TMDs in two-photon production

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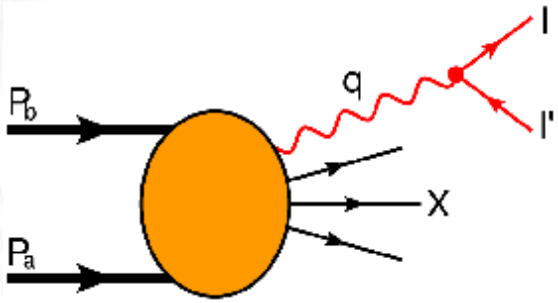
”Progress in High-p_T Physics at RHIC”, BNL

The Drell-Yan process

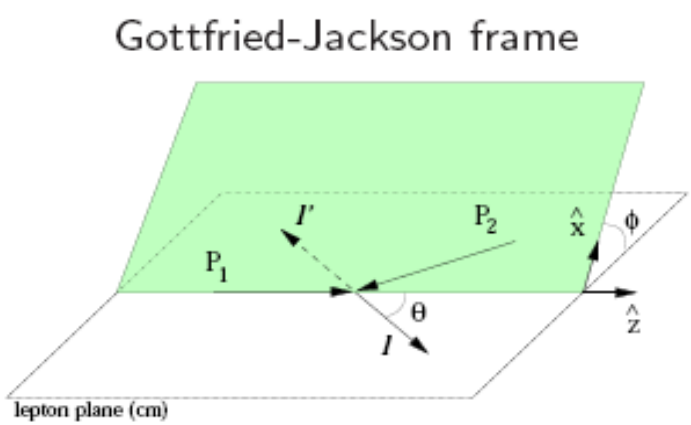
Kinematics (lepton pair produced by one decaying gauge boson):

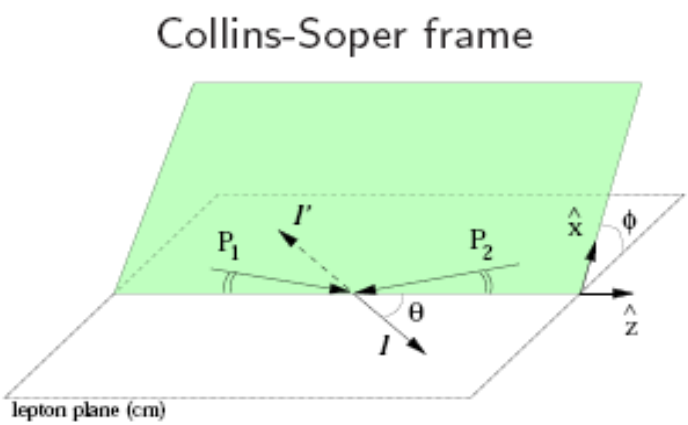


$$\frac{d\sigma}{d^4l\,d^4l'} = \frac{d\sigma}{d^4q\,d^4l} \propto \frac{\delta^+(l^2)\delta^+((q-l)^2)}{4F} \sum_X |M|^2 \delta^{(4)}(P_a + P_b - q - P_X)$$

Disentangling the δ-functions → Dilepton rest frame

Gottfried-Jackson frame and Collins-Soper frame:





Lepton angles:

$$d^4l \to d\Omega = d\phi\, d\cos\theta$$

Diff. CS including angular dependences:

$$\frac{d^6\sigma}{d^4q\,d\Omega} = 2\frac{d^6\sigma}{dy\,dQ^2\,d^2\vec{q}_T\,d\Omega}$$

Angular structure functions

Separation of the leptonic part (generated by one photon):

$$\frac{d\sigma}{d^4q d\Omega} \propto L_{\mu\nu} W^{\mu\nu}$$

with: $L_{\mu\nu} = 4\left(l_\mu l'_\nu + l_\nu l'_\mu - \frac{Q^2}{2} g_{\mu\nu}\right)$ → **Limited number of structure function**

Hadronic Tensor:

$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = \int \frac{d^4x}{(2\pi)^4} \, \mathrm{e}^{iq\cdot x} \langle a, b | J^\mu(0) J^\nu(x) | a, b \rangle$$

Parameterization constraint by current conservation, hermiticity and parity

Decomposition into 4 + 8 +8 + 28 = 48 structure functions $F(x_a, x_b, q_T^2, Q^2)$

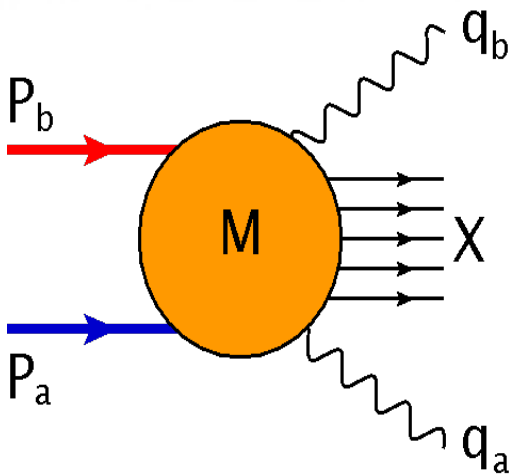
[Arnold, Metz, M.S., PRD 79, 034005] **e.g. unpolarized Drell-Yan**

$$\frac{d\sigma_{UU}}{dx_a \, dx_b \, d^2q_T \, d\Omega} = \frac{\alpha^2 s}{2Q^2 F}\left((1+\cos^2\Theta) F_{UU}^1 + (1-\cos^2\Theta) F_{UU}^2 + \sin 2\Theta \cos\phi F_{UU}^{\cos\phi} + \sin^2\Theta \cos 2\phi F_{UU}^{\cos 2\phi}\right)$$

Classification of structure functions helpful for data analysis

→ *Parton model*: 24 leading twist structure functions

Diphoton production



Two highly energetic real photons produced with

$$q \equiv q_a + q_b$$

$$\frac{d\sigma}{d^4q_a\, d^4q_b} = \frac{d\sigma}{d^4q\, d^4q_a} \propto \frac{\delta^+(q_a^2)\delta^+((q-q_a)^2)}{2\times 4F} \sum_X |M|^2 \delta^{(4)}(P_a + P_b - q - P_X)$$

Convenient choice: Diphoton rest frame → **Collins-Soper frame** $\dfrac{d^6\sigma}{dy\, dQ^2\, d^2\vec{q}_T\, d\Omega_a}$

Unfortunately: No separation into ***hadronic – photonic*** parts possible!
→ *all* angular modulations are allowed, in principle.

$$\frac{d^6\sigma}{dy\, dQ^2\, d^2\vec{q}_T\, d\Omega_a} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_{lm}(y, Q^2, q_T^2)\, Y_{lm}(\Omega_a)$$

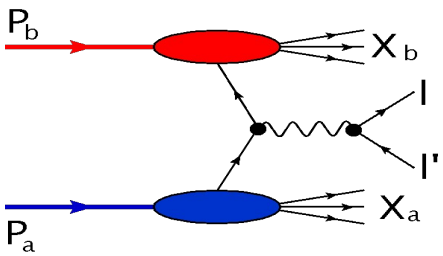
$$C_{00} = \frac{d^4\sigma}{dy\, dQ^2\, d^2q_T}, \ldots$$

However, we can calculate the cross section in the parton model.

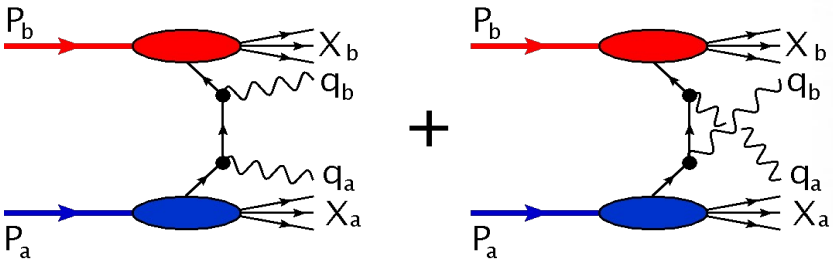
TMD tree-level formalism

Parton model tree-level at $O(\alpha_s^0)$:

Drell-Yan dilepton production:



Diphoton production:



Only relevant at very small q_T: $\Lambda_{QCD} \sim q_T \ll Q$

$$\left(\frac{d\sigma}{d^4q\, d\Omega}\right) \propto \int d^2k_{aT} \int d^2k_{bT}\, \delta^{(2)}(\vec{k}_{aT} + \vec{k}_{bT} - \vec{q}_T)\, \mathrm{Tr}\left[\Phi\,(x_a, \vec{k}_{aT})\, H(x_a, x_b, q_a, q_b)\, \bar{\Phi}\,(x_b, \vec{k}_{bT})\, H^\dagger\right] + \mathcal{O}(\tfrac{M}{Q})$$

k_T - correlator: $$\Phi_{ij}(x, \vec{k}_T) = \int \frac{dz^- d^2z_T}{(2\pi)^2}\, \mathrm{e}^{ik\cdot z} \langle P, S|\, \bar{q}_j(0)\, \mathcal{W}^{?/DY}[0\,;\, z]\, q_j(z)\, |P, S\rangle\Big|_{z^+=0}$$

→ can be parameterized in terms of TMDs according to quark / nucleon spin

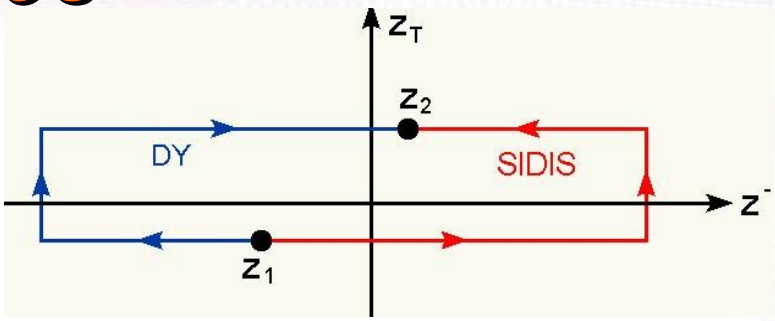
Main result of the TMD tree-level formalism:

$$\left(\frac{d^6\sigma^{hh\to\gamma\gamma X}}{dy\, dQ^2\, d^2q_T\, d\Omega}\right)(\Lambda \sim q_T \ll Q) = \frac{2}{\sin^2\theta}\left(\frac{d\sigma^{hh\to l^+l^-X}}{dy\, dQ^2\, d^2q_T\, d\Omega}\right)(\Lambda \sim q_T \ll Q\,|\, e_q \to e_q^2)$$

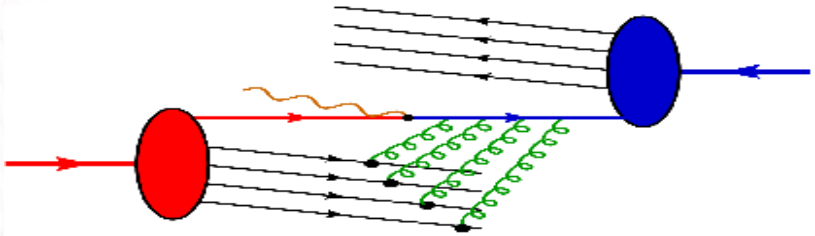
Wilson lines

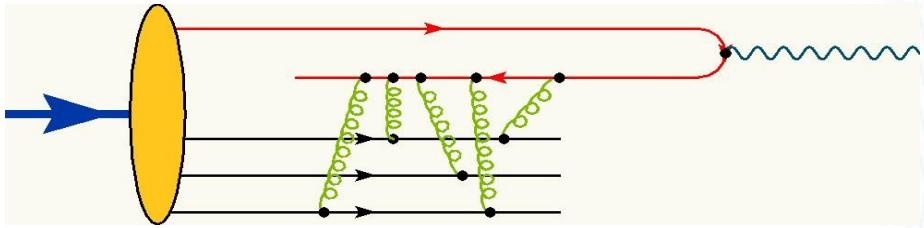
Wilson line process-dependent in DY/SIDIS:

$$\mathcal{W}[z_1; z_2] = \mathcal{P}\mathrm{e}^{-ig\int_{z_1}^{z_2} ds\cdot A(s)}$$

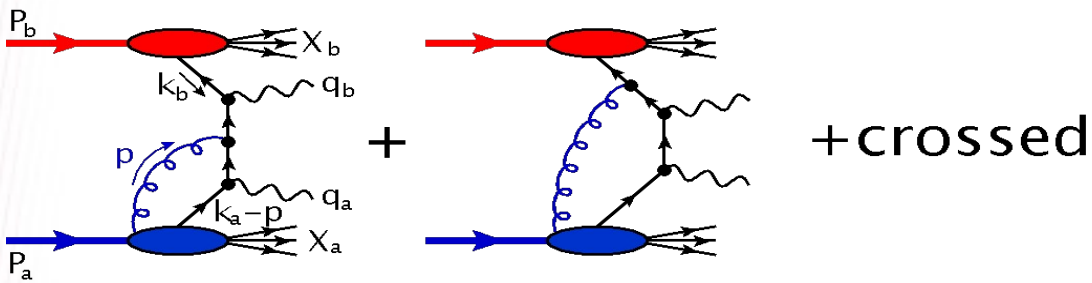


Physics: Initial / Final state interactions





Wilson line in diphoton production:



Check for A^+, $A_T^i(z^- = -\infty)$

Diagrams topologically different to DY, *but* cancellations between diagrams

$$\mathcal{W}^{\gamma\gamma}[0; z]\Big|_{z^+=0} = 1 - ig\int_0^{-\infty} d\lambda\, A^+(\lambda\, n) - ig\int_{0_T}^{z_T} d\vec{y}_T\cdot\vec{A}_T(-\infty, 0, \vec{y}_T) - ig\int_{-\infty}^{0} d\lambda\, A^+(\lambda\, n + z_T) + \mathcal{O}(g^2) = \mathcal{W}^{DY}[\bullet; z]\Big|_{z^+=\bullet}$$

Sivers effect

k_T – correlator for unpolarized quarks:

$$\frac{1}{2}\mathrm{Tr}[\Phi(x,\vec{k}_T)\,\gamma^+] = f_1(x,\vec{k}_T^2) - \frac{\epsilon_T^{ij}\,k_T^i\,S_T^j}{M} f_{1T}^{\perp}(x,\vec{k}_T^2)$$

Sivers function → time-reversal odd → sign switch:

$$f_{1T}^{\perp}\Big|_{DIS} = -f_{1T}^{\perp}\Big|_{DY}$$

Can be determined from SIDIS data of a transverse target SSA (HERMES, COMPASS):

$$A_{UT}^{Siv} \sim \frac{f_{1T}^{\perp} \otimes D_1}{f_1 \otimes D_1}$$

k_T – deconvolution through Gaussian ansatz

$$f(x,\vec{k}_T^{\prime}) = f(x)\exp\left[-\vec{k}_T^{\prime}/\langle k_T^{\prime}\rangle\right]$$

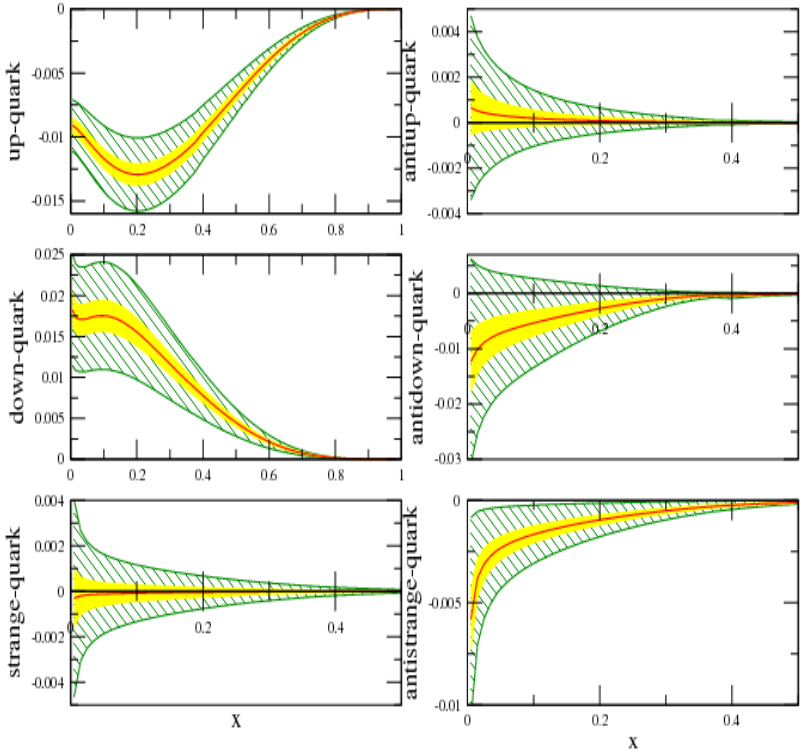
<u>Fit of the Sivers function to data:</u>

[Anselmino et al., EPJA 39, 89],
[Schweitzer, M.S., in preparation]

$$f_{1T}^{\perp,q}(x) = A^q f_1^q(x)$$

- u- and d-Sivers roughly equal, but opposite sign
- non-zero sea-quark Sivers functions.
- small statistical errors, χ^2 / d.o.f. ~ 1.07
- Fits not constraint for x > 0.4...

First transverse Moment of Sivers functions



Sivers effect in photon pair production

Assume azimuthal symmetry:

$$\int_0^{2\pi} d\phi \, \frac{d\sigma_{TU}^{DP/DY}}{d^4q \, d\Omega} = \frac{\alpha^2}{2SQ^2} |\vec{S}_{aT}| \sin\phi_a \left[\frac{2}{\sin^2\theta}\right] (1+\cos^2\theta) F_{TU}^{DP/DY} + \mathcal{O}(M/Q)$$

$$F_{TU}^{DP/DY} = -\sum_{q,\bar{q}} e_q^{4/2} \frac{1}{N_c} \int d^2k_{aT} \int d^2k_{bT} \, \delta^{(2)}(\vec{k}_{aT} + \vec{k}_{bT} - \vec{q}_T) \frac{\vec{q}_T \cdot \vec{k}_{aT}}{|\vec{q}_T| M} f_{1T}^{\perp,q}(x_a, \vec{k}_{aT}^2) f_1^{\bar{q}}(x_b, \vec{k}_{bT}^2)$$

Apply Gaussian ansatz with sign switch:

$$f_{1T}^{\perp}\Big|_{DIS} = -f_{1T}^{\perp}\Big|_{DY}$$

Work at larger rapidity

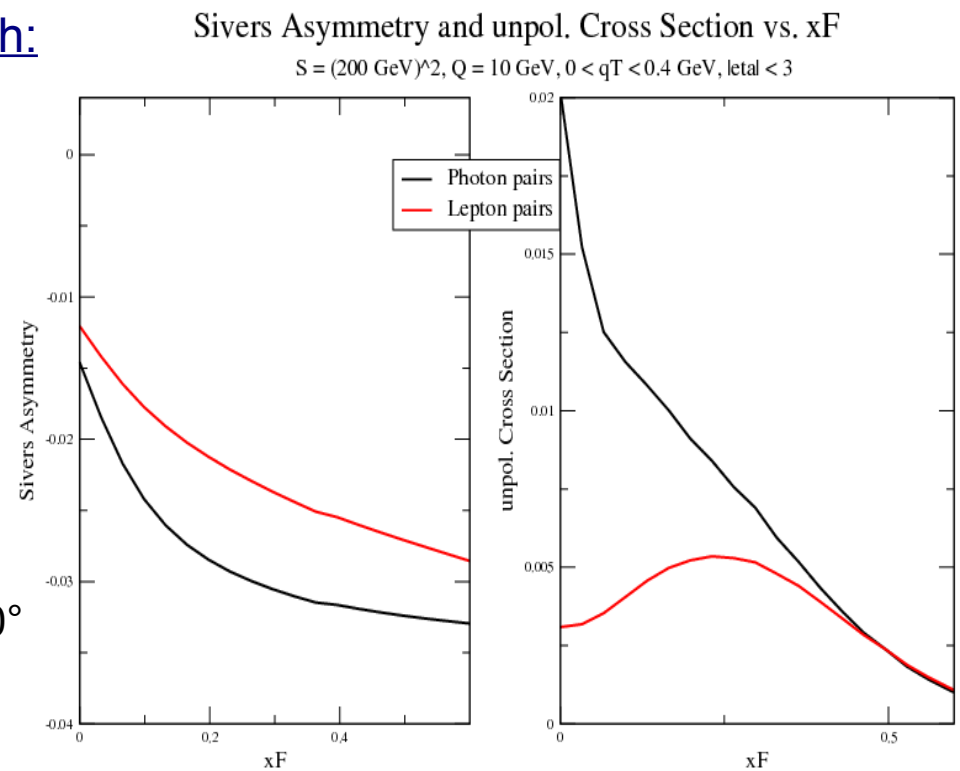
$$|\eta_{a,b}| < ۳, \; \sqrt{S} = ۲۰۰ \, \mathrm{GeV}$$

TMD-range: $0\,\mathrm{GeV} < q_T < 0.4\,\mathrm{GeV}$

Find ~ 1.5 – 2 larger effect

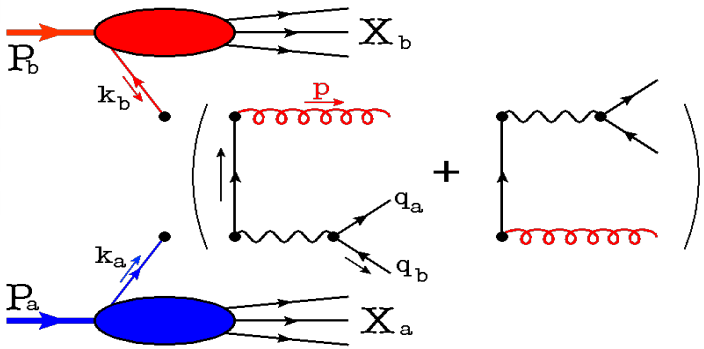
$\frac{2}{\sin^2\theta}$ → factor of 4 at θ = 30°, 33 at θ = 10°

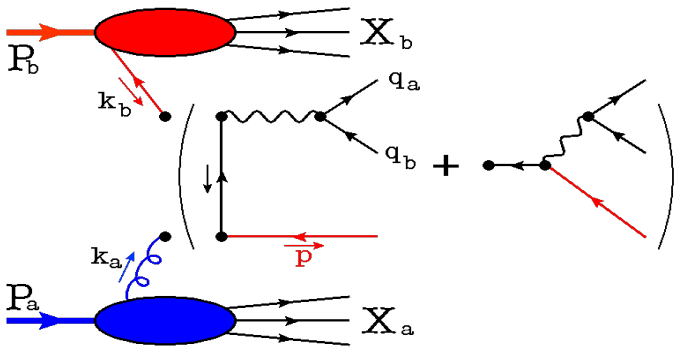
$e_q^2 \to e_q^4$ u-quark dominance



High - q_T behaviour

At large $q_T \sim Q \rightarrow$ transverse momentum generated by gluon radiation





Unpolarized angular dependencies in Drell-Yan:

$$\frac{d\sigma_{UU}}{dx_a\, dx_b\, d^2q_T\, d\Omega} = \frac{\alpha^2 s}{2Q^2 F}\left((1+\cos^2\Theta)F^1_{UU} + (1-\cos^2\Theta)F^2_{UU} + \sin 2\Theta \cos\phi F^{\cos\phi}_{UU} + \sin^2\Theta \cos 2\phi F^{\cos 2\phi}_{UU}\right)$$

Collinear parton model applicable [Boer, Vogelsang, PRD74, 014004]

$$\frac{F^i}{x_a x_b} \sim \int \frac{d\xi_a}{\xi_a} \int \frac{d\xi_b}{\xi_b}\, \delta\left((\xi_a - x_a)(\xi_b - x_b) - \frac{q_T^2}{s})\right) \sum_{q,\bar{q}} e_q^2 \left(f_1^q(\xi_a) f_1^{\bar{q}}(\xi_b) \hat{F}^i_{q\bar{q}} + f_1^q(\xi_a) f_1^g(\xi_b) \hat{F}^i_{qg} + f_1^g(\xi_a) f_1^{\bar{q}}(\xi_b) \hat{F}^i_{g\bar{q}}\right)$$

Partonic structure functions automatically obey angular decomposition

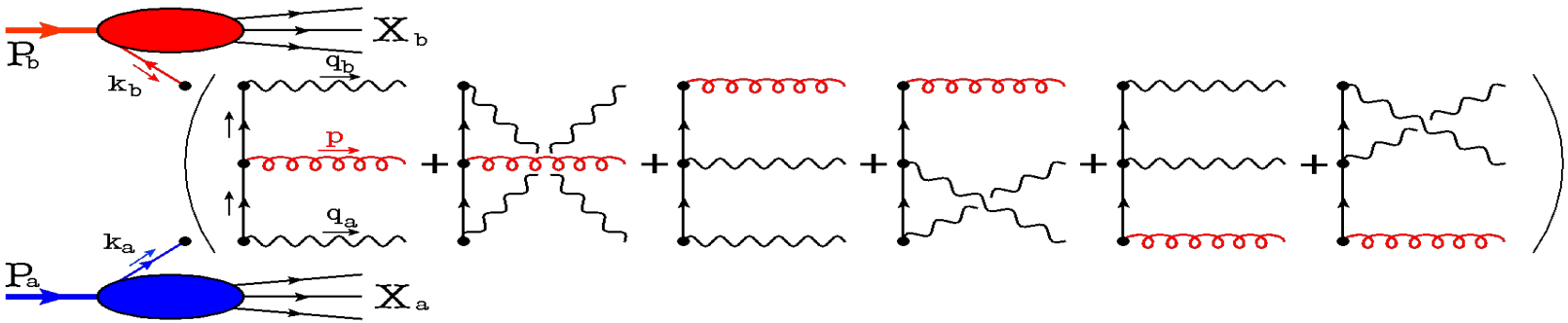
Lam-Tung relation in pQCD $\quad F^2_{UU} = 2F^{\cos 2\phi}_{UU} + \mathcal{O}(\alpha_s^2)$

Similar to Callan-Gross relation in DIS

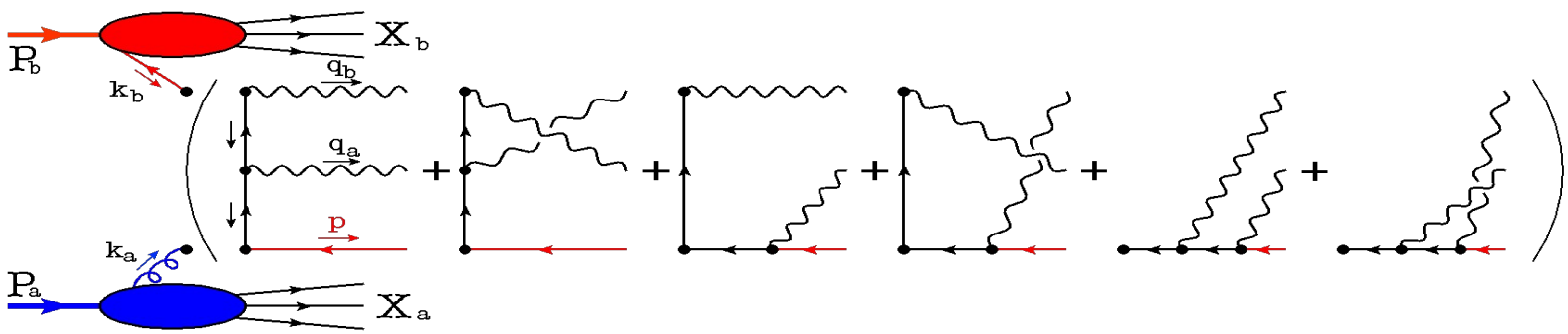
High - q_T of the photon pair

Same strategy as for DY:

quark – antiquark scattering:



quark – gluon scattering:



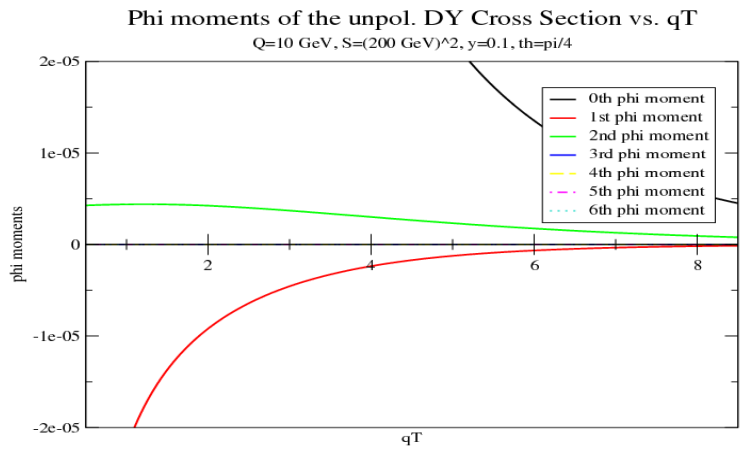
However: No model-independent angular decomposition!

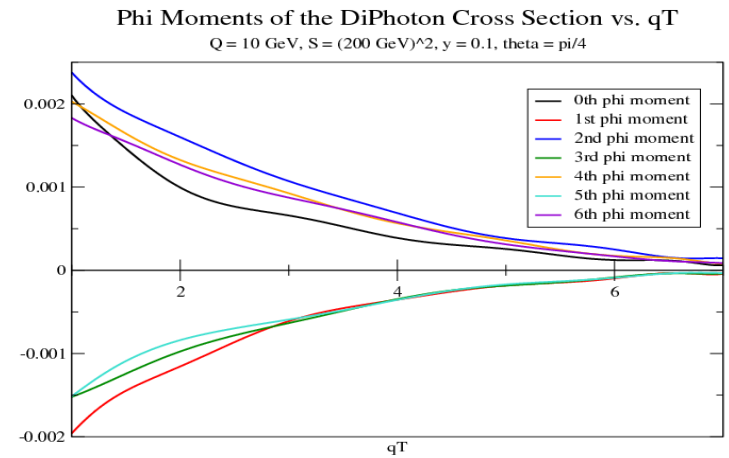
Diphoton angles enter the partonic cross section in numerator and denominator
→ All angular depedencies are allowed.

Numerical results

Define phi moments:

$$\langle \cos(n\phi) \rangle = \int_0^{2\pi} d\phi \cos(n\phi) \frac{d\sigma}{dy\, dQ^2\, d^2q_T\, d\Omega}$$

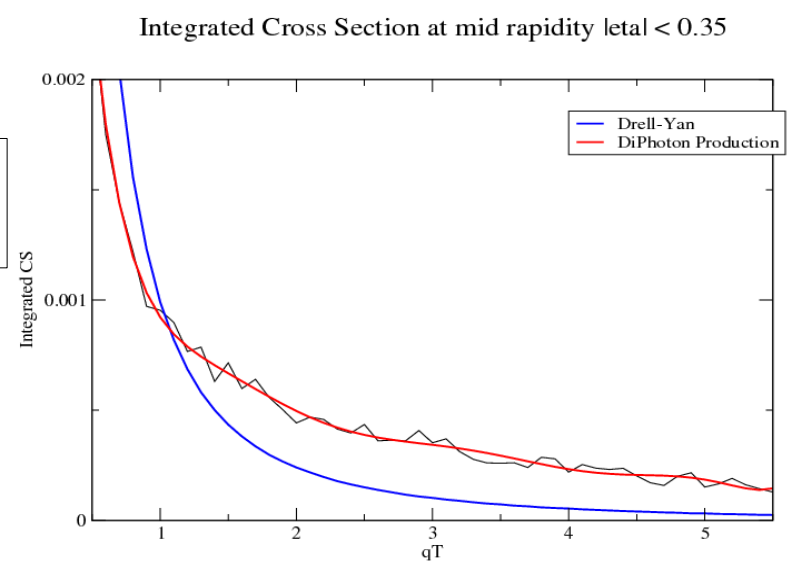




Cross Section at mid rapidity:

$$\int dy \int d\Omega \frac{d\sigma}{dy\, dQ^2\, d^2q_T\, d\Omega} \Theta\left(|\eta_{a,b}| < 0.35\right)$$

At larger qT:
→ DiPhoton cross section larger



Analytical results

What happens if q_T becomes smaller? → Expansion in $1/q_T$...

Drell-Yan:

$$d\sigma_{UU} \sim \left((1+\cos^2\Theta) F^1_{UU} + (1-\cos^2\Theta) F^2_{UU} + \sin 2\Theta \cos\phi F^{\cos\phi}_{UU} + \sin^2\Theta \cos 2\phi F^{\cos 2\phi}_{UU} \right)$$

Order $1/q_T^2$:

$$F^1_{UU} \propto \frac{\alpha_s}{q_T^2} \left[-C_F (2 \ln \frac{q_T^2}{Q^2} + 3) q(x_1)\bar{q}(x_2) + q(x_1)(P_{qq} \otimes \bar{q} + P_{qg} \otimes g)(x_2) + \{x_1 \leftrightarrow x_2\} \right]$$

Order $1/q_T$:

$$F^{\cos\phi}_{UU} \propto \frac{\alpha_s}{q_T} \left[q(x_1)(\tilde{P}_{qq} \otimes \bar{q} + \tilde{P}_{qg} \otimes g)(x_2) - \{x_1 \leftrightarrow x_2\} \right]$$

Order $1/q_T^0$:

$$F^2_{UU} = 2F^{\cos 2\phi}_{UU} = -C_F (2 \ln \frac{q_T^2}{Q^2} + 3) q(x_1)\bar{q}(x_2) + q(x_1)(P_{qq} \otimes \bar{q} + P'_{qg} \otimes \bar{q})(x_2) + \{x_1 \leftrightarrow x_2\}$$

DiPhoton production:

leading order follows "TMD - rule"

$$\sigma^{DP} = \frac{2}{\sin^2\theta} \sigma^{DY}(e_q \to e_q^2) + \mathcal{O}(1/q_T)$$

Order $1/q_T$:

$$\sigma^{DP} \sim (\sin 2\theta \cos\phi) \frac{\alpha_s}{q_T} \left[\frac{4}{\sin^4\theta} q(x_1)(\tilde{P}_{qq} \otimes \bar{q} + \tilde{P}_{qg} \otimes g)(x_2) + \frac{2}{\sin^2\theta} q(x_1)(\tilde{P}_{qq} \otimes \bar{q} + \tilde{P}^a_{qg} \otimes g)(x_2) - \{x_1 \leftrightarrow x_2\} \right]$$

Order $1/q_T^0$:

quark – antiquark contr. to diphoton production

$$\sigma^{DP}_{q\bar{q}} \propto \frac{(4 - \cos^2\theta \sin^2\theta) \cos 2\phi + (3\sin^2\theta + 2\cos^2\theta)}{\sin^4\theta} F^{2,DY}_{UU,q\bar{q}}$$

quark – gluon scattering → work in progress...

Summary:

- Drell-Yan cross section can be decomposed model-independently into angular structure function, not possible for photon pair production
- TMD-factorization at low q_T: Photon pair production similar to Drell-Yan
- Sivers effect similar in Photon pair production, but higher production rate
- Collinear factorization at larger q_T: all azimuthal modulations possible
- Expansion to smaller q_T: Partly recover azimuthal behaviour

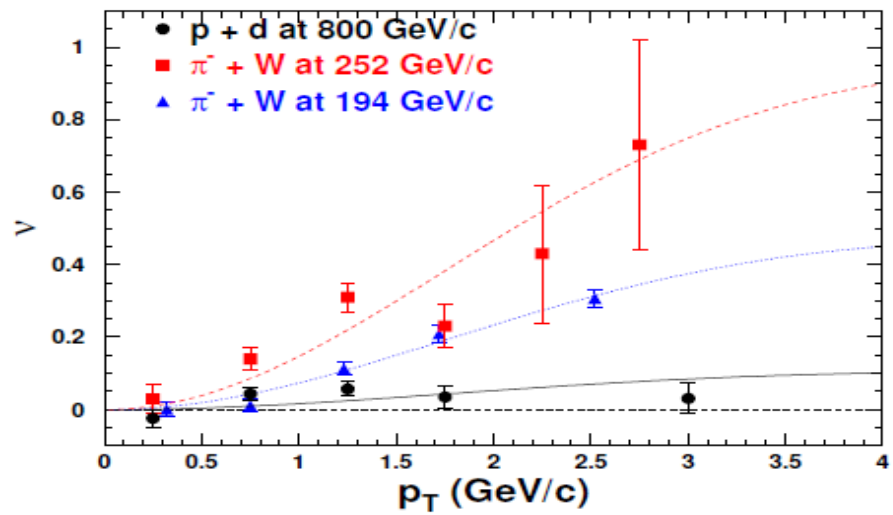
DY-data for the Lam-Tung relation

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q d\Omega}\right) \Big/ \left(\frac{d\sigma}{d^4q}\right) = \frac{3}{4\pi}\frac{1}{\lambda+3}\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

Lam-Tung relation: $1-\lambda-2\nu=0 \iff F_{UU}^{2}=2F_{UU}^{\cos 2\phi}$

LT-relation even valid after resummation (Extension of coll. parton model to lower q_T)

ν has been measured in $\pi^- N \to \mu^+ \mu^- X$ [NA10 Coll. ('86/'88) & E615 Coll. ('89)]



Large angular distribution violates Lam-Tung relation, far from pQCD result.

However, recent FermiLab data on proton-deuterium DY agree with LT-relation [FNAL-E866/NuSea Coll.]

Possible explanation through TMDs: *"Boer-Mulders effect"*

$$\nu(q_T \ll Q) \propto \frac{h_1^{\perp,q} \otimes h_1^{\perp,\bar{q}}}{f_1^q \otimes f_1^{\bar{q}}}$$