# SSAs in hadron-hadron collisions and TMDs

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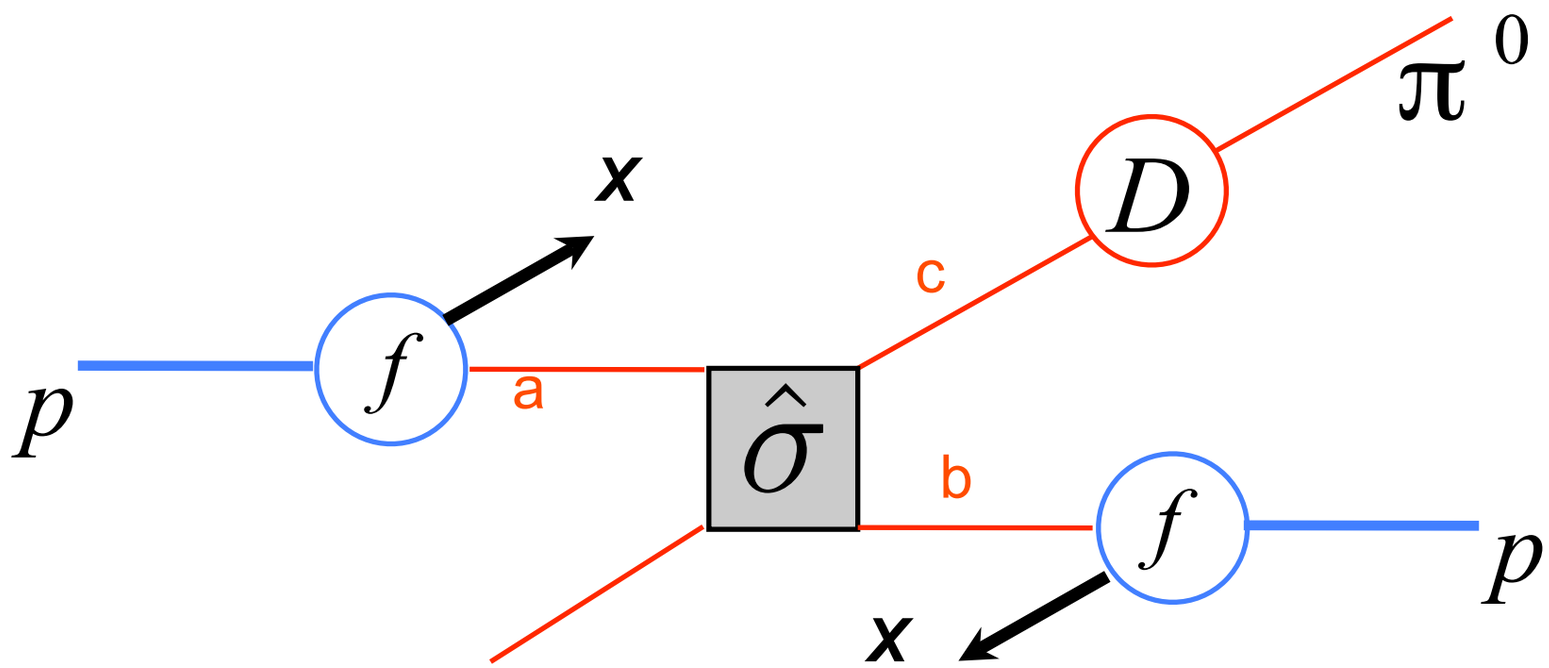
based on work with M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia, A. Prokudin

Progress in High-$p_T$ Physics at RHIC
RBRC Workshop, 17-19 March 2010

# TMDs and SSAs in hadronic processes

Cross section for $p\,p \to \pi^0\,X$ in pQCD, only one scale, $P_T$

based on factorization theorem
(in collinear configuration)

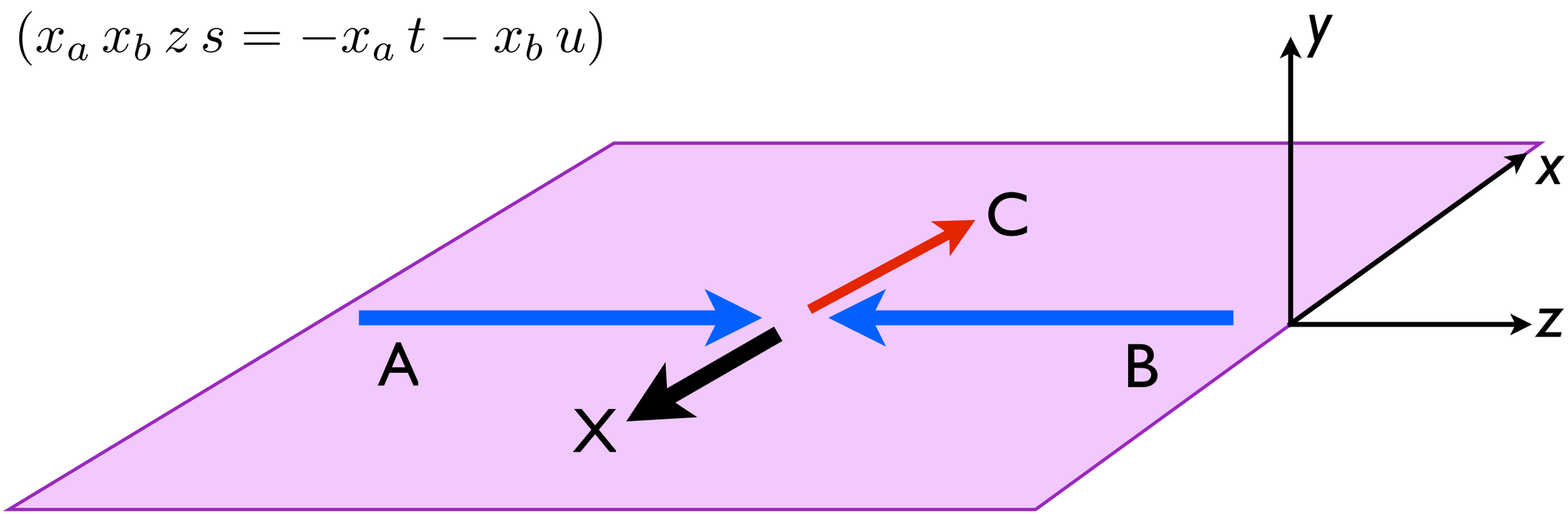



$$\mathrm{d}\sigma = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{f_{a/p}(x_a) \otimes f_{b/p}(x_b)}_{\text{PDF}} \otimes \mathrm{d}\hat{\sigma}^{ab\to cd} \otimes \underbrace{D_{\pi/c}(z)}_{\text{FF}}$$

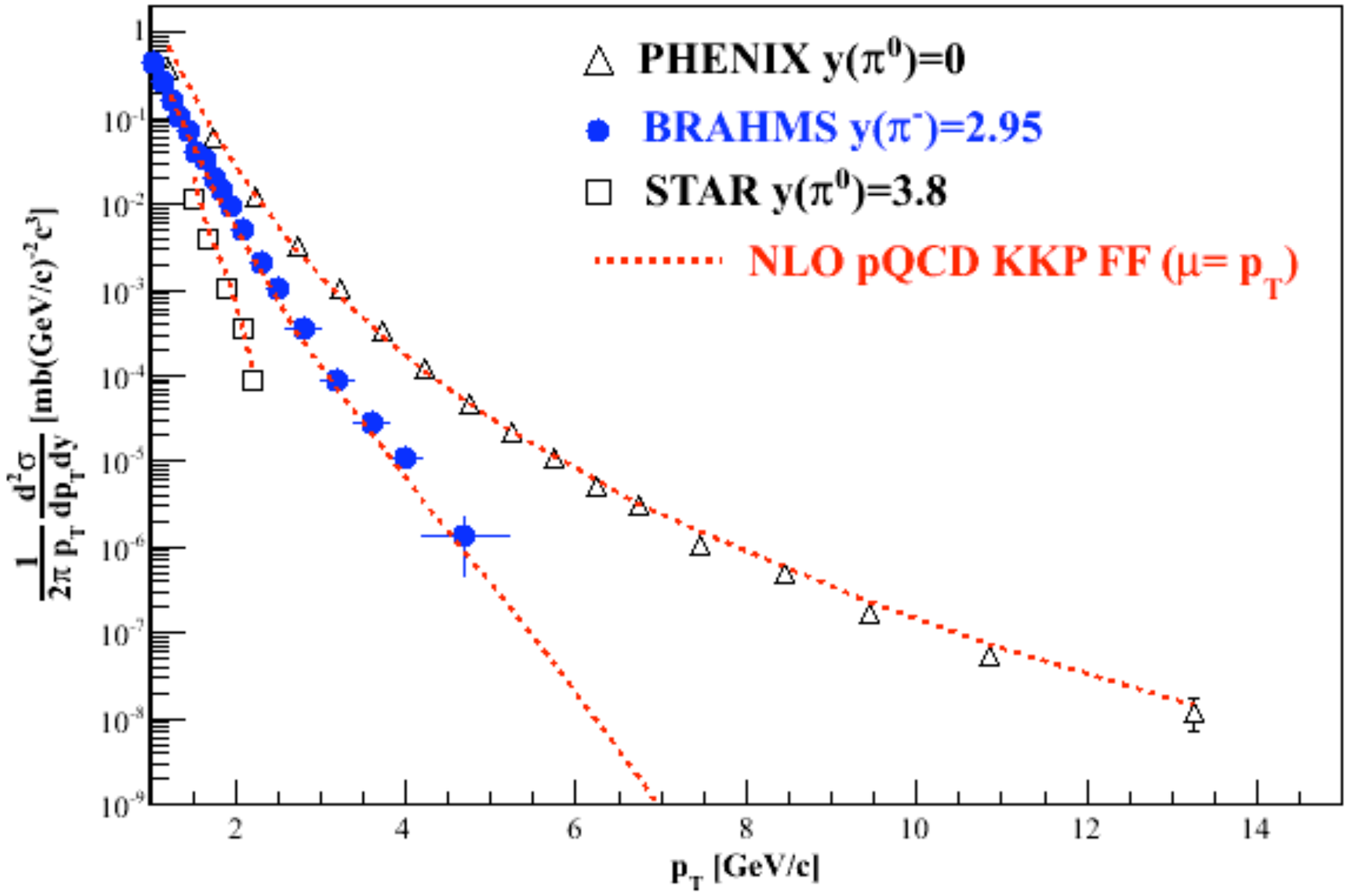
pQCD elementary interactions

$$
\begin{aligned}
\frac{E_C\, d\sigma^{AB\to CX}}{d^3\boldsymbol{p}_C} &= \sum_{a,b,c,d} \int dx_a\, dx_b\, dz\, f_{a/A}(x_a, Q^2)\, f_{b/B}(x_b, Q^2) \\
&\times \frac{\hat{s}}{\pi z^2}\, \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b)\, \delta(\hat{s} + \hat{t} + \hat{u})\, D_{C/c}(z, Q^2) \\
&= \sum_{a,b,c,d} \int dx_a\, dx_b\, f_{a/A}(x_a, Q^2)\, f_{b/B}(x_b, Q^2) \\
&\times \frac{1}{\pi z}\, \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b)\, D_{C/c}(z, Q^2)
\end{aligned}
$$

$$(x_a\, x_b\, z\, s = -x_a\, t - x_b\, u)$$

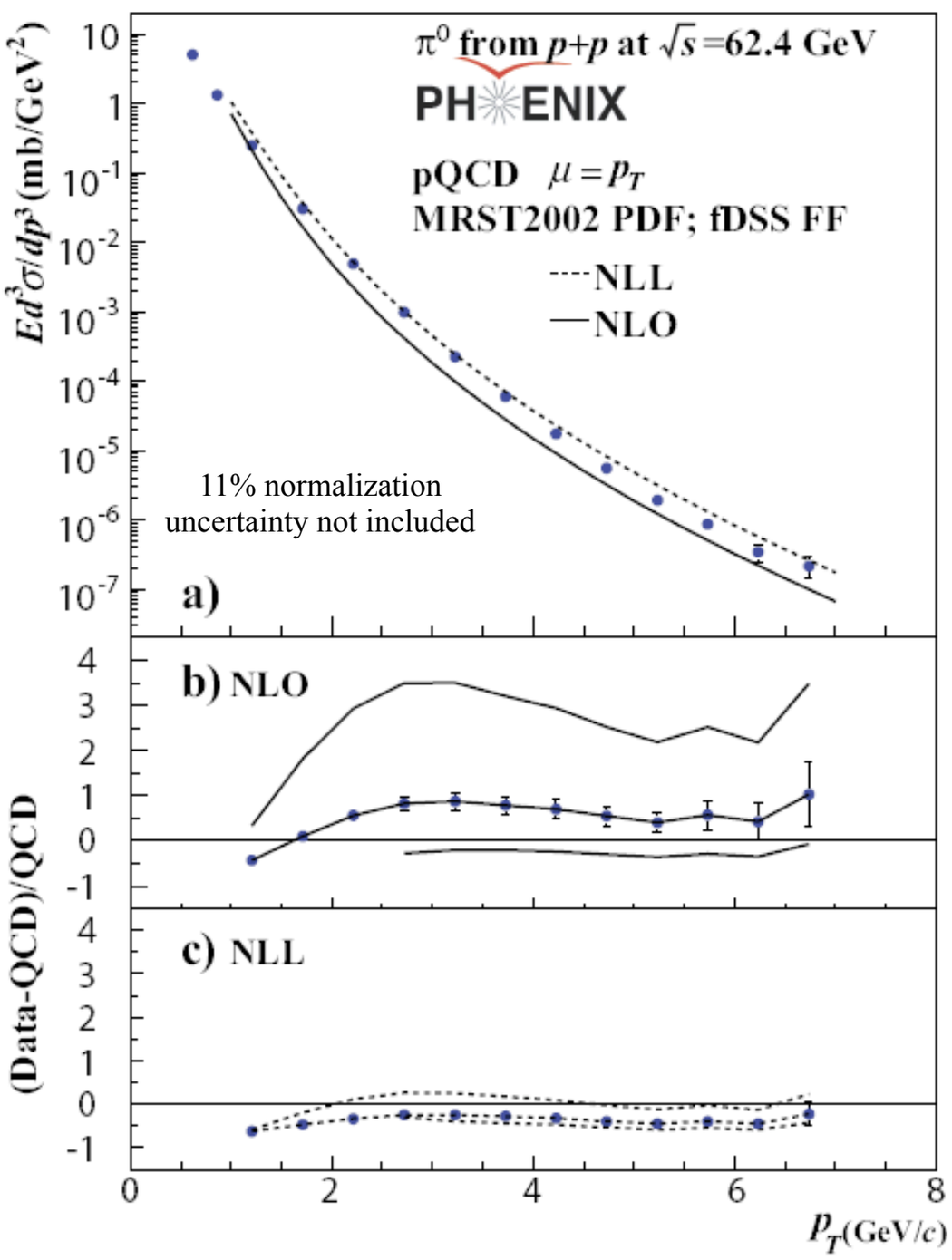


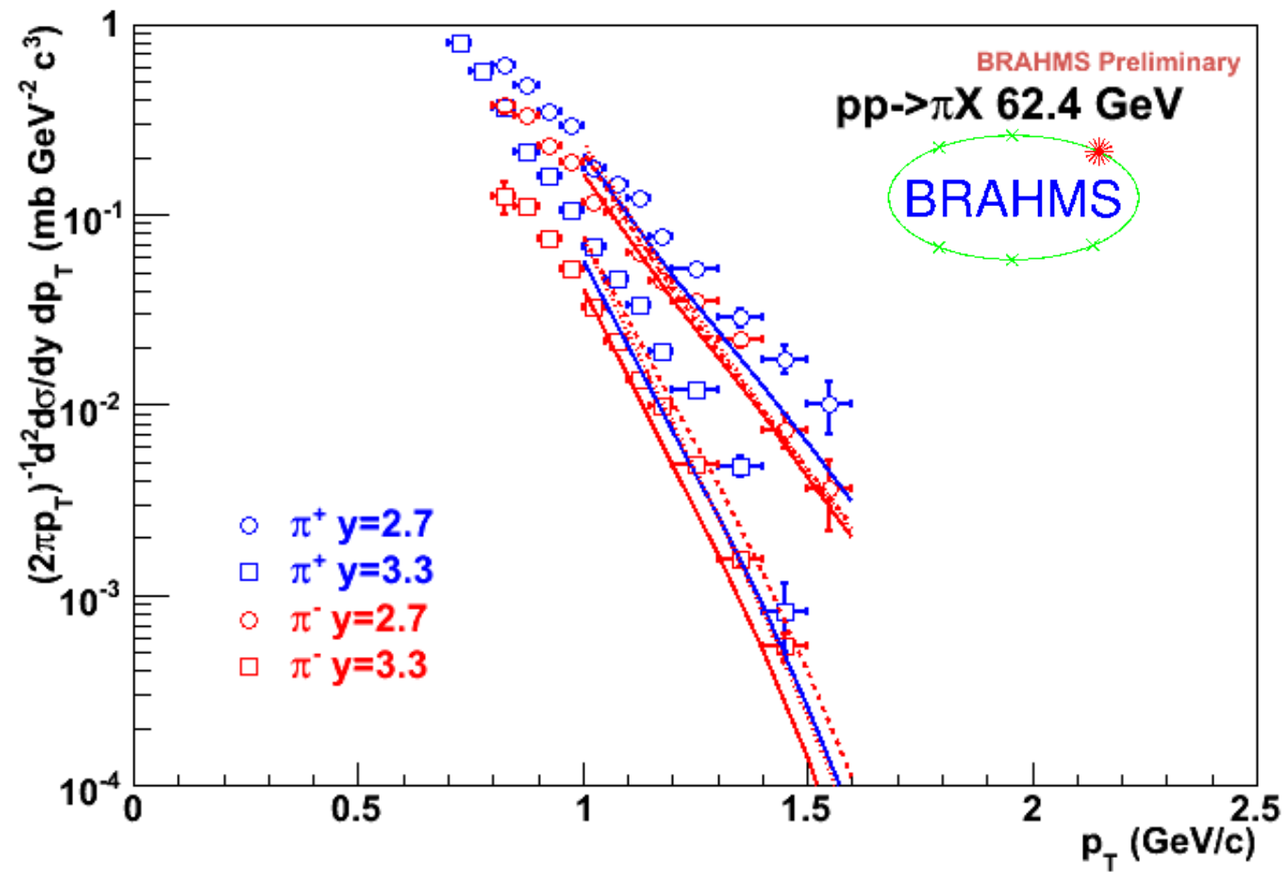
# Polarization-averaged cross sections at $\sqrt{s}$=200 GeV




good pQCD description of data at 200 GeV, at all rapidities, down to $p_T$ of 1-2 GeV/c

# rather good agreement even at √s=62.4 GeV

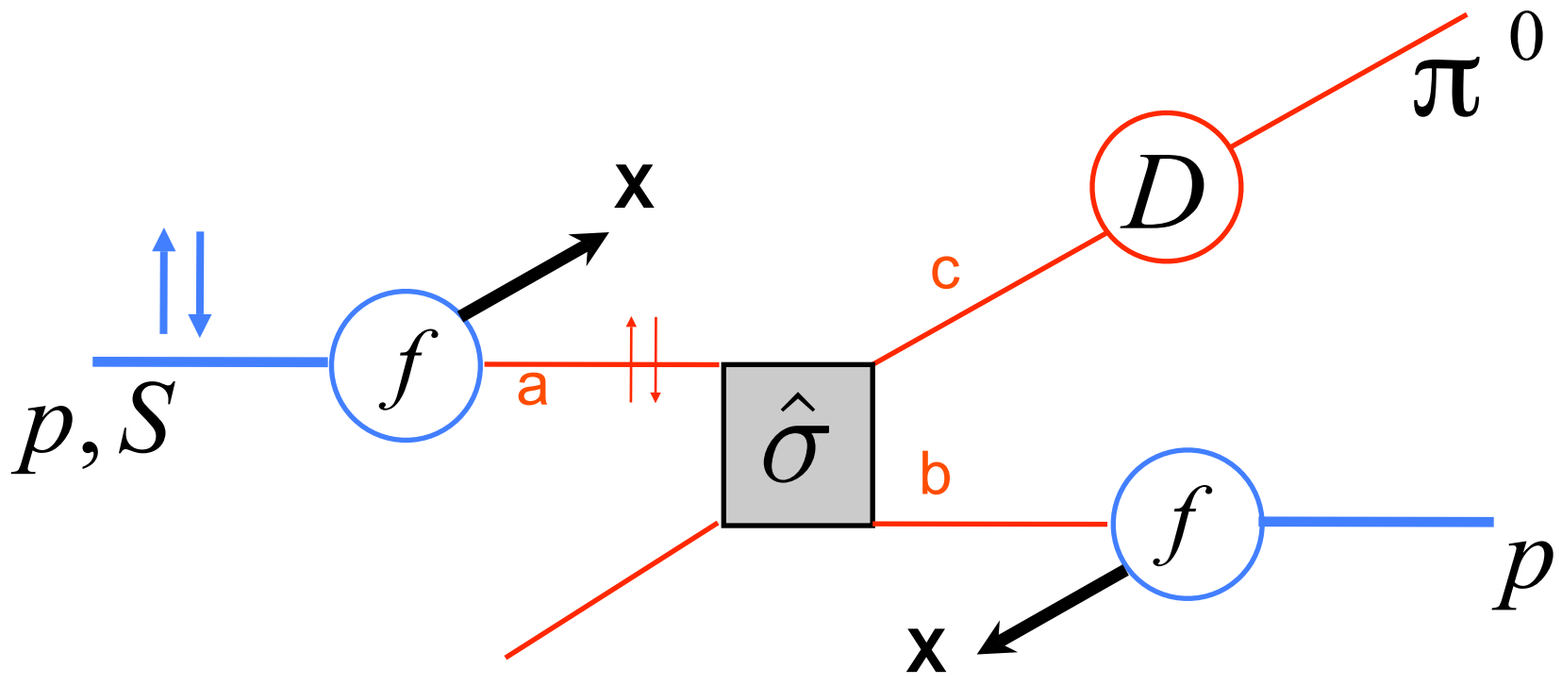



Comparison of NLO pQCD calculations with BRAHMS $\pi$ data at high rapidity. The calculations are for a scale factor of $\mu = p_T$, KKP (solid) and DSS (dashed) . with CTEQ5 and CTEQ6.5.

mid-rapidity pions

# SSA?

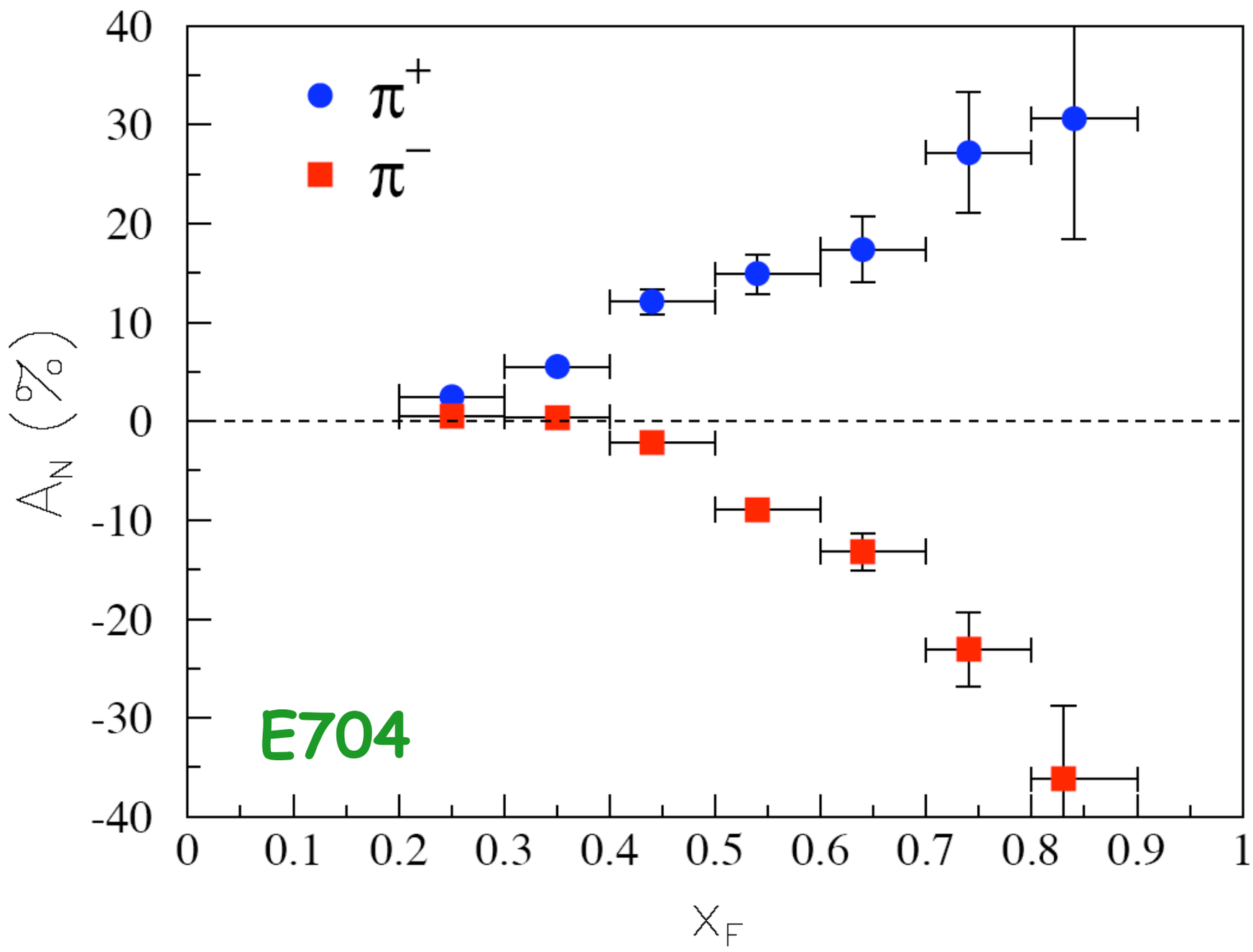



$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{\Delta_T f_a}_{\text{transversity}} \otimes f_b \otimes \underbrace{[\mathrm{d}\hat{\sigma}^{\uparrow} - \mathrm{d}\hat{\sigma}^{\downarrow}]}_{\text{pQCD elementary SSA}} \otimes \underbrace{D_{\pi/c}}_{\text{FF}}$$

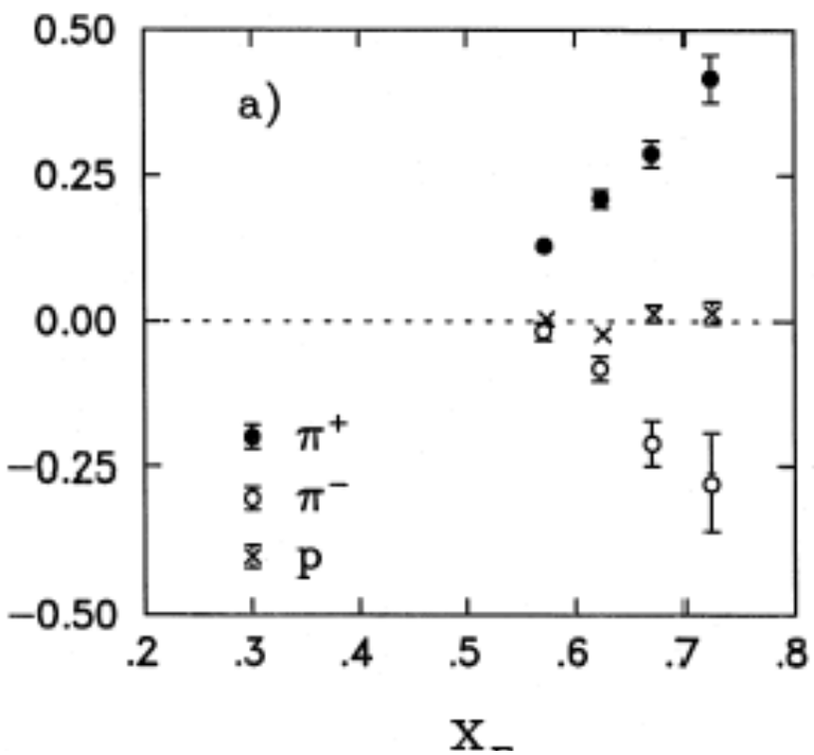
$$A_N = \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} \propto \hat{a}_N \propto \frac{m_q}{E_q}\,\alpha_s$$

was considered almost a theorem

# but, .... where it all started from



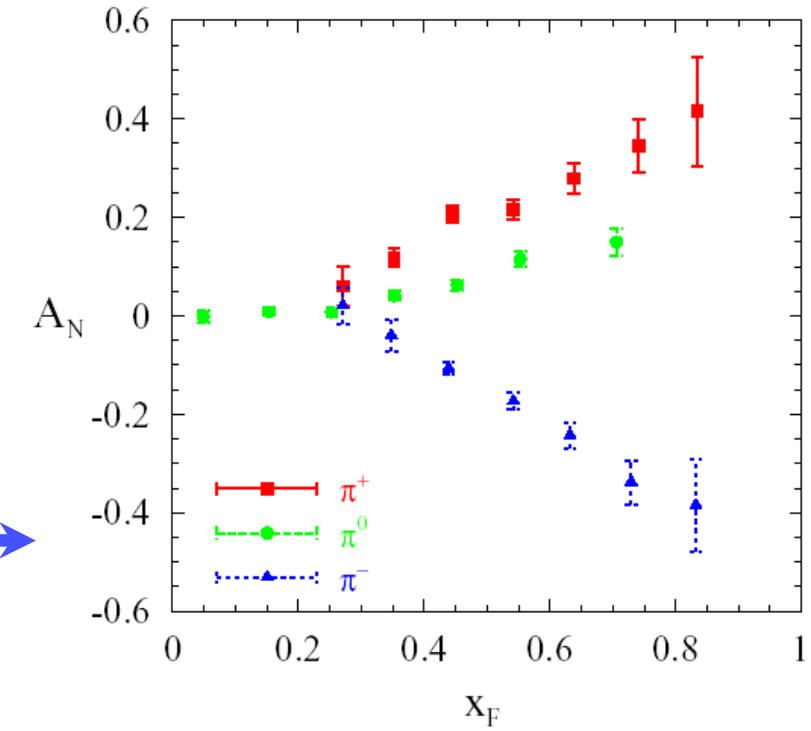

E704 $\sqrt{s}$ = 20 GeV  $0.7 < p_T < 2.0$



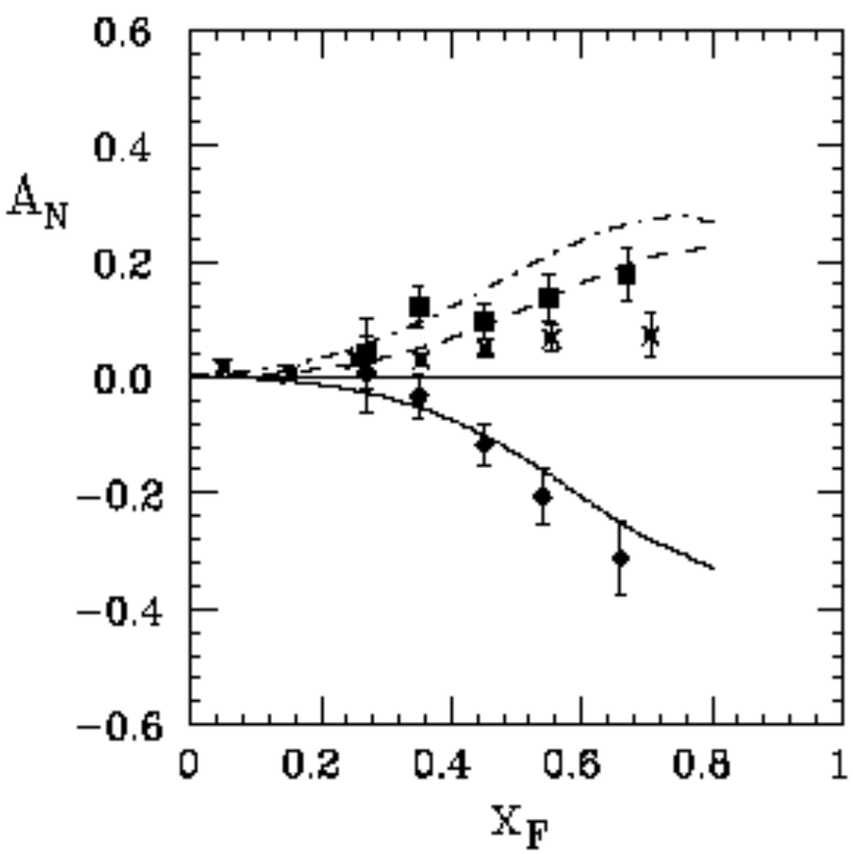

**BNL-AGS $\sqrt{s}$ = 6.6 GeV**
**$0.6 < p_T < 1.2$**

$$p^{\uparrow} p \rightarrow \pi X$$

**E704 $\sqrt{s}$ = 20 GeV 0.7**
**$< p_T < 2.0$**




observed transverse Single Spin Asymmetries




**E704 $\sqrt{s}$ = 20 GeV 0.7**
**$< p_T < 2.0$**

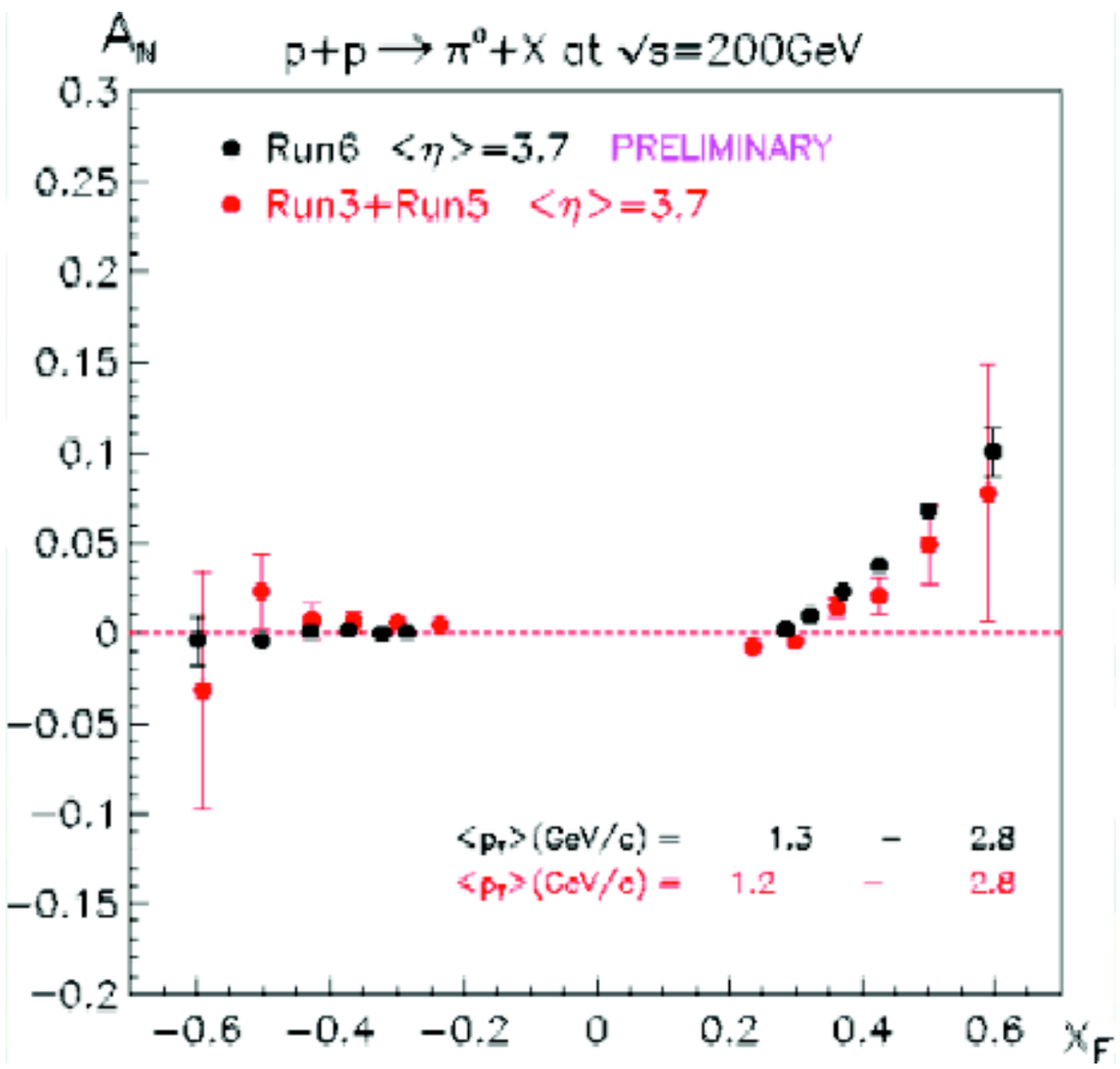
$$\bar{p}^{\uparrow} p \rightarrow \pi X$$

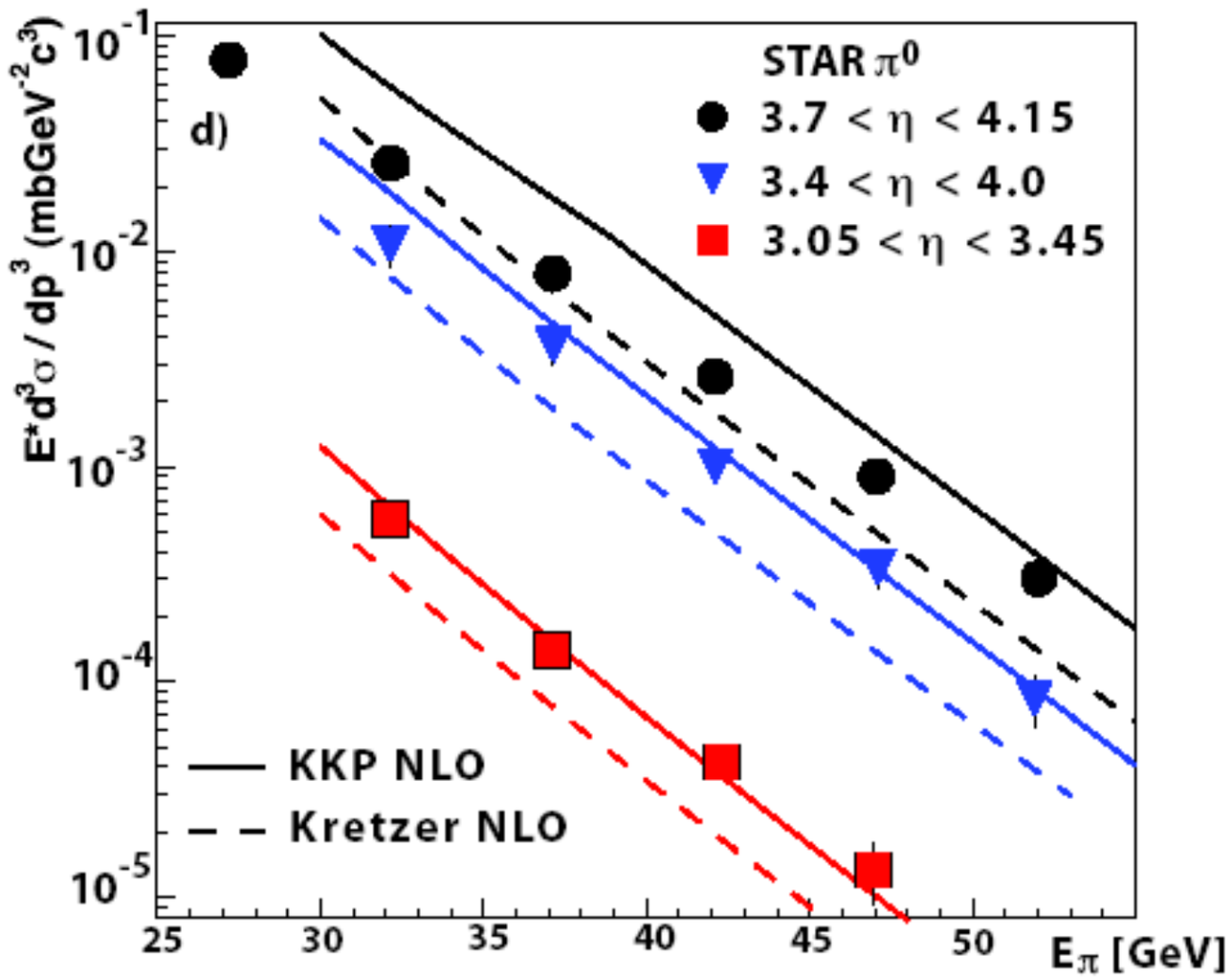
$$A_N \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

$$x_F = x_a - x_b$$

large $x_F \Rightarrow x_a$ valence region

# RHIC: good description of unpolarized cross-section, with collinear factorization. And $A_N$ ... ?

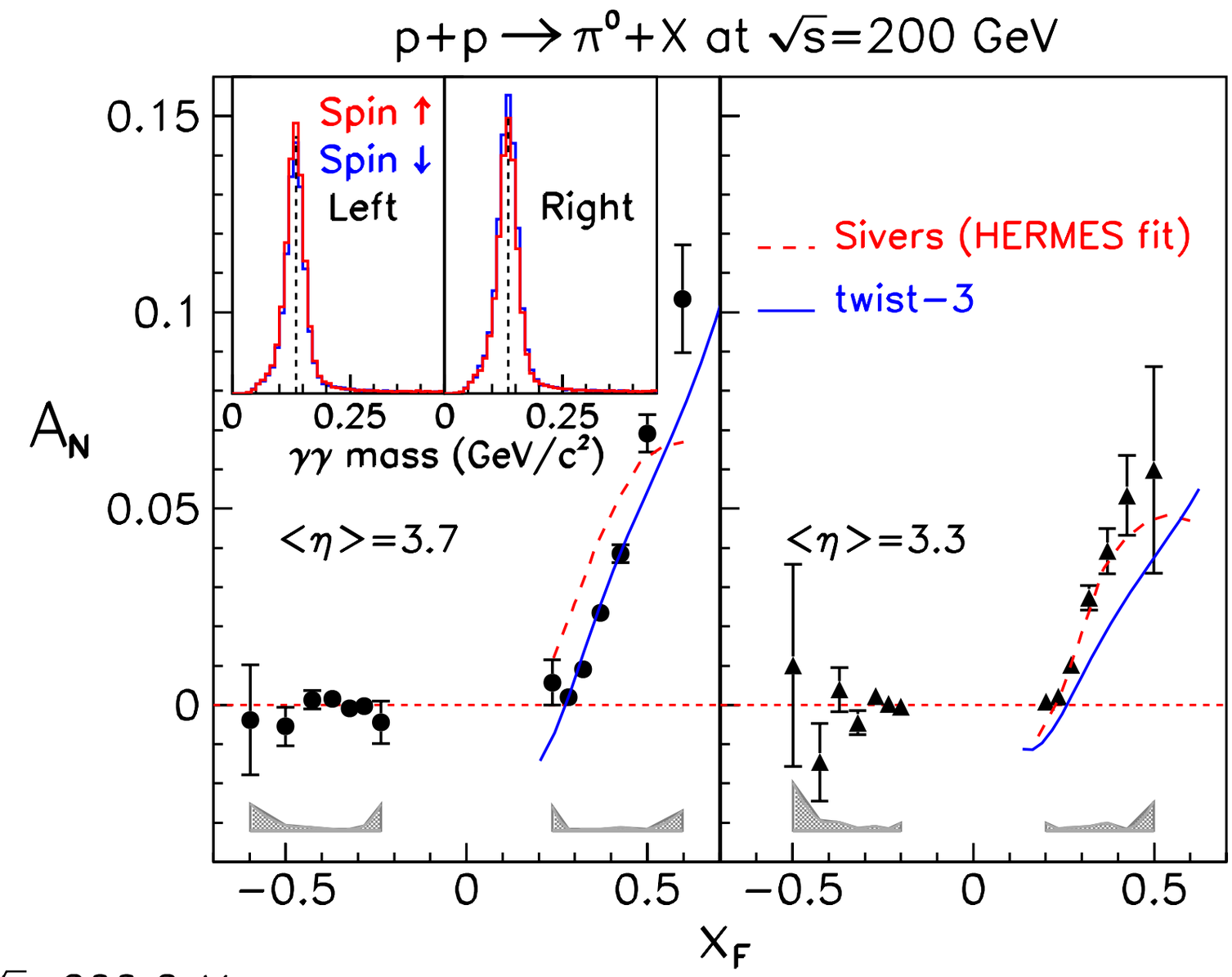


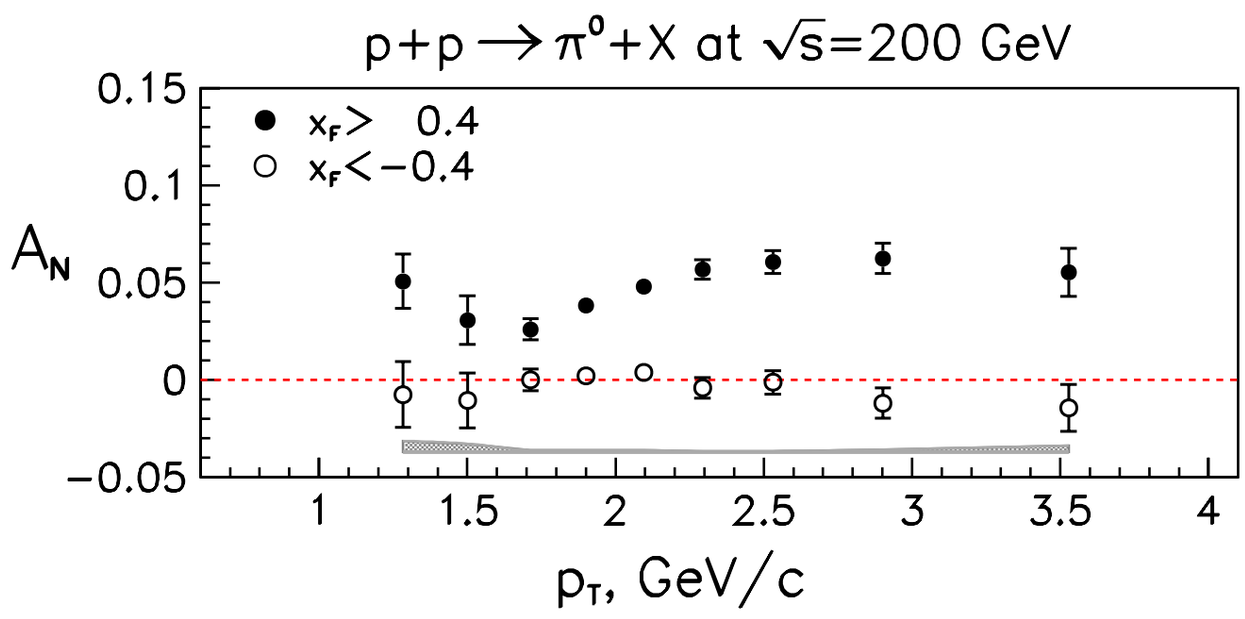




STAR-RHIC $\sqrt{s}$ = 200 GeV  $1.2 < p_T < 2.8$

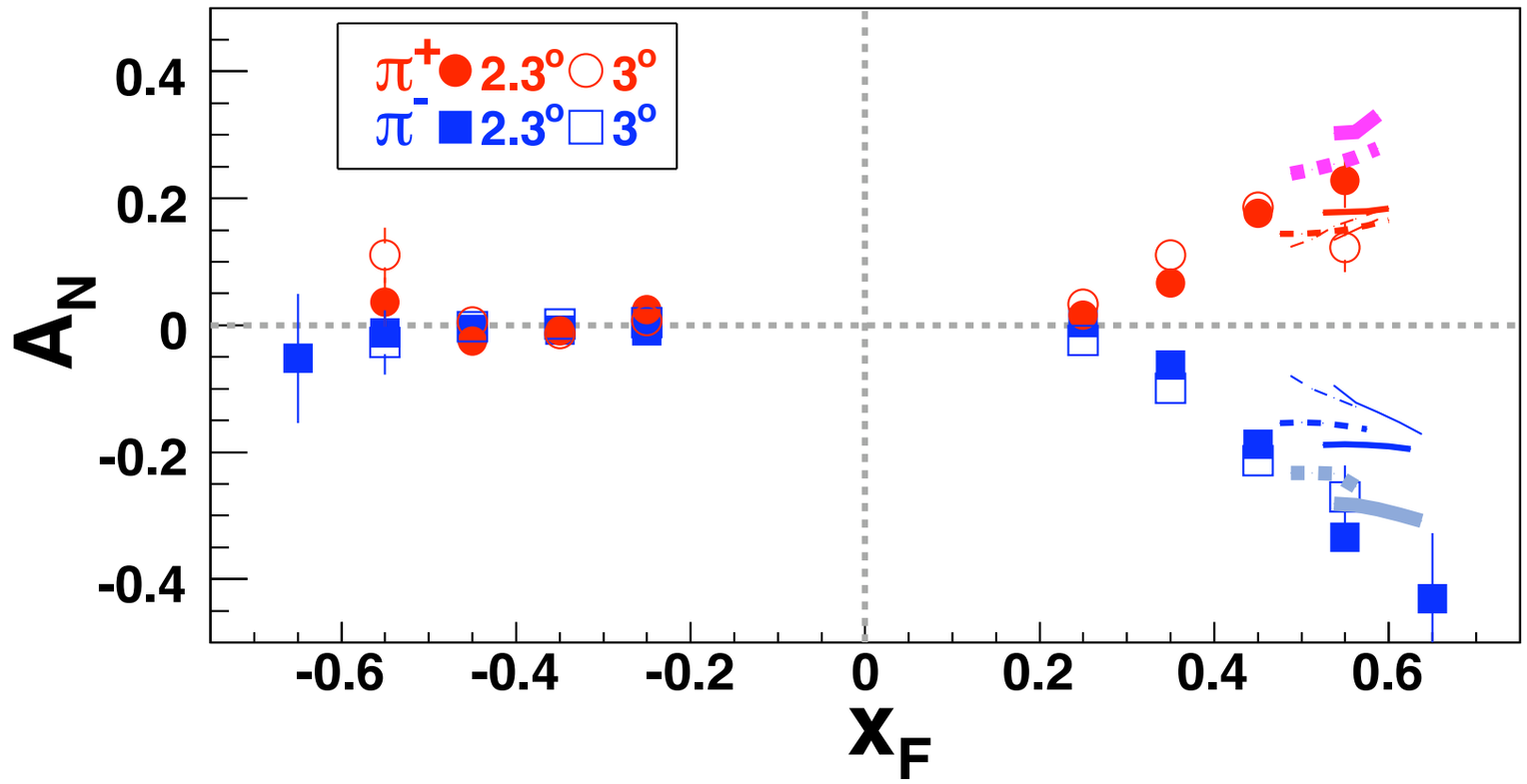
# STAR data

PRL 101, 222001 (2008)



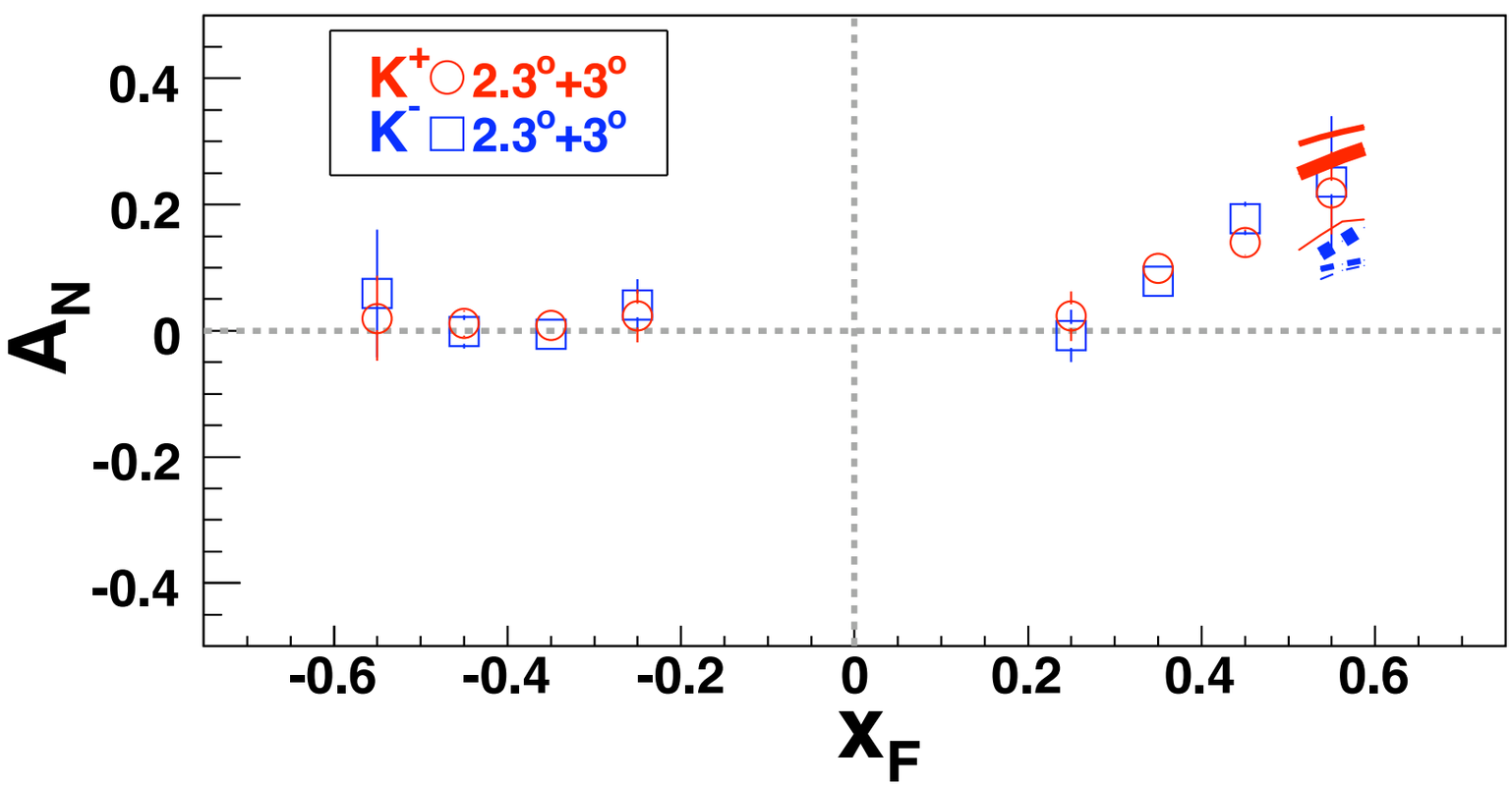

interesting $P_T$ dependence




data keep coming ...

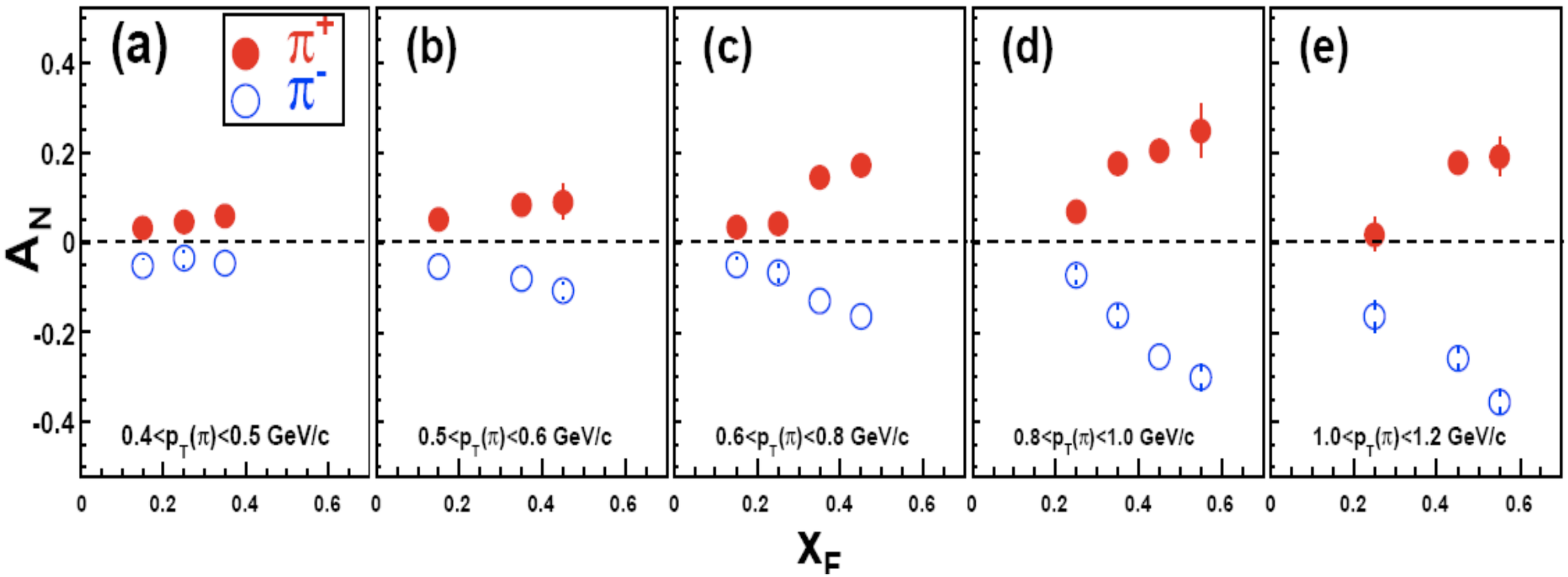
BRAHMS

PRL 101, 042001 (2008)

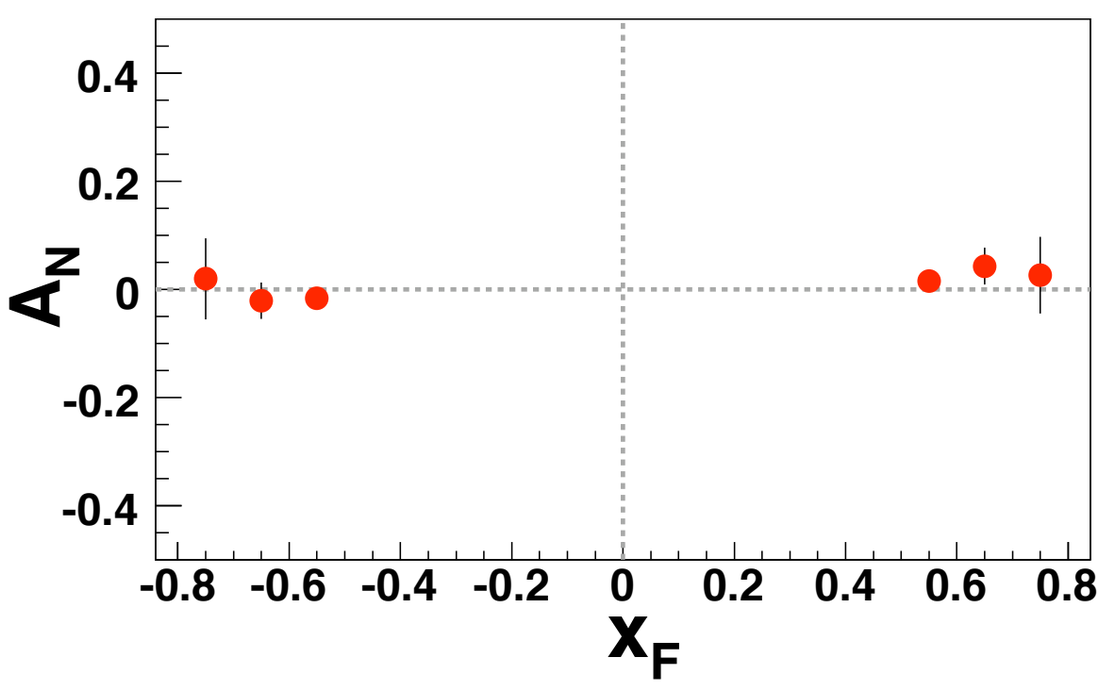



pion and Kaon SSA, measured by BRAHMS at $\sqrt{s}$ = 62.4 GeV

# $A_N$ $x_F$-$p_T$ dependence at $\sqrt{s}$ = 62.4 GeV




PRL 101, 042001 (2008)

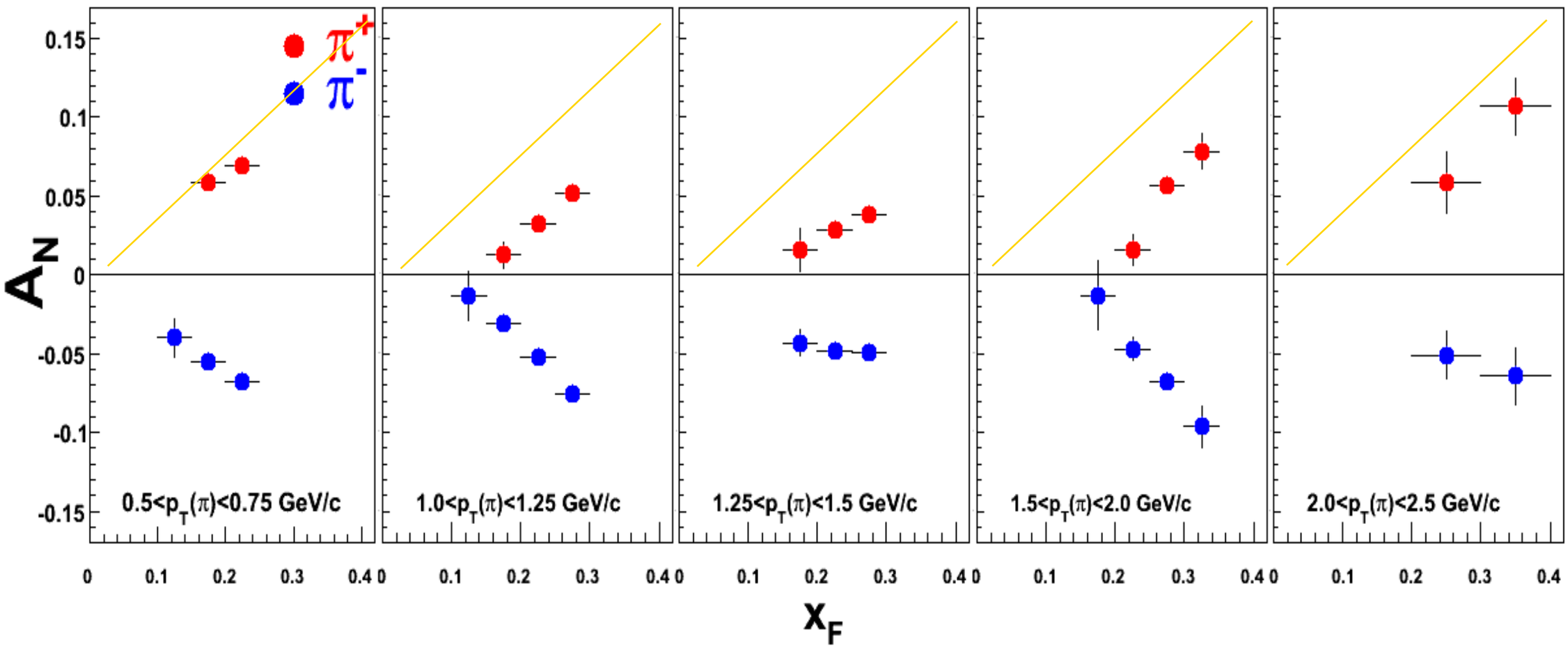


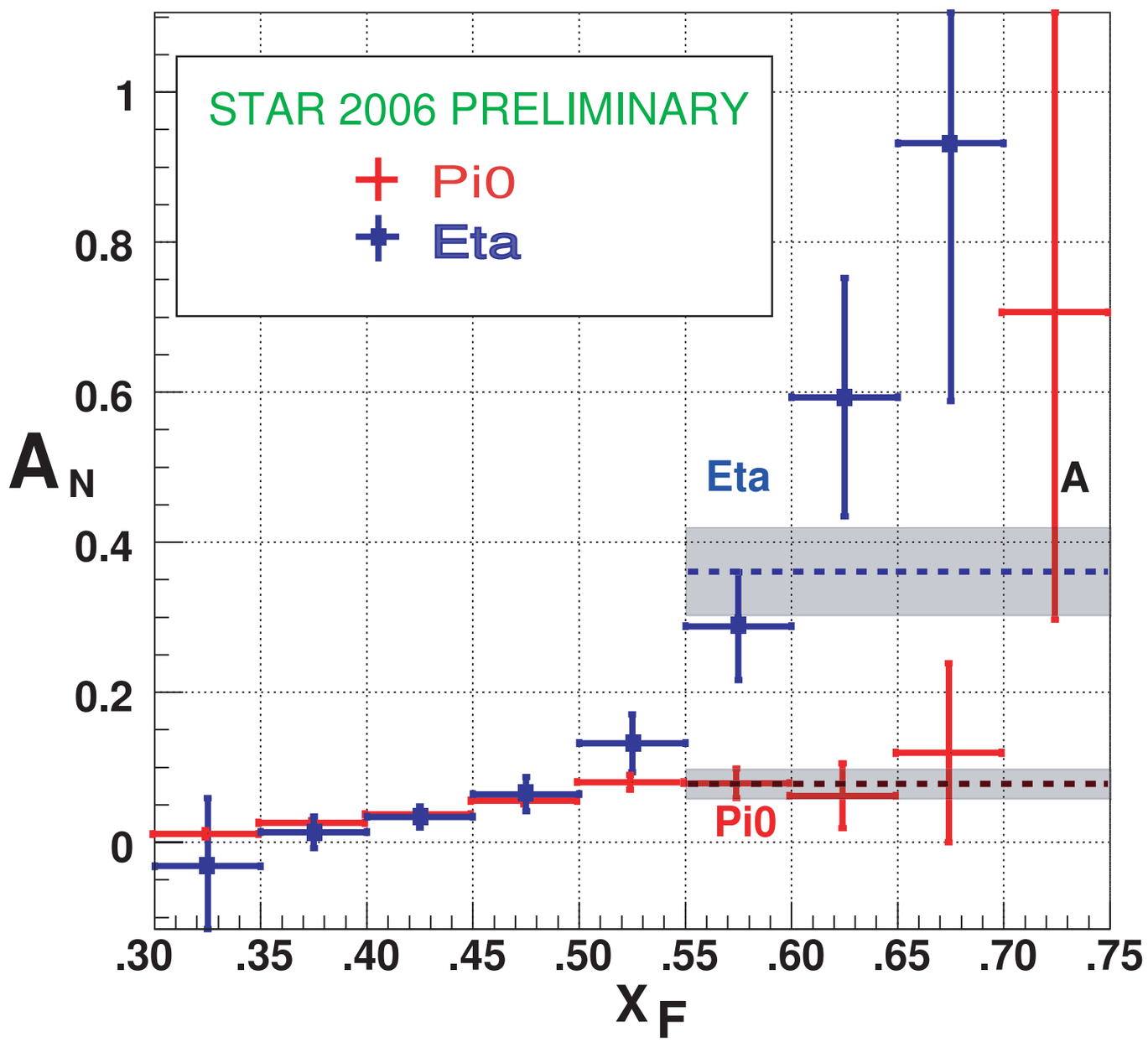

$A_N \approx 0$ for proton

BRAHMS

# $A_N$ $x_F$-dependence in $p_T$ slices, $\sqrt{s}$ = 200 GeV

(C. Aidala talk at Transversity 2008)



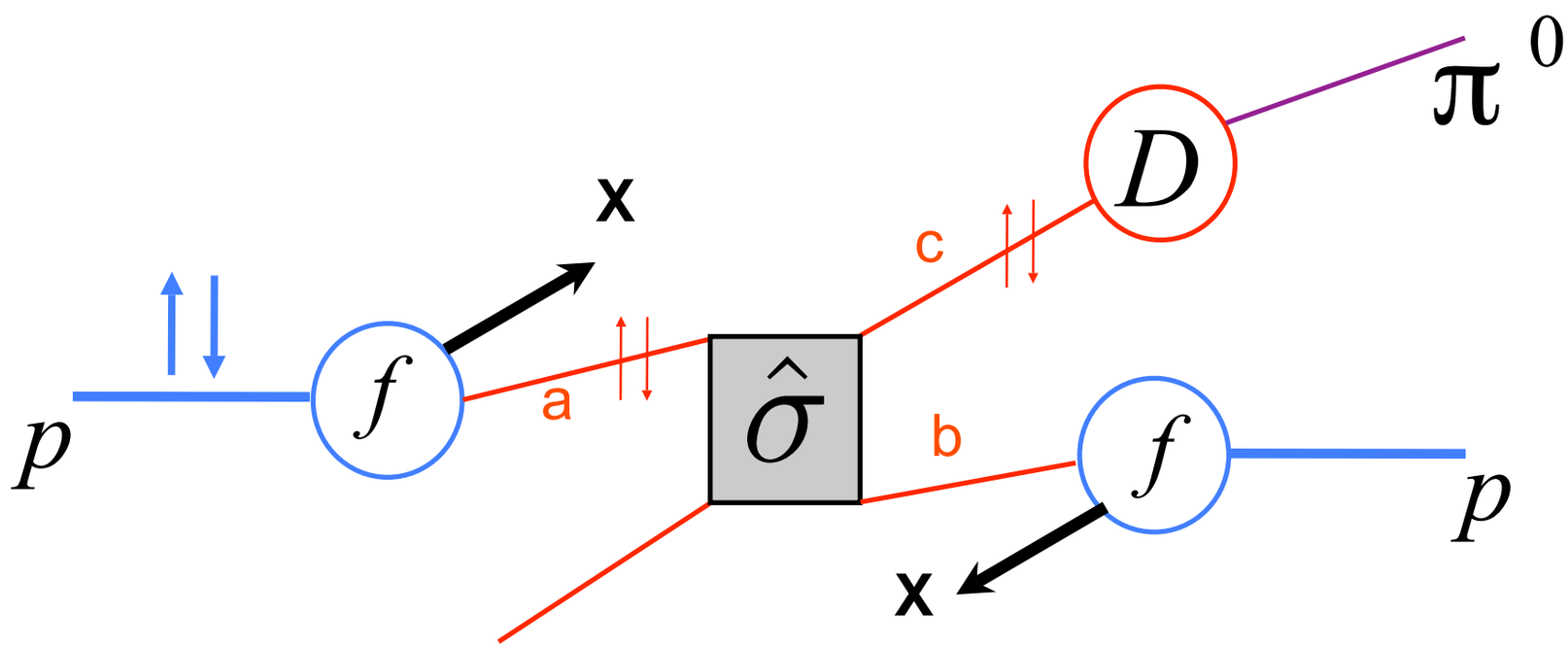



STAR amazing data on $A_N$ for eta production

# SSA in hadronic processes: TMDs, higher-twist correlations?

## Two main different (?) approaches

### 1. Generalization of collinear scheme
(assuming factorization)




$$\mathrm{d}\sigma^{\uparrow} = \sum_{a,b,c=q,\bar{q},g} \underbrace{f_{a/p^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})} \otimes \underbrace{f_{b/p}(x_b, \boldsymbol{k}_{\perp b})} \otimes \mathrm{d}\hat{\sigma}^{ab\to cd}(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \underbrace{D_{\pi/c}(z, \boldsymbol{p}_{\perp \pi})}$$

single spin effects in TMDs

M.A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia, A. Prokudin, ...
(first proposed by Field-Feynman in unpolarized case)

# General formalism with helicity amplitudes

## (Cagliari-Torino group)

$$\mathrm{d}\sigma^{(A,S_A)+(B,S_B)\to C+X} = \sum \overbrace{\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\ \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a}) \otimes \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\ \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b})}^{\text{PDFs for polarized partons inside polarized hadrons}}$$

$$\otimes\ \underbrace{\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\ \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}(\boldsymbol{k}_{\perp a},\boldsymbol{k}_{\perp b})}_{\text{polarized pQCD processes}}\ \underbrace{\hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})}_{\text{polarized FFs}}$$

Take into account all intrinsic motions: in parton distributions, fragmentation and elementary interactions.
It brings dependence on plenty of phases...

M.A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia,
PR **D71**, 014002 (2005), PR **D73**, 014020 (2006)

# cross-section

$$\frac{E_C\, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d,\{\lambda\}} \int \frac{dx_a\, dx_b\, dz}{16\pi^2 x_a x_b z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_{\perp C}\, \delta(\boldsymbol{k}_{\perp C}\cdot\hat{\boldsymbol{p}}_c)\, J(\boldsymbol{k}_{\perp C})$$

$$\times \quad \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\, \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\, \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b})$$

$$\times \quad \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\, \delta(\hat{s}+\hat{t}+\hat{u})\, \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})\,,$$

## parton helicity density matrix

$$\rho^{a/A,S_A}_{\lambda_a,\lambda'_a} = \begin{pmatrix} \rho^a_{++} & \rho^a_{+-} \\ \rho^a_{-+} & \rho^a_{--} \end{pmatrix}_{A,S_A} = \frac{1}{2}\begin{pmatrix} 1+P^a_z & P^a_x - iP^a_y \\ P^a_x + iP^a_y & 1-P^a_z \end{pmatrix}_{A,S_A}$$

x,y and z refer to parton helicity frame

# Origin of phases and quark correlator

$$\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\ \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a}) = \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A} \sum\!\!\!\!\!\!\int_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}\, \hat{\mathcal{F}}^*_{\lambda'_a,\lambda_{X_A};\lambda'_A}$$

$$\equiv \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A}\, \hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}$$

$\hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a,\boldsymbol{k}_{\perp a})$ helicity amplitude for the "process":

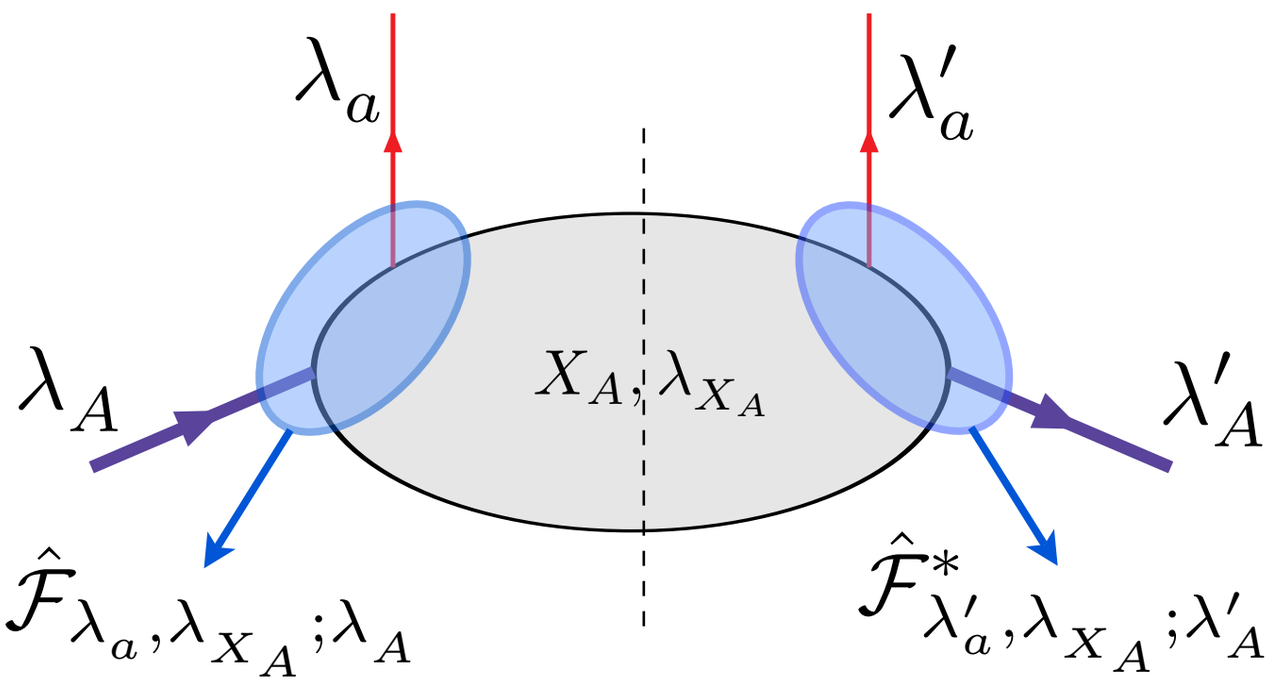
$$A \to a + X$$

from general properties of helicity amplitudes:

$$\hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a,\boldsymbol{k}_{\perp a}) = \mathcal{F}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a,k_{\perp a})\, e^{i\lambda_A\phi_a}$$

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,\boldsymbol{k}_{\perp a}) = F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,k_{\perp a})\, e^{(\lambda_A-\lambda'_A)\phi_a}$$

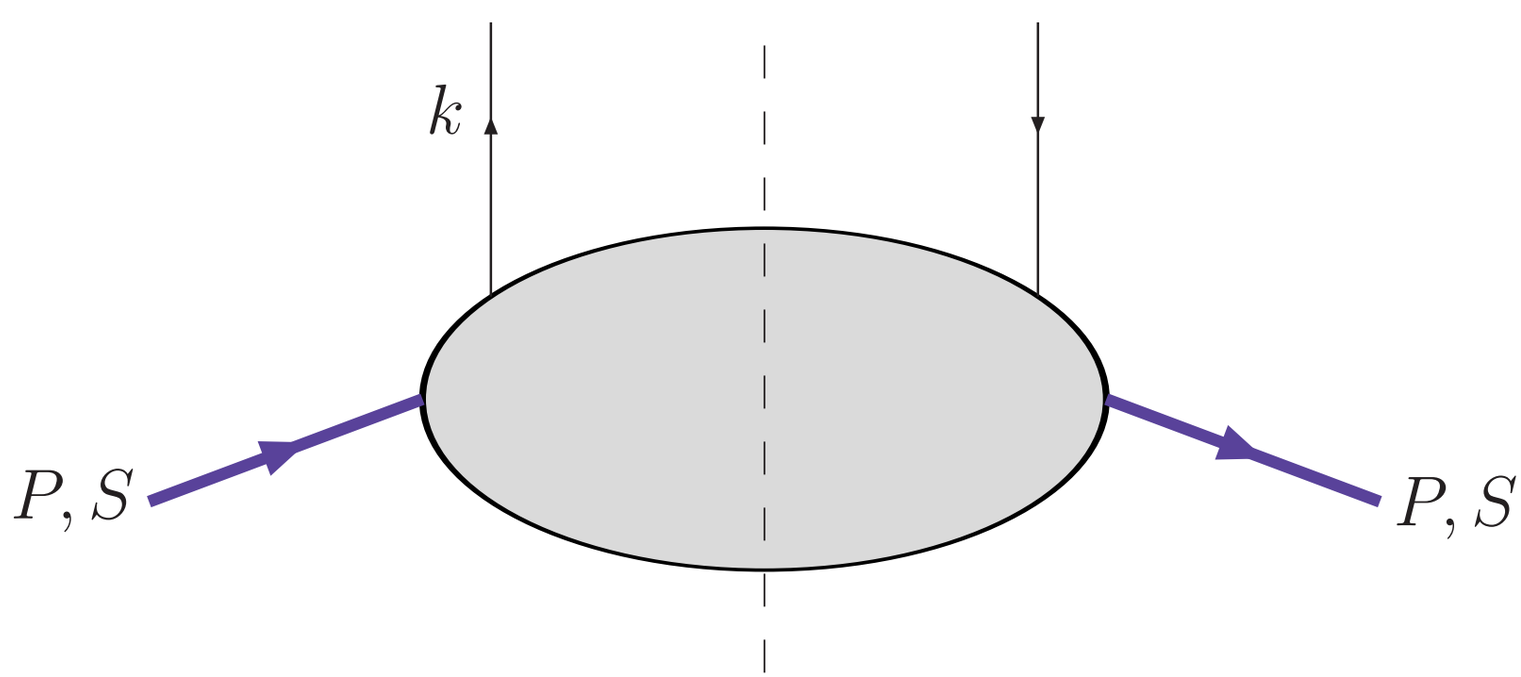
$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a, \boldsymbol{k}_{\perp a})$ is the quark correlator



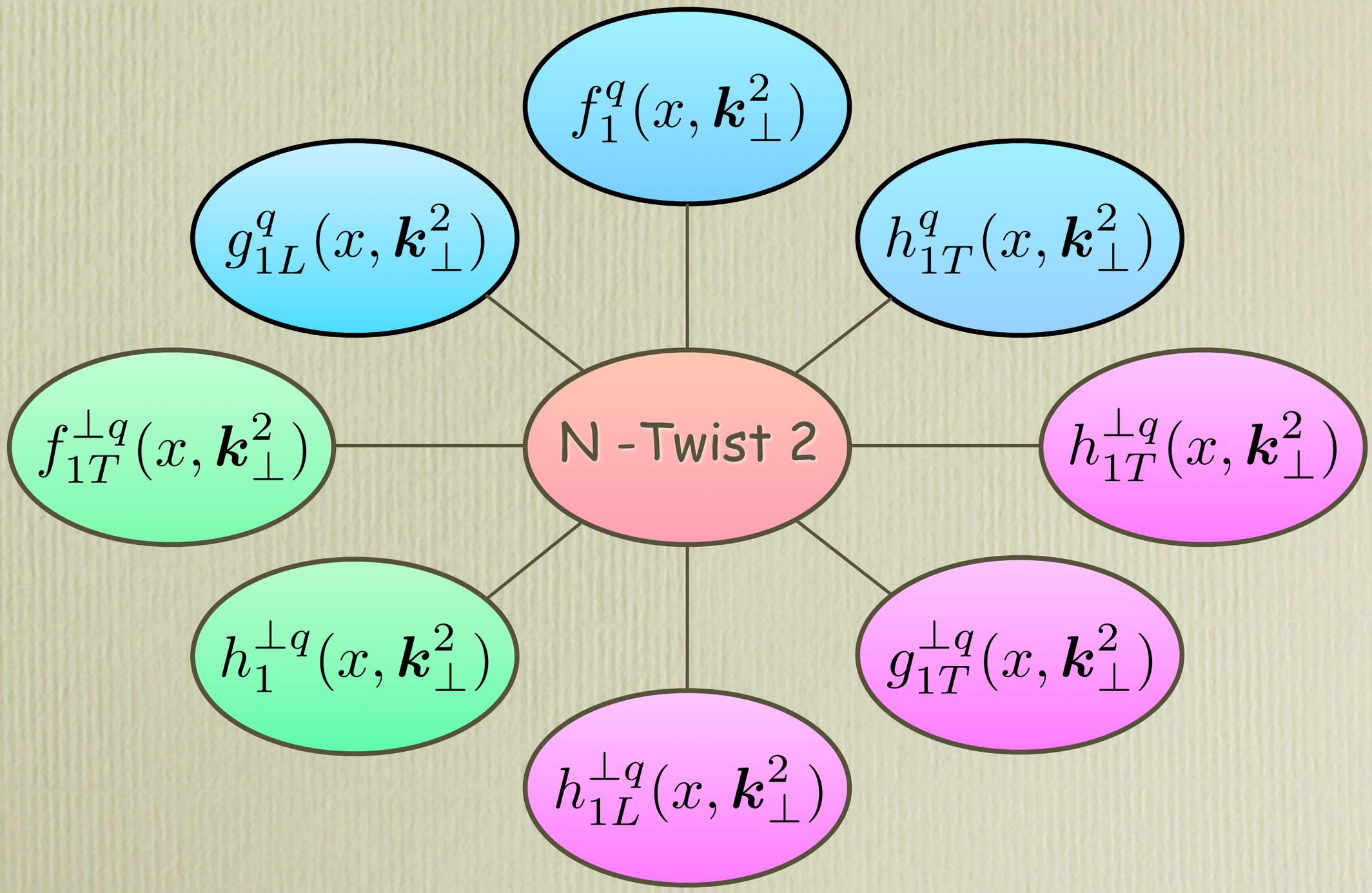

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A} = \sum\!\!\!\!\!\!\int_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}\, \hat{\mathcal{F}}^*_{\lambda'_a,\lambda_{X_A};\lambda'_A}$$

# The leading-twist correlator, with intrinsic $k_\perp$, contains eight independent functions with partonic interpretation

$$
\begin{aligned}
\Phi(x, \boldsymbol{k}_\perp) \;=\; & \frac{1}{2}\Bigg[ f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left( S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \right) \gamma^5 \not{n}_+ \\
& + h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \left( S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \right) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} \\
& + h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Bigg]
\end{aligned}
$$



# The nucleon at twist-2,
# 8 TMDs



and there are eight independent $F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}$

$$\underbrace{F^{++}_{++}\,,\ F^{++}_{--}}_{\text{real}}\,,\ \underbrace{F^{+-}_{+-}\,,\ F^{-+}_{+-}}_{\text{real for quarks}}\,,\ \underbrace{F^{++}_{+-}\,,\ F^{+-}_{++}}_{\text{complex}}$$

$$\begin{aligned}
\hat{f}_{a/A} &= \hat{f}_{a/A,S_L} = \left(F^{++}_{++} + F^{++}_{--}\right) \\
\hat{f}_{a/A,S_T} &= \left(F^{++}_{++} + F^{++}_{--}\right) + 2\,\mathrm{Im}F^{++}_{+-}\sin(\phi_{S_A} - \phi_a) \\
P^a_x\,\hat{f}_{a/A,S_L} &= 2\,\mathrm{Re}F^{+-}_{++} \\
P^a_x\,\hat{f}_{a/A,S_T} &= \left(F^{+-}_{+-} + F^{-+}_{+-}\right)\cos(\phi_{S_A} - \phi_a) \\
P^a_y\,\hat{f}_{a/A,S_L} &= P^a_{y\ a/A} = -2\,\mathrm{Im}F^{+-}_{++} \\
P^a_y\,\hat{f}_{a/A,S_T} &= -2\,\mathrm{Im}F^{+-}_{++} + \left(F^{+-}_{+-} - F^{-+}_{+-}\right)\sin(\phi_{S_A} - \phi_a) \\
P^a_z\,\hat{f}_{a/A,S_L} &= \left(F^{++}_{++} - F^{++}_{--}\right) \\
P^a_z\,\hat{f}_{a/A,S_T} &= 2\,\mathrm{Re}F^{++}_{+-}\cos(\phi_{S_A} - \phi_a)
\end{aligned}$$

# relations between different notations

$$
\begin{aligned}
f_1(x_a, k_{\perp a}) &= F_{++}^{++} + F_{--}^{++} = f_{a/A} \\
\frac{k_{\perp a}}{M}\, f_{1T}^{\perp}(x_a, k_{\perp a}) &= -2\,\mathrm{Im} F_{+-}^{++} \\
g_{1L}(x_a, k_{\perp a}) &= F_{++}^{++} - F_{--}^{++} \\
\frac{k_{\perp a}}{M}\, g_{1T}^{\perp}(x_a, k_{\perp a}) &= 2\,\mathrm{Re} F_{+-}^{++} \\
\frac{k_{\perp a}}{M}\, h_{1L}^{\perp}(x_a, k_{\perp a}) &= 2\,\mathrm{Re} F_{++}^{+-} \\
\frac{k_{\perp a}}{M}\, h_{1}^{\perp}(x_a, k_{\perp a}) &= 2\,\mathrm{Im} F_{++}^{+-} \\
h_1(x_a, k_{\perp a}) &= F_{+-}^{+-} \\
\left(\frac{k_{\perp a}}{M}\right)^2 h_{1T}^{\perp}(x_a, k_{\perp a}) &= 2\, F_{+-}^{-+}
\end{aligned}
$$

# examples

$$\Delta^N \hat{f}_{a/A^\uparrow} = 4\,\mathrm{Im} F_{+-}^{++} = 4\,\mathrm{Im} \sum\!\!\!\!\!\!\int_{X_A,\lambda_{X_A}} \mathcal{F}_{+,\lambda_{X_A};+}\,\mathcal{F}^*_{+,\lambda_{X_A};-}$$

Sivers function: needs interference between two amplitudes with nucleon helicity flip (chiral-even)

$$\Delta^N \hat{f}_{a^\uparrow/A} = -2\,\mathrm{Im} F_{++}^{+-} = -2\,\mathrm{Im} \sum\!\!\!\!\!\!\int_{X_A,\lambda_{X_A}} \mathcal{F}_{+,\lambda_{X_A};+}\,\mathcal{F}^*_{-,\lambda_{X_A};+}$$

Boer-Mulders function: needs interference between two amplitudes with quark helicity flip (chiral-odd)

similar situation with fragmentation functions

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = \sum\!\!\!\!\!\!\int_{X,\lambda_X} \hat{\mathcal{D}}_{\lambda_C,\lambda_X;\lambda_c}(z,\boldsymbol{k}_{\perp C})\,\hat{\mathcal{D}}^*_{\lambda'_C,\lambda_X;\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

$\hat{\mathcal{D}}_{\lambda_C,\lambda_X;\lambda_c}$ helicity amplitude for the "process":

$$c \to C + X$$

from general properties of helicity amplitudes:

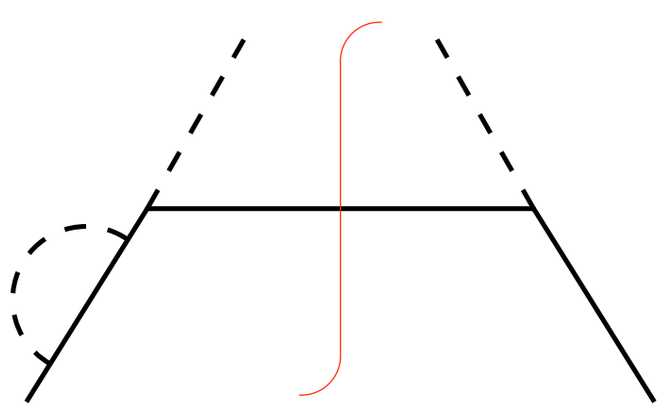
$$\hat{\mathcal{D}}_{\lambda_C,\lambda_X;\lambda_c}(z,\boldsymbol{k}_{\perp C}) = \mathcal{D}_{\lambda_C,\lambda_X;\lambda_c}(z,k_{\perp C})\,e^{i\lambda_c\phi^H_C}$$

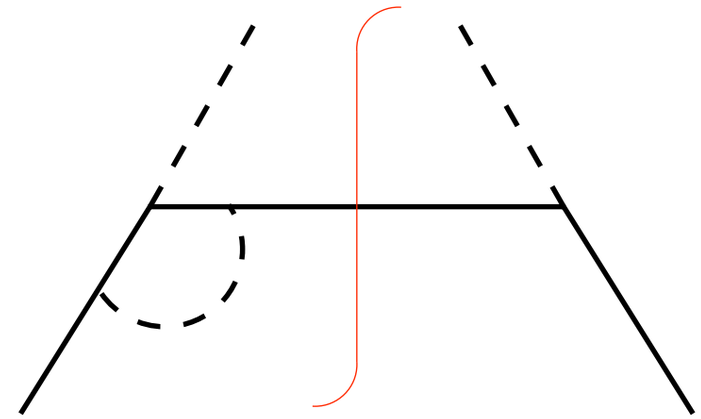
$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = D^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})\,e^{i(\lambda_c-\lambda'_c)\phi^H_C}$$

Collins function (unpolarized final particles)

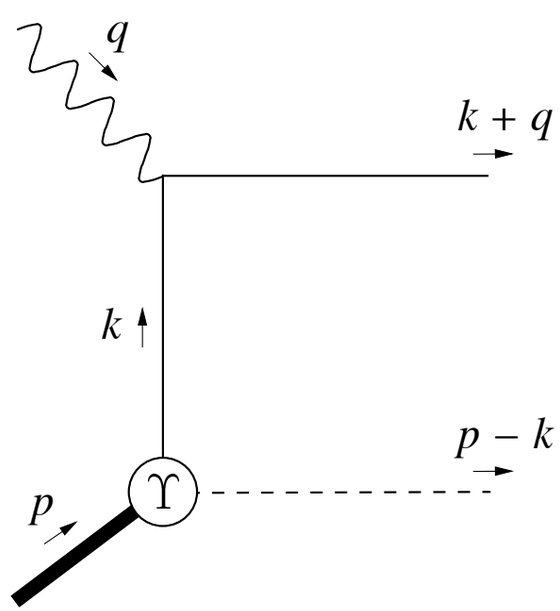
$$-2i\,D^{C/q}_{+-}(z,k_{\perp C}) = 2\,\mathrm{Im}\,D^{C/q}_{+-}(z,k_{\perp C}) \equiv \Delta^N\hat{D}_{C/}(z,k_{\perp C})$$

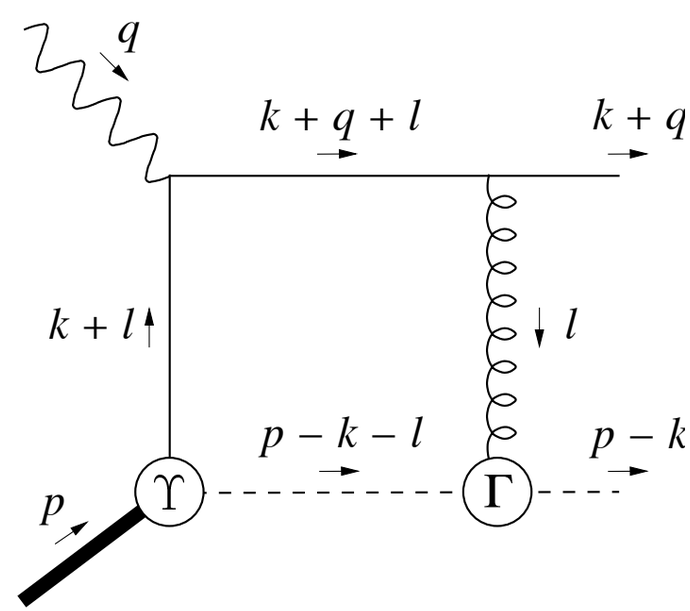
# models for Collins function





# and for Sivers function

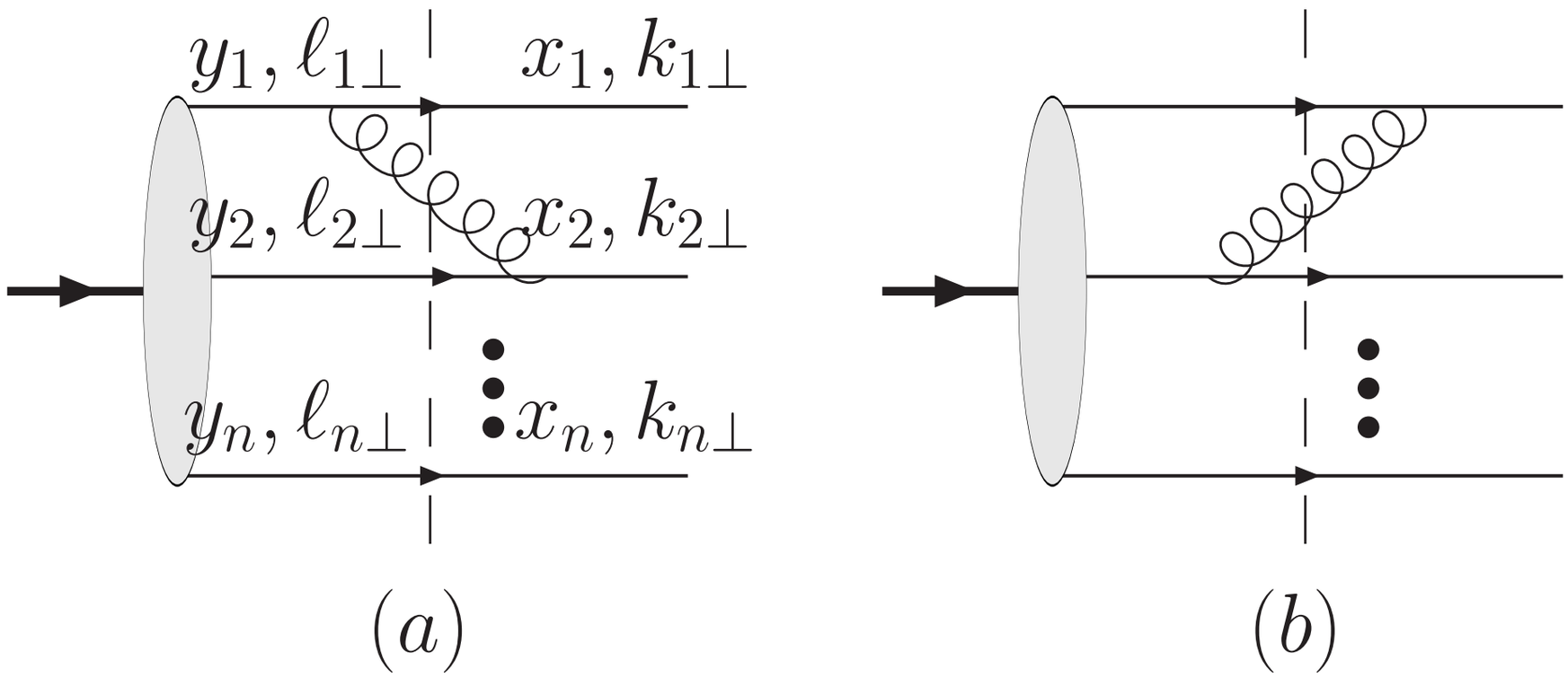



Bacchetta, Gamberg, Goldstein, Mukherjee, Metz, Amrath, Schaefer, Yang, Brodsky, Schmidt, Hwang, Scopetta, Courtoy, Frattini, Vento, Pasquini, Xiao, Yuan ....

# Sivers function from light-front wave function

Brodsky, Pasquini, Xiao, Yuan, arXiv:1001.1163
Pasquini, Yuan, arXiv:1001.5398

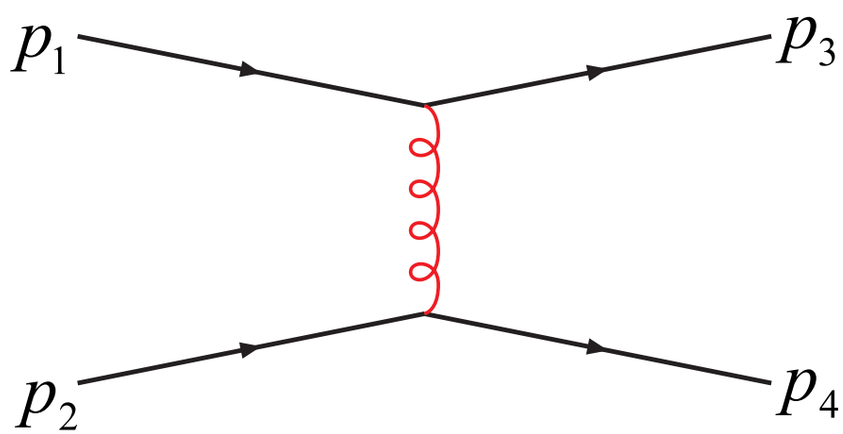



in all models one has:

$$[f_{1T}^{q\perp}]_{\rm SIDIS} = -[f_{1T}^{q\perp}]_{\rm DY}$$

see also Hwang, arXiv:1003.0867 - incorporation of final state interactions into the light-cone wave function

# phases in (non-planar) helicity amplitudes




$$\begin{aligned} \hat{M} &\propto \bar{u}(p_3,\lambda_3)\,\gamma^\mu\,u(p_1,\lambda_1)\,\bar{u}(p_4,\lambda_4)\,\gamma_\mu\,u(p_2,\lambda_2) \\ &= \hat{M}^0\,e^{i\varphi} \end{aligned}$$

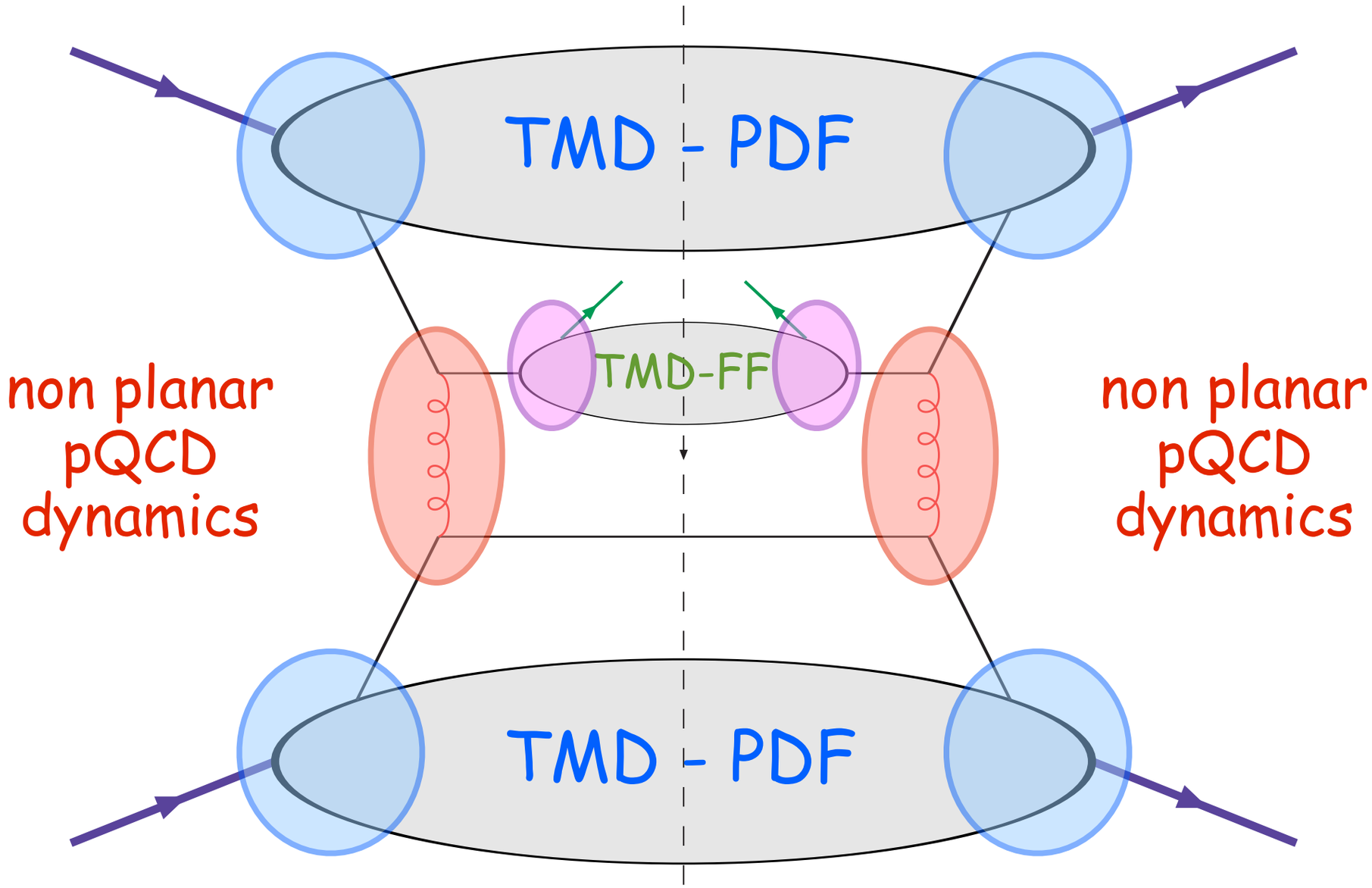
Dirac-Pauli helicity spinors, $p_i = (p_i^0,\ \boldsymbol{p}_i)$

$$u(p_i,\,\lambda_i) = \sqrt{p_i^0}\begin{pmatrix}1\\ \lambda_i\end{pmatrix}\chi_{\lambda_i}(\hat{\boldsymbol{p}}_i) \qquad \hat{\boldsymbol{p}}_i = (\sin\theta_i\cos\phi_i,\ \sin\theta_i\sin\phi_i,\ \cos\theta_i)$$

$$\chi_+(\hat{\boldsymbol{p}}_i) = \begin{pmatrix}\cos(\theta_i/2)\,e^{-i\phi_i/2}\\ \sin(\theta_i/2)\,e^{i\phi_i/2}\end{pmatrix} \qquad \chi_-(\hat{\boldsymbol{p}}_i) = \begin{pmatrix}-\sin(\theta_i/2)\,e^{-i\phi_i/2}\\ \cos(\theta_i/2)\,e^{i\phi_i/2}\end{pmatrix}$$

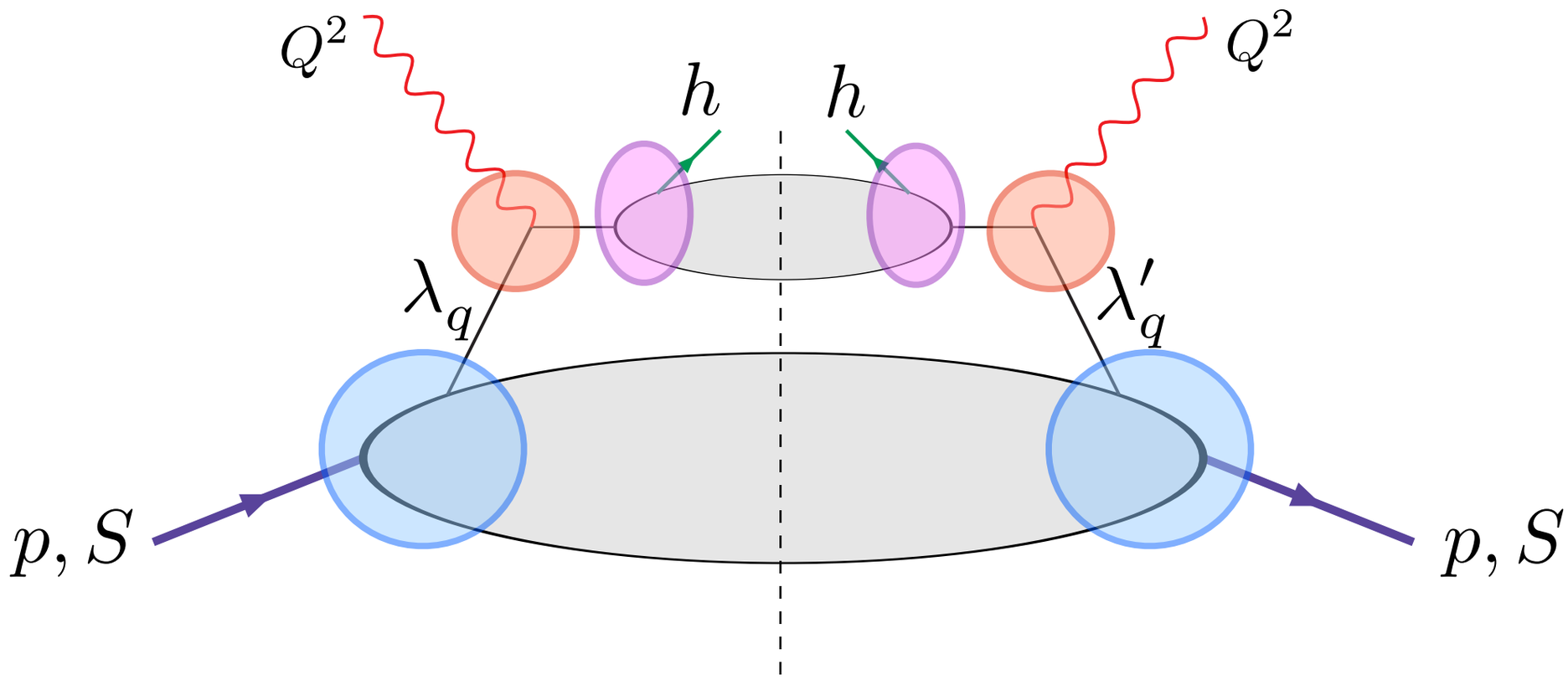
if scattering is not planar all phases are different and remain in the amplitudes; they suppress the results when integrating over $\mathbf{k}_\perp$

# TMD factorization




factorization assumed

# particular case: SIDIS




factorization holds at large $Q^2$, and $P_T \approx k_\perp \approx \Lambda_{\rm QCD}$

Two scales: $P_T \ll Q^2$

$$\mathrm{d}\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes \mathrm{d}\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

$$\frac{d\sigma^{\ell(S_\ell)+p(S)\to\ell'+h+X}}{dx_B\,dQ^2\,dz_h\,d^2\boldsymbol{P}_T\,d\phi_S}$$

$$= \rho^{\ell,S_\ell}_{\lambda_\ell,\lambda'_\ell} \otimes \rho^{q/p,S}_{\lambda_q,\lambda'_q}\,\hat{f}_{q/p,S}(x,\boldsymbol{k}_\perp) \otimes \hat{M}_{\lambda_\ell,\lambda_q;\lambda_\ell,\lambda_q}\,\hat{M}^*_{\lambda'_\ell,\lambda'_q;\lambda'_\ell,\lambda'_q} \otimes \hat{D}^h_{\lambda_q,\lambda'_q}(z,\boldsymbol{p}_\perp)$$

TMD-PDF hard scattering TMD-FF

keeping into account all phases one obtains

$$\frac{d\sigma^{\ell p^\uparrow\to\ell' hX} - d\sigma^{\ell p^\downarrow\to\ell' hX}}{dx_B\,dQ^2\,dz_h\,d^2\boldsymbol{P}_T\,d\phi_S} =$$

$$\sum_q \int d^2\boldsymbol{k}_\perp \frac{4e_q^2e^4}{y^2}[-4\mathrm{Im}F^{++}_{+-}(x,k_\perp)]\sin(\phi_S-\phi_\perp)\left(\frac{1+(1-y)^2}{2} - 2(1-y)\sqrt{1-y}\frac{k_\perp}{Q}\cos\phi_\perp\right)D^h_q(z,P_T)$$

$$-\sum_q \int d^2\boldsymbol{k}_\perp \frac{4e_q^2e^4}{y^2}\Bigg\{F^{+-}_{+-}(x,k_\perp)\Bigg[(1-y)\left(\sin(\phi_S+\phi_h) - z_h\frac{k_\perp}{P_T}\sin(\phi_S+\phi_\perp)\right)$$

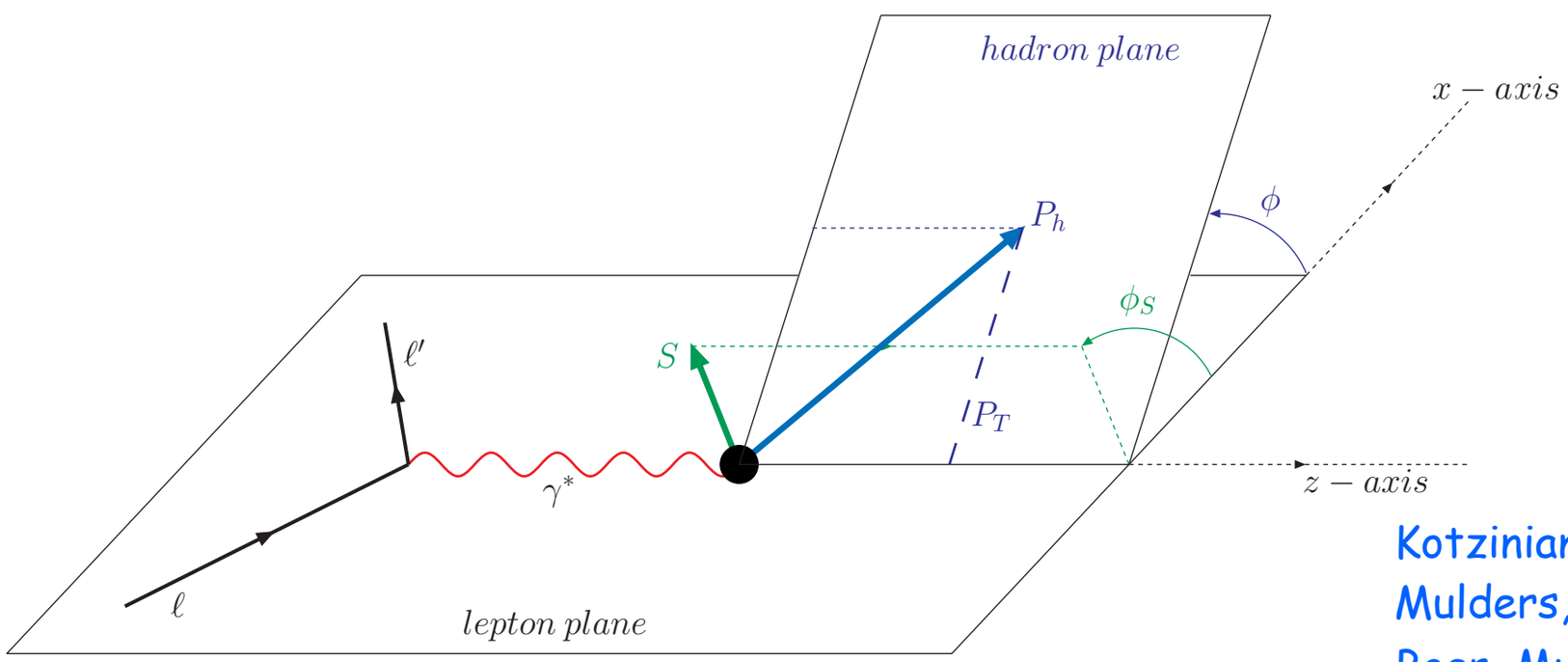
$$-2(2-y)\sqrt{1-y}\frac{k_\perp}{Q}\left(\sin(\phi_S+\phi_h-\phi_\perp) - z_h\frac{k_\perp}{P_T}\sin(\phi_S)\right)\Bigg]$$

$$-F^{-+}_{+-}(x,k_\perp)\Bigg[(1-y)\left(\sin(\phi_S-\phi_h-2\phi_\perp) - z_h\frac{k_\perp}{P_T}\sin(\phi_S-3\phi_\perp)\right)$$

$$-2(2-y)\sqrt{1-y}\frac{k_\perp}{Q}\left(\sin(\phi_S-\phi_h-\phi_\perp) - z_h\frac{k_\perp}{P_T}\sin(\phi_S-2\phi_\perp)\right)\Bigg]\Bigg\}\frac{P_T}{p_\perp}\Delta^N D^h_q(z,P_T),$$

which exactly corresponds to the usual expression

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} &= F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\, \cos\phi\, F_{UU}^{\cos\phi} + \lambda\, \frac{1}{Q}\, \sin\phi\, F_{LU}^{\sin\phi} \\
&+ S_L \left\{ \sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\, \sin\phi\, F_{UL}^{\sin\phi} + \lambda \left[ F_{LL} + \frac{1}{Q}\, \cos\phi\, F_{LL}^{\cos\phi} \right] \right\} \\
&+ S_T \Big\{ \sin(\phi - \phi_S)\, F_{UT}^{\sin(\phi-\phi_S)} + \sin(\phi + \phi_S)\, F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi - \phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \\
&+ \frac{1}{Q} \left[ \sin(2\phi - \phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S} \right] \\
&+ \lambda \left[ \cos(\phi - \phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q} \left( \cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi - \phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \Big\}
\end{aligned}
$$





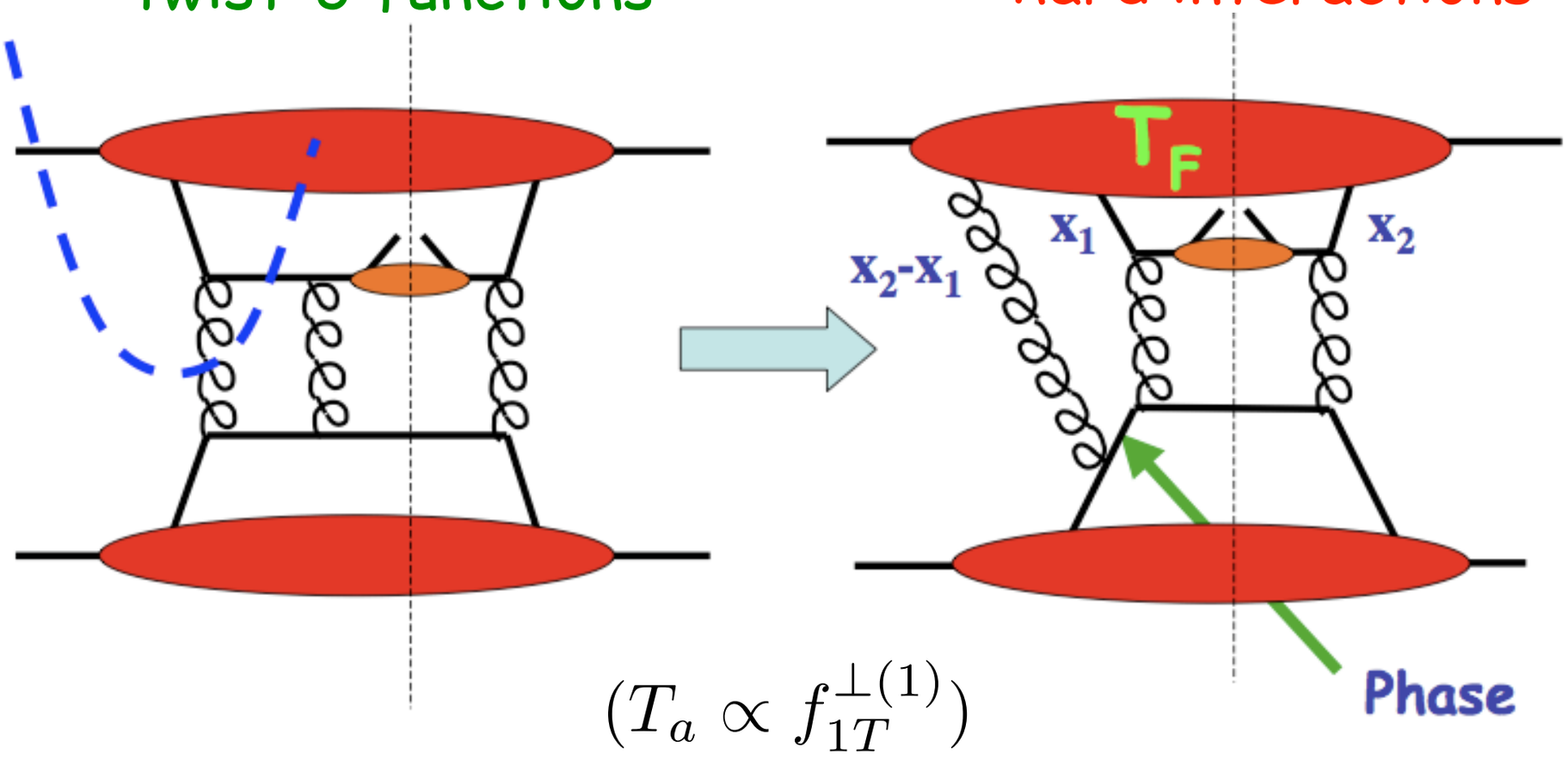
Kotzinian, **NP B441** (1995) 234
Mulders,Tangermann, **NP B461** (1996) 197
Boer, Mulders, **PR D57** (1998) 5780
Bacchetta et al., **PL B595** (2004) 309
Bacchetta et al., **JHEP 0702** (2007) 093

# 2. Higher-twist partonic correlations

(Efremov, Teryaev; Qiu, Sterman; Kouvaris, Vogelsang, Yuan; Bacchetta, Bomhof, Mulders, Pijlman; Koike ... )

contribution to SSA $(A^{\uparrow} B \to h\, X)$

$$\mathrm{d}\Delta\sigma \propto \sum_{a,b,c} \underbrace{T_a(k_1, k_2, \boldsymbol{S}_\perp)}_{\text{twist-3 functions}} \otimes f_{b/B}(x_b) \otimes \underbrace{H^{ab\to c}(k_1, k_2)}_{\text{hard interactions}} \otimes D_{h/c}(z)$$




$$(T_a \propto f_{1T}^{\perp(1)})$$

factorization established

# TMD vs. higher-twist collinear factorization

processes with two scales: SIDIS and Drell-Yan

Yuan, Zhou, Liang; Ji, Qiu, Vogelsang, ...

TMD factorization holds at $\Lambda_{\rm QCD} \simeq k_\perp \ll Q$

higher-twist factorization at $\Lambda_{\rm QCD} \ll k_\perp \ll Q$

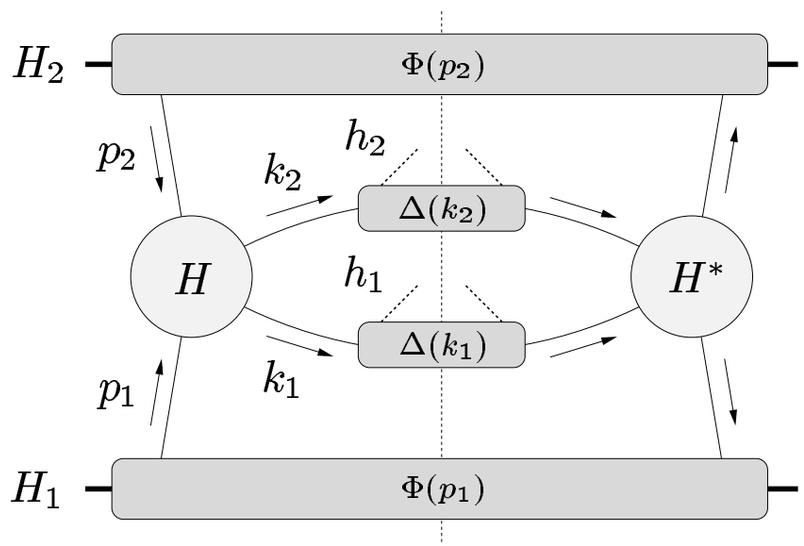
well established relationships between TMDs and quark-gluon correlators

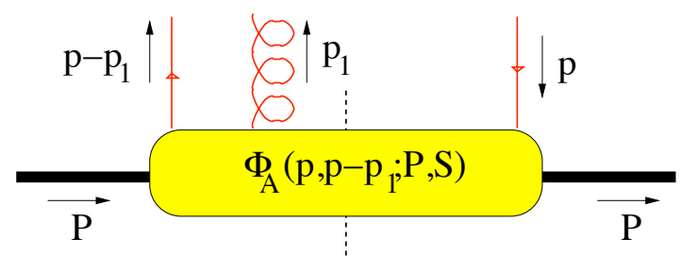
the two approaches are consistent in the overlapping intermediate $k_\perp$ region

TMD factorization not proven for processes with one (large) scale only

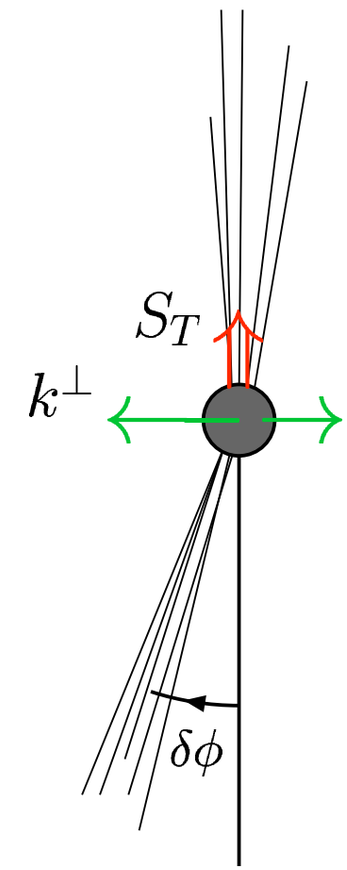
# SSA in pp → jet + jet + X, $H_1 H_2 \to h_1 h_2 X$

Bacchetta, Bomhof, Mulders, Pijlman; Boer,Vogelsang, Yuan; Ratcliffe, Teryaev




gluonic pole




$k^\perp$ = jet pair transverse momentum

Sivers contribution to SSA $(T_a \propto f_{1T}^{\perp(1)})$

$$\mathrm{d}\Delta\sigma \propto \sum_{a,b,c} f_{1T}^{\perp(1)}(x_1) \otimes f_{b/H_2}(x_2) \otimes \mathrm{d}\hat{\sigma}_{[a]b\to cd} \otimes D_{h_1/c}(z_1)\, D_{h_2/d}(z_2)$$

gluonic pole cross sections take into account gauge links

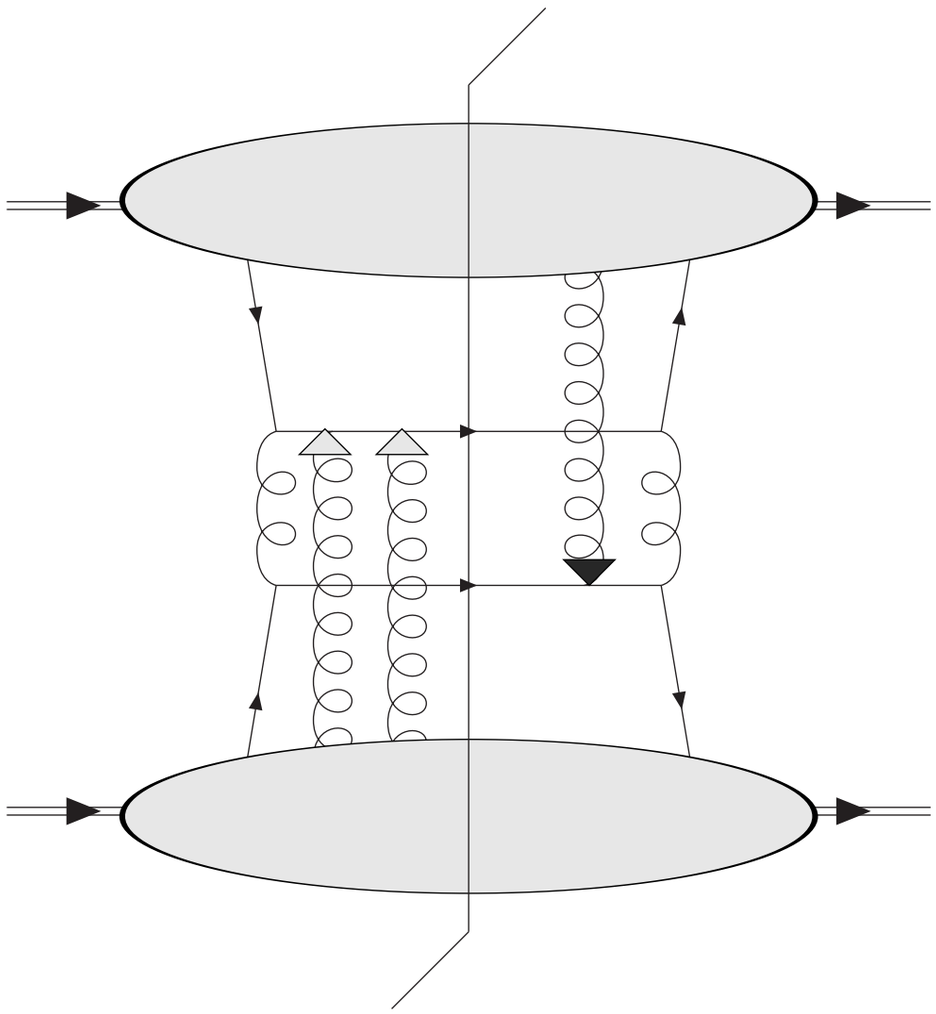
$$\mathrm{d}\hat{\sigma}_{[a]b\to cd} = \sum_D C_G^{[D]}\, \mathrm{d}\hat{\sigma}^D_{ab\to cd}$$

$C_G^{[D]}$ Diagram dependent Gauge link Colour factors

(breaking of factorization?)

# but, recent paper casts doubts on factorization for di-jet production ...

Rogers, Mulders, arXiv: 1001.2977



example of graph expected to violate TMD-factorization

see also Collins, Qiu, arXiv: 0705.2141

# Phenomenology - TMD factorization

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

main contribution from Sivers and Collins effects

$$d\sigma^\uparrow - d\sigma^\downarrow \equiv \frac{E_\pi\, d\sigma^{\,p\to\pi\,X}}{d^3\boldsymbol{p}_\pi} - \frac{E_\pi\, d\sigma^{\,p\to\pi\,X}}{d^3\boldsymbol{p}_\pi} = [d\sigma^\uparrow - d\sigma^\downarrow]_{\rm Sivers} + [d\sigma^\uparrow - d\sigma^\downarrow]_{\rm Collins}$$

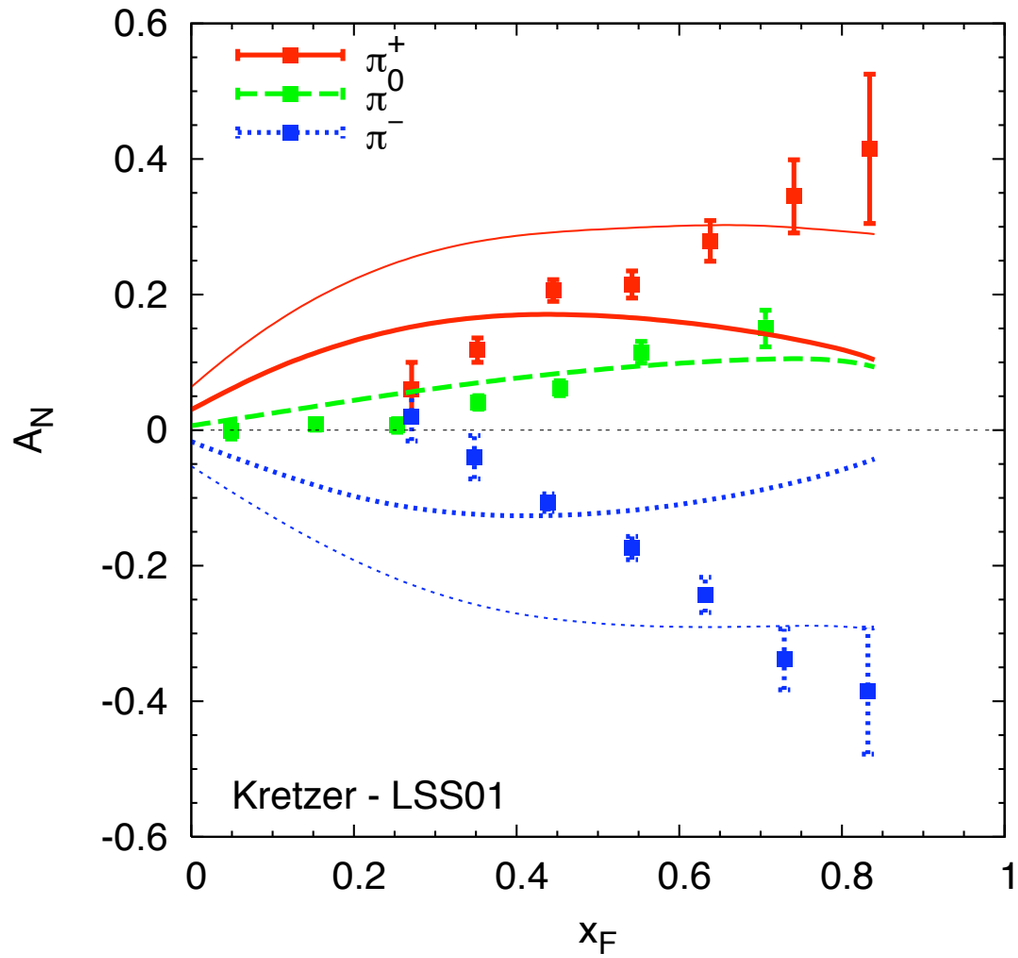
$$\begin{aligned}
[d\sigma^\uparrow - d\sigma^\downarrow]_{\rm Sivers} \;=\; & \sum_{q_a,b,q_c,d} \int \frac{dx_a\, dx_b\, dz}{16\,\pi^2\, x_a\, x_b\, z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{p}_\perp\, \delta(\boldsymbol{p}_\perp\cdot\hat{\boldsymbol{p}}_c)\, J(p_\perp)\, \delta(\hat{s}+\hat{t}+\hat{u}) \\
\times\; & \Delta^N f_{a/}(x_a, k_{\perp a}) \cos\phi_a \\
\times\; & f_{b/p}(x_b, k_{\perp b})\, \frac{1}{2}\, \Big[|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 + |\hat{M}_3^0|^2\Big]_{ab\to cd}\, D_{\pi/c}(z, p_\perp)
\end{aligned}$$

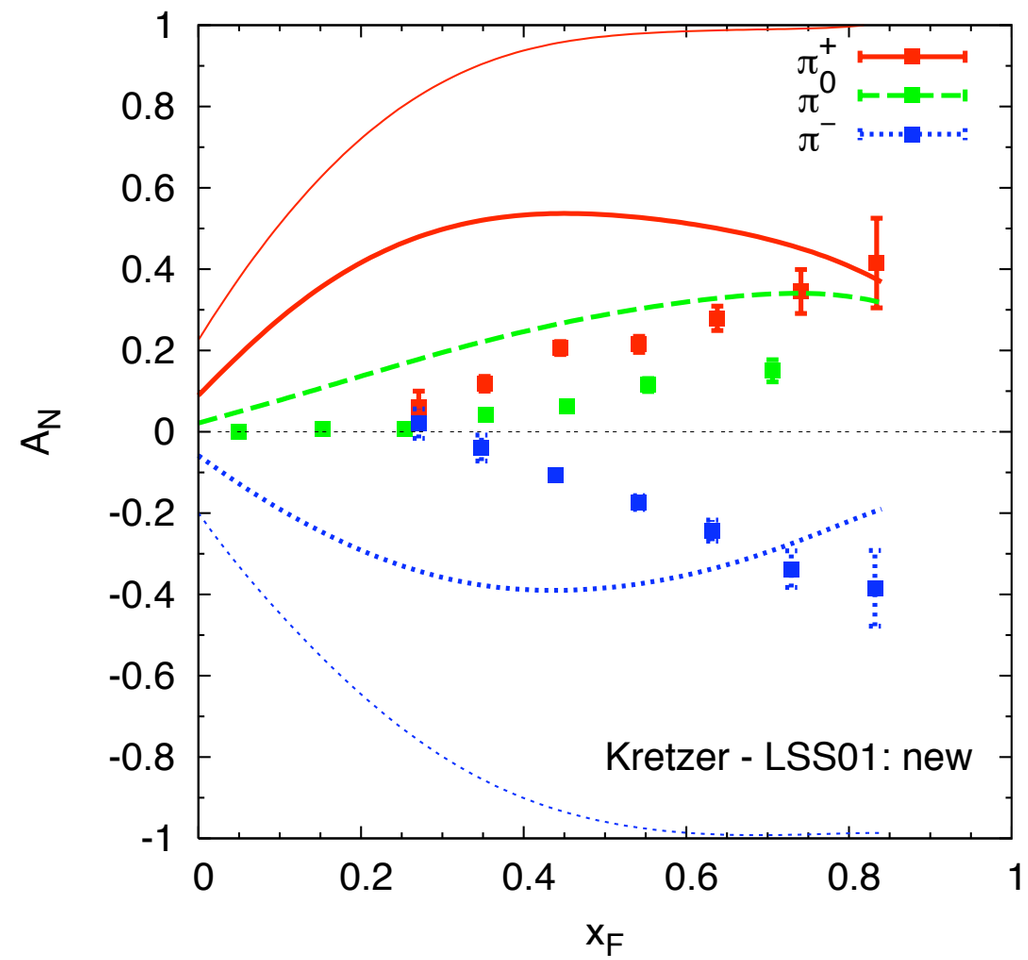
Sivers phase

$$\begin{aligned}
[d\sigma^\uparrow - d\sigma^\downarrow]_{\rm Collins} \;=\; & \sum_{q_a,b,q_c,d} \int \frac{dx_a\, dx_b\, dz}{16\,\pi^2\, x_a\, x_b\, z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{p}_\perp\, \delta(\boldsymbol{p}_\perp\cdot\hat{\boldsymbol{p}}_c)\, J(p_\perp)\, \delta(\hat{s}+\hat{t}+\hat{u}) \\
\times\; & \Delta_T q_a(x_a, k_{\perp a}) \cos(\phi_a + \varphi_1 - \varphi_2 + \phi_\pi^H) \\
\times\; & f_{b/p}(x_b, k_{\perp b})\, \Big[\hat{M}_1^0\, \hat{M}_2^0\Big]_{q_a b\to q_c d}\, \Delta^N D_{\pi/q_c}(z, p_\perp)
\end{aligned}$$

Collins + scattering phases

negligible contributions from other TMDs




maximum possible contribution of Collins effect

(thin lines = all phases to zero)

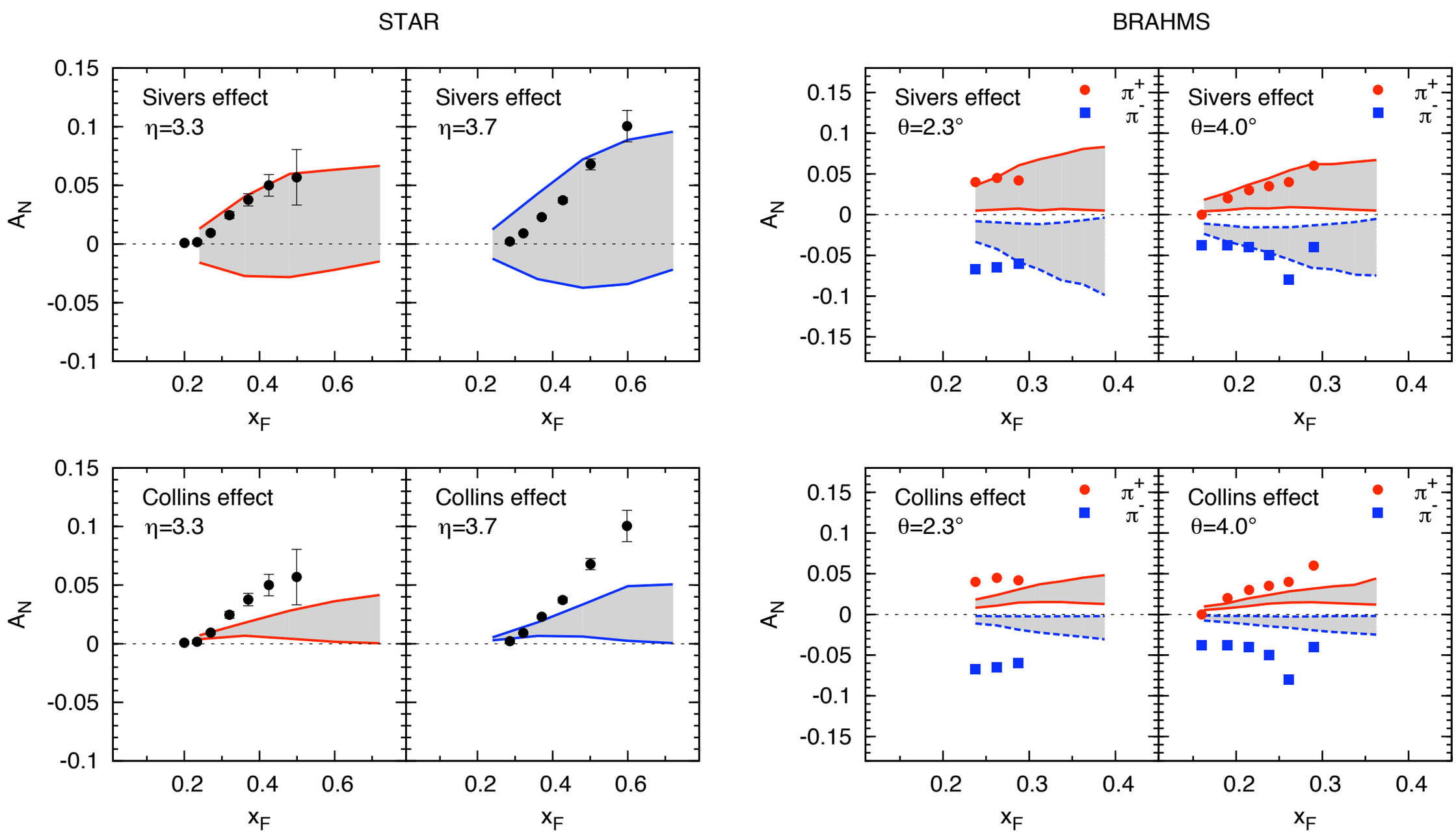
there was a mistake in previous evaluation (wrong sign in one spin transfer cross-section)

PRD 71:014002,2005

in principle, Collins effect might be large (depending on Collins function and transversity)

in preparation

# contributions to A_N of SIDIS extracted Sivers, Collins and transversity distributions




a combination of Sivers and Collins effect might explain data

(great uncertainty in actual knowledge of Sivers functions at large x)

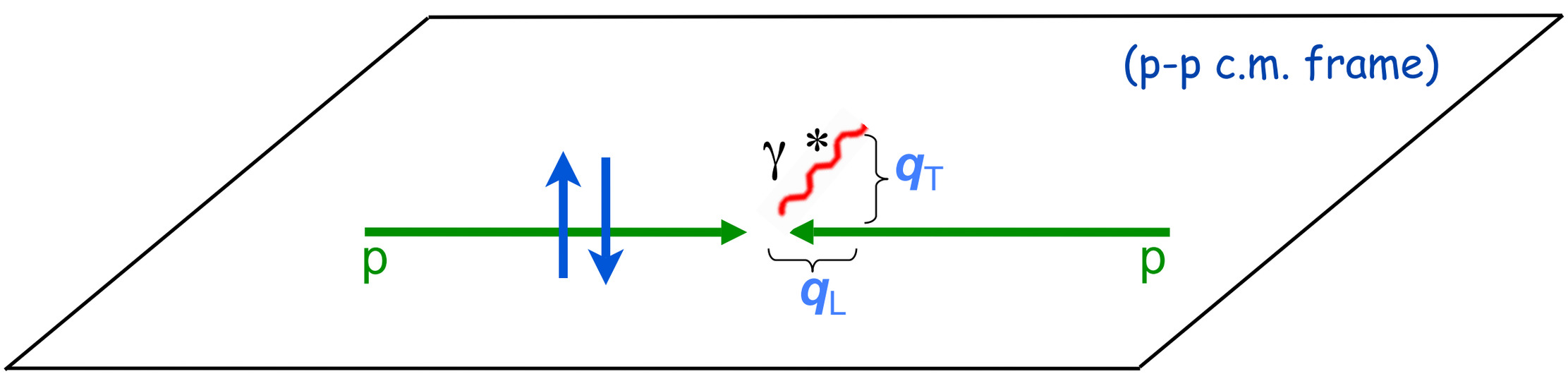
# Sivers effect in D-Y processes

By looking at the $d^4\sigma/d^4q$ cross section one can single out the Sivers effect in D-Y processes

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} \propto \sum_q \Delta^N f_{q/p^{\uparrow}}(x_1, \boldsymbol{k}_{\perp}) \otimes f_{\bar{q}/p}(x_2) \otimes \mathrm{d}\hat{\sigma}$$

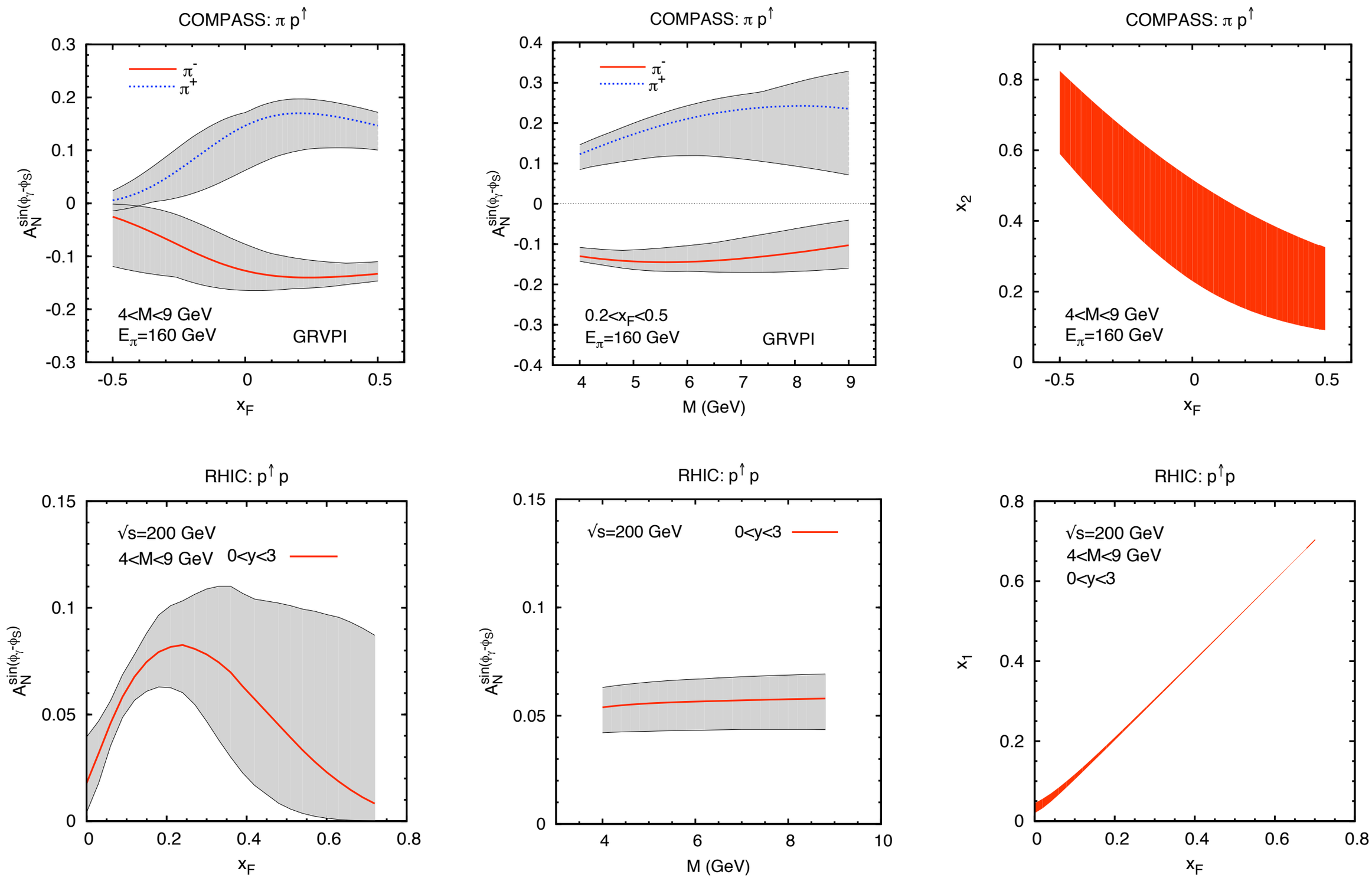
$$q = u, \bar{u}, d, \bar{d}, s, \bar{s}$$

$$A_N^{\sin(\phi_S - \phi_\gamma)} \equiv \frac{2\int_0^{2\pi} \mathrm{d}\phi_\gamma \left[\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}\right] \sin(\phi_S - \phi_\gamma)}{\int_0^{2\pi} \mathrm{d}\phi_\gamma \left[\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}\right]}$$



# Predictions for $A_N$

## Sivers functions as extracted from SIDIS data, **with opposite sign**




M.A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, e-Print: arXiv:0901.3078

# Conclusions

SSAs in inclusive hadronic interactions keep being large and measured

TMDs, Sivers and Collins mechanisms, might explain data on $A_N$. Difficult phenomenology

TMD vs. higher-twist collinear factorization: much improvement for two-scale processes...

Factorization in one-scale processes.
Universality of Collins function.
(Non) universality of Sivers function.
QCD evolution ...