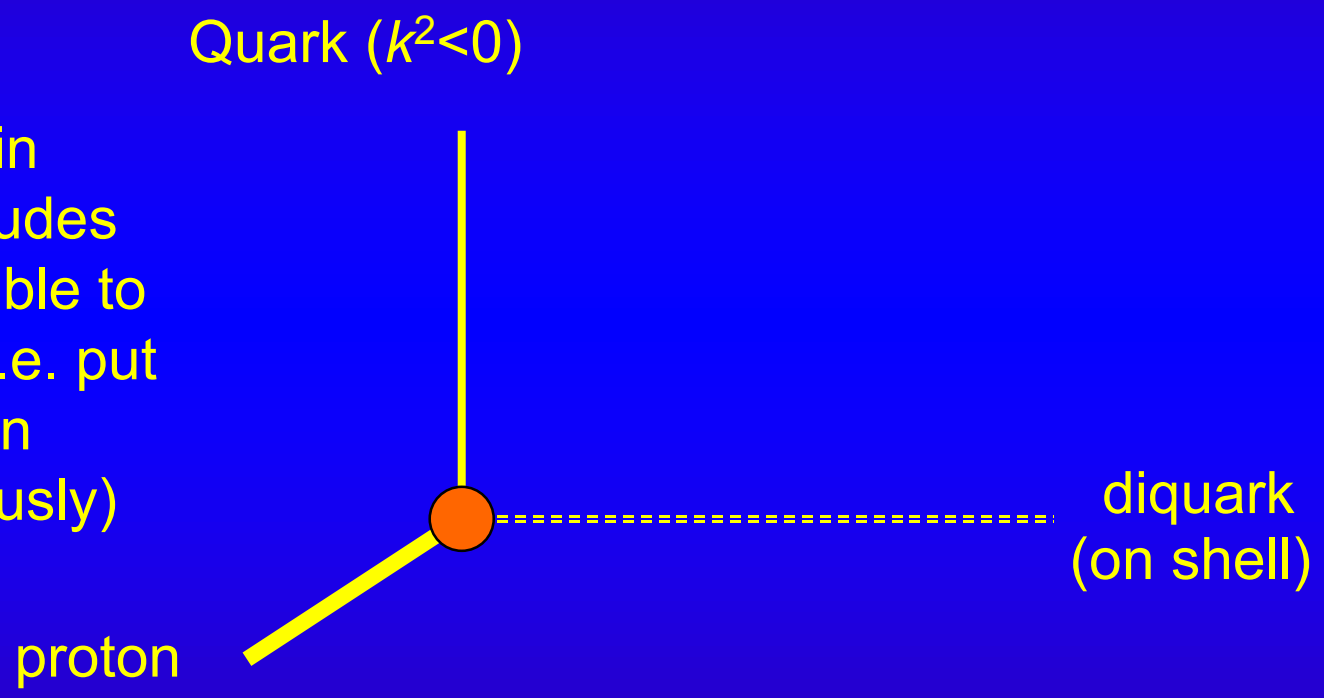


No imaginary parts.

No T-odd functions.

Imaginary parts and Cutkosky rules

Imaginary parts in Feynman amplitudes arise if it is possible to "cut" the loops (i.e. put some particles on shell simultaneously)



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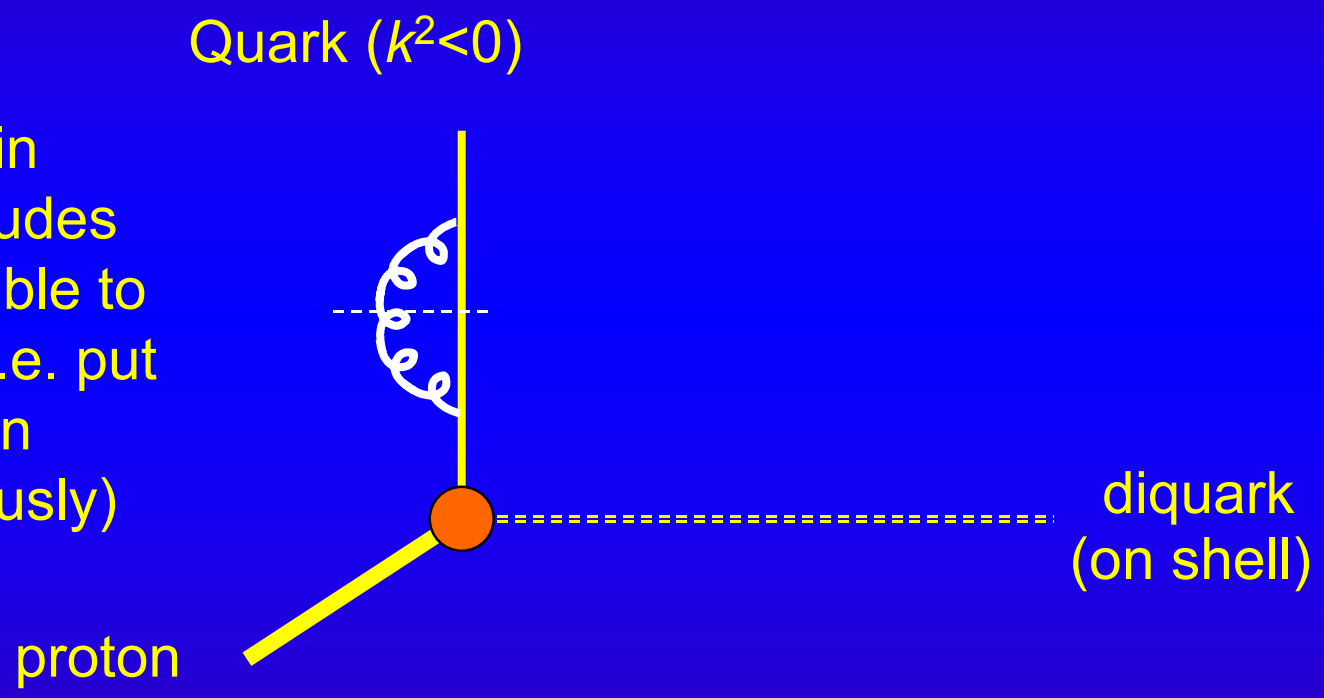

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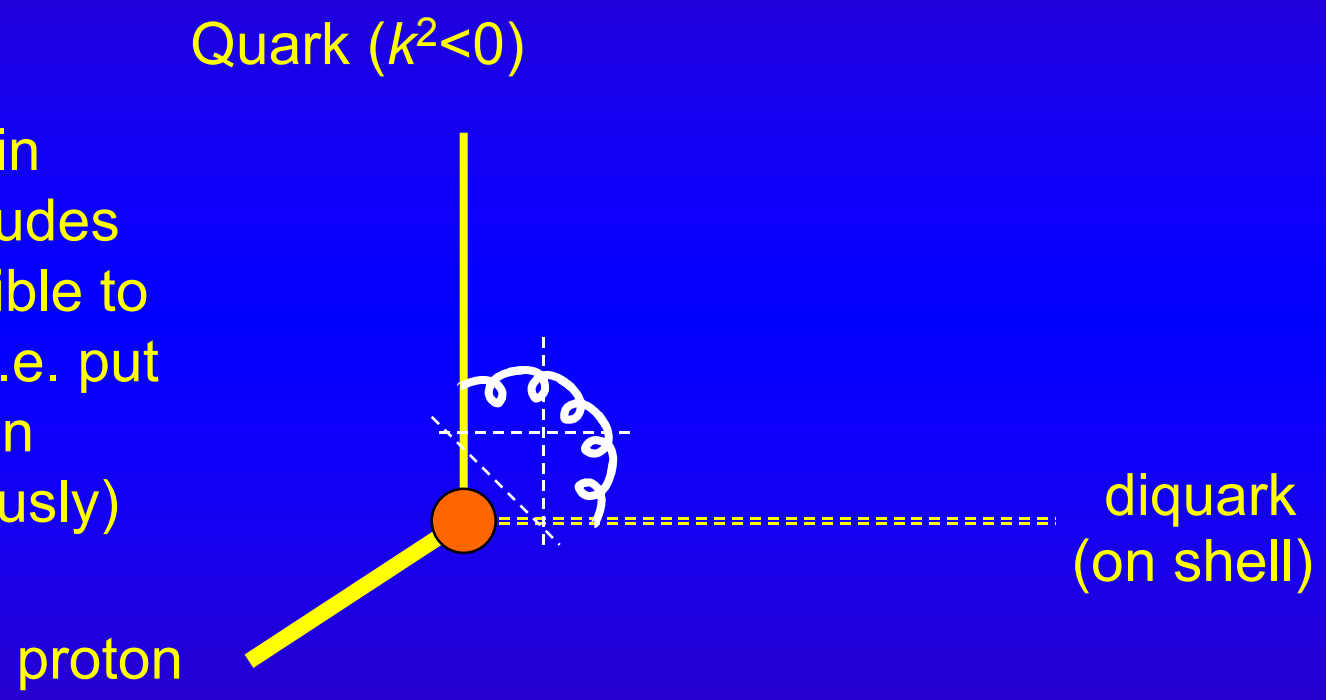

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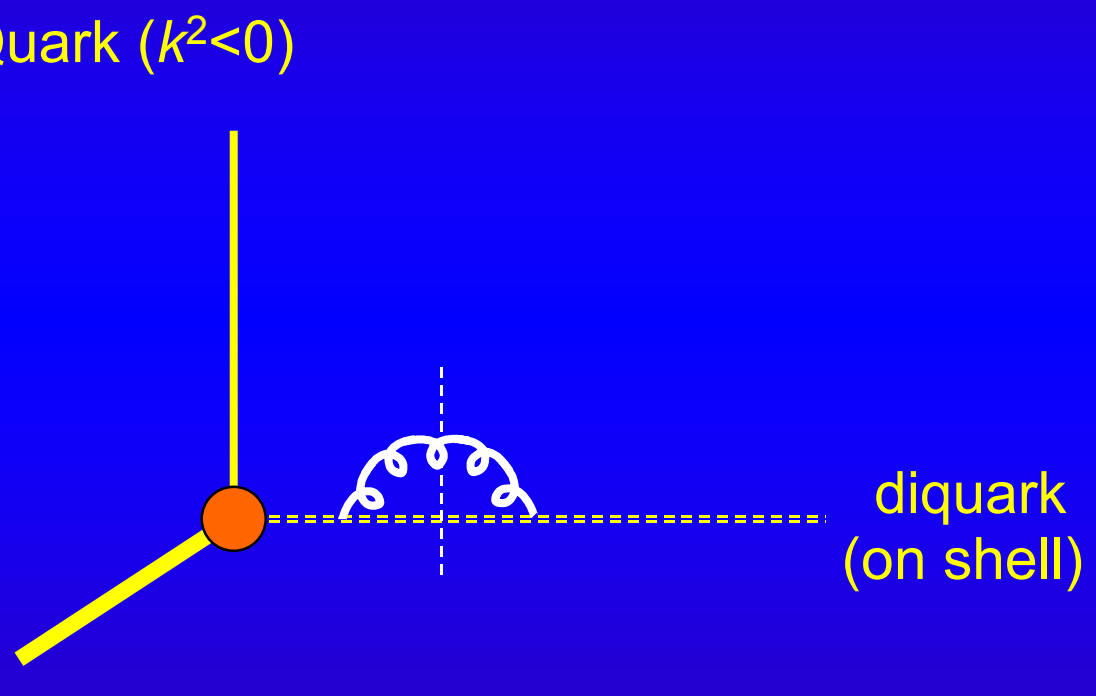

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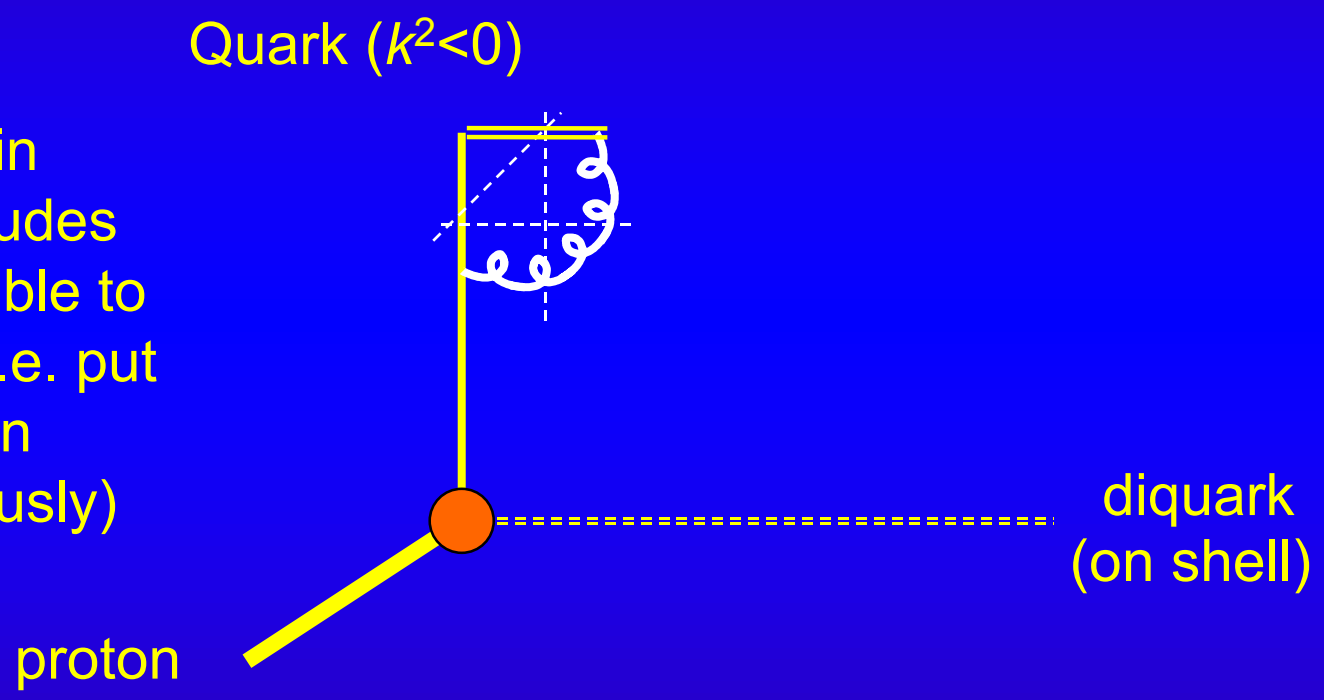

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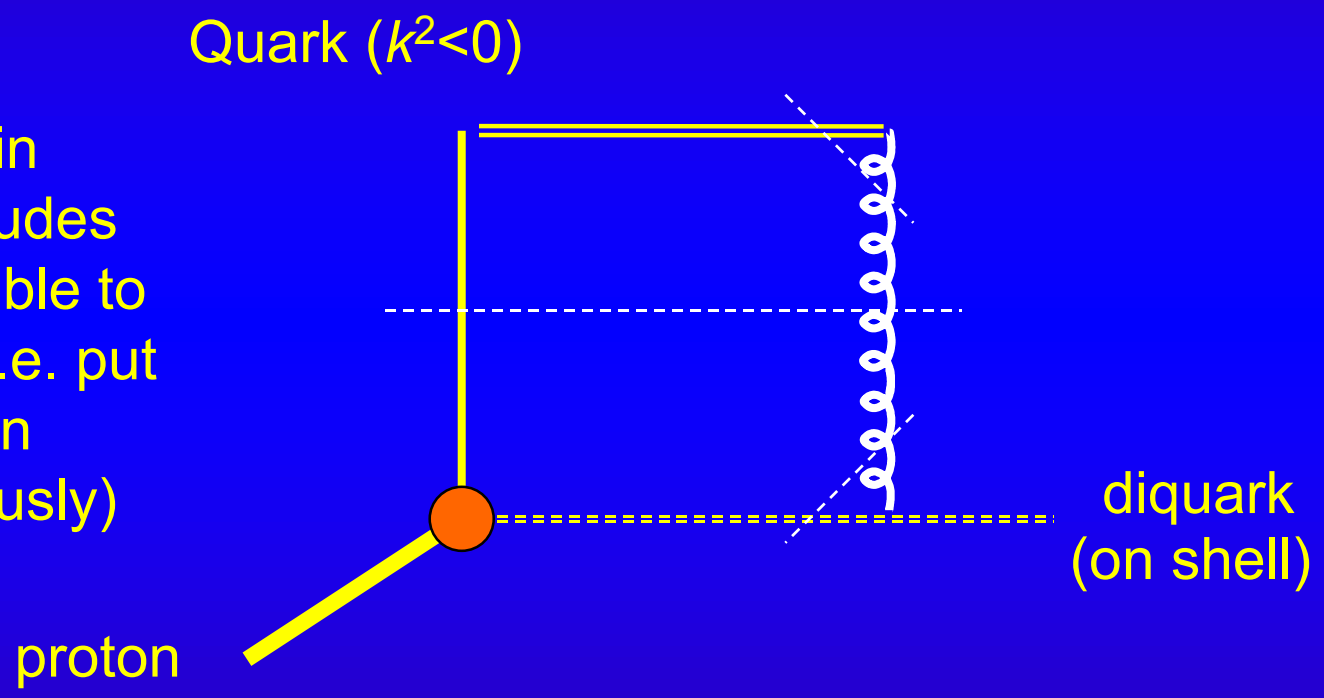

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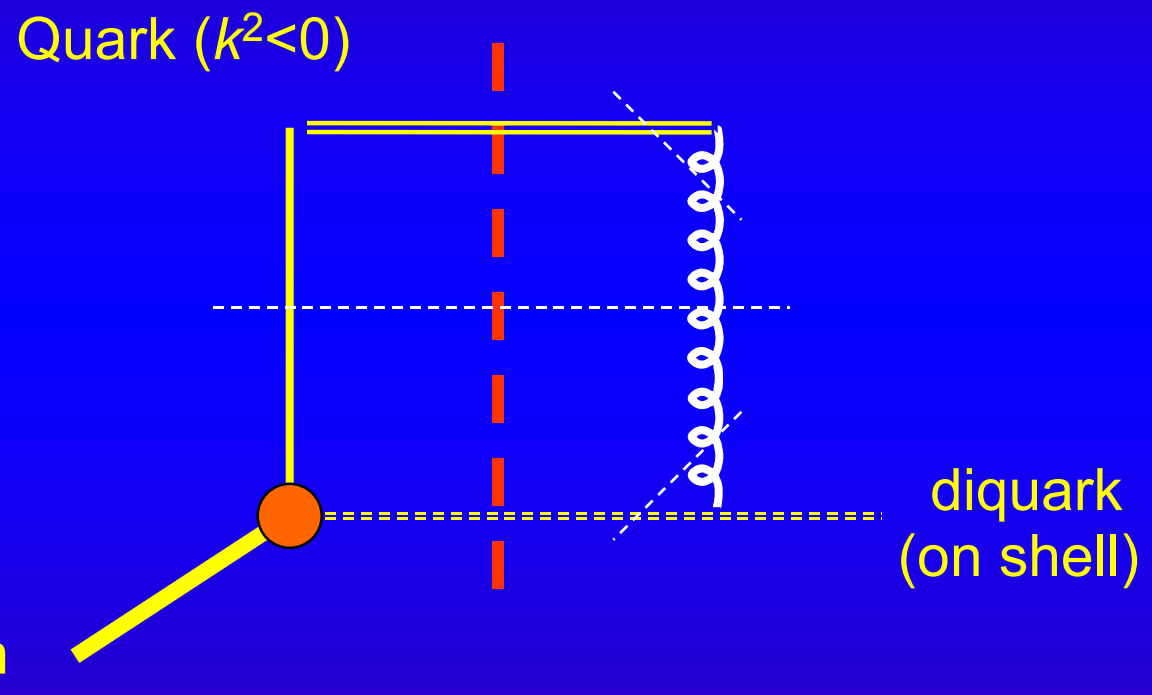

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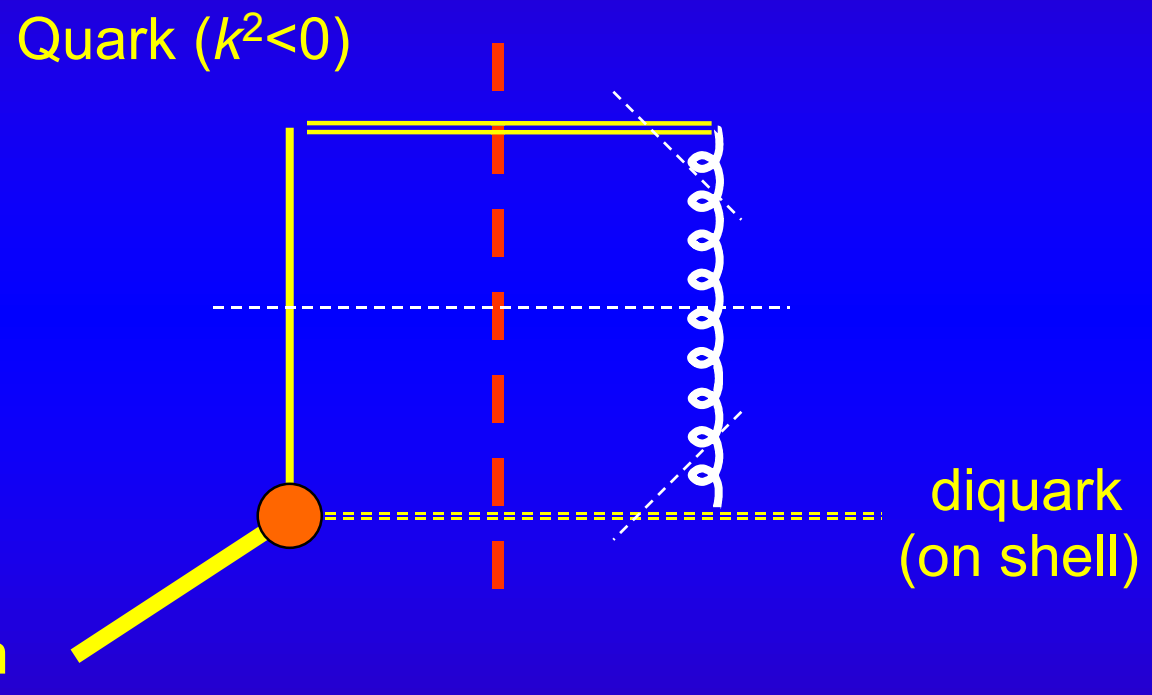
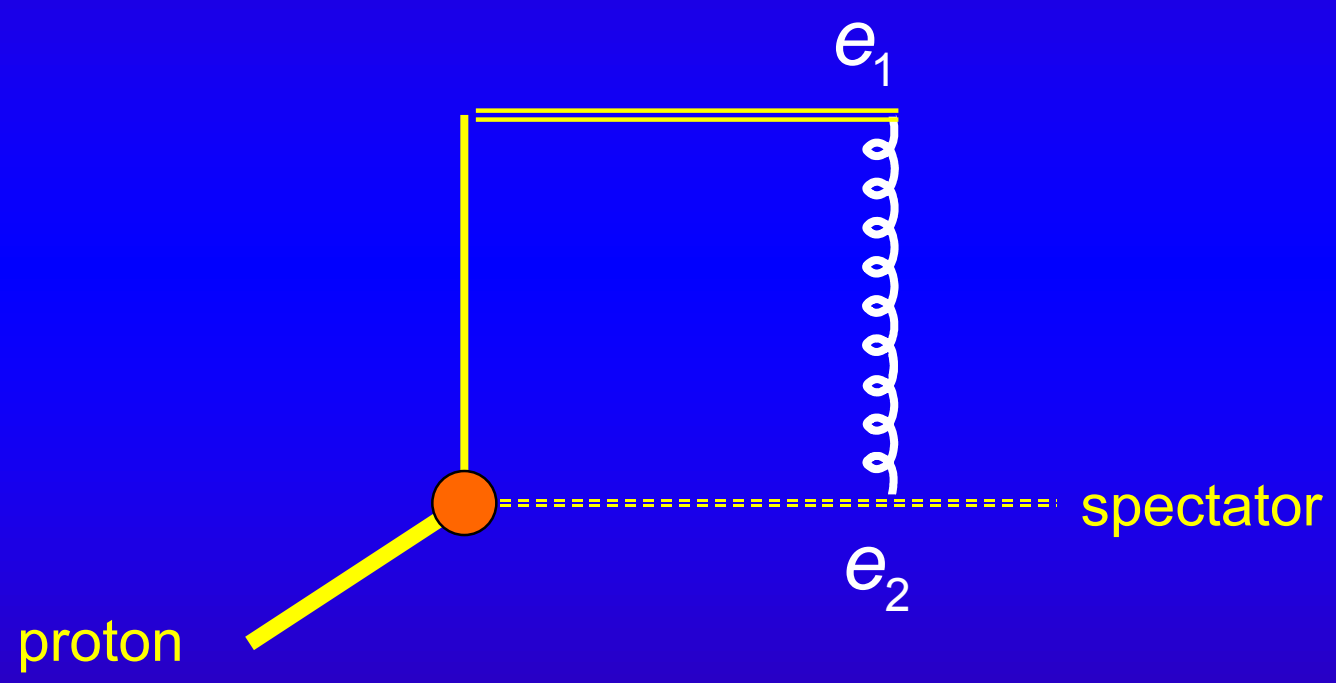
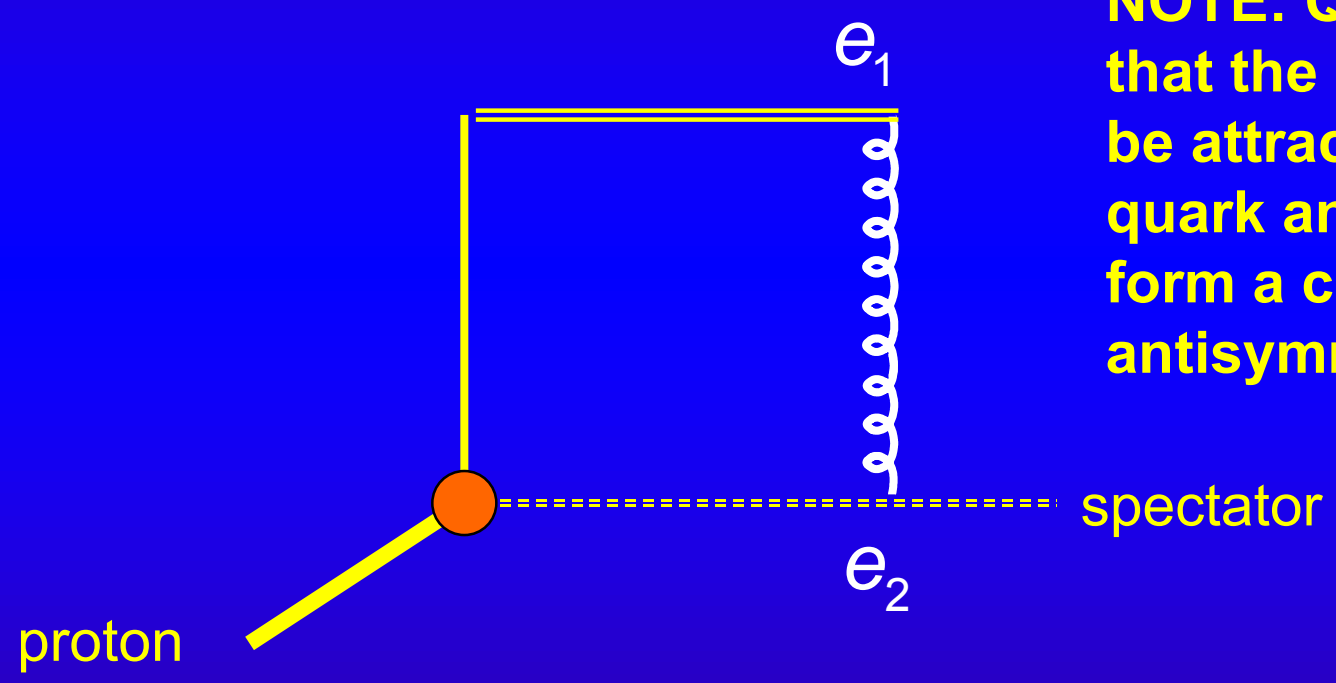


The imaginary part comes from the gauge link

The sign of the amplitude

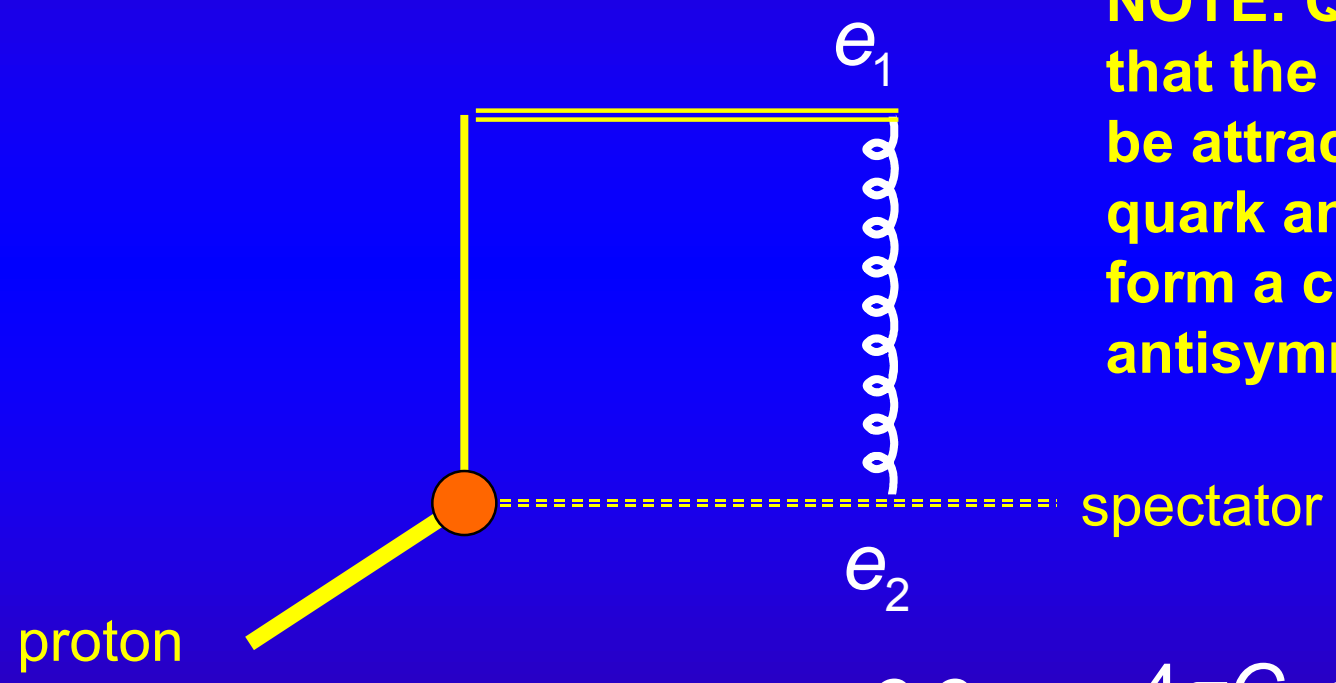


The sign of the amplitude



NOTE: QCD tells us that the FSI has to be attractive, since quark and spectator form a color antisymmetric state

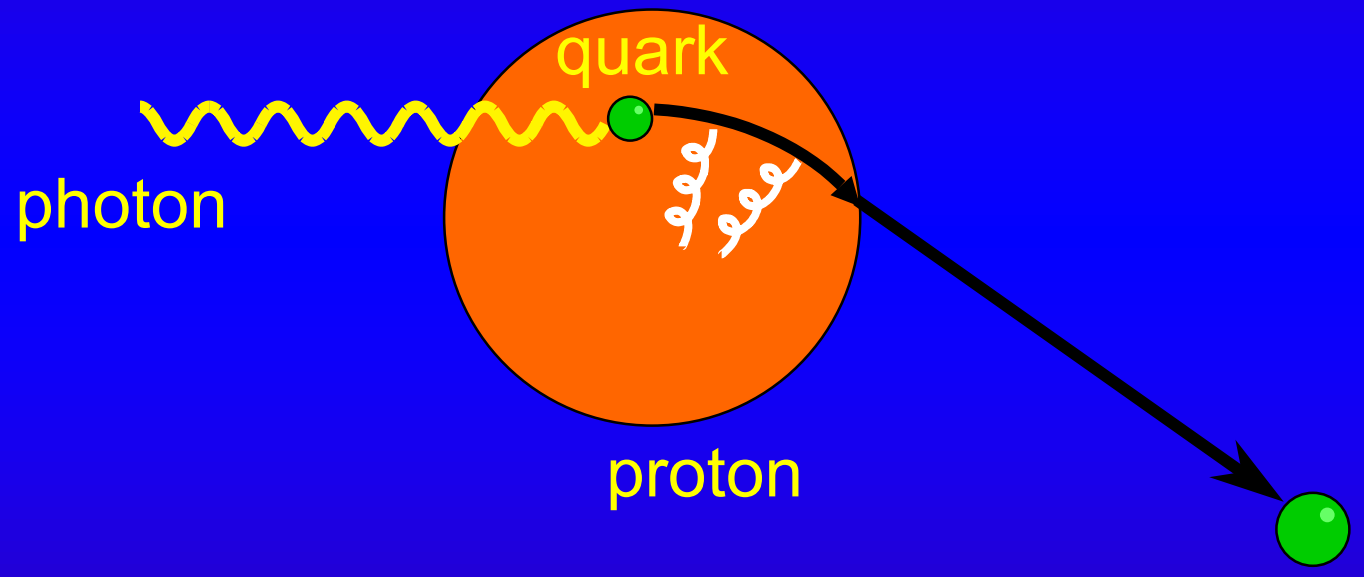
The sign of the amplitude



NOTE: QCD tells us that the FSI has to be attractive, since quark and spectator form a color antisymmetric state

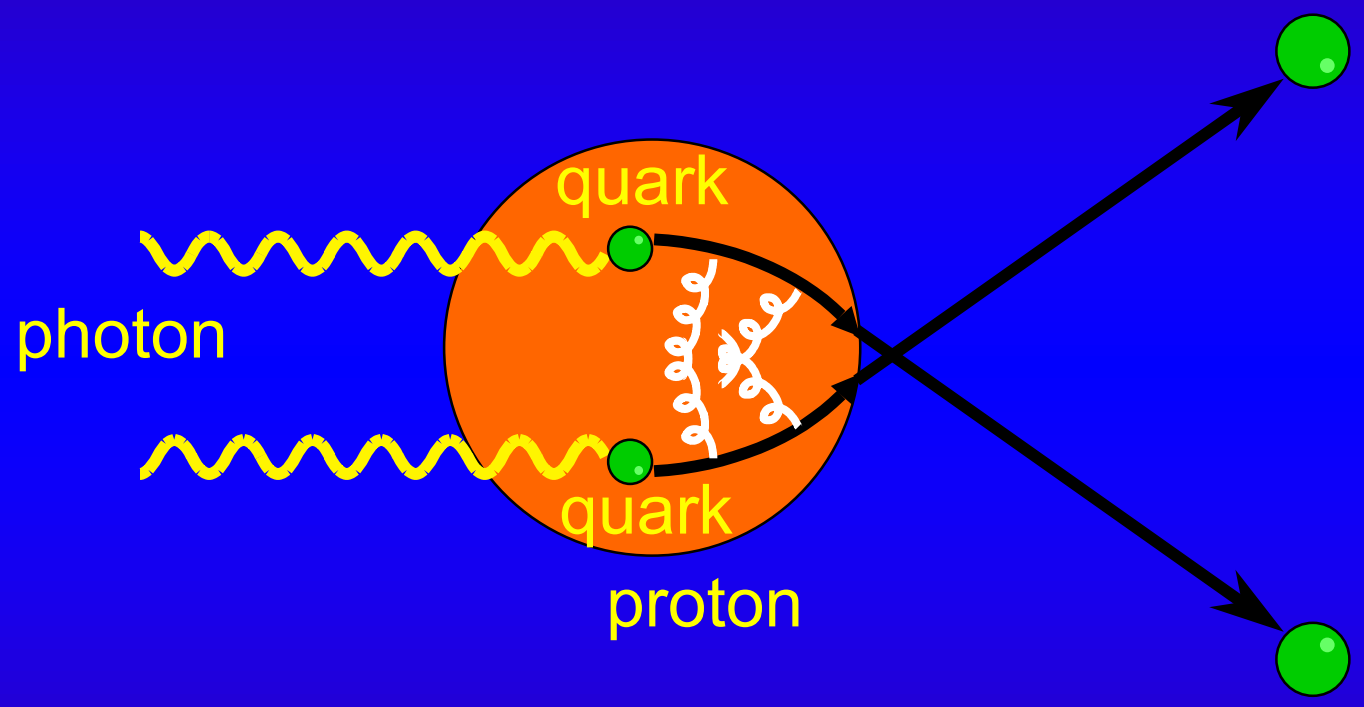
$$e_1 e_2 \approx -4\pi C_F \alpha_S \approx -5$$

An intuitive picture



Burkardt, PRD 66 (02)

An intuitive picture



Burkardt, PRD 66 (02)

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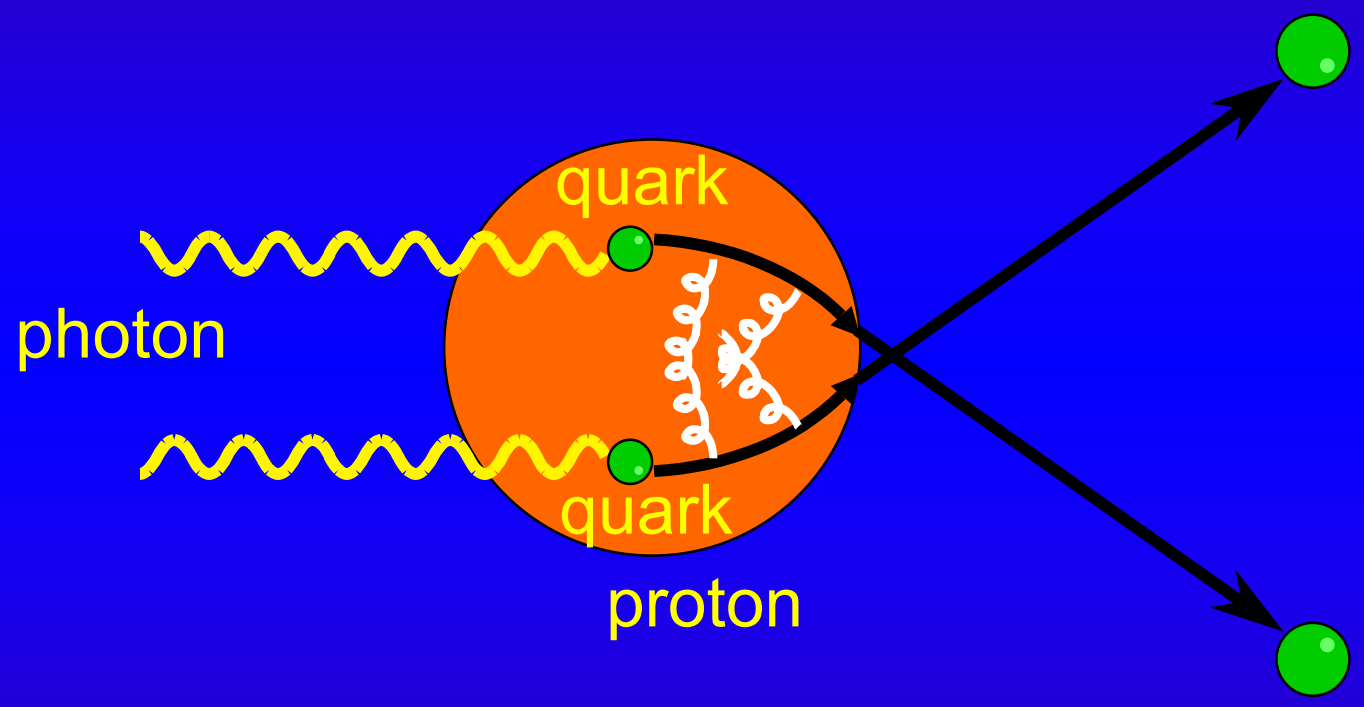
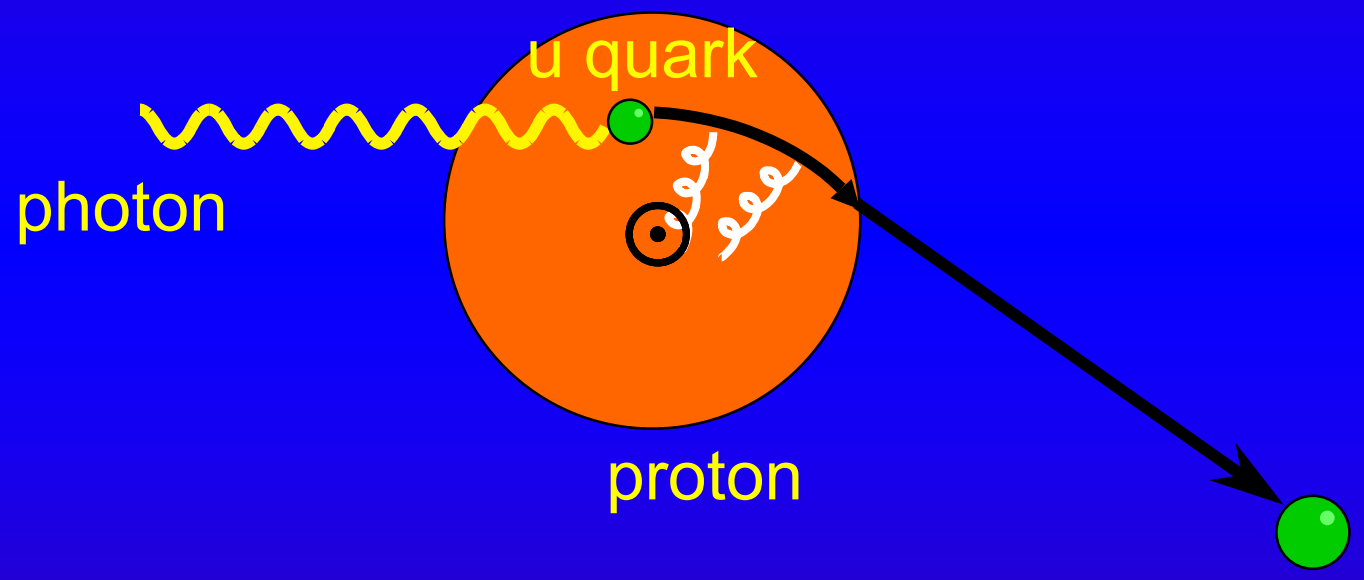


Chromodynamic lensing

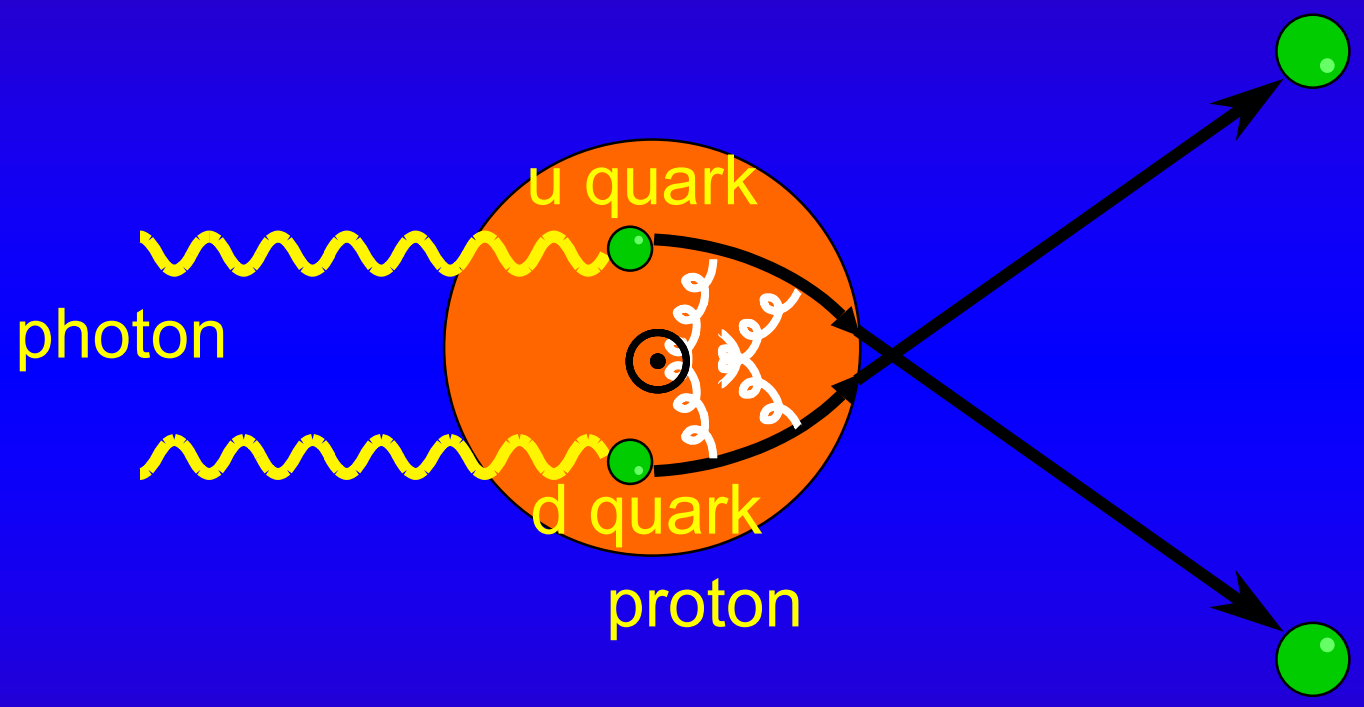
Burkardt, PRD 66 (02)

An intuitive picture



A distortion in the distribution of quarks in transverse space can give rise to a single-spin asymmetry

An intuitive picture



A distortion in the distribution of quarks in transverse space can give rise to a single-spin asymmetry

Model results

Brodsky, Hwang, Schmidt, PLB 530 (02)

Brodsky, Hwang, Schmidt, PLB 530 (02)

- scalar spectator,
- pointlike form factor,
- sign error (pointed out in Burkardt, Hwang, PRD 69),
- negative u Sivers function (after correcting the error)

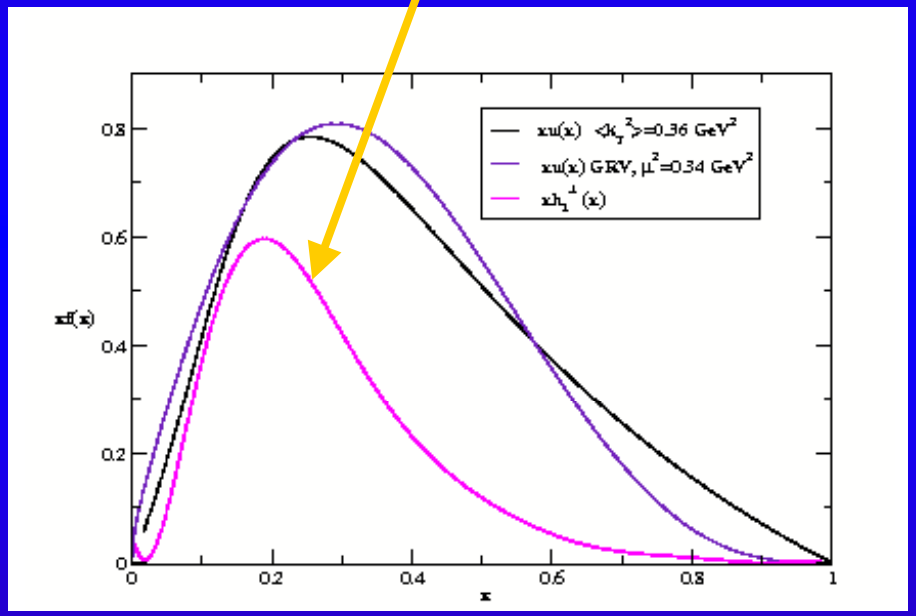
Brodsky, Hwang, Schmidt, PLB 530 (02)

- scalar spectator,
- pointlike form factor,
- sign error (pointed out in Burkardt, Hwang, PRD 69),
- negative u Sivers function (after correcting the error)

- Boer, Brodsky, Hwang, PRD 67 (03)
  - same as above (including sign error)
  - Boer-Mulders = Sivers

Gamberg, Goldstein, Oganessyan, PRD67 (03)

$$xf_{1T}^{\perp(1/2)u} = xh_1^{\perp(1/2)u}$$



x

Gamberg, talk at DIS2005

Gamberg, Goldstein, Oganessyan, PRD67 (03)

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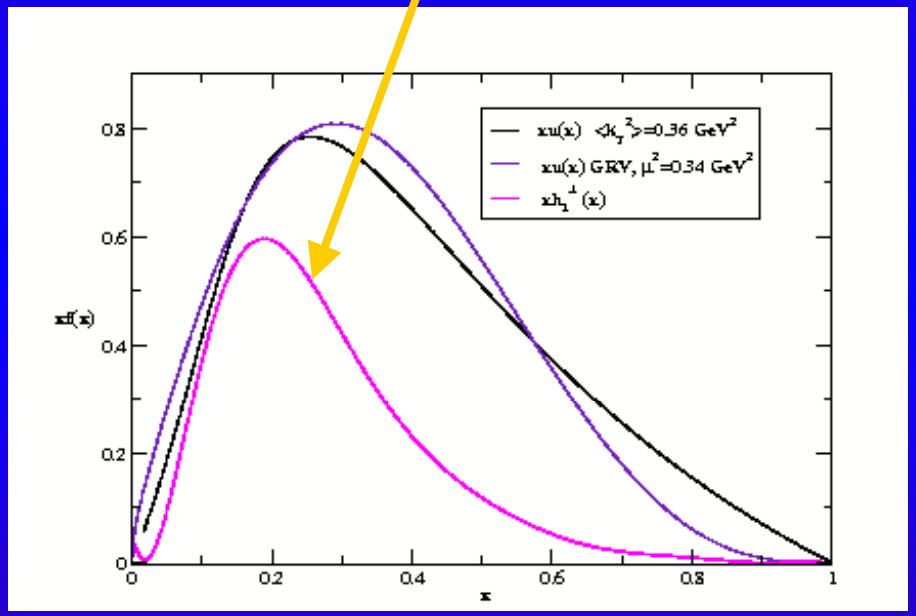
- scalar spectator



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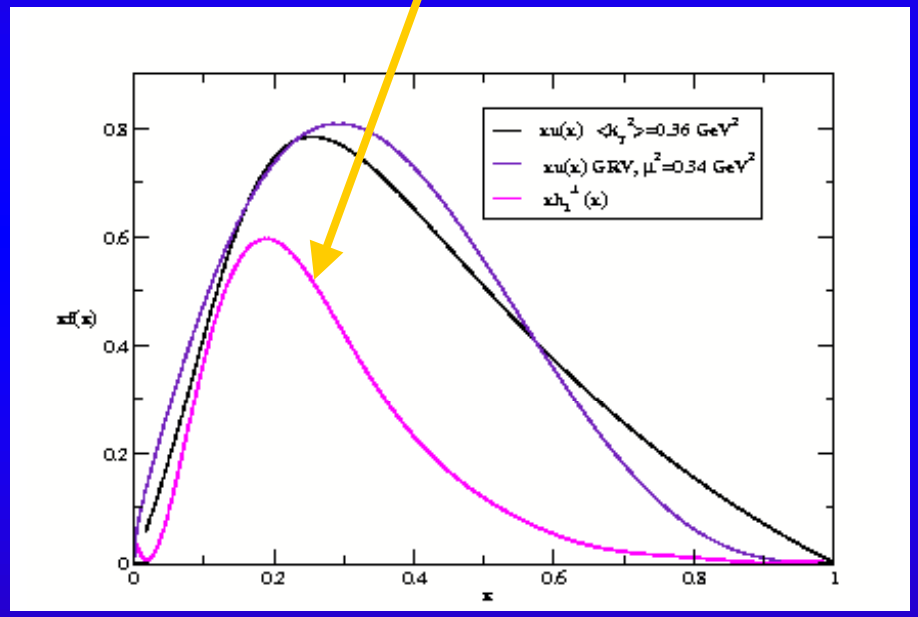

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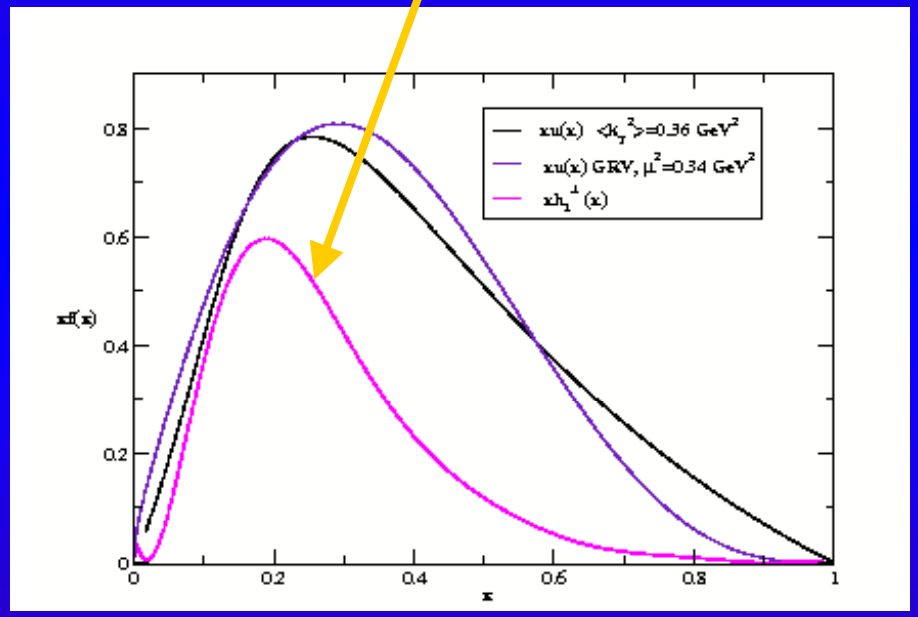

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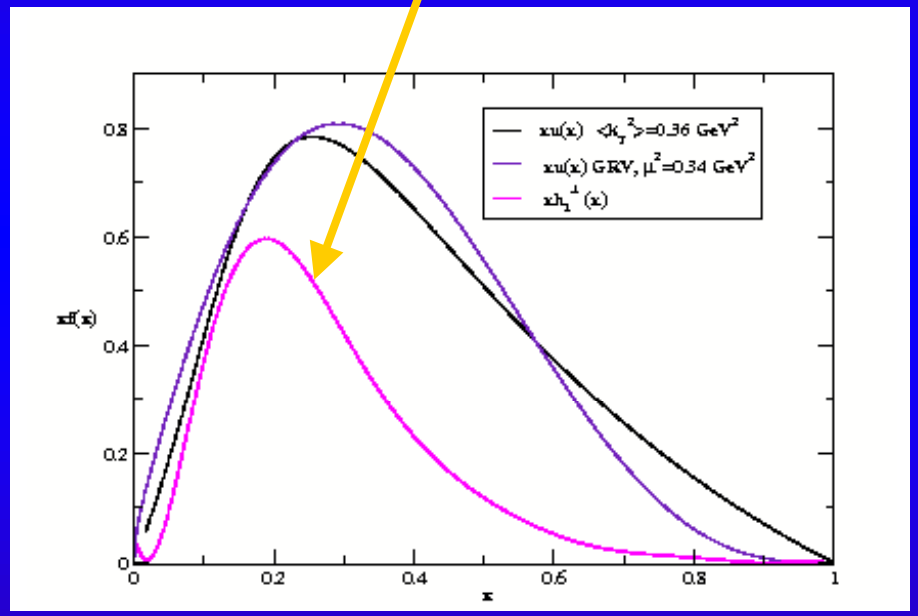
- sign should be reversed because of attractive FSI



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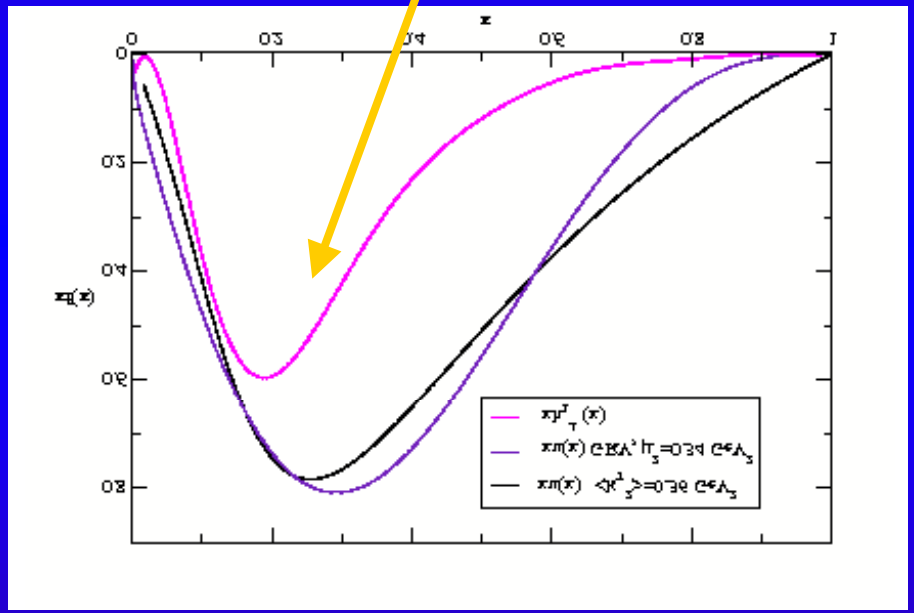
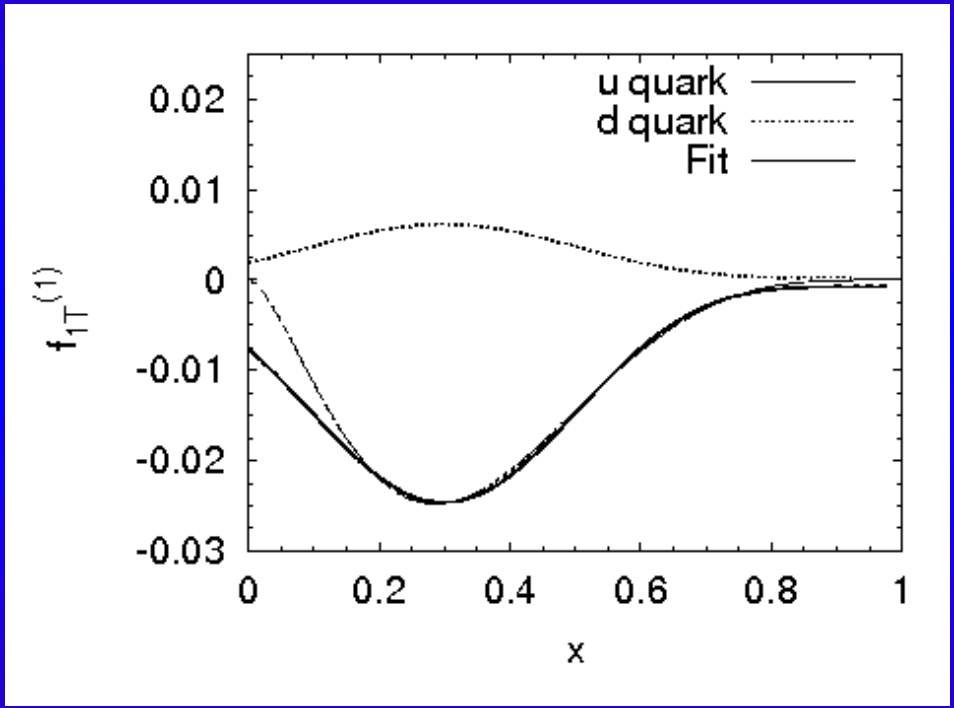


x

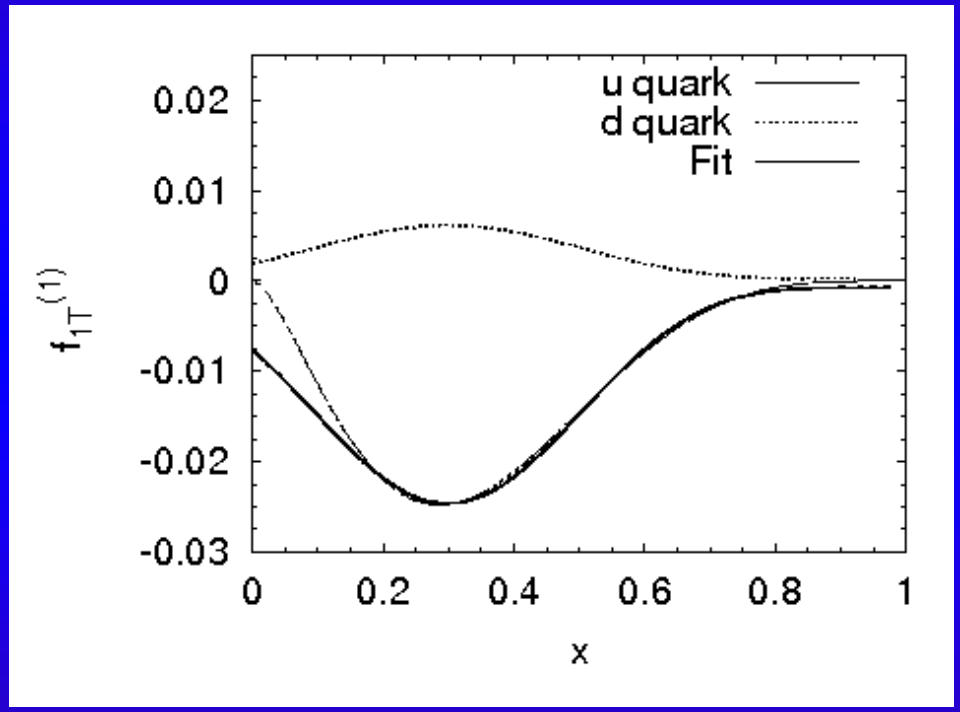
Gamberg, talk at DIS2005

Yuan, PRD67 (03) - MIT bag



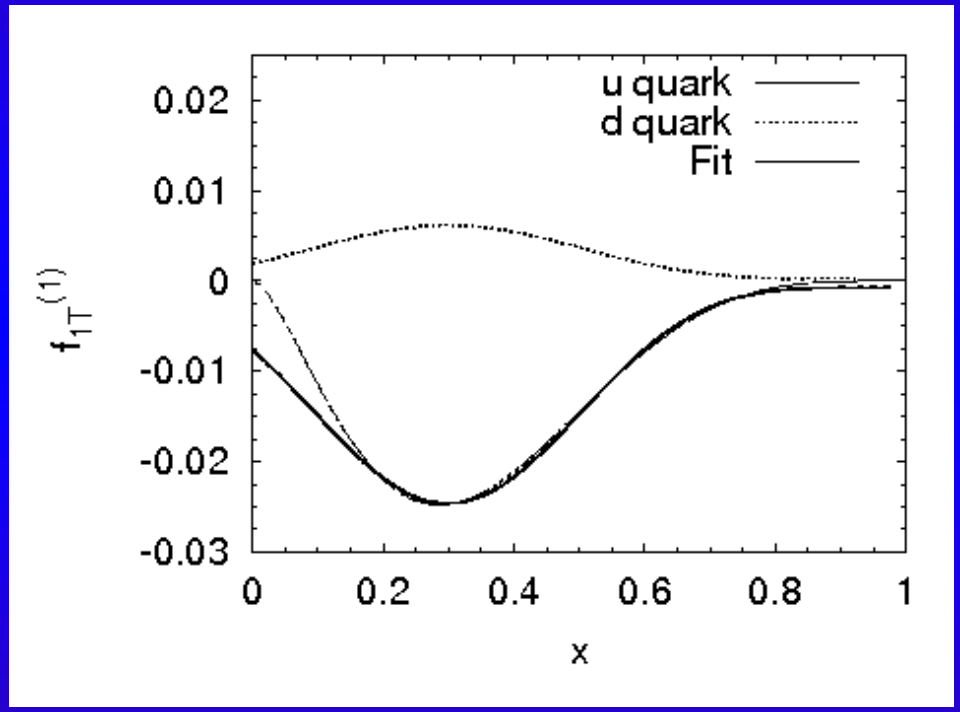
Yuan, PRD67 (03) - MIT bag

- up and down quarks



Yuan, PRD67 (03) - MIT bag

- up and down quarks
- Sivers d = -1/4 Sivers u



Yuan, PRD67 (03) - MIT bag

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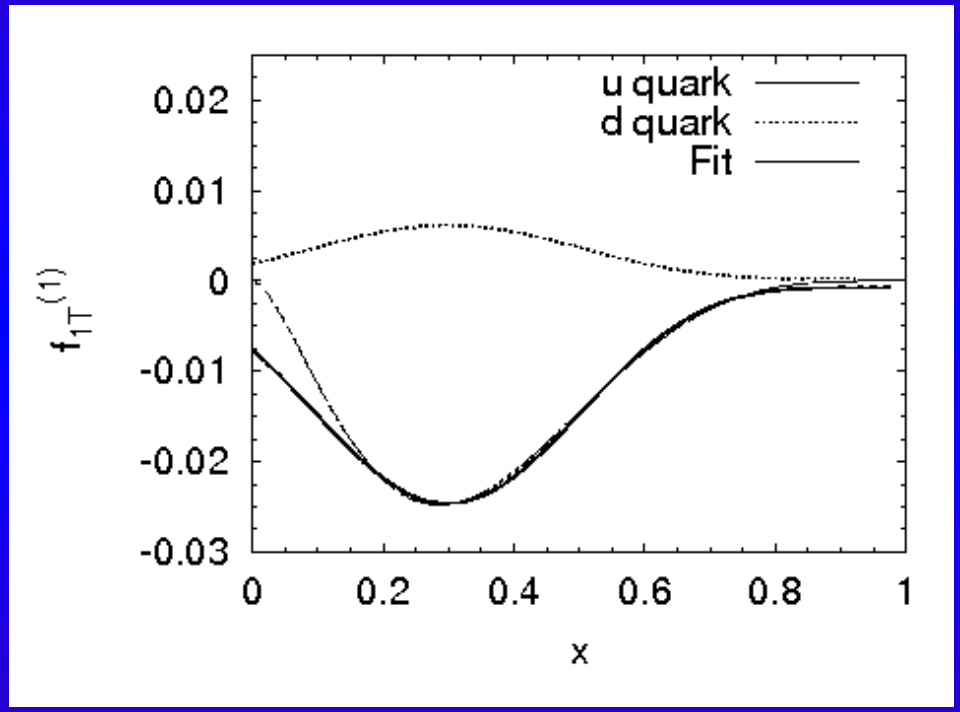
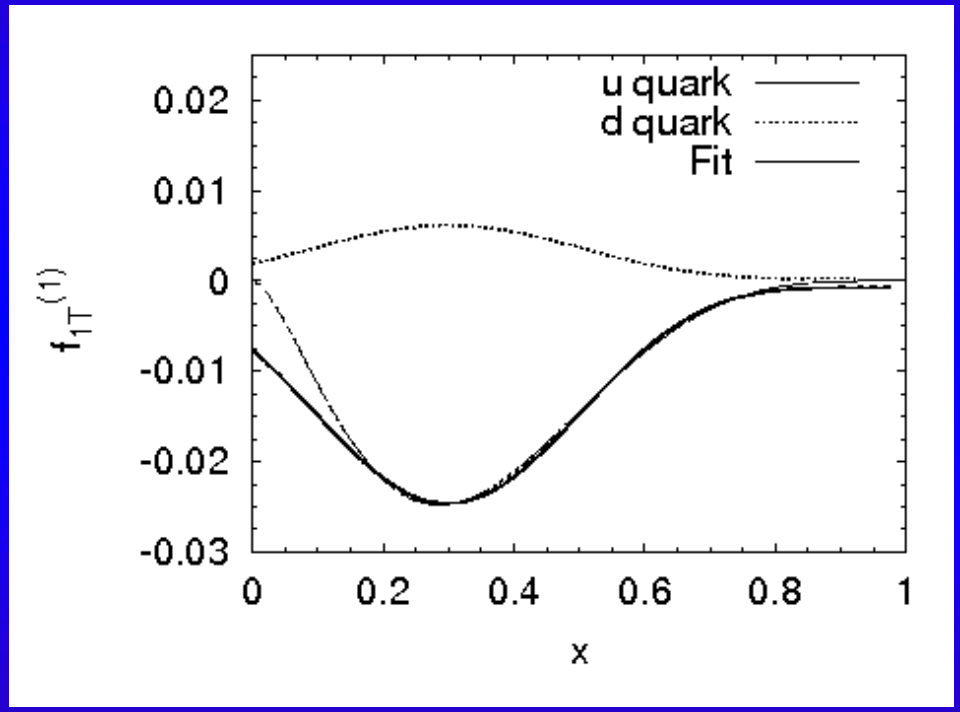


Yuan, PRD67 (03) - MIT bag

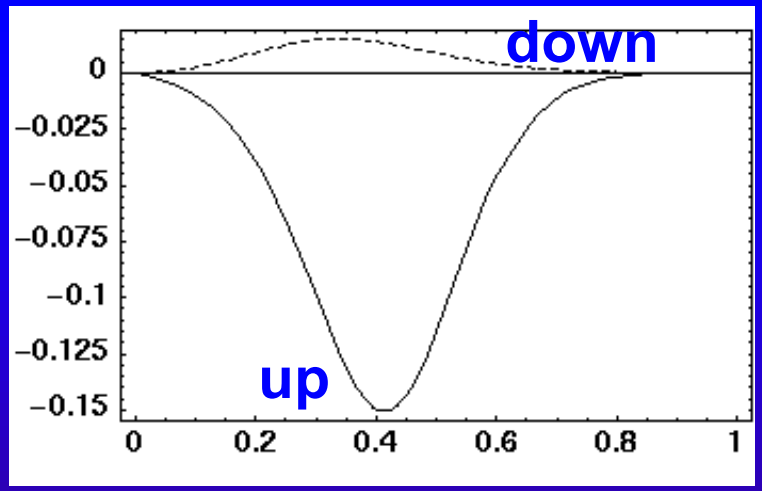
- up and down quarks
- Sivers d = -1/4 Sivers u
- Boer-Mulders u = 3/2 Sivers u
- Boer-Mulders d = -3 Sivers d



A. B., Schäfer, Yang, PLB 578 (04)

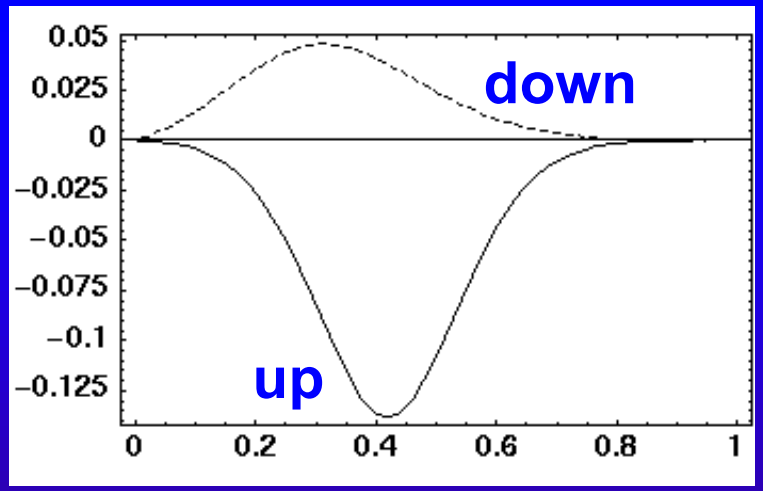
- scalar and vector diquarks,
- up and down quarks
- Sivers d opposite sign and much smaller than Sivers u
- Boer-Mulders u = Sivers u, Boer-Mulders d ≈ Sivers d

$xf_{1T}^{\perp(1/2)}$



x

$xh_1^{\perp(1/2)}$

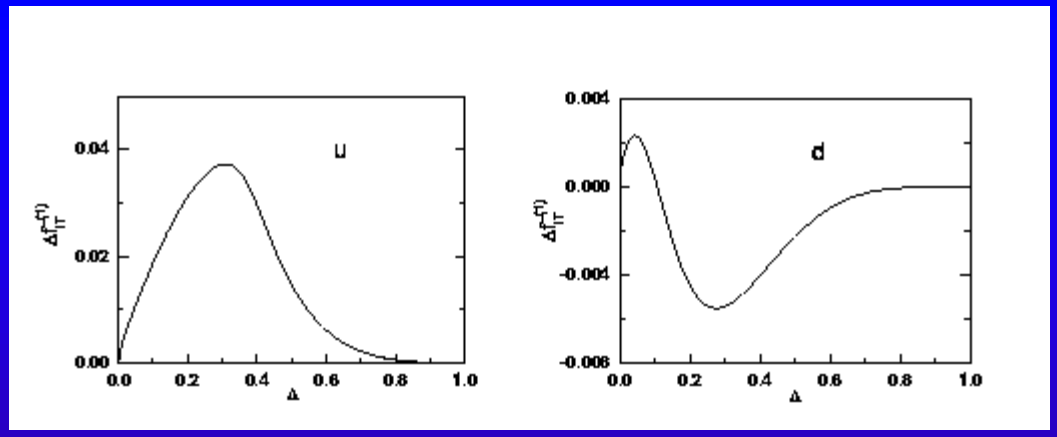


x

Lu, Ma, NPA 741 (04)

- scalar and vector diquarks
- up and down quarks
- Sivers d opposite sign and much smaller than Sivers u
- sign difference due to usual problem

$f_{1T}^{\perp(1)}$



x

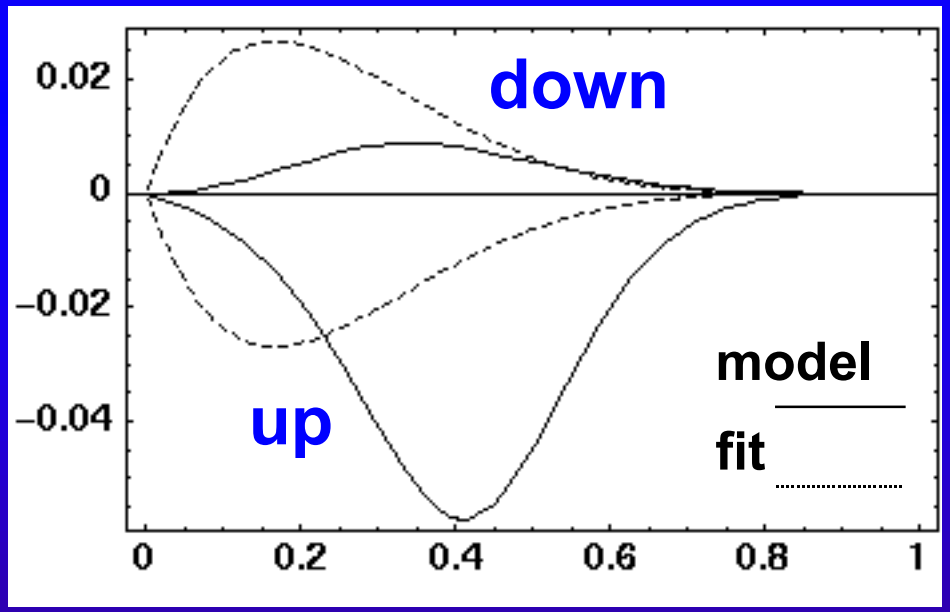
Comparison with phenomenology

Data: *HERMES, PRL94 (05)*

Fit from: *Efremov et al, hep-ph/0412353*

More sofisticated fit: *Anselmino et al, hep-ph/0501196*

$xf_{1T}^{\perp(1)}$



x

Boer-Mulders for pions

- ***Lu, Ma, hep-ph/0504184***
- ***Lu, Ma, PRD 70 (04)***

Conclusions 1

- Models give the same signs for Sivers functions and agree with signs of phenom. extractions
- Discrepancy in sign Boer-Mulders down
- Sivers down is significantly smaller than Sivers up in different models, but this looks different from phenomenology
- The size of the functions varies a lot and mostly overestimates phenomenology

To be done

- Check sign of Boer-Mulders down
- Investigate REAL parts of one gluon amplitudes
- Introduce a second loop?
- Gluon Sivers function

T-odd fragmentation functions

The correlation function

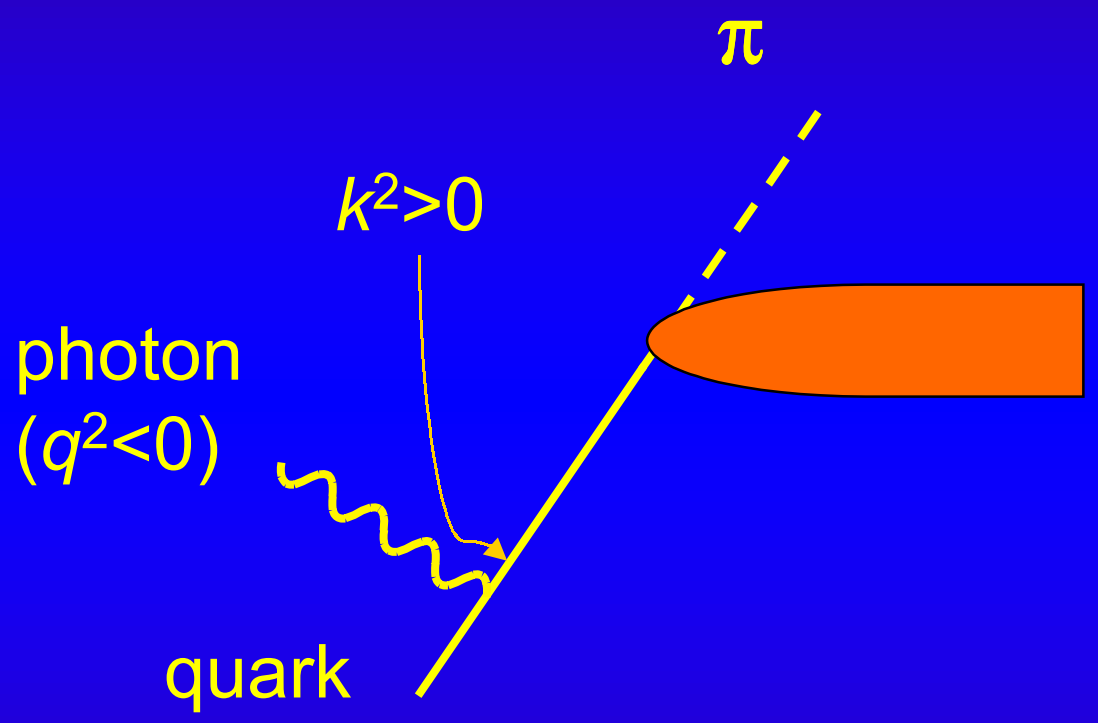
$$\Delta \simeq \frac{1}{4}\left(D_1 + i\, H_1^{\perp} \frac{\not{k}_T}{M_h} \right)\gamma^+$$

Collins

T-odd

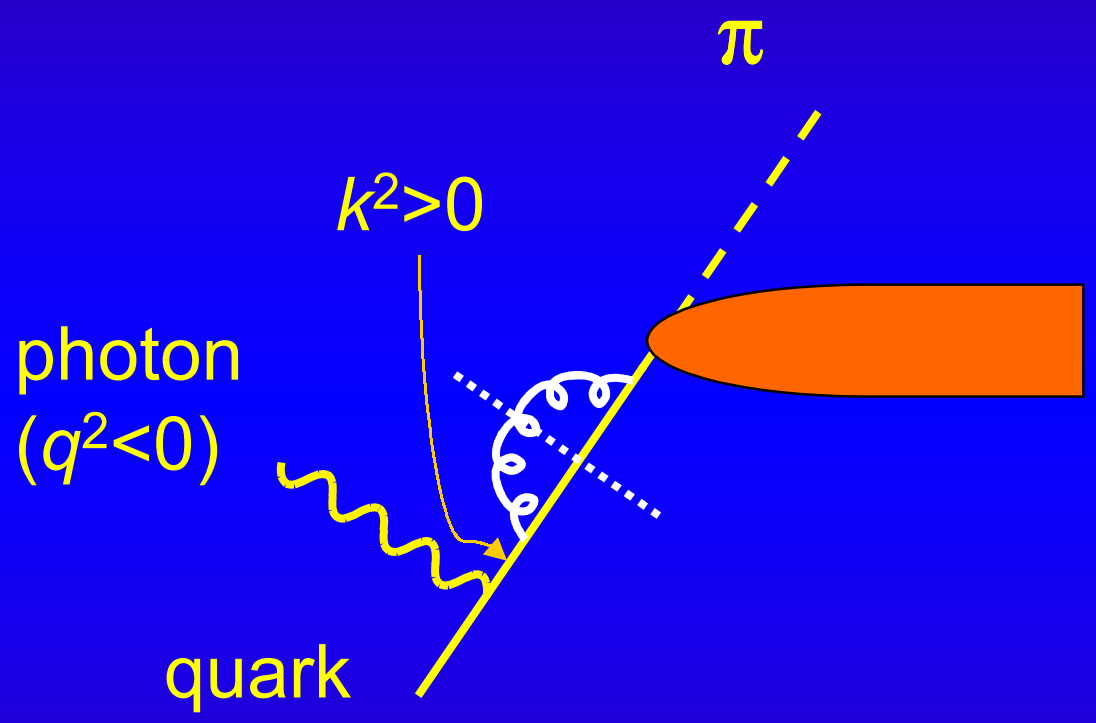
$$\left(i\, H_1^{\perp} \frac{\not{k}_T}{M_h} \gamma^+ \right)^* = -i\, \gamma^1 \gamma^3 \left(i\, H_1^{\perp} \frac{(-\not{k}_T)}{M_h} \gamma^- \right) i \gamma^1 \gamma^3$$

Imaginary parts in SIDIS



A.B, Metz, Yang, PLB574 (03)
Amrath, A. B., Metz, hep-ph/0504124

Imaginary parts in SIDIS



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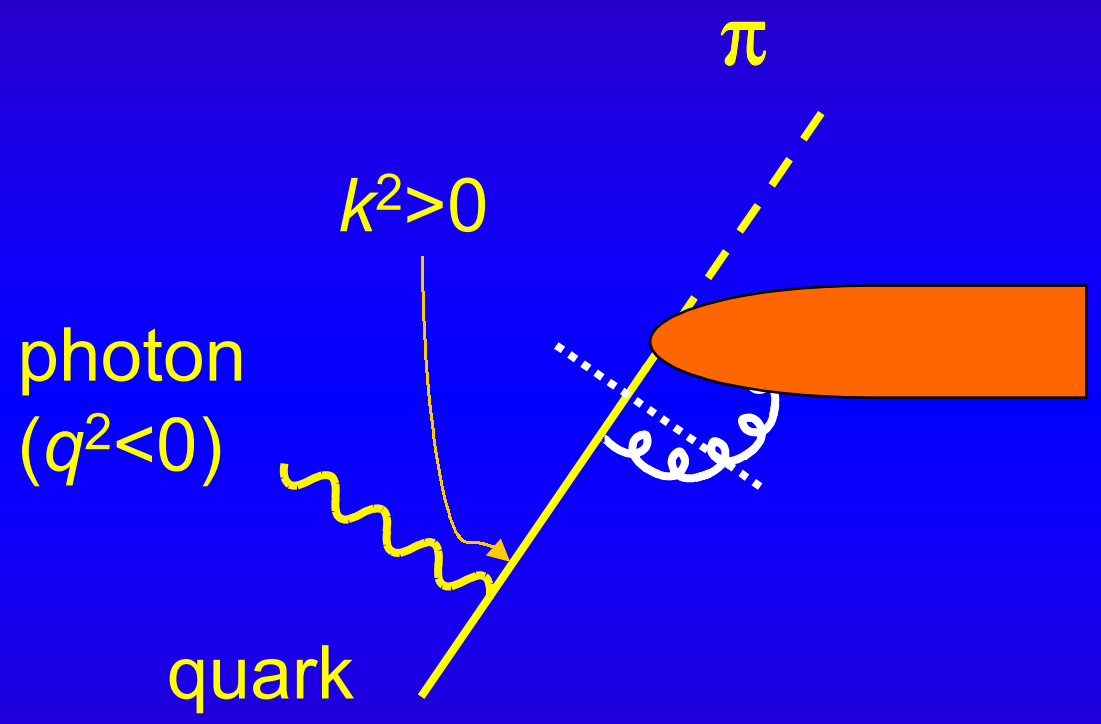

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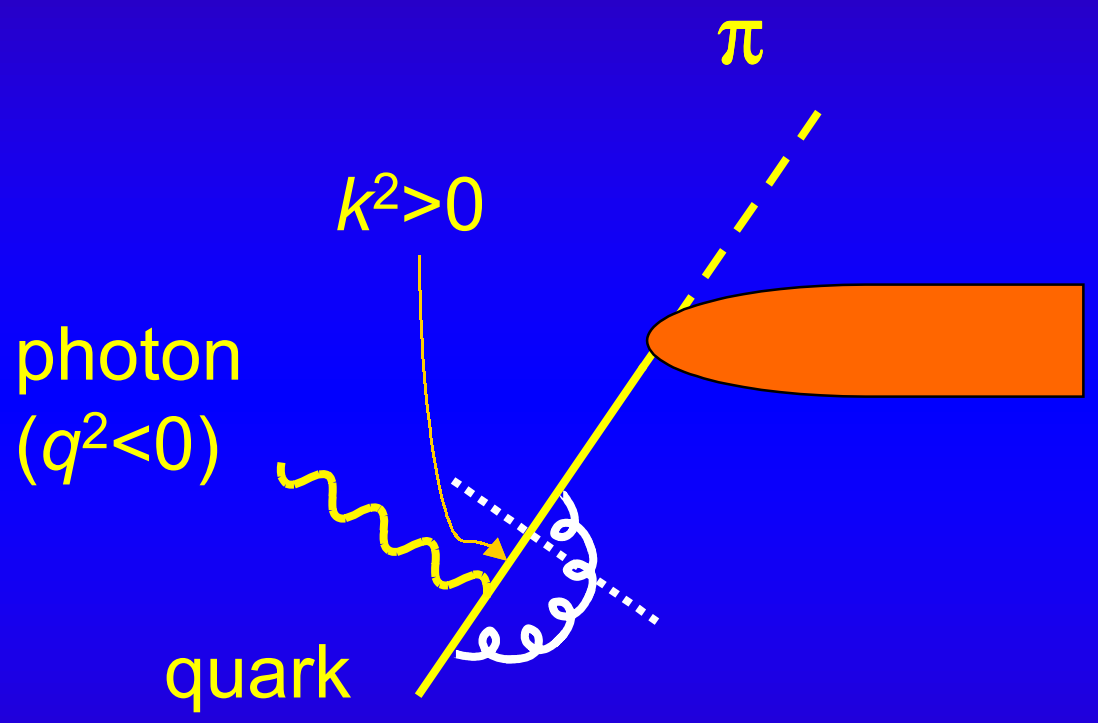

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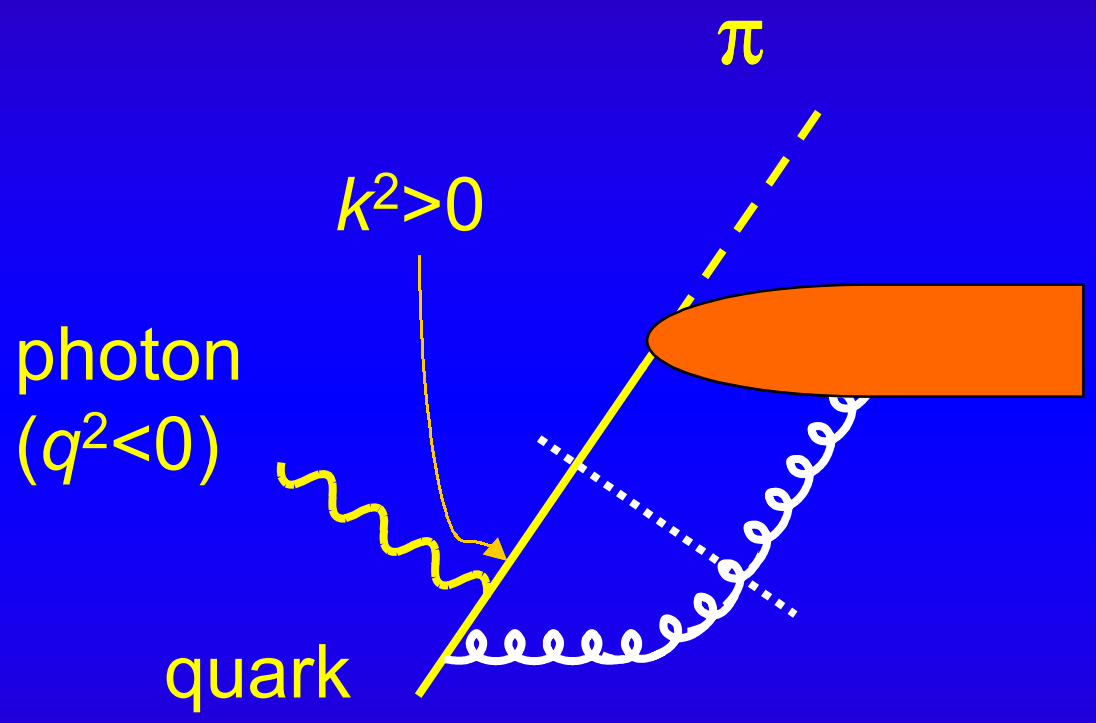
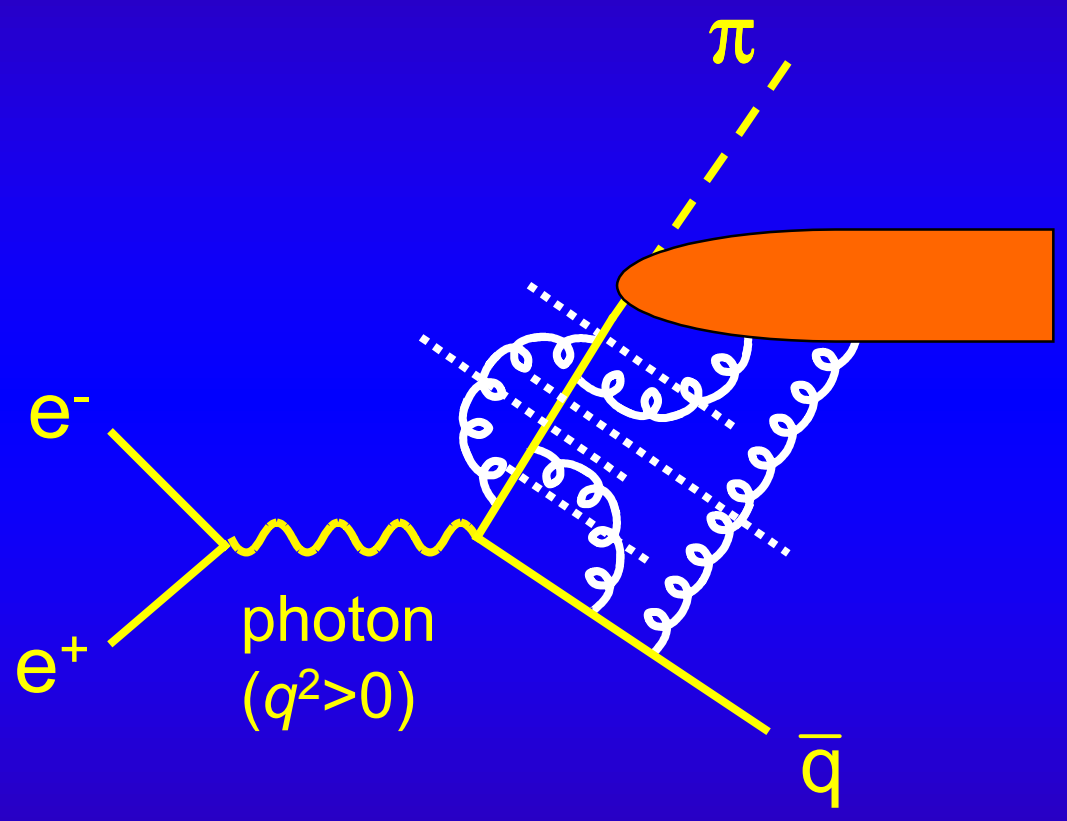
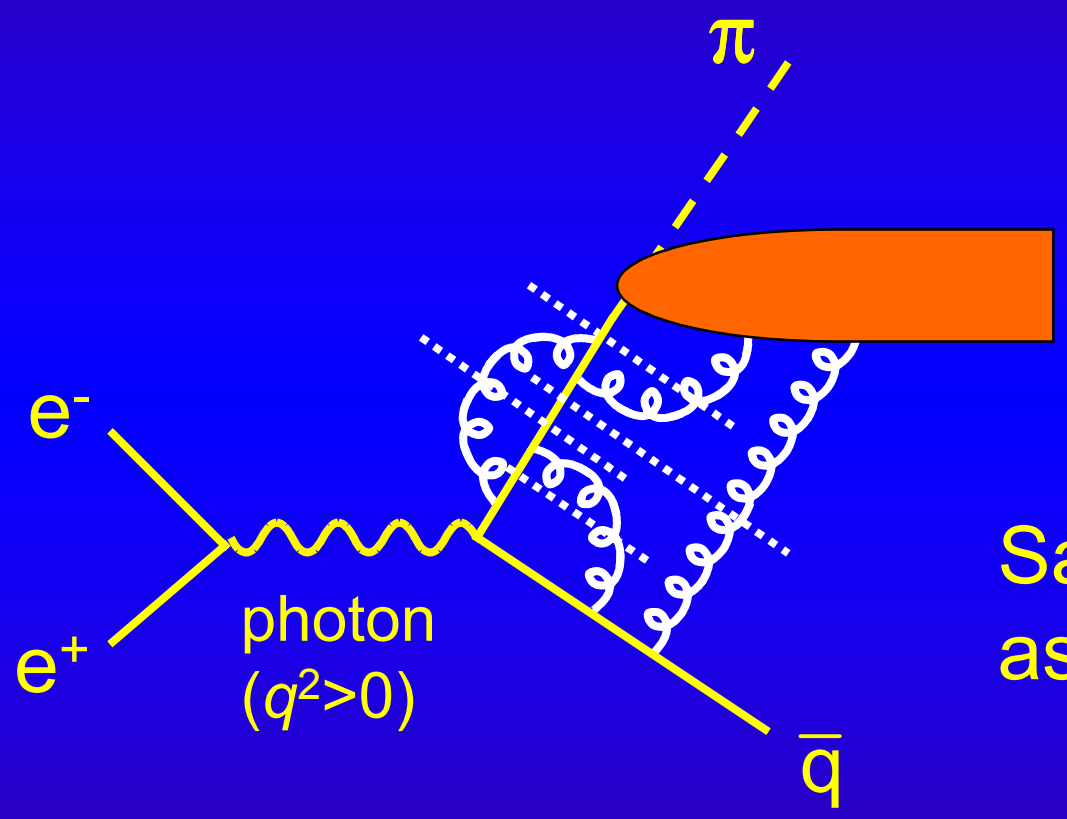


A.B, Metz, Yang, PLB574 (03)
Amrath, A. B., Metz, hep-ph/0504124

Imaginary parts in annihilation

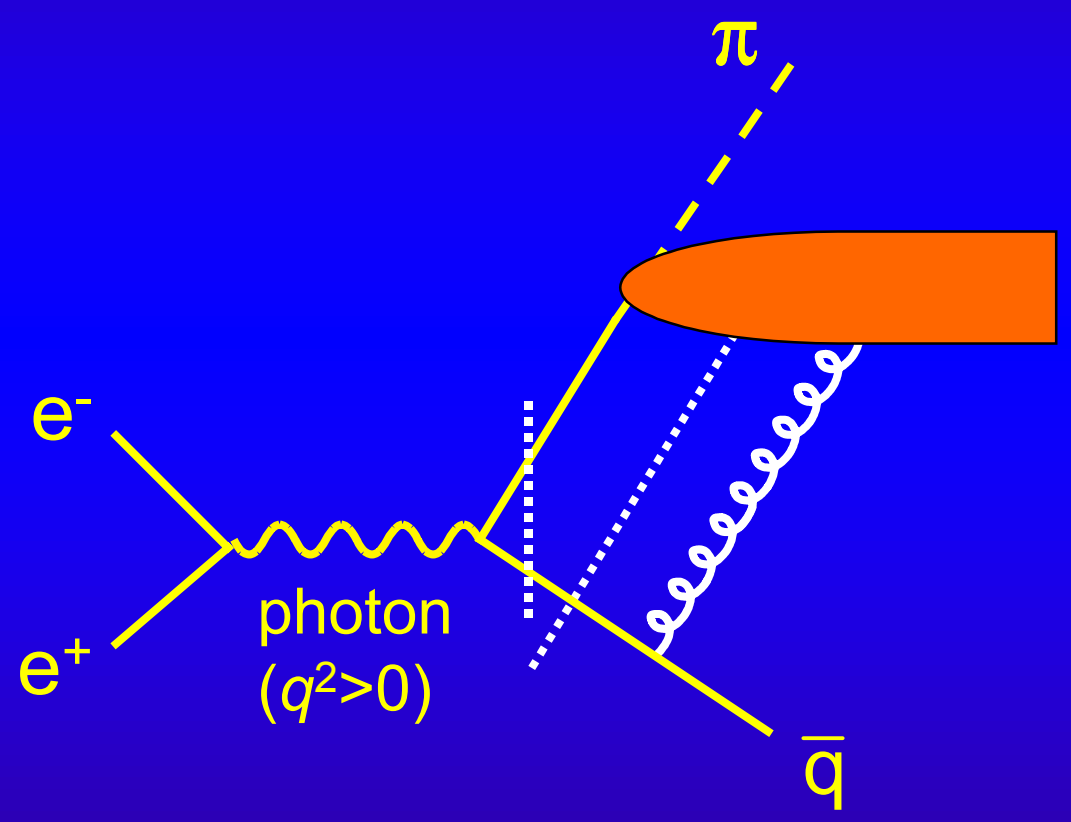


Imaginary parts in annihilation



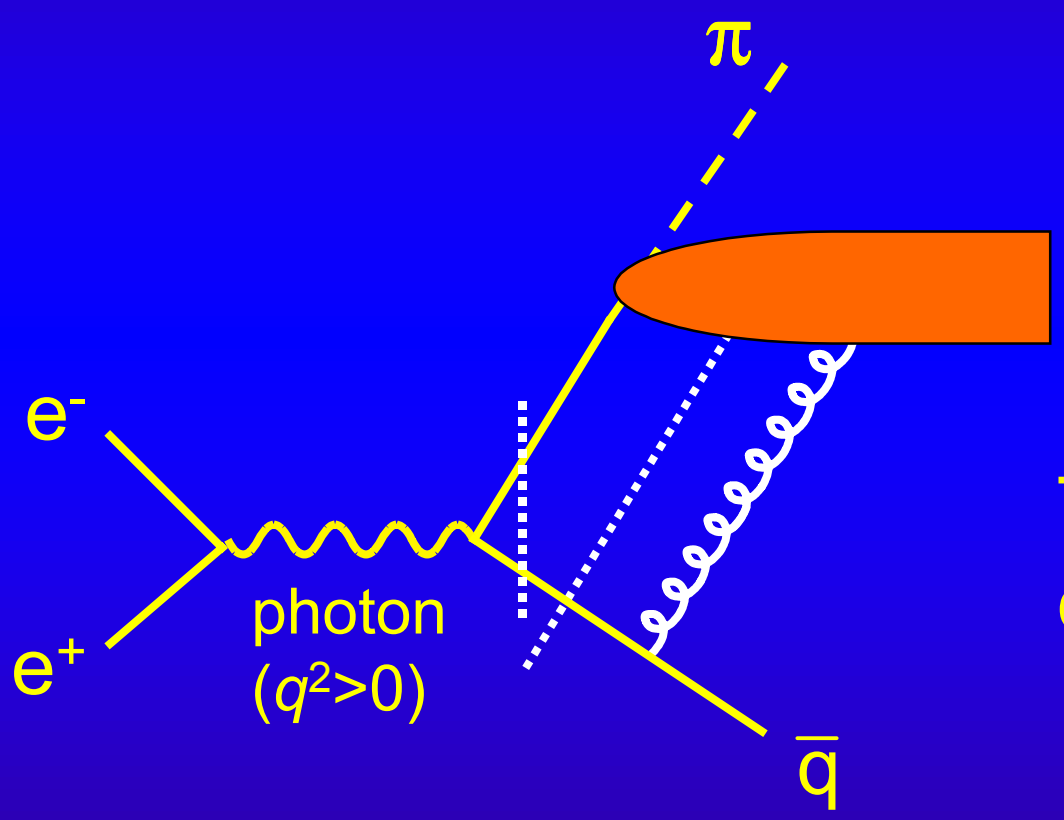
Same contributions as before...

Extra contributions



Metz, PLB 549 (02)

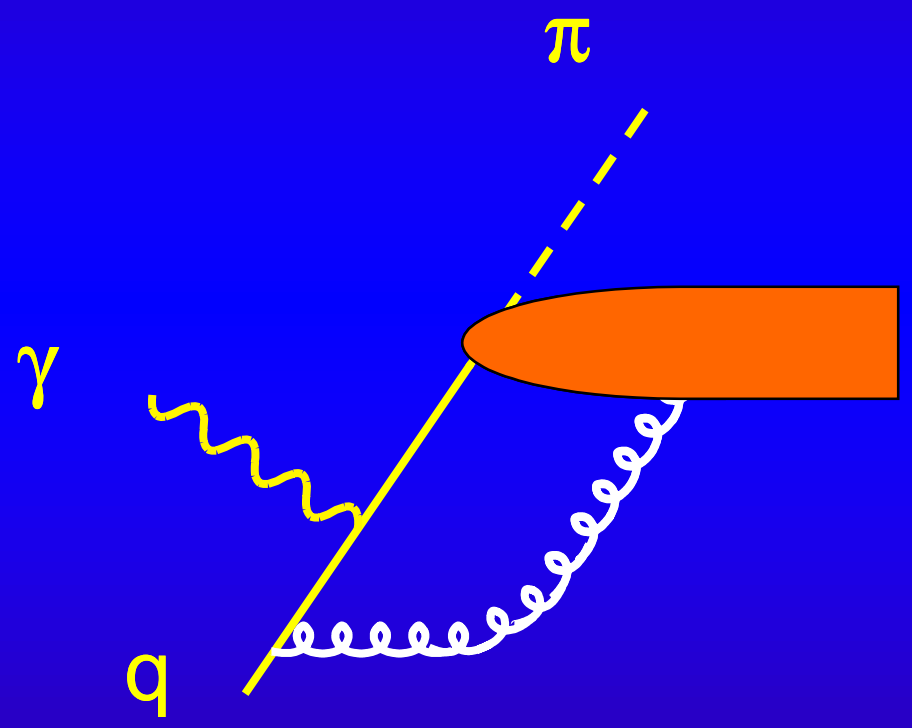
Extra contributions



These two cuts cancel each other!

Metz, PLB 549 (02)

Gauge link



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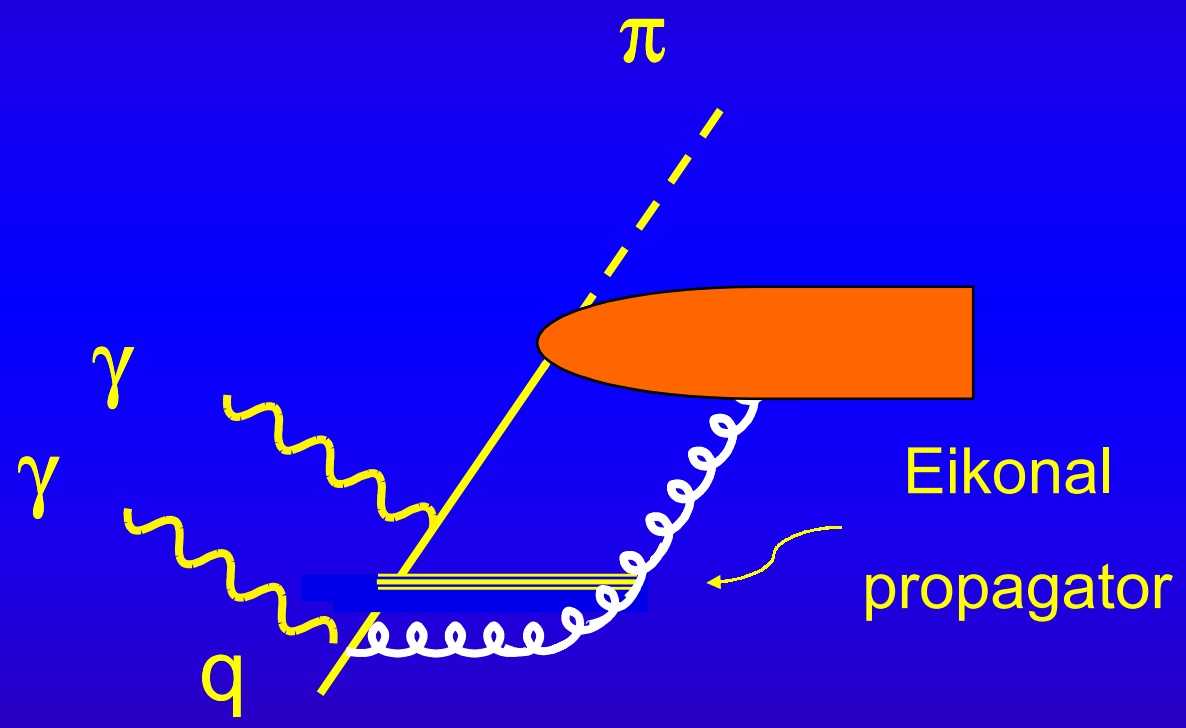
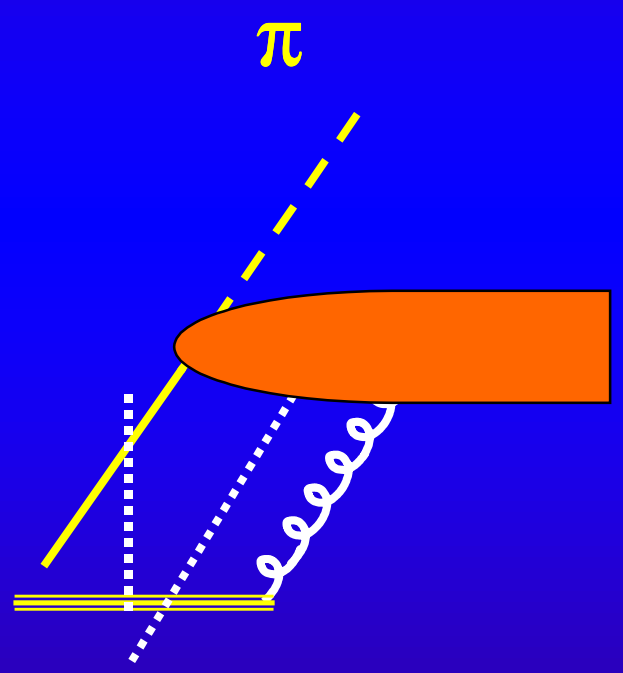
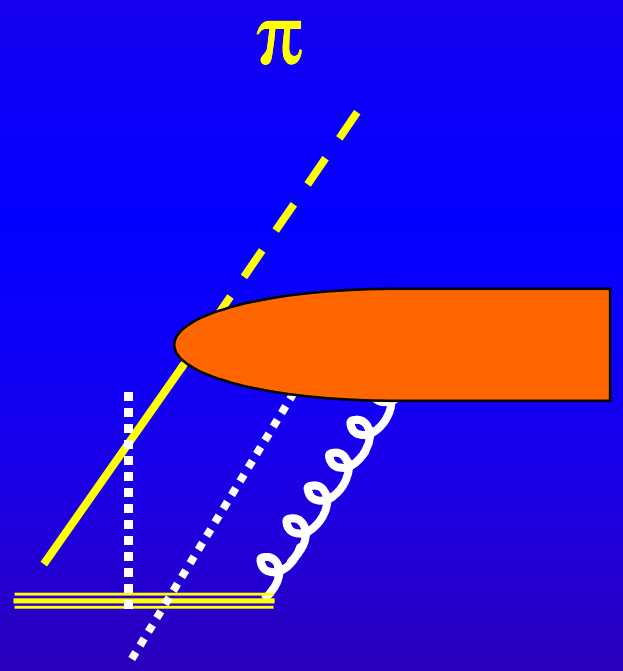


Universality of fragmentation functions



Collins, Metz, PRL 93 (04)

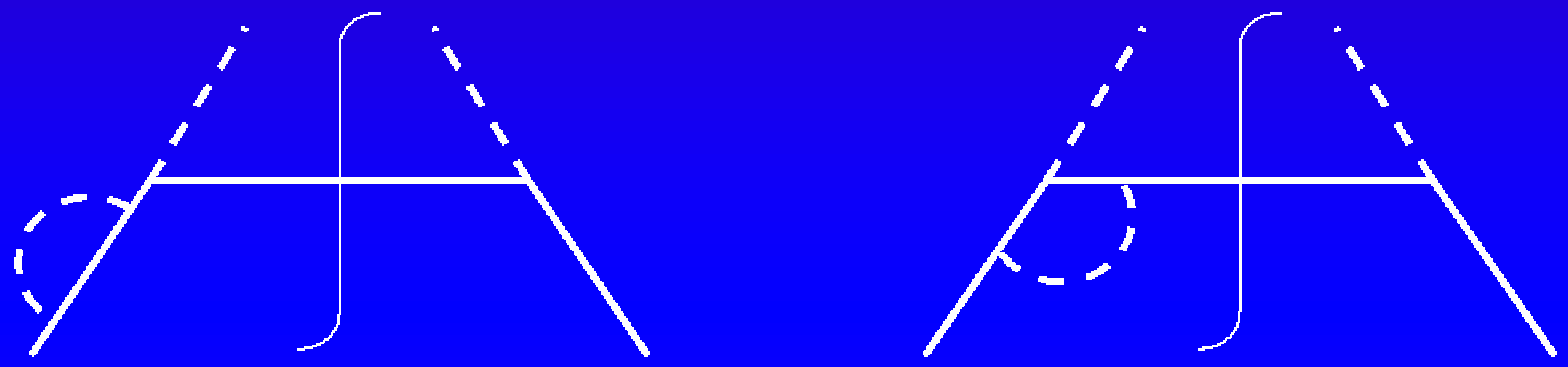
Universality of fragmentation functions



No contribution from imaginary part of eikonal propagator

Collins, Metz, PRL 93 (04)

Non-gluonic loops



The first ones to be ever calculated!

A.B., Kundu, Metz, Mulders, PLB506 (01)

Model results

Literature

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- ***Amrath, A. B., Metz, hep-ph/0504124***
  - review the above and correct errors

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  - pion loops, pseudovector pion-quark coupling
- ***Gamberg, Goldstein, Oganessyan, PRD68 (03)*** **wrong imaginary part**
  - gluon loops, scalar pion-quark coupling
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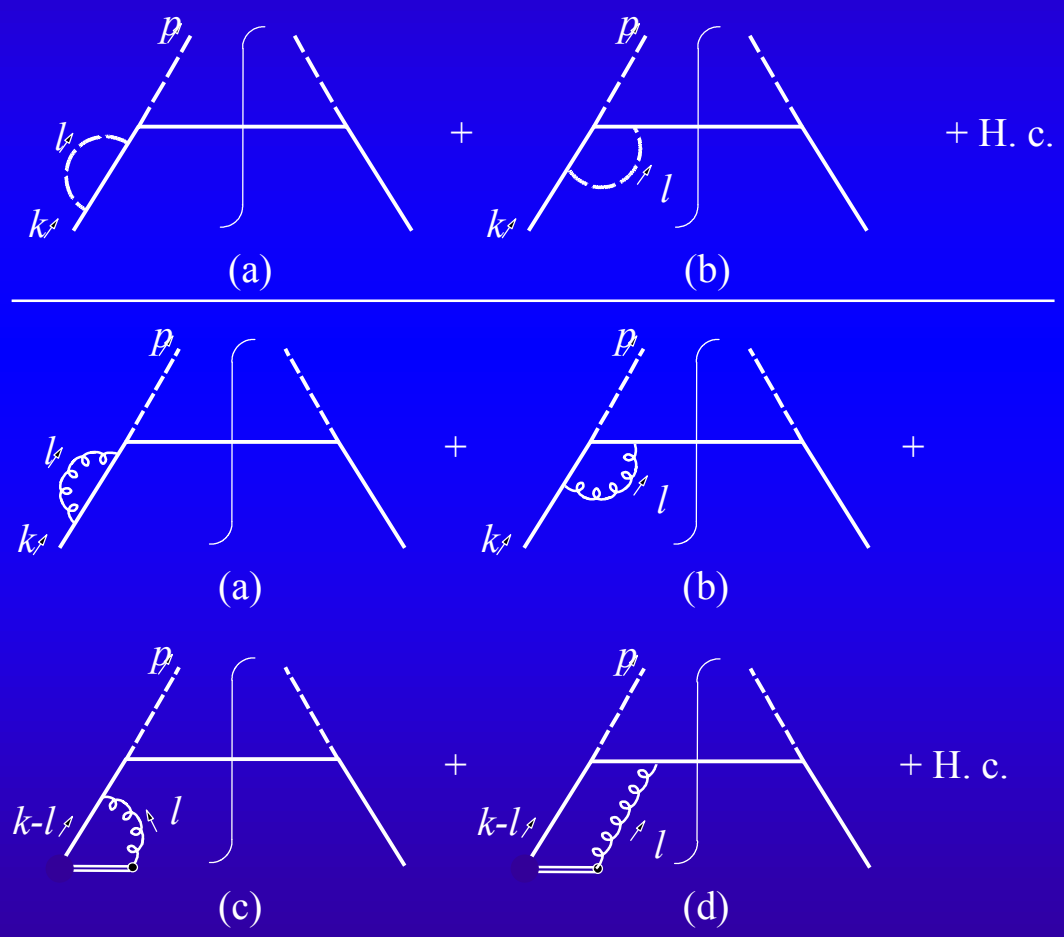
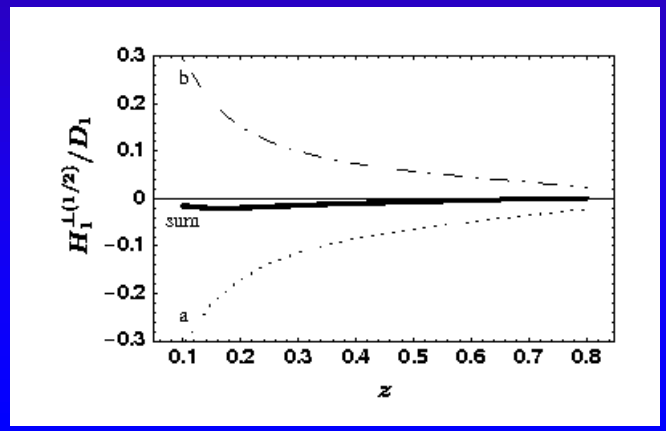
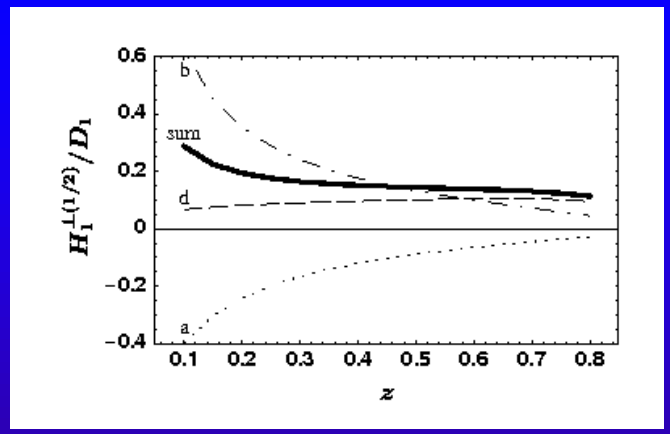
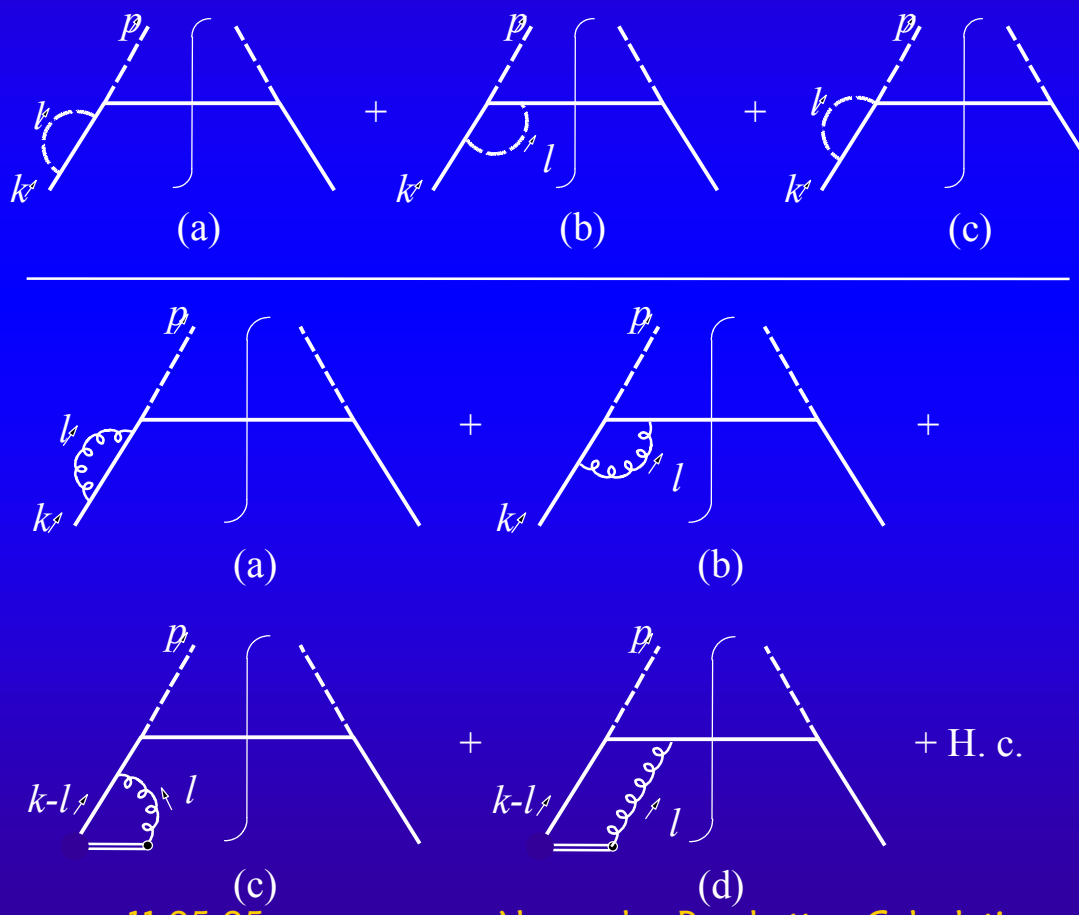
Pseudoscalar coupling

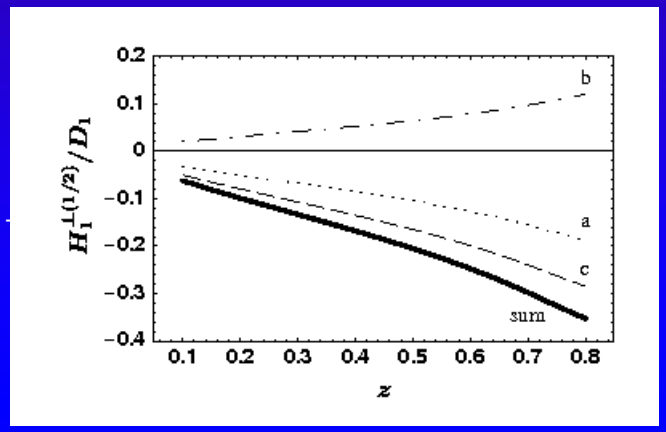


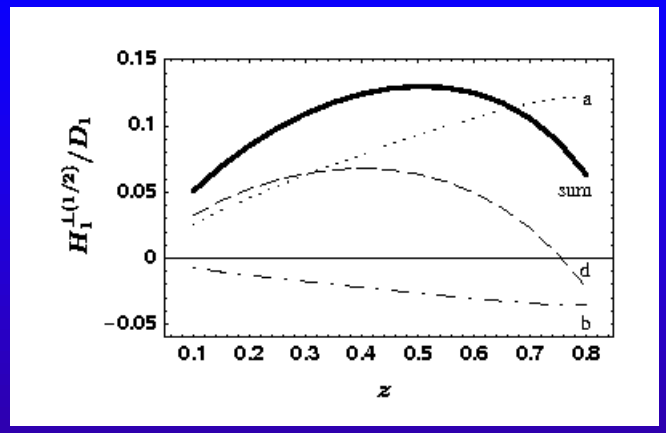




Pseudovector coupling



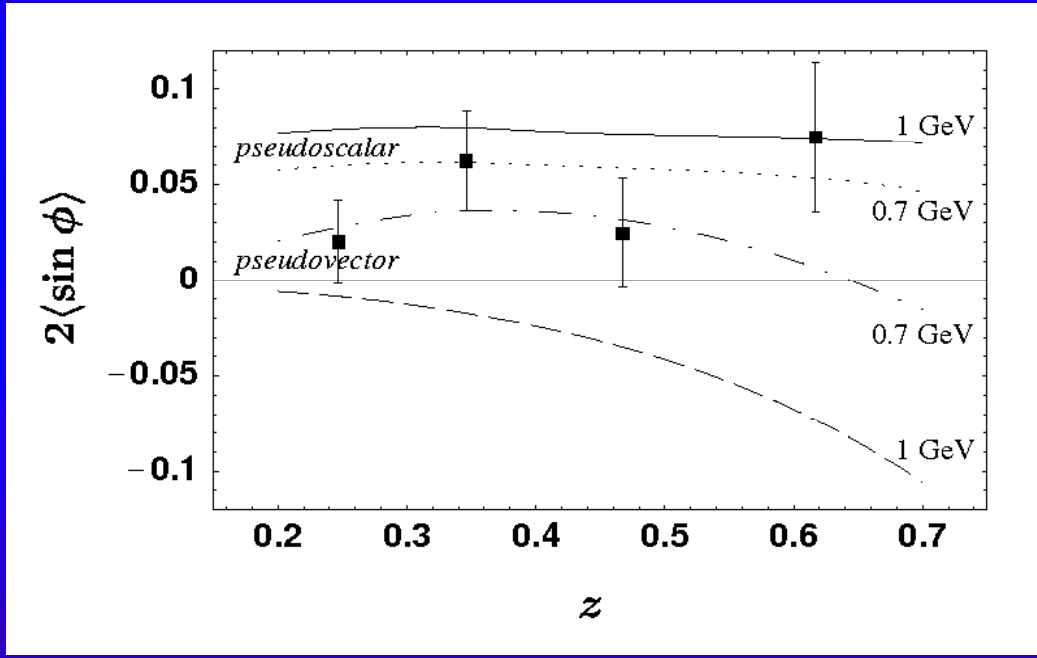




Results

Data: *HERMES, PRL94 (05)*

π^+



Conclusions 2

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- Models can give different signs for Collins function (no semi-classical interpretation)
- Pseudoscalar model: negligible pion contribution, close to phenomenology
- Pseudovector: pion contribution big and wrong sign. Cancellation with gluons brings model closer to phenomenology.

To be done

- Find a way to calculate DISFAVORED Collins function
- Investigate contributions from real parts of the diagrams
- Play around with form factors